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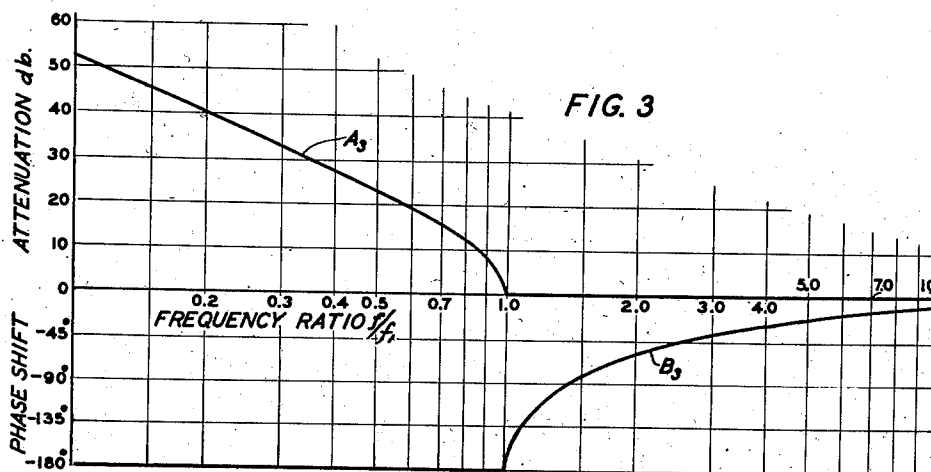
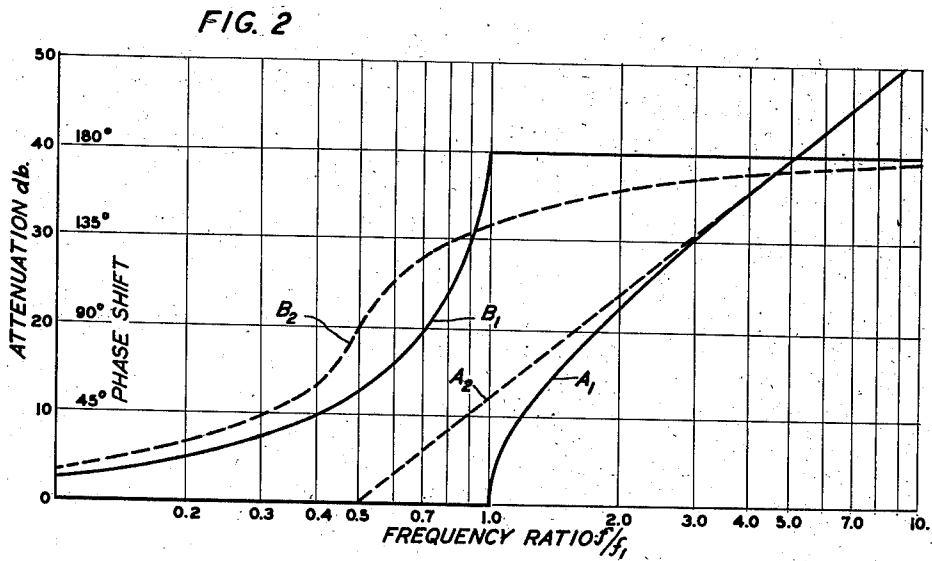
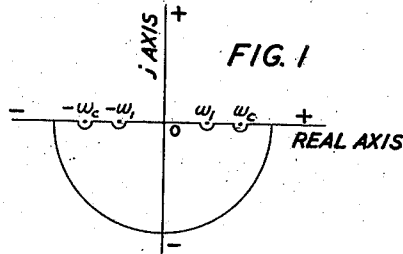
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2,123,178

AMPLIFIER

Filed June 22, 1937

5 Sheets-Sheet 1



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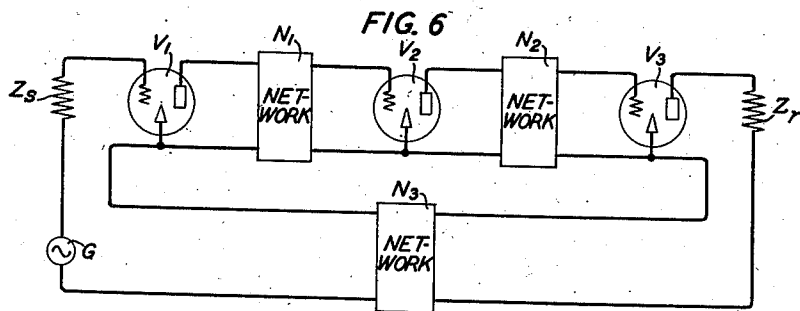
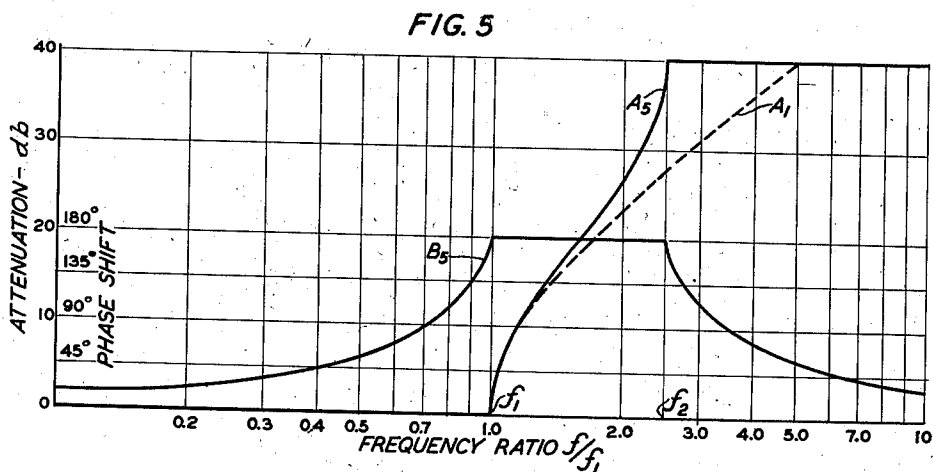
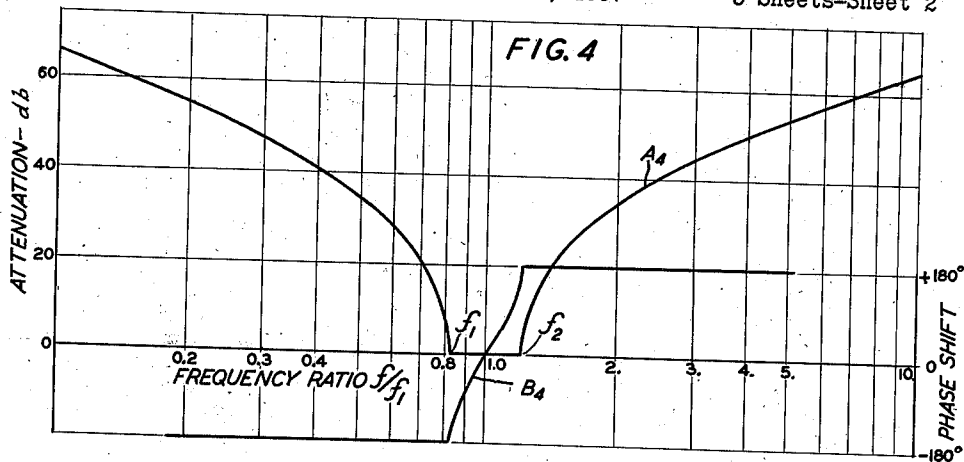
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AMPLIFIER

Filed June 22, 1937

5 Sheets-Sheet 2



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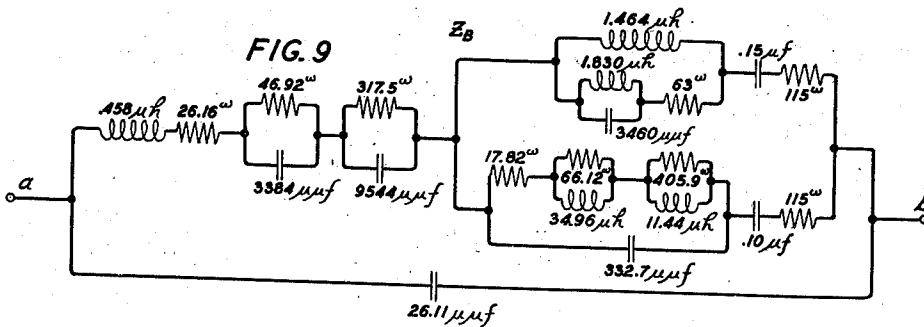
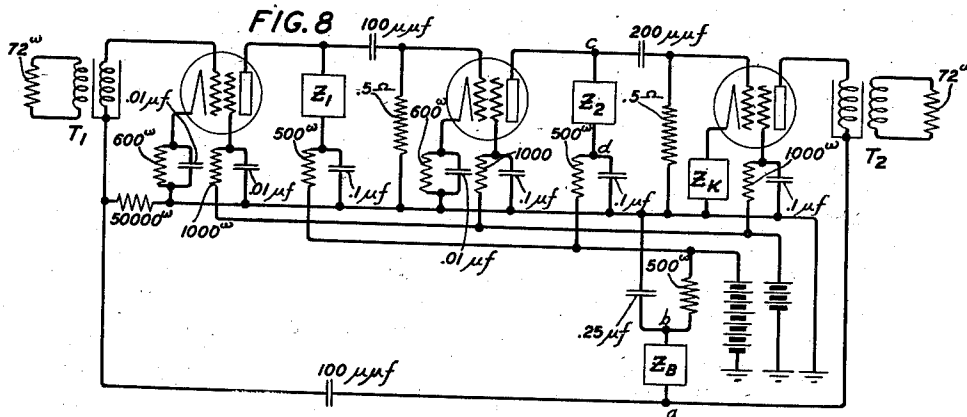
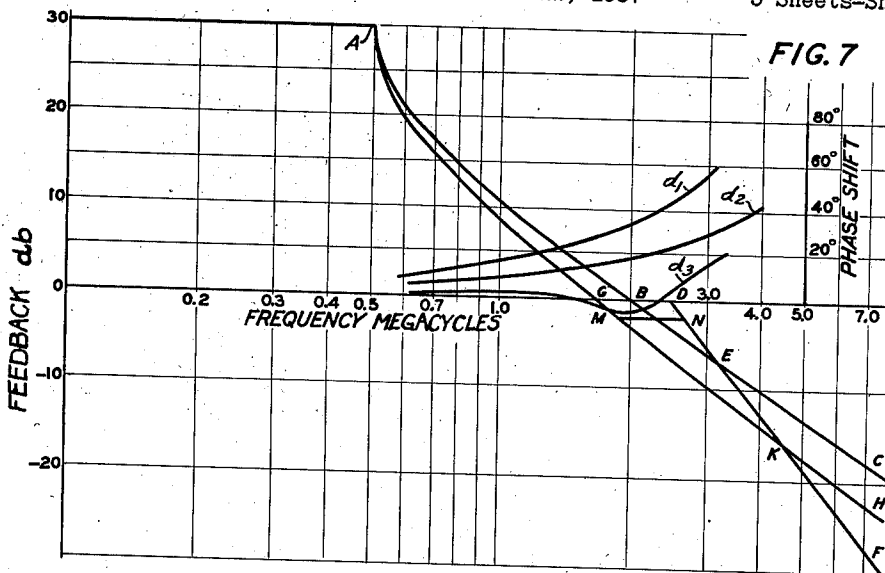
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Filed June 22, 1937

