May 3, 1938. E. L. NORTON ET AL 2,115,826

IMPEDANCE TRANSFORMER

Filed Sept. 30, 1936 2 Sheets-Sheet 1

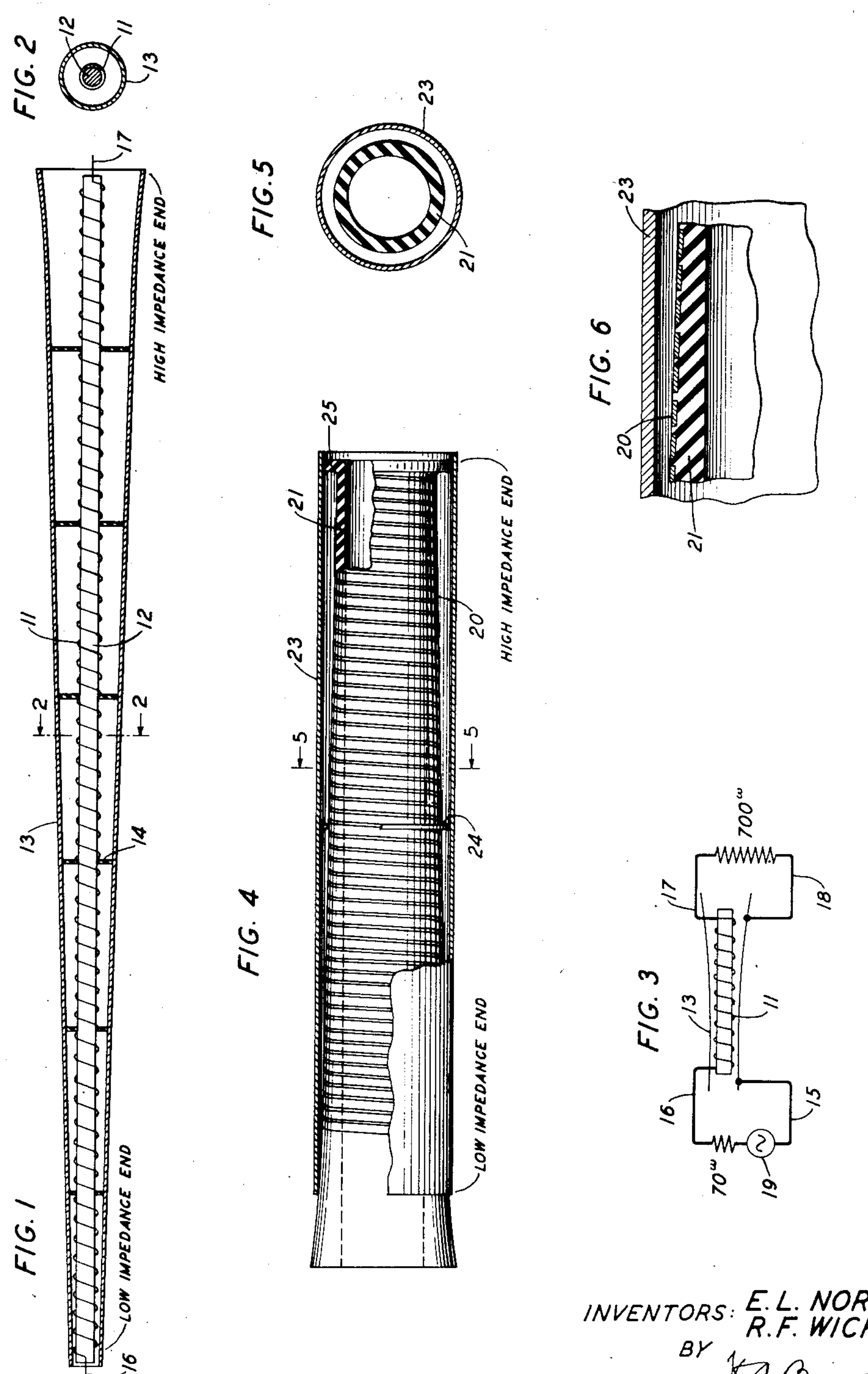

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FIG. 7

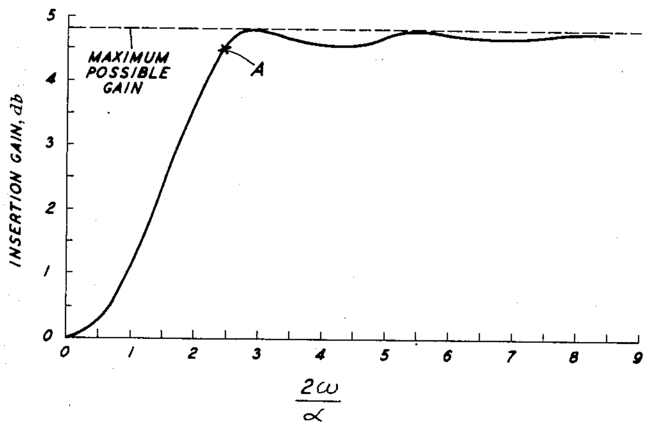


FIG. 8

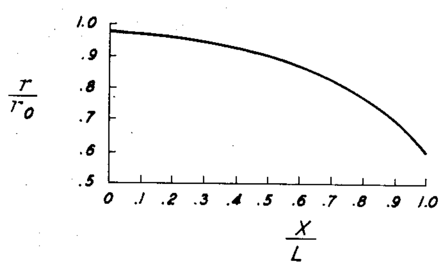
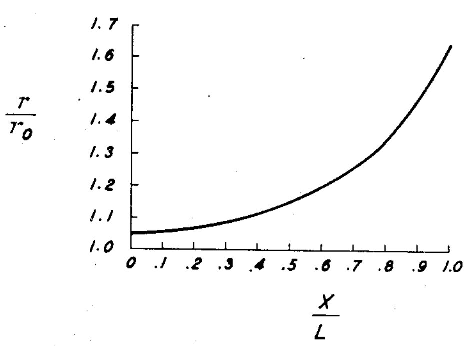


FIG. 9



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IMPEDANCE TRANSFORMER

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Application September 30, 1936, Serial No. 103,316

16 Claims. (Cl. 178—44)

This invention relates to impedance transformers for coupling lines or electrical devices of unequal impedance and has for an object to provide an impedance transformer which will be substantially independent of the frequency over a wide range of high frequencies.

In the preferred embodiment the coupling device takes the form of a tapered transmission line comprising a long cylindrical coil of a large number of turns surrounded by a concentric metallic sheath of circular cross-section in which the sheath or the coil is tapered in radius. The dimensions of the coil and sheath and the rate of taper have values dependent upon certain factors such as the ratio of the two impedances to be connected by the device and the minimum frequency to be transmitted as will be explained hereinafter. In employing such a properly designed device as a high frequency transformer between a low impedance line and a high impedance line one terminal of the low impedance line is connected to the low impedance end of the coil, one terminal of the high impedance line is connected to the high impedance end of the coil, and the other two terminals of the two lines are connected to opposite ends of the surrounding metallic sheath. The parameters of such a tapered line may be readily computed in order that its impedance at each end will be substantially equal to the impedance of the line connected thereto. The device constructed in this manner will act efficiently as a step-up transformer over a wide range of frequencies and the device is of particular interest in the frequency range above one million cycles per second.