5 Sheets-Sheet 3



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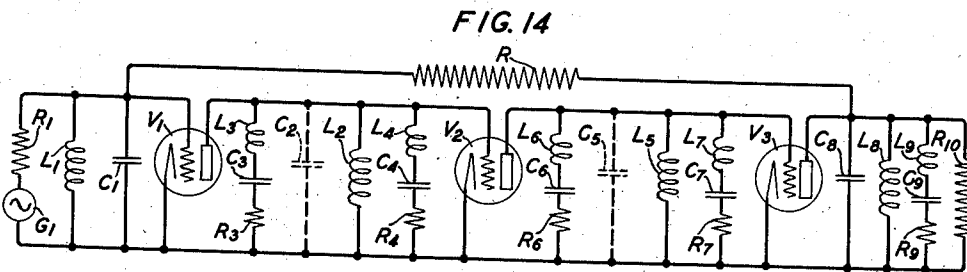
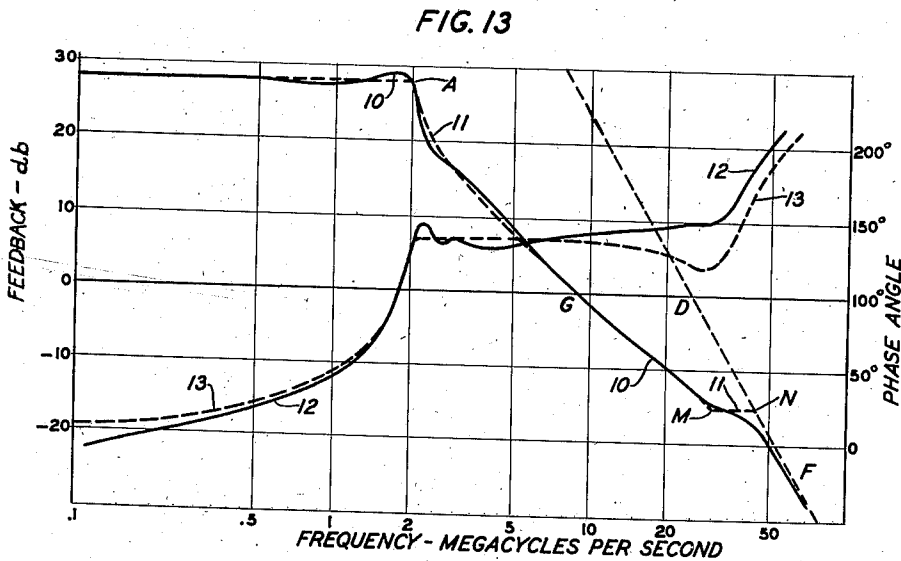
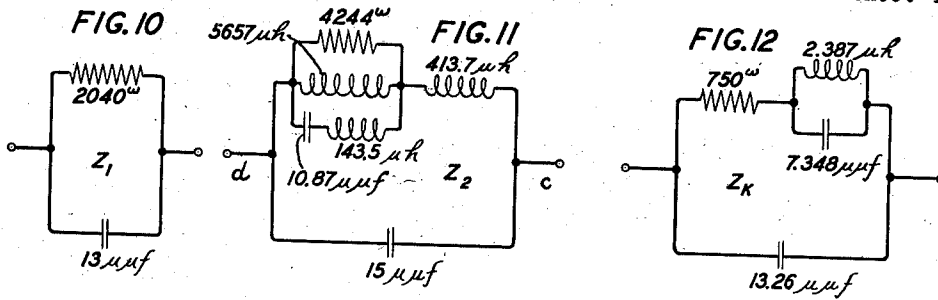
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AMPLIFIER

Filed June 22, 1937

5 Sheets—Sheet 4



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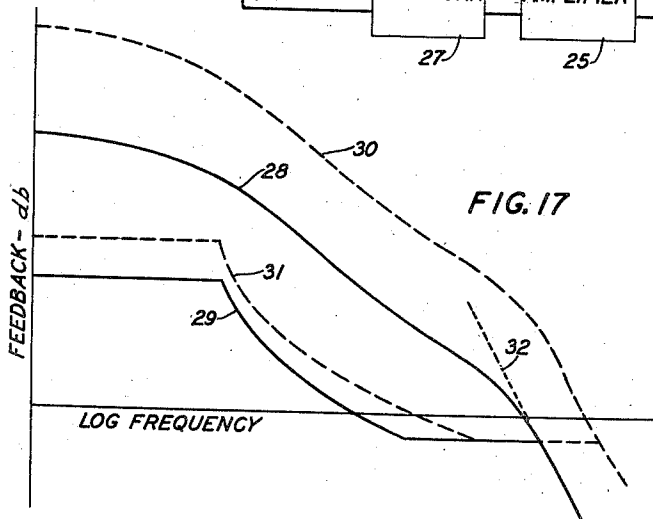
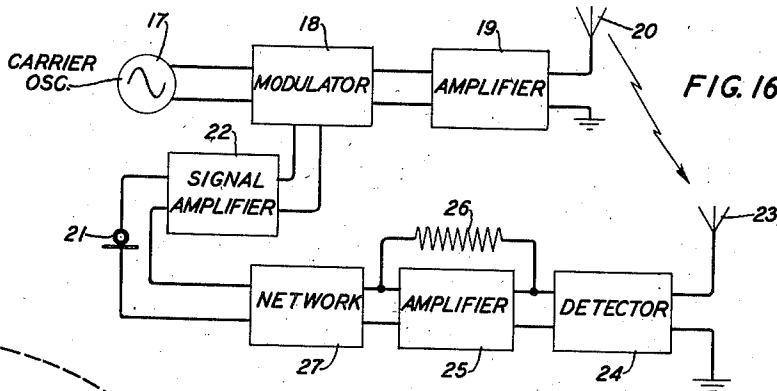
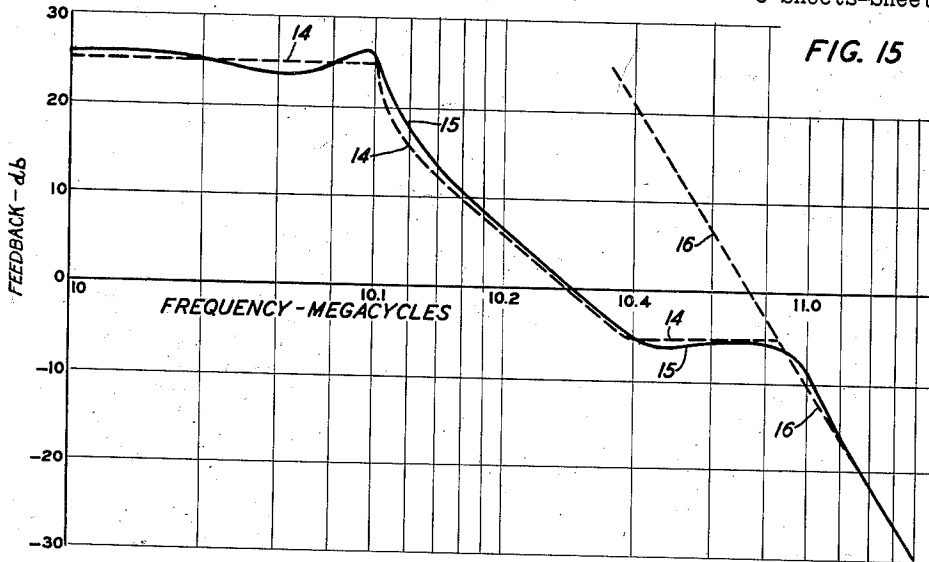
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2,123,178

AMPLIFIER

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UNITED STATES PATENT OFFICE

2,123,178

AMPLIFIER

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Application June 22, 1937, Serial No. 149,565

15 Claims. (Cl. 179—171)

One of the problems encountered in the design and construction of feedback amplifiers is that of preventing singing at frequencies outside the operating range of the amplifier. This difficulty arises from the tendency of the phase of the feedback voltage to change progressively as the frequency departs from the operating range, until it finally comes to an aiding or regenerative relationship which produces a condition of instability. The expedients heretofore adopted for the prevention of singing have, in general, required some sacrifice of the effective amplification or some reduction of the frequency range in which useful amplification is obtained. They have also limited the amount of feedback that could be used and hence have prevented the fullest realization of the benefits to be derived therefrom.

By this invention, improved feedback systems are provided which are completely stabilized against singing and which at the same time permit the utilization of feedback to the maximum degree consistent with certain basic limitations imposed by inherent properties of the apparatus elements. A maximum extension of the frequency range of useful amplification is also made possible.

The benefits to be derived from the use of feedback are described in an article by H. S. Black on Stabilized feedback amplifiers, Bell System Technical Journal, January 1934. Principal among these are the stabilization of the effective amplification in the presence of variations of the energizing potentials applied to the amplifier vacuum tubes and the diminution of the effects of non-linear distortion. These benefits are obtained when the magnitude and phase of the voltage fed back to the amplifier input are such as to produce a diminution of the effective amplification and the extent to which the benefits are realized is substantially proportional to the amount by which the amplification is diminished.

The degenerative action is provided for by so designing the amplifier and the feedback circuit that, when the frequency has some convenient value near the middle of the desired operating range, the effective input voltage is in exact phase opposition to the feedback voltage. The effective input voltage is reduced in comparison with the voltage from the input wave source and the net amplification of the system is diminished in proportion. As the frequency increases, the phase of the feedback voltage changes progressively until at some high frequency, usually well above the operating range, it comes into phase coincidence with the effective input voltage. At

this point, the amplifier may become unstable and develop oscillations. Whether it does so or not, depends upon the relative values of the feedback voltage and the input voltage. If the feedback voltage is equal to or greater than the effective input voltage, the amplifier is unstable, but if the feedback voltage is less than the input voltage, there is no instability. An exceptional case is illustrated in Nyquist Patent 1,915,440, issued June 27, 1933. However, the type of stability therein disclosed is not absolute and such systems tend to become unstable when the amplifier gain is reduced.

To insure complete stability of the amplifier against singing it is therefore necessary to so proportion the different parts of the circuit that, before the condition of phase coincidence is reached, the feedback voltage shall have been attenuated to a value less than the effective input voltage. This requirement may be expressed in terms of certain transmission parameters of the circuit as follows: If the voltage gain, or transfer ratio, from the input to the output of the amplifier be denoted by μ and the voltage transfer ratio of the feedback path in the reverse direction by β then the vector ratio of the feedback voltage to the resultant input voltage on the amplifier is equal to the product $\mu\beta$. For complete stability it is necessary that the magnitude of $\mu\beta$ become less than unity before its phase angle becomes zero.

The vector quantity $\mu\beta$ defines the net change in amplitude and phase experienced by a voltage wave in traversing the amplifier and the feedback path in sequence. This change is made up, essentially, of two parts, first a constant amplification and a constant phase shift representing the effect of the amplification factors of the vacuum tubes and, second, a variable diminution of the amplitude and a variable phase shift representing the transmission loss in the passive networks of the system. Since the part contributed by the vacuum tubes is constant, all of the variation of $\mu\beta$ is represented by the variation of the transmission loss of the passive networks.

By using only pure resistances for the feedback and coupling networks it would be possible, theoretically, to make $\mu\beta$ constant in both amplitude and phase at all frequencies and thereby provide for any desired amount of feedback and an unlimited operating frequency range. In practice, however, this is impossible because of the presence of unavoidable parasitic reactances in the system. At very high and very low frequencies these parasitic reactances become the sole factors determining the magnitude and the vari-

ation of the feedback and are generally productive of phase changes sufficiently great to cause instability. To avoid this, it is necessary that the magnitude of the feedback should be reduced to less than unity before the limiting frequency ranges are reached and suitable frequency intervals must be allowed for this reduction.

It is desirable that the cut-off ranges in which the magnitude of the feedback is systematically reduced should be as small as possible in order that the greatest operating frequency range may be conserved. I have discovered that the phase shift component of $\mu\beta$ is greatly influenced by the course that its magnitude follows in the cut-off range and by the rate at which the magnitude diminishes. In the amplifiers of the invention, the diminution of $\mu\beta$ is made to follow certain preferred courses which represent optimum characteristics in the sense that they permit the use of maximum amounts of feedback and involve the minimum loss of useful frequency range, while at the same time insuring complete stability against singing.

The optimum characteristics referred to above are based on certain fundamental relationships which I have discovered between the phase shift and the attenuation components of the insertion loss characteristics of passive networks. These theorems show that, when the attenuation characteristic is known, the minimum phase shift possible at any given frequency becomes determined and, also, that physical networks can be constructed which will provide this minimum phase shift.

The nature of the invention will be more fully understood from the following detailed description and by reference to the attached drawings of which:

Fig. 1 is a diagram used in the development of mathematical principles;

Figs. 2 to 5 represent transmission characteristics in accordance with the invention;

Fig. 6 is a generalized schematic of the amplifying system of the invention;

Fig. 7 is a group of curves further illustrating the principles of the invention;

Figs. 8 to 12 show the circuit and details thereof of an amplifier in accordance with the invention;

Fig. 13 shows characteristics of the amplifier of Fig. 8;

Figs. 14 and 15 illustrate a modified form of the invention and its characteristics; and

Figs. 16 and 17 illustrate the application of the invention in radio transmitters.

Mathematical theory

The principles on which the invention is based may be developed from the mathematical theory of wave transmission in four-terminal passive networks. The general expression for the voltage transfer ratio, or the voltage insertion loss, of a four-terminal passive network when connected between resistive terminal impedances takes the form of the ratio of two polynomials in frequency. In the most typical case, the polynomials are of the same degree in frequency and so have the same number of roots. In terms of the roots of the polynomials, the insertion loss may be written in the form

$$e^{-\theta} = K \frac{(p-a_1)(p-a_2) \dots (p-a_n)}{(p-b_1)(p-b_2) \dots (p-b_n)} \quad (1)$$

where θ is the insertion loss in logarithmic units, or nepers, K is a constant, p is the variable quantity $j2\pi f$, f , being the frequency, and the a 's

and b 's are the roots of the two polynomials. In order that Equation (1) may represent only such characteristics as are obtainable with physically possible networks and that it may include all of these characteristics, certain restrictions are necessary. These are the following: Both the a 's and the b 's must be real or must occur in conjugate complex pairs. The real parts of the b 's must be negative and not zero. The real parts of the a 's may be positive or negative or zero.

The significance of the restriction on the b 's that is on the roots of the denominator, is that a passive network is necessarily stable. If the real part of any one of these roots could be zero, there would then be some frequency at which the output voltage would be infinite for any finite value, no matter how small, of the input voltage. The damping would be zero and a transient oscillation of this frequency would be sustained indefinitely. If the real part could be positive, the network would have negative damping and a transient oscillation would increase instead of dying out.

Equation (1) may be modified, without changing the value of the insertion loss, by introducing additional frequency factors equally into the numerator and the denominator of the right-hand side. The modified equation then becomes:

$$e^{-\theta} = K \left[\frac{(p-c_1) \dots (p-c_r)}{(p-c_1) \dots (p-c_r)} \right] \left[\frac{(p-a_1) \dots (p-a_n)}{(p-b_1) \dots (p-b_n)} \right] \quad (2)$$

where the c 's are arbitrarily chosen roots. Since the numerator of Equation (1) may include roots having positive real parts, the number of the c factors introduced may be chosen to correspond to the number of such roots and the values of the c 's may be made respectively equal to the negatives of the corresponding a 's. For example, if the roots a_2 and a_3 have positive real parts, the c factors introduced would be

$$(p-c_2) = p - (-a_2)$$

and

$$(p-c_3) = p - (-a_3)$$

the c 's being given the same subscripts as the corresponding a 's for convenience.

By choosing the c factors in the above manner and regrouping the factors, Equation (2) may be rewritten in the form

$$e^{-\theta} = K \left[\frac{(p-a_1)(p-c_2)(p-c_3) \dots (p-a_n)}{(p-b_1)(p-b_2)(p-b_3) \dots (p-b_n)} \right] \left[\frac{(p-a_2)(p-a_3)}{(p-c_2)(p-c_3)} \right] = KP_1P_2 \quad (3)$$

where P_1 denotes the first group of factors within squared brackets and P_2 the second group. Equation (3) is particular to the case in which only two of the a 's, namely a_2 and a_3 , have positive real parts. In the general case, the number of the c factors is not restricted.

The group of factors P_2 has the characteristic property that for each factor in the numerator there is a corresponding factor in the denominator with the sign of the root reversed. Since the a 's are either real or occur in complex pairs, this correspondence between the numerator and the denominator factors results in the group having a constant absolute value, or modulus, equal to unity at all real frequencies. The part P_2 of the loss expression therefore contributes nothing to the attenuation and modifies only the phase shift. It corresponds to the insertion loss of an all-pass constant resistance network properly terminated.

In the product P_1 the a 's, b 's and c 's still conform to the general restrictions for physically realizable networks and the group therefore represents the insertion loss of a physical network.

The factors K and P_1 account for the whole of the attenuation prescribed by Equation (1), together with a phase shift corresponding to that specified by the original equation diminished by the phase shift represented by P_2 .