A uniform transmission line is characterized by an impedance,

$$z=\sqrt{\frac{L}{C}}$$

and a velocity constant,

$$a=\frac{1}{\sqrt{LC}}$$

where L and C are the inductance and capacity per unit length of the line. It is possible so to construct a line that L and C vary along its length, giving thus a tapered line. A flexible and practical arrangement consists of a solenoid enclosed in a conducting sheath. The ratio of sheath diameter to coil diameter at any point then determines C, while L is determined by this ratio, the number of turns per unit length, and the diameter of the coil, at that point. The sheath affects the inductance, of course, because of currents induced in it by the coil. While this effect could be eliminated or reduced to a small value by slitting the sheath, it is desirable because it permits a much greater rate of change in z than would otherwise obtain, and also because it acts as an electromagnetic shield for the coil. The tapering effect can be controlled by varying the diameter of sheath or of coil or both, or by varying the number of turns per unit length of the coil. This latter could be accomplished by varying the pitch of the winding or by using a multilayer winding and varying the number of layers. In the embodiments hereinafter described, single layer coils of constant pitch have been used, with either the coil diameter uniform and sheath diameter variable, or vice versa. When a tapered line is so constructed that its impedance varies from some value, z_1, at one end to a different value, z_2, at the other, it behaves like a transformer at sufficiently high frequencies. It is this property with which we are here concerned.

Referring to the drawings,

Fig. 1 represents a longitudinal section and Fig. 2 a cross-section of an impedance transformer in which the coil is of constant radius and the surrounding sheath is tapered;

Fig. 3 represents the manner of connecting the impedance transformer of Fig. 1 in a transmission circuit;

Figs. 4 and 5 represent another type of impedance transformer in which the coil is of tapered diameter and the surrounding sheath is of constant radius;

Fig. 6 is an enlarged longitudinal section of the tapered line of Fig. 4;

Fig. 7 represents the frequency transmission characteristic of the tapered line of Fig. 1;

Fig. 8 represents the contour of the tapered coil line of Fig. 4; and

Fig. 9 represents the contour of the tapered sheath line of Fig. 1.

In determining the parameters of such tapered line used as an impedance transformer advantage is taken of the observation that when the differential equations are expressed in terms of the time required for a wave to travel along the line, instead of in terms of the distance, the velocity function drops out. That is, the electrical characteristics of a tapered line depend solely on the way in which the impedance varies with the time of transmission. Thus all the electrical characteristics obtainable with tapered lines can be studied by considering different functions for this one parameter. A particular function for the impedance having been chosen, the actual shape of the line, for any desired type of construction can be found by direct means.

We begin the mathematical discussion by writing down the fundamental differential equation for a non-dissipative transmission line with a sinusoidal applied voltage of frequency

$$\frac{\omega}{2\pi}$$

$$-\frac{dv}{dX}=j\omega Li$$

$$-\frac{di}{dX}=j\omega Cv$$

in which

v=voltage at distance X from one end of the line

i=current at same point

L=inductance per unit length at this point

C=capacity per unit length at this point

$j=\sqrt{-1}$

Expressed in terms of the impedance,

$$z=\sqrt{\frac{L}{C}}$$

and the velocity,

$$a=\frac{1}{\sqrt{LC}}$$

these equations are:

$$-\frac{dv}{dX}=j\omega\frac{z}{a}i \qquad (1)$$

$$-\frac{di}{dX}=j\omega\frac{1}{za}v \qquad (2)$$

The time required for a wave to travel to this point from the end where X=0 is

$$t=\int_0^X\frac{dX}{a}$$

from which

$$\frac{dt}{dX}=\frac{1}{a}$$

Since

$$\frac{d}{dX}=\frac{dt}{dX}\frac{d}{dt}=\frac{1}{a}\frac{d}{dt}$$

Equations (1) and (2) can be written

$$-\frac{dv}{dt}=j\omega zi \qquad (1')$$

$$-\frac{di}{dt}=j\omega\frac{v}{z} \qquad (2')$$

The velocity, a, has canceled out, showing that the electrical characteristics of the line depend only on the impedance z regarded as a function of the time t.

Differentiating Equation (2') once with respect to t and using (1') and (2') to eliminate v and

$$\frac{dv}{dt}$$

we get

$$\frac{d^2i}{dt^2}+\frac{1}{z}\frac{dz}{dt}\frac{di}{dt}+\omega^2i=0 \qquad (3)$$

In order to study the various possible electrical characteristics it is now necessary to assume a particular functional form for $z(t)$. One of the simplest is

$$z(t)=z_1\left(1+\frac{t}{t_1}\right)^m \qquad (4)$$

The rate of taper is fixed by the pure number m. This function leads to solutions of (3) which, for general values of m, contain Bessel functions. When m is an even integer, the solutions are in terms of sines and cosines. In Equation (4), z_1 is the value of the line impedance at the low impedance end, say the left-hand end. The physical significance of t_1 is explained as follows: Suppose the line were extended to the left until a point was reached where the line impedance was zero. Then t_1 is the time required for a wave, started at this end of the line, to reach that part of the line where $z=z_1$ (the actual extremity of the physical line).

Another type of characteristic is the exponential:

$$z(t)=z_1e^{\alpha t} \qquad (5)$$

in which the rate of taper is determined by α. This is a limiting case of (4) obtained by allowing m and t_1 to approach infinity in such a way that

$$\frac{m}{t_1}=\alpha$$

remains constant.

When the function (4) is used for $z(t)$, Equation (3) becomes

$$\frac{d^2i}{dt^2}+\frac{m}{t+t_1}\frac{di}{dt}+\omega^2i=0 \qquad (6)$$

Put

$$s=\omega(t+t_1)$$

and define W by

$$i=Wy^{\frac{1-m}{2}}$$

Then Equation (6) becomes

$$s^2\frac{d^2W}{ds^2}+s\frac{dW}{ds}+\left[s^2-\left(\frac{m-1}{2}\right)^2\right]W=0 \qquad (7)$$

This is Bessel's equation and the general solution for the current in a tapered line having the type of taper given by Equation (4) is thus

$$i=A\left(J_{\frac{m-1}{2}}[\omega(t+t_1)]+BY_{\frac{m-1}{2}}[\omega(t+t_1)]\right) \qquad (8)$$

where

$$J_{\frac{m-1}{2}}[\omega(t+t_1)] \text{ and } Y_{\frac{m-1}{2}}[\omega(t+t_1)]$$

are Bessel functions of first and second kind, respectively, and of order

$$\frac{m-1}{2}$$

and argument $\omega(t+t_1)$. A and B are constants which are to be determined by the boundary conditions. The voltage can be found from the current by Equation (2').

When a tapered line is used as a transformer the values of the line impedance z, at the ends of the line are made equal to the terminal impedances at the respective ends. The measure of the electrical performance we take to be the "insertion gain of the line between the terminal resistances", which we define as

$$\text{Insertion gain}=20\log_{10}\frac{I_2}{I_2'}db$$

where I_2 is the current in the receiving resistance when an E. M. F. is applied to the other end of the line through the terminal resistance appropriate to that end, and I_2' is the current which would flow in the receiving resistance if the tapered line were removed and the input connected directly to the receiving termination.

The boundary conditions are:

$$\left.\begin{array}{c}\text{At the low impedance end,}\\ t=0 \text{ and } E=z_1i(0)+v(0);\\ \text{at the high impedance end,}\\ t=t_2\text{, say, and }\dfrac{v(t_2)}{i(t_2)}=z_2\end{array}\right\} \qquad (9)$$

E is the generator E. M. F.; t_2 is the time required for a wave to travel the length of the line; $i(0)$, $v(0)$ are the current and voltage at the low impedance end of the line; $i(t_2)$, $v(t_2)$ are the current and voltage at the high impedance end; and z_1, z_2 are the values of the terminal impedances. The constant t_2 is determined in terms of z_1, z_2, t_1, and m by Equation (4):

$$z_2=z_1\left(1+\frac{t_2}{t_1}\right)^m$$

or

$$t_2=t_1\left[\left(\frac{z_2}{z_1}\right)^{\frac{1}{m}}-1\right]$$

Equations (9) thus permit the determination of the constants A and B, and finally the insertion gain. The insertion gain will now be given for different rates of taper of the impedance by first, making $m=1$ (so-called linear line); second, making $m=2$ (so-called conical line); and third, making m as well as t_1 approach infinity in such a way that

$$\frac{m}{t_1}$$

remains constant (so-called exponential line).