Since all of the a 's having positive real parts are included in the group P_2 , that part represents the largest all-pass network that can be taken out of the system or the largest amount by which the phase shift may be reduced without affecting the attenuation. It follows then that the product KP_1 represents a network having the minimum possible phase shift for a given attenuation. Networks characterized by expressions of the type of P_1 may be termed minimum phase-shift networks. As compared with the most general type of physically realizable network, they are subject to the further restriction, that the roots of the polynomial in the numerator of the insertion loss expression can have negative or zero real parts only.

The minimum phase networks, besides giving the smallest phase shift for a given attenuation, are also capable of providing any physically possible attenuation characteristic. This follows from the fact that Equation (1), subject to the original restrictions, is completely general and the part KP_1 of Equation (3) represents the same attenuation as the original equation.

Thus far the existence has been shown of a class of networks which can provide all possible attenuation characteristics with the minimum amount of phase shift. These networks include the well-known ladder or series-shunt configurations and other familiar forms including most of the constant resistance attenuating networks. It may also be shown that any of the physically possible attenuation characteristics defined by the product KP_1 may be realized by a single constant resistance attenuating network of appropriate configuration and complexity together with an ideal transformer, the constant resistance impedance of the network being preferably equal to one or other of the terminating resistances. A chain of simple constant resistance networks may also be used, but in that case the generality of the system is subject to the condition that the constant K must not be greater than some ascertainable maximum value. This restriction still permits the realization of all possible attenuation variations, but in certain cases requires the presence of a finite constant attenuation.

I have found that for the minimum phase-shift networks there exists a definite relationship between the phase-shift component of the insertion loss and the attenuation component such that the one may be determined from a knowledge of the other. The nature of this relationship will now be examined.

The insertion loss θ , which is the logarithm of the expression on the right-hand side of Equation (1), has the general form

$$\theta = A + jB \quad (4)$$

where A is the attenuation in nepers and B is the phase shift in radians. The characteristics of primary interest are the values of A and B at real frequencies, that is for positive imaginary values of the quantity p in Equation (1). However, the development of the phase attenuation relationship requires a consideration of the values of θ for

all values of p including those corresponding to imaginary and complex frequencies. If the insertion loss be expressed in terms of ω , or 2π times frequency, instead of p , the restrictions defining the minimum phase-shift networks may be summarized by the statement that none of the roots of the insertion loss expression can lie below the real axis of the complex plane of ω , or that θ has no singularities in this region. In addition, since θ is the logarithm of the factored expression in Equation (1), any singularities on the real frequency axis are at most logarithmic. It also follows that A is an even function and B an odd function of frequency.

In the development of the relationship between attenuation and phase, use is made of the general theorem that the integral of any function of a complex variable around a closed contour in the complex plane is equal to zero if no singular points are included within the contour. The function chosen is denoted here by W and is expressed by

$$W = (\theta - A_c) \left(\frac{1}{\omega - \omega_c} - \frac{1}{\omega + \omega_c} \right) \quad (5)$$

$$= (A - A_c) \frac{2\omega_c}{\omega^2 - \omega_c^2} + jB \frac{2\omega_c}{\omega^2 - \omega_c^2} \quad (5a)$$

where ω_c corresponds to some arbitrary real frequency for which the phase angle is to be determined, and A_c is the attenuation at that frequency. This function has singularities at the points $+\omega_c$ and $-\omega_c$.

The contour around which the integration is taken is illustrated in Fig. 1, which represents the complex plane for ω . Singularities exist at $\pm\omega_c$ on the real axis and, for illustrative purposes are assumed to occur also at another pair of points at $+\omega_1$ and $-\omega_1$. With reference to Equation (1) the points $+\omega_1$ and $-\omega_1$ correspond to one conjugate pair of pure imaginary roots of the numerator of the loss function, that is, roots at positive and negative real frequencies. The path of the integration comprises a large semicircle of radius ρ , corresponding to constant large magnitude of the complex frequency ω , the intercept of the real axis of ω between the values $-\rho$ and $+\rho$, and small semicircles about the four singular points as indicated in the diagram. The integration is evaluated for the limiting case in which the radius of the large semicircle becomes infinitely great and the radii of the small semicircles become infinitesimally small.

It may be shown that the integral over the large semicircle cannot be greater than $\log \rho + \rho$ and therefore becomes zero as ρ becomes infinitely large. Similarly, since the singularities at $+\omega_1$ and $-\omega_1$ are logarithmic, it may be shown that the integrals over the small semicircles about the points $\pm\omega$ are also zero. This leaves to be considered only the path along the real axis between plus infinity and minus infinity and the semicircles about the points $\pm\omega_c$. Since the integral around the whole closed contour is zero, it follows that the integral over the two remaining parts of the contour is also zero. Since the phase shift B is an odd function of frequency, the integral of the second term on the right-hand side of Equation (5a) along the real axis is zero and the integral of W along the real axis is therefore simply the integral of the first term. This is finite since the first term remains finite at $\omega = \pm\omega_c$. Over the semicircles around the points $+\omega_c$ and $-\omega_c$ it may be shown that the integral of W has the value $-2\pi B_c$, where B_c is the value

of the phase shift at ω_c . From these relationships it follows, finally, that

$$2\omega_c \int_{-\infty}^{+\infty} \frac{A-A_c}{\omega^2-\omega_c^2} d\omega - 2\pi B_c = 0 \quad (6)$$

or, since the attenuation is an even function of frequency

$$B_c = \frac{2\omega_c}{\pi} \int_0^{\infty} \frac{A-A_c}{\omega^2-\omega_c^2} d\omega \quad (7)$$

Equation (7) shows that if the attenuation of a minimum phase-shift network is known for all real frequencies from zero to infinity, the phase shift at any given frequency is definitely fixed and may be computed from the attenuation characteristic. This equation was arrived at by starting with an arbitrarily chosen function of the complex frequency which exhibited singularities at the points $\pm\omega_c$. By choosing other functions having this property different mathematical formulations of the attenuation, phase-shift relationship may be obtained, but these would be broadly equivalent to the relationship expressed by Equation (7).

By mathematical transformations of standard types various modifications of Equation (7) can be obtained. Changing the variable from ω to u , where

$$u = \log_e \frac{\omega}{\omega_c} \quad (8)$$

Equation (7) takes the form

$$B_c = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{A-A_0}{\sinh u} du \quad (9)$$

In this equation A_0 denotes the attenuation at the point $u=0$ or $\omega=\omega_c$. Integration of Equation (9) by parts leads to the final relationship

$$B_c = \frac{\pi}{2} \left(\frac{dA}{du} \right)_0 + \int_{-\infty}^{+\infty} \left[\left(\frac{dA}{du} \right) - \left(\frac{dA}{du} \right)_0 \right] \log \coth \frac{|u|}{2} du \quad (10)$$

wherein $\log \coth \frac{|u|}{2}$

denotes the real part of $\log \coth \frac{u}{2}$

which is complex when u is negative. In the above equation the derivatives are the rates of change of the attenuation with the logarithm of the frequency, or the slopes of the attenuation characteristic plotted on a logarithmic frequency scale. The subscript zero indicates that the derivative is taken at the reference point ω_c or the point $u=0$. The attenuation slopes are measured in nepers for unit change in u , that is, for a frequency ratio of 1:2.718. For practical purposes it is more convenient to measure the slope in decibels per octave, a rate of change of one neper per unit change of u being equal to a rate of six decibels per octave.

Equations (9) and (10) are in forms suitable for the evaluation of the phase angle in many simple cases and also for the computation by graphic methods of integration. Equation (10), moreover, is in such form as to permit a ready comprehension of some of the salient characteristics of the phase-shift, attenuation relation-

ship. The first term on the right-hand side is dependent only on the attenuation slope at the reference frequency for which the phase shift is to be determined and represents an angle directly proportional to the slope at this point. In the second term, the integrand is proportional to the product of the symmetrical function

$$\log \coth \frac{|u|}{2}$$

and the difference of the attenuation slopes at the reference frequency and elsewhere. Its value is zero at the reference frequency and converges to zero for large values of u . From these considerations it follows that the value of the integral depends upon the degree of dissymmetry of the attenuation slope characteristic about the reference frequency. It will thus be seen that, when the slope characteristic is symmetrical about the reference frequency, the phase shift at that point is determined wholly by the first term of the equation. For example, if the slope is everywhere uniform, the second term is evidently zero and the phase shift has the constant value at all frequencies given by the first term. If the slope does not change rapidly in a fairly wide range on both sides of the reference frequency, most of the phase shift is given by the first term.

Basic attenuation characteristics

The characteristics described in this section correspond to the optimum $\mu\beta$ characteristics provided in the amplifiers of the invention. They include low-pass, high-pass, and band-pass types, but all are similar in respect of the law of variation that the attenuation follows in the cut-off region adjacent the transmission band.

The first basic characteristic is of the low-pass type, the attenuation having the values

$$A_1 = 0 \quad (11)$$

at frequencies less than a limiting frequency f_1 and

$$A_1 = k \log_e \left[\sqrt{\left(\frac{f}{f_1} \right)^2 - 1} + \frac{f}{f_1} \right] \quad (12)$$

at frequencies greater than f_1 . In this equation k is a numerical constant which may have any value less than two and which is preferably greater than 1.5. The frequency f_1 is the upper limit of the useful operating range of the amplifier or the upper limit of the range in which $\mu\beta$ has a uniform magnitude. By means of Equation (7) it may be shown that the minimum phase shift corresponding to this attenuation characteristic has the values

$$B_1 = k \sin^{-1} \frac{f}{f_1} \quad (13)$$

when f is less than f_1 , and the constant value

$$B_1 = k \frac{\pi}{2} \quad (14)$$

at all frequencies above f_1 .