For the linear line ($m=1$):

$$\text{Insertion gain}=20\log_{10}\frac{1+\frac{z_2}{z_1}}{2\sqrt{\frac{z_2}{z_1}}}$$

In the first two cases when

$$\frac{t_2}{t_1}$$

is fixed the insertion gain may be plotted as a function of ωt_1. The ratio

$$\frac{t_2}{t_1}$$

is found from the impedance ratio by Equation (4):

$$\frac{t_2}{t_1}=\frac{z_2}{z_1}-1, \text{ for linear line,}$$

$$=\sqrt{\frac{z_2}{z_1}}-1 \text{ for conical line.}$$

In the exponential case, when αt_2 is fixed the insertion gain may be plotted as a function of

$$\frac{2\omega}{\alpha}$$

The parameter αt_2 is determined from the impedance ratio by Equation (5):

$$\frac{z_2}{z_1}=e^{\alpha t_2}$$

$$\alpha t_2=\log\frac{z_2}{z_1}$$

The quantity t_1, in the case of linear and conical lines, or α, in the case of exponential lines, is

$$-20\log_{10}\frac{\sqrt{\left[1+\frac{\pi^2}{4}\omega^2 t_1^2\left(1+\frac{t_2}{t_1}\right)(D_{00}^2+D_{01}D_{10})\right]^2+\frac{\pi^4}{16}\omega^4 t_1^4\left(1+\frac{t_2}{t_1}\right)^2 D_{00}^2(D_{01}-D_{10})^2}}{\omega t_1\pi D_{00}\sqrt{1+\frac{t_2}{t_1}}} \qquad (10)$$

in which

$$D_{00}\equiv J_0(\omega t_1)Y_0\left[\omega t_1\left(1+\frac{t_2}{t_1}\right)\right]-J_0\left[\omega t_1\left(1+\frac{t_2}{t_1}\right)\right]Y_0(\omega t_1)$$

$$D_{01}\equiv J_0(\omega t_1)Y_1\left[\omega t_1\left(1+\frac{t_2}{t_1}\right)\right]-J_1\left[\omega t_1\left(1+\frac{t_2}{t_1}\right)\right]Y_0(\omega t_1)$$

$$D_{10}\equiv J_1(\omega t_1)Y_0\left[\omega t_1\left(1+\frac{t_2}{t_1}\right)\right]-J_0\left[\omega t_1\left(1+\frac{t_2}{t_1}\right)\right]Y_1(\omega t_1)$$

$$D_{11}\equiv J_1(\omega t_1)Y_1\left[\omega t_1\left(1+\frac{t_2}{t_1}\right)\right]-J_1\left[\omega t_1\left(1+\frac{t_2}{t_1}\right)\right]Y_1(\omega t_1)$$

The J- and Y-functions are Bessel functions of the first and second kind, respectively, and satisfy the same recurrence formulae.

For the conical line ($m=2$):

$$\text{Insertion gain}=20\log_{10}\frac{1+\frac{z_2}{z_1}}{2\sqrt{\frac{z_2}{z_1}}}$$

determined in any specific case by the lowest frequency it is desired to transmit. The "cut-off frequency" of a tapered line is not, in general, sharply defined. Consequently, one should plot the insertion gain for the impedance ratio desired before deciding upon the value of t_1 (or α).