The attenuation and phase characteristics are illustrated by curves A_1 and B_1 respectively in Fig. 2 for the particular case in which k is equal to 2. The attenuation increases sharply at first at frequencies just above f_1 , but at higher frequencies the rate of increase converges rapidly toward the uniform value of 12 decibels per octave. The phase shift increases to 180 degrees in the range below f_1 and remains constant at that value at higher frequencies. Since a change in the phase of $\mu\beta$ of 180 degrees would bring any given amplifier into a potentially unstable con-

dition, these curves evidently represent a limiting condition which it is desirable to avoid. By giving the constant k a smaller value the total phase shift will be proportionately less than 180 degrees. For example, if k has the value 1.75 the maximum phase shift will be 157.5 degrees, which provides a margin of 22.5 degrees against instability.

The advantageous properties of the characteristic described above may be seen by comparing the curves A_1 and B_1 of Fig. 2 with curves A_2 and B_2 representing the case in which the attenuation increases at the uniform rate of 12 decibels per octave at frequencies above an assigned value. The phase shift characteristic B_2 approaches the same final value of 180 degrees. For the same degree of attenuation at high frequencies it will be seen that the range in which the attenuation is zero is reduced by fifty per cent as compared with the optimum characteristic. A high-pass characteristic of similar type to the low-pass characteristic illustrated by curves A_1 and B_1 is shown by the curves A_3 and B_3 of Fig. 3. In this case the attenuation has the values

$$A_3 = k \log_e \left[\sqrt{\left(\frac{f_1}{f}\right)^2 - 1} + \frac{f_1}{f} \right] \quad (15)$$

for frequencies less than f_1 and zero for frequencies greater than f_1 . The phase shift has the constant value of

$$-k \frac{\pi}{2}$$

in the lower frequency range and at frequencies above f_1 has the value

$$B_3 = -k \sin^{-1} \frac{f_1}{f} \quad (16)$$

The curves are drawn for the case where k has the value 2.

Networks providing attenuation characteristics of the types illustrated by curves A_1 and A_3 are described in U. S. patent to Zobel 1,724,987, August 20, 1929, the low-pass type being shown in Fig. 7 of the patent. These networks are minimum phase-shift types.

A band-pass characteristic can be obtained by combining a low-pass characteristic of the type defined by Equations (11) and (12) with a high-pass characteristic in accordance with Equation (15) having a lower cut-off frequency. This method is convenient when the band width is great, since it permits the impedances determining the two cut-offs to be designed and constructed separately. However, when the band is narrow it is preferable to design the networks as definite band-pass structures in which the impedances control both cut-off characteristics simultaneously.

The optimum band-pass characteristic has an attenuation which is zero within the pass band and which, at frequencies outside the band, has the value

$$A_4 = k \log_e [\sqrt{|q|^2 - 1} + |q|] \quad (17)$$

where

$$q = \frac{f^2 - f_1 f_2}{f(f_2 - f_1)}$$

and $|q|$ denotes the absolute magnitude of q , f_2 and f_1 being the upper and lower cut-off frequencies respectively. It is to be noted that Equation (17) becomes equivalent to Equation (12) when the lower cut-off frequency is reduced to zero and becomes equivalent to Equation (15) when the upper cut-off frequency is made infinite. The low-pass and the high-pass characteristics are, therefore, special cases of the general band-pass characteristic.

The corresponding phase shift has the value

$$B_4 = k \sin^{-1} q \quad (18)$$

at frequencies within the transmission band and has the constant values

$$k \frac{\pi}{2} \text{ and } -k \frac{\pi}{2}$$

in the ranges above and below the band respectively. The attenuation and phase-shift characteristics for this case are shown by the curves of Fig. 4 for the limiting value of k equal to 2. Curve A_4 represents the attenuation and curve B_4 the corresponding minimum phase shift.

If the band width is small in comparison with the limiting frequencies, the value of q in Equation (17) is very nearly equal to

$$\frac{f - f_m}{f_2 - f_m}$$

or

$$\frac{f_m - f}{f_m - f_1}$$

where f_m denotes the geometric mean of f_1 and f_2 . It follows then that, in the frequency ranges adjacent the transmission band, the upper and lower branches of the attenuation characteristic defined by Equation (17) closely approximate the low-pass and high-pass characteristics of Equations (12) and (15) respectively when the variable is taken as the departure of the frequency from the geometric mean frequency f_m . Since the geometric mean frequency for the low-pass case is zero and is infinite for the high-pass case, it is evident that the characteristics in all three cases correspond in terms of the variable defined by the departure from the mean frequency.

A fourth type of optimum characteristic is illustrated by the curves of Fig. 5 of which A_5 is the attenuation characteristic and B_5 the associated minimum phase shift. The attenuation is zero below a limiting frequency f_1 , is constant above a preassigned higher frequency f_2 , and in the range between f_1 and f_2 has the value

$$A_4 = k \log \frac{\left[\sqrt{\left(\frac{f}{f_1}\right)^2 - 1} + \frac{f}{f_1} \right]}{\left[\sqrt{\left(\frac{f_2}{f}\right)^2 - 1} + \frac{f_2}{f} \right] \frac{f}{2f_2}} \quad (19)$$

The phase shift has the values given by Equations (13) and (14), respectively, in the ranges below f_1 and between f_1 and f_2 . At frequencies above f_2 it has the value

$$B_4 = k \sin^{-1} \frac{f_2}{f} \quad (20)$$

The curves of Fig. 5 correspond to the case for which the constant k has the value 2 and in which the frequencies f_2 and f_1 are in the ratio of 2.5 to 1. For values of k less than 2 the heights of the ordinates would be proportionately reduced. The attenuation characteristic corresponding to Equation (12) is reproduced by the curve A_1 for comparison purposes. It will be noted that the final value of the attenuation is reached at a frequency one octave lower than the frequency at which the characteristic A_1 reaches the same value and that, at the frequency f_2 where this final value is reached the attenuation is substantially 12 decibels greater than that provided by the characteristic A_1 . The relatively small cut-off interval makes this type of characteristic particularly useful when a sharp cut-off is desired and the effects of parasitic impedances are negligible in the cut-off range.

Examination of the expression in Equation (19) for the attenuation characteristic A_s shows that it may be considered as being arrived at by starting with a low-pass characteristic of the type A_1 and subtracting from it a high-pass characteristic of the type A_2 having the same coefficient k , but having a cut-off at frequency f_2 , and also subtracting a linear characteristic having a constant slope at all frequencies corresponding to the asymptotic slope of A_3 . The phase shift due to the combination of the three characteristics is the sum of the individual phase shifts. By combining the logarithmic characteristics with linear characteristics in other manners, various useful characteristics may be obtained, certain of which are described later in connection with particular embodiments of the invention.

The arbitrary characteristics arrived at in the above manner can always be approximated as closely as may be desired by ascertainable minimum phase-shift networks. A method of design is described in an article by O. J. Zobel on Distortion correction in electrical circuits with constant resistance networks, Bell System Technical Journal, Vol. VII, No. 3, July 1928.

Application to feedback amplifiers

A typical three-stage feedback amplifier is shown in general schematic form in Fig. 6. It comprises vacuum tubes V_1 , V_2 and V_3 connected in tandem by interstage networks N_1 and N_2 , and a feedback network N_3 inserted in a path between the output of the last vacuum tube and the input of the first. The input wave source is represented by the generator G and the source and load impedances are designated Z_s and Z_r , respectively. A series connection of the feedback path is shown in the diagram but the specific character of the connection is unimportant.

The feed back ratio $\mu\beta$ is made up of the amplification factors of the three tubes, the voltage transfer ratios of the interstage networks and the voltage transfer ratio of the feedback network including the source of load impedances. Its value may be expressed in the form

$$\mu\beta = \mu_1\mu_2\mu_3r_1r_2r_3 \quad (21)$$

where μ_1 , μ_2 and μ_3 are the vector amplification factors of the vacuum tubes and r_1 , r_2 and r_3 are the voltage transfer ratios of the networks N_1 , N_2 , and the complete feedback path respectively. The voltage transfer ratios are defined as the ratios of the voltages between the grids and cathodes of the successive tubes to the voltages appearing in the plate circuits of the immediately preceding tubes in the closed loop circuit. Equation (21) may be written in the logarithmic form

$$\log \mu\beta = P - A + j(n\pi - B) \quad (22)$$

where P is the logarithm of the product of the moduli of the amplification factors, A and B are respectively the attenuation and the phase shift of the combined coupling networks, and n is the number of vacuum tube stages. The term $jn\pi$ takes into account the fact that the vector amplification factor of each vacuum tube includes a phase shift of π radians.

Since the amplification factors are constant, the variable part of $\mu\beta$ is represented wholly by the quantities A and B in Equation (22), that is, by the insertion loss of the passive networks. If each of these networks is of the minimum phase-

shift type, the phase angle of $\mu\beta$ will have the minimum variation consistent with the magnitude characteristic of $\mu\beta$.

Having selected an appropriate characteristic for the variation of the magnitude of $\mu\beta$, the insertion loss may be divided among the several networks in any desired manner. Frequently, the part to be assigned to the feedback circuit will be fixed by the over-all amplification characteristic that is wanted, since, with large amounts of feedback, the over-all amplification is substantially equal to the loss in the feedback path. The other parts of the circuit are then designed to make up the difference between the desired $\mu\beta$ characteristic and the part assigned to the feedback network.