The above discussion was concerned with the study of possible electrical characteristics of tapered lines without regard to the mechanical

$$-20\log_{10}\sqrt{1+\frac{1-\left[1+4\omega^2 t_1^2\left(1+\frac{t_2}{t_1}\right)\right]\cos 2\frac{t_2}{t_1}\omega t_1-2\frac{t_2}{t_1}\omega t_1\sin 2\frac{t_2}{t_1}\omega t_1}{8\omega^4 t_1^4\left(1+\frac{t_2}{t_1}\right)^2}} \qquad (11)$$

For the exponential line

$$\left(m \text{ and } t_1=\infty, \frac{m}{t_1}=\alpha\right)$$

$$\text{Insertion gain}=20\log_{10}\frac{1+\frac{z_2}{z_1}}{2\sqrt{\frac{z_2}{z_1}}}$$

$$-20\log_{10}\sqrt{1+\frac{\sin^2\left[\frac{\alpha t_2}{2}\sqrt{\left(\frac{2\omega}{\alpha}\right)^2-1}\right]}{\left(\frac{2\omega}{\alpha}\right)^2-1}} \qquad (12)$$

construction. We shall now show how to determine the shape, size, and number of turns of wire for two particular kinds of mechanical construction, first, where a tapered sheath and a constant radius coil are employed and, second, where a tapered coil and a constant radius sheath are employed.

I—Development of equations for transformer with tapered sheath

First, let the tapered line be made up of a long cylindrical coil of circular cross-section having a constant radius r_0 and N turns per unit length, surrounded by a shield with a variable radius r. The inductance per unit length is:

$$L=4\pi^2 r_0^2 N^2\left(1-\frac{r_0^2}{r^2}\right)10^{-9} \tag{13}$$

and the capacity per unit length:

$$C=\frac{10^9}{2c^2\log\frac{r}{r_0}} \tag{14}$$

where $c=3\times10^{10}$ centimeters per second, the velocity of light.

The factor

$$1-\frac{r_0^2}{r^2}$$

in the inductance formula takes account of the "short-circuiting" effect of the sheath which permits taper in the inductance. The impedance and velocity are given by

$$z=\sqrt{\frac{L}{C}}=2\pi r_0 N c 10^{-9}\sqrt{2\left(1-\frac{r_0^2}{r^2}\right)\log\frac{r}{r_0}} \tag{15}$$

$$a=\frac{1}{\sqrt{LC}}=\frac{c}{2\pi r_0 N}\sqrt{\frac{2\log\frac{r}{r_0}}{1-\frac{r_0^2}{r^2}}} \tag{16}$$

By definition let

$$\left.\begin{aligned} y&=2\log\frac{r}{r_0}\\ a_0&=\frac{c}{2\pi r_0 N}\end{aligned}\right\} \tag{17}$$

and let the subscripts 1, 2 refer to the low and high impedance ends of the tapered line, respectively. Then

$$z=\frac{c^2 10^{-9}}{a_0}\sqrt{y(1-e^{-y})} \tag{18}$$

or

$$z=z_1\sqrt{\frac{y(1-e^{-y})}{y_1\left(1-e^{-y_1}\right)}} \tag{19}$$

$$a=a_0\sqrt{\frac{y}{1-e^{-y}}} \tag{20}$$

We want a relation between X, the axial distance, and y.

By definition of t,

$$\frac{dt}{dX}=\frac{1}{a}$$

and from (20)

$$\frac{dt}{dX}=\frac{1}{a_0\sqrt{\frac{y}{1-e^{-y}}}} \tag{21}$$

Suppose we consider a perfectly general variation of z with respect to t and write it

$$z=z_1 f(t)$$

Later, the specific forms of $f(t)$ given by Equations (4) and (5) will be taken up. Differentiating (22) we have

$$\frac{dz}{dt}=z_1 f'(t)$$

Where

$$f'(t)=\frac{df}{dt}$$

But

$$\frac{dz}{dt}=\frac{dz}{dy}\frac{dy