In accordance with the invention the cut-off characteristics of the feedback are made to conform to one or other of the optimum attenuation characteristics hereinbefore described. For example, if the operating frequency range is very wide, the upper cut-off characteristic may conform to the low-pass characteristic defined by Equation (12) and the lower cut-off characteristic to the high-pass characteristic of Equation (15). If the operating range is narrow the two cut-offs may be made to conform to the band-pass characteristic defined by Equation (17).

In the case of amplifiers intended for operation in a relatively narrow low frequency range the optimum characteristics defined by the equations may be adopted without any modification. But, when a maximum operating range is required or when the amplifier has to operate at very high frequencies, some modification becomes necessary because of the fact that parasitic impedances of the apparatus elements become important and tend to limit the frequency range available for the cut-off characteristic. The effect of the parasitic impedances is strongly marked at very high and at very low frequencies, but the problem presented is generally more serious in the high frequency cut-off region.

The parasitic impedances principally effective at high frequencies are the inherent shunt capacities of the vacuum tubes and the unavoidable shunt capacities of the network impedance elements. At very high frequencies the impedances of these capacities become dominant and each of the coupling networks ultimately degenerates to a single shunt capacity. The final loss characteristic of the combined networks is then determined solely by the capacities and it exhibits an attenuation which increases uniformly at the rate of 6 decibels per octave for each amplifier stage. A three-stage amplifier will thus exhibit a final loss characteristic which increases at the rate of 18 decibels per octave, or 6 decibels per octave for each of the interstage networks and for the feedback path. This results in a phase shift of 270 degrees which is more than sufficient to bring the amplifier into an unstable condition. At very low frequencies a similar effect may be produced when the circuit contains transformers or series condensers or shunt inductances. However, in this range it frequently happens that the resistances of the circuit elements suffice to limit the rate of rise of the attenuation to a value that is insufficient to cause instability.

In certain cases the leakage inductances of transformers included in the system may appear as parasitic impedances and modify the asymptotic characteristic in important frequency

ranges. The effect of such inductive impedances can usually be computed by standard processes in any particular case.

The high frequency asymptotic characteristic appears as a straight line on the logarithmic frequency scale. Its position depends upon the magnitudes of the parasitic capacities and is controllable only to a limited extent. In narrow band amplifiers a substantial part of the parasitic capacities may be incorporated into the frequency selective network impedances, but in broad band amplifiers, such as are used in multiplex carrier telephone systems, this is rarely possible. The position of the asymptote in the frequency scale may be estimated quite closely from a skeletonized schematic of the amplifier circuit in which only the parasitic capacities appear. Since the parasitic capacity of any particular type of impedance element is substantially independent of the magnitude of the impedance coefficient, the determination of the asymptote requires only a knowledge of the circuit configuration to be used. Subsequent changes in the element values produce little or no change in the asymptote.

The curves of Fig. 7 show the asymptotic characteristic for a particular amplifier and illustrate the manner in which the course of the feedback characteristic may be adjusted to the asymptote to avoid instability. The case illustrated is that of a three-stage amplifier having a useful operating range from substantially zero frequency to 0.5 megacycle. The asymptotic characteristic is represented by the straight line DEKF which meets the axis of zero feedback at a frequency of 2.46 megacycles. In the operating range the value of $\mu\beta$ is constant at 30 decibels and in the cut-off range it falls off in accordance with the optimum characteristic corresponding to Equation (12). This equation defines the attenuation variation of the network portions of the circuit and therefore also the diminution of $\mu\beta$. Two cut-off characteristics are illustrated by curves ABEC and AGKH respectively, the first corresponding to a value of 1.666 for the coefficient k and the second to a value of 1.833. Since at very high frequencies the $\mu\beta$ characteristic is determined wholly by the parasitic impedances, the total characteristic in either of the above cases is a combination of the logarithmic curve and the asymptotic characteristic. Thus, in the first case, the total characteristic will be substantially the broken curve ABEC and in the second case by the broken curve AGKF. Actually there will be a rounding off of the characteristics at the points E and K where the logarithmic curves meet the asymptotic characteristic, but for practical purposes this may be disregarded.

The logarithmic characteristics by themselves give rise the constant phase shifts for all frequencies above the operating range, as shown in Fig. 2, the constant value depending on the coefficient k . For the value of k equal to 1.666, the constant phase shift is 150 degrees, which, with the phase shift of 180 degrees produced by the three-amplifier tubes, provides a margin of 30 degrees against singing. The slope of the characteristic at frequencies remote from the cut-off is 10 decibels per octave. At the point E there is a sudden increase in the slope amounting to 8 decibels per octave, the effect of which is to increase the total phase shift and to reduce the phase margin of stability. The amount of the added phase shift is shown by the curve d_1 the ordinates of which are measured by the scale on

the right-hand side of the figure. This curve is similar to the curve B_2 of Fig. 2 and represents the phase shift for an attenuation which is zero up to the frequency of the point E and then increases at the rate of 8 decibels per octave. At the point B where $\log \mu\beta$ becomes zero, or $\mu\beta$ unity, the added phase shift is 35 degrees. This is greater than the original phase margin of 30 degrees and the circuit is therefore unstable.

By increasing the coefficient k to 1.833, the constant phase margin is reduced to 15 degrees, but the effect of the asymptotic characteristic is diminished by an amount which more than compensates for this reduction. The new characteristic cuts the axis of zero feedback at the point G at a substantially lower frequency and meets the asymptote at a substantially higher frequency than in the previous case. Its final slope is also increased by about one decibel per octave so that the change of slope at the point K where the curve meets the asymptote is now only 7 decibels per octave. All of these factors combine to diminish the added phase shift, the values of which are shown by the curve d_2 . At the crossover point G, the added phase shift is just less than 15 degrees and the circuit is completely stable.

Further improvement in the phase margin of stability can be obtained by making the cut-off characteristic follow a course such as the broken curve AMNF, the portion MN being substantially horizontal and located below the zero feedback axis. The added phase shift in this case is produced by the combination of two linear attenuation characteristics of the same type as shown by the line A_2 in Fig. 2, one starting at the frequency corresponding to the point M and having a positive slope of 11 decibels per octave, and the other starting at the frequency corresponding to point N and having the slope of the asymptotic characteristic, namely 18 decibels per octave. If the frequency limits of the horizontal portion MN are held in about the same ratio as the slopes of the two linear characteristics, the added phase shifts substantially neutralize each other over the critical part of the frequency range. Curve d_3 shows the net added phase shift for the case illustrated. It is substantially zero or negative at all frequencies up to the point D where the asymptote DKF meets the zero axis.

From an examination of the limiting case, for which the logarithmic curve corresponds to a value of 2 for the constant k , giving a maximum phase shift of 180 degrees, and in which the horizontal portion MN coincides with the zero axis, simple expressions are found for the maximum amount of feedback that can be used. For a three-stage amplifier, the maximum feedback is approximately, $12n+5$ decibels, where n is the number of octaves between the edge of the useful band and the frequency at which the asymptote intercepts the zero axis. In this case, the frequency limits of the horizontal portion should be in the ratio of 3 to 2. For amplifiers of two and four stages the corresponding values are $12n+12$ decibels and $12n$ decibels respectively. For the example illustrated in Fig. 7, the interval between the frequencies corresponding to points A and D is 2.25 octaves giving a limiting value of 32 decibels for the maximum feedback.

Fig. 8 shows the circuit of a feedback amplifier embodying the invention which was designed for operation in a frequency range between 60 and 2000 kilocycles per second and to have a uniform feedback of 28 decibels throughout that range.

The amplifier comprises three-vacuum tube stages, each stage employing two tubes in parallel. The tubes used in this particular amplifier were similar in design and construction to the R. C. A. type 954 tube for ultra-high frequency operation, the amplification factor being approximately 2000 and the mutual conductance about 2000 micromhos. The amplifier was designed to operate as a repeater between lines having resistive characteristic impedances of 72 ohms, coupling to the lines being made through shielded input and output transformers T_1 and T_2 . The principal parts of the circuit entering into the $\mu\beta$ characteristic at high frequencies are the coupling impedances Z_1 and Z_2 and the feedback impedance Z_B together with the transformers T_1 and T_2 . The impedance Z_k connected in the cathode circuit of the third vacuum tube provides a local feedback for that tube, but does not materially affect the over-all feedback. The remaining elements of the circuit comprise resistance-capacity filters in the power supply leads to the anodes and screens, cathode resistances and by-pass condensers for providing grid bias potentials, and series coupling condensers and grid leak resistances in the input circuits of the several tubes. These resistance-capacity combinations are effective also in controlling the low-frequency, cut-off characteristic and provide the principal control for that purpose. The low-frequency asymptotic characteristic is established by the grid leak resistances and the associated coupling condensers and the approach of the feedback characteristic to the asymptote is controlled mainly by the cathode impedances and the resistance-capacity filters in the power supply leads to the anodes. The element values are indicated in the figure, the CR product having different values in the several combinations so that each impedance may affect a different portion of the cut-off characteristic.

The impedances Z_B , Z_1 , and Z_2 , which control the feedback in the operating range and at high frequencies, and the local feedback impedance Z_k , are shown in detail in Figs. 9, 10, 11 and 12 respectively. The character and the magnitudes of the various elements are indicated in the figures and need not be further described. Impedances Z_B and Z_2 , which are of complex configuration, are preferably connected into the circuit as indicated by the designations of their terminals. When so connected most of the parasitic shunt capacities are located close to the grounded side of the system and so have a minimum shunting effect on the total impedance.

The feedback characteristics for the above amplifier are shown by the curves of Fig. 13, the optimum characteristics being shown by the dotted lines and the characteristics actually obtained being shown by the continuous curves. The magnitude of $\mu\beta$ is represented by curve 10 and the corresponding optimum characteristic by curve 11. The actual and the optimum phase characteristics are represented by curves 12 and 13 respectively. The agreement between the actual and the optimum characteristics is very close throughout the whole frequency range. The phase characteristics give the phase shift in the passive networks of the system, the difference between this and 180 degrees being the phase angle of the feed back. The design provides a phase margin of stability of approximately 45 degrees.

The asymptote of the feedback is represented by the straight line DNF which has a slope of

18 decibels per octave and intersects the axis of zero feedback at a frequency of about 25 megacycles. The high frequency at which the asymptote meets the zero axis results in part from the smallness of the parasitic capacities and the high gain of the vacuum tubes and also in part from the arrangement of the elements in the coupling impedances so that their parasitic capacities have a minimum effect. The use of single shunt impedances for the coupling and feedback networks is also helpful in raising the frequency of the asymptote.

The relatively large margin of stability indicated by the curves of Fig. 13 makes it possible to increase the feedback of the amplifier by about 10 decibels without risk of instability. This can be done by adjustment of the screen and anode voltages of the vacuum tubes and also by adjusting the resistance of the impedance Z_1 . Increasing the resistance of this impedance results in an increased amplifier gain and a correspondingly increased feedback which follows a new optimum characteristic with a somewhat reduced phase margin. The adjustment, in effect, produces a change in the value of the constant k in Equation (12).

The application of the invention in a high-frequency amplifier having a narrow band selective characteristic is illustrated by Figs. 14 and 15. The amplifier circuit as shown in the figure includes only the elements entering into the feedback and the gain characteristics, the current and voltage supply circuits for the vacuum tubes being omitted for the sake of clearness. The circuit comprises three vacuum tube stages V_1 , V_2 , and V_3 , coupled in tandem by frequency selective shunt impedances and connected through tuned input and output circuits to a wave source G of resistance R_1 , and a resistance load R_{10} . The load resistance R_{10} may, for example, be that of a high-frequency line leading to an antenna. The feedback path extends from the plate of the last tube V_3 to the grid of the first tube and includes a variable resistance R by which the amount of feedback is controlled. Generally the feedback resistance will be large compared with the interstage coupling impedances and the input and output circuit impedances, consequently changes in its value will affect only the amount of feedback and will not modify the shape of the feedback characteristic or of the net amplifier gain characteristic.

The passive networks of the feedback loop comprise the two interstage impedances and the input and output circuits separated from each other by the series feedback resistance. The parasitic capacities of the system appear as shunt capacities associated with the coupling impedances and enter into the tuning of these. If all of the coupling impedances comprised simple anti-resonant circuits tuned to the operating frequency, the parasitic capacities being included, the gain of the amplifier and the gain around the feedback loop would be a maximum at the operating frequency and would fall off symmetrically on both sides thereof. At frequencies not far removed from the operating frequency the diminution of the gain produced by each tuned circuit would assume the rate of six decibels per octave of departure from the operating frequency value and the phase shift from the phase at the operating frequency would be 90 degrees for each circuit. Since, in the feedback loop, the input and output tuned circuits are separated by the feedback resistance, the circuit illustrated contains four

tuned circuits which are separately effective in determining the characteristics. If all of these were simple anti-resonant circuits the rate of diminution of the feedback would be 24 decibels per octave of departure and the total phase shift 360 degrees. Evidently this condition would greatly limit the amount of feedback that could be used. In the circuit illustrated, this limitation is overcome in the following manner.

The maximum gain and the broadest transmission range of the amplifier would be obtained if the coupling impedances consisted only of the parasitic shunt capacities and shunt inductances large enough to resonate with these capacities at the operating frequency. The limiting characteristics obtained with such impedances are symmetrical about the operating frequency and may be regarded as the asymptotic characteristics of the system. In accordance with the invention the limiting asymptotes are first established by resonating the parasitic capacities as described above and then supplementary impedances are added to control the course of the feedback characteristic within the range between the asymptotes.

The amplifier circuit is designed to give a uniform feedback of 25 decibels throughout an assigned narrow band and to follow a course corresponding to Equation (17) in the cut-off ranges, the constant k having the value 1.75 which provides a phase margin of stability of 22.5 degrees. The ideal feedback characteristic and the characteristic provided by the circuit of Fig. 14 are shown respectively by curves 14 and 15 of Fig. 15. Only one side of the characteristic is shown, but the other side is symmetrical therewith about the mid-frequency of the band. The asymptotic characteristic is represented by the straight line 16 which has a slope of 24 decibels per octave of departure from the band center. For the circuit illustrated the center of the operating range lies at a frequency of 10 megacycles and the limits of the range lie at frequencies 100 kilocycles above and below this point. The band width is 200 kilocycles which is two per cent of the mean frequency. The cut-off characteristic follows the logarithmic course from the band limits to frequencies 380 kilocycles above and below the band and thereafter follows a horizontal course until it meets the asymptotes. Zero feedback gain occurs at frequencies about 280 kilocycles above and below the center of the band. The design characteristic follows the ideal characteristic with minor undulating departures therefrom, the agreement with the ideal characteristic being obtained by appropriate tuning of the various resonant impedance combinations and by proportioning the resistances thereof.

Referring to Fig. 14, the several impedances shown therein are tuned as follows. The input circuit L_1C_1 is tuned to the mid-frequency of 10 megacycles. This combination may be regarded as including the parasitic capacity and its tuning inductance together with an additional anti-resonant circuit of lower impedance proportioned to confine the response within the assigned band limits. In the first interstage circuit C_2 is the total parasitic capacity and L_2 is an inductance tuning with this capacity at 10 megacycles. Resonant circuits L_3C_3 and L_4C_4 are tuned respectively to frequencies just beyond the band limits of 9.90 and 10.1 megacycles, and in combination are anti-resonant at 10 megacycles. The effect of these circuits is to sharpen the cut-off at the band limits, the degree of sharpening being controlled by

the L/C ratios of the circuits and their resistances R_3 and R_4 . These resistances may be separate elements or they may be provided by the winding resistances of the inductances. In the second interstage circuit C_5 is the total parasitic capacity and L_5 the tuning inductance therefor. Resonant circuits L_6C_6 and L_7C_7 are tuned to frequencies 380 kilocycles above and below 10 megacycles, respectively, and to be anti-resonant in combination at 10 megacycles. These circuits provide the flattening of the feedback characteristic as it approaches the asymptotes, the shape being controlled by adjustment of the damping by resistances R_6 and R_7 . The combination L_8C_8 in the output circuit includes the parasitic capacity and is tuned to be anti-resonant at 10 megacycles with a response similar to that of L_1C_1 . Circuit L_9C_9 is resonant also at 10 megacycles and is further proportioned so that the total output impedance exhibits anti-resonances at the band limits, 10.1 megacycles and 9.90 megacycles. The output combination acts to sustain the amplifier gain and the feedback at a uniform level up to the band limits and, in conjunction with the first interstage coupling impedance, imparts the correct shape to the feedback cut-off characteristic close to the band limits. As in the case of the other coupling impedances, the effect of the output circuit impedance may be controlled by varying the L/C ratios of the tuned circuits and by adjustment of the damping resistance R_9 .

The manner in which the invention may be applied to amplifying systems including frequency changing circuits is illustrated diagrammatically in Figs. 16 and 17. The circuit shown in the block schematic of Fig. 16 is that of a narrow band radio transmitter, for example, a radio broadcast transmitter. The transmitter proper comprises a carrier wave generator 17, modulator 18, power amplifier 19, antenna 20, and a speech input circuit including signal source 21 and signal amplifier 22. The feedback path comprises an antenna 23 or other pick-up means, preferably located fairly close to the transmitting antenna, linear rectifier 24, linear amplifier 25 stabilized by feedback resistance 26, and feedback network 27. The feedback path terminates at the input of the signal amplifier 22, the feedback voltage being there superimposed upon the signal voltage from source 21. Instead of using an antenna to pick up the waves radiated from the transmitter, direct coupling of the feedback path to a point in the output circuits of the power amplifier may be employed if desired. Also, if desired, the stabilized amplifier 25 may be omitted, but in systems such as broadcast transmitters where only a single speech channel is used a considerable advantage may be realized by the use of this amplifier without noticeably diminishing the benefits obtained by the use of feedback.

The feedback network 27 provides the control of the feedback characteristic. Because of the large number of vacuum tube stages usually present in a transmitter of large power, it is usually preferable to design this network in accordance with data from experimental measurements of the feedback loop gain in any particular case. This may be obtained, for example, by opening the feedback path at the output of detector 24 and comparing the magnitude of the detected signal voltage at that point with the signal voltage at the input of signal amplifier 22. In making these measurements it is necessary that the portions of the feedback circuit on the two sides of the interruption be closed through impedances

substantially equal to the impedances into which they normally work.

Referring to Fig. 17, curve 28 represents a typical feedback characteristic obtained by measurement in the manner described above. It reproduces at low frequencies the variations imposed by the high frequency selective circuits and at some relatively high frequency merges into an asymptotic characteristic 32, the steepness of which depends on the number of amplifier stages included. Having determined the asymptotic characteristic, an optimum characteristic in accordance with the invention may be adjusted thereto as described in connection with Fig. 7. Such a characteristic is shown by curve 29 and represents substantially the maximum feedback that can be applied with the given loop gain. Network 27 may then be designed in accordance with standard methods to have an attenuation characteristic equal to the difference between curves 28 and 29. The completion of the feedback circuit through this network will then bring the feedback characteristic into accord with the desired optimum.

Curves 28 and 29 refer to the circuit with the amplifier 25 omitted. With the relatively sharp tuning used in the high frequency circuits of broadcast transmitters and the like, the feedback loop gain, measured in the audio frequency portion of the system will generally drop to zero at a frequency of the order of 20 to 30 kilocycles. Since it is necessary to provide for a signal band extending from about 100 cycles per second to about 7500 cycles per second it is evident that the frequency range available for the feedback cut-off is quite limited and the amount of feedback that can be used is correspondingly limited. In typical instances it has been found that not more than about 15 decibels feedback could be used. By the inclusion of stabilized amplifier 25 in the feedback path a considerable increase in the amount of feedback is made possible.

Ordinarily it is undesirable to include vacuum tubes or other non-linear elements in the feedback path, since the linearity and freedom from distortion of the whole system is no greater than that of the feedback path. However, by the use of stabilizing feedback in the amplifier 25 and by designing this amplifier to have an operating frequency range extending well beyond the effective feedback range of the other parts of the system, a large amount of amplification can be introduced without noticeably diminishing the quality of the transmission. The amplifier may be designed and constructed in accordance with the principles described in the article by H. S. Black on stabilized feedback amplifiers hereinbefore mentioned and may readily be made to have a uniform gain of 30 decibels or more throughout a frequency range from 100 cycles per second to 60 kilocycles per second or greater. Because of the action of the feedback the phase shift in the amplifier 25 is negligibly small in the operating range and the net effect of the addition of the amplifier is to increase the feedback loop gain without adding to the phase shift or introducing distortion.

The effect of adding to the gain of the feedback path in the above manner is illustrated by curves 30 and 31 of Fig. 17. Curve 30 corresponds to curve 28 moved upward by the uniform amount equal to the gain of amplifier 25. The location of the high frequency asymptotic characteristic is shifted upward on the frequency scale, thereby providing a greater frequency range for the final

feedback cut-off characteristic and, therefore, permitting a larger amount of feedback to be used as indicated by curve 31. It will be evident from an inspection of the form of the curves of Fig. 17 that not all of the gain of amplifier 25 goes to increasing the feedback. The net increase in the feedback depends on the slope of the asymptotic characteristic, which determines the frequency shift in its location brought about by the use of the amplifier. When the slope is great it may be found that the net increase of feedback may be about one-sixth of the amplifier gain.

What is claimed is:

1. A wave amplifying system comprising an amplifier, a feedback path extending between the output and input circuits of said amplifier and forming therewith a closed loop circuit, and frequency selective impedance networks included in said loop circuit, said networks being proportioned to provide in combination with the gain of the amplifier a relatively large loop gain between two assigned frequencies f_1 and f_2 determining the limits of a utilization range and to diminish the loop gain at frequencies outside the utilization range in accordance with the relationship

$$A = k \log_e [\sqrt{q^2 - 1} + q]$$

in which A denotes the diminution of the loop gain in nepers, q denotes the absolute magnitude of the frequency function

$$\frac{f^2 - f_1 f_2}{f(f_2 - f_1)}$$

and k is a numerical factor of value not greater than two.

2. A wave amplifying system comprising an amplifier, a feedback path extending between the output and input circuits of said amplifier and forming therewith a closed loop circuit, and frequency selective networks included in said loop circuit, said networks being proportioned to provide in combination with the gain of said amplifier a substantially uniform loop gain between two assigned frequencies f_1 and f_2 determining the limits of a utilization range and to diminish the loop gain at frequencies outside the utilization range in accordance with the relationship

$$A = k \log_e [\sqrt{q^2 - 1} + q]$$

in which A denotes the diminution of the loop gain in nepers, q denotes the absolute magnitude of the frequency function

$$\frac{f^2 - f_1 f_2}{f(f_2 - f_1)}$$

and k is a numerical factor not greater than two.

3. A wave amplifying system comprising a plurality of vacuum tube amplifying devices, impedance networks coupling said tubes in tandem, a feedback path extending between the output circuit of the last of said tandem connected tubes and the input circuit of the first of said tubes and forming therewith a closed loop circuit, and an impedance network included in said feedback path, said impedance networks having a combined attenuation which is low relatively to the combined gain of said amplifying devices at frequencies between two assigned values f_1 and f_2 determining the limits of a utilization range and which increases at frequencies outside of said range in accordance with the relationship

$$A = k \log_e [\sqrt{q^2 - 1} + q]$$

in which A denotes the increase of the attenua-

tion in nepers, q is the absolute magnitude of the frequency function

$$\frac{f^2 - f_1 f_2}{f(f_2 - f_1)}$$

and k is a numerical factor of value not greater than two.

4. An amplifying system in accordance with claim 1 in which the said selective impedances are proportioned to make the limiting frequency f_1 of the utilization range sufficiently low compared with the upper limiting frequency f_2 so that the ratio of f_1 to f_2 is a negligibly small quantity and the loop gain above the frequency f_2 diminishes in accordance with the relationship

$$A = k \log_e \left[\sqrt{\left(\frac{f}{f_1}\right)^2 - 1} + \frac{f}{f_1} \right]$$

where A denotes the diminution of the loop circuit gain in nepers.

5. A wave amplifying system comprising a plurality of vacuum tube amplifying devices, impedance networks coupling said tubes in tandem, a feedback path coupling the output circuit of the last of said tandem connected tubes with the input circuit of the first of said tubes, and an impedance network included in said feedback path, said impedance networks having a combined attenuation which is substantially constant and small relatively to the combined gain of said amplifying devices at frequencies in an assigned operating range and which increases with frequency above said operating range at a rate which is high at frequencies adjacent said operating range and diminishes to a substantially constant rate between the limits of nine and twelve decibels per octave at frequencies more remote.

6. An amplifying system in accordance with claim 3 in which one of the said vacuum tube stages includes frequency changing means and in which the feedback path includes complementary frequency changing means.

7. An amplifying system in accordance with claim 1 in which there is included in the said feedback path a degenerative feedback amplifier having uniform gain over a frequency range which is large compared with the assigned utilization range of the system.

8. An amplifying system in accordance with claim 3 in which one of the said vacuum tube stages includes frequency changing means and in which the said feedback path includes complementary frequency changing means and a degenerative feed-back amplifier having substantially zero phase shift in the assigned utilization frequency range of the system.

9. A wave amplifying system comprising an amplifier, a feedback path extending between the output and input of said amplifier and forming therewith a closed loop circuit, and frequency selective impedance networks included in said loop circuit, said networks being proportioned to provide in combination with the gain of the amplifier a relatively large loop gain at frequencies below an assigned frequency f_1 and to diminish the loop gain at frequencies above f_1 in accordance with the relationship

$$A = k \log_e \left[\sqrt{\left(\frac{f}{f_1}\right)^2 - 1} + \frac{f}{f_1} \right]$$

where A denotes the diminution of the loop gain in nepers, and k is a numerical factor of value between 1 and 2.

10. A wave amplifying system comprising a plurality of vacuum tube amplifying devices, impedance networks coupling said vacuum tubes in

tandem, a feedback path extending between the output circuit of the last of said tandem connected tubes and the input circuit of the first of said tubes, and an impedance network included in said feedback path, said impedance networks having a combined insertion loss characterized by uniform low attenuation and progressively changing phase at frequencies in an assigned operating range and by progressively increasing attenuation and constant phase at frequencies outside said range, the rate of diminution of the attenuation having a value between 9 and 12 decibels per octave at frequencies remote from said operating range.

11. In a carrier wave transmitter comprising a carrier wave source, a signal source, modulating means, means for impressing oscillations from said carrier source and said signal source upon said modulating means, and a high frequency amplifier coupled to said modulating means for amplifying the modulated carrier wave produced therein, a feedback path extending from the output circuit of said amplifier to the input circuit of said signal source, a rectifier in said feedback path and a degenerative feedback amplifier included in said feedback path between said rectifier and said signal source, said amplifier having uniform gain over a frequency range which includes the frequency range of the signals from said source and is large compared therewith.

12. A wave amplifying system in accordance with claim 2 in which the said frequency selective networks included in the loop circuit are all of such configurations as to provide phase shifts which are uniquely determined by their attenuation characteristics.

13. A wave amplifying system in accordance with claim 5 in which the said coupling networks and the said network included in the feedback path are all of such configurations as to produce the minimum phase shifts consistent with their attenuation characteristics.

14. An amplification system in accordance with claim 2 in which the networks included in the said closed loop circuit are proportioned to diminish the loop gain at frequencies outside the utilization range in accordance with the relationship

$$A = k \log_e [\sqrt{q^2 - 1} + q]$$

until the gain in nepers attains a negative value not greatly different from zero, and are further proportioned to maintain the loop gain substantially constant at the said negative value as the frequency departs further from the utilization range, whereby the effect of parasitic impedances in the system upon the phase shift of the feedback is substantially neutralized at the frequency of zero loop gain.

15. An amplifying system in accordance with claim 9 in which the networks included in the said closed loop circuit are proportioned to diminish the loop gain at frequencies greater than the assigned frequency f_1 in accordance with the relationship

$$A = k \log_e \left[\sqrt{\left(\frac{f}{f_1}\right)^2 - 1} + \frac{f}{f_1} \right]$$

until the gain in nepers attains a negative value not greatly different from zero, and are further proportioned to maintain the loop gain substantially constant at the said negative value at higher frequencies, whereby the effect of parasitic impedances in the system upon the phase shift of the feedback is substantially neutralized at the frequency of zero gain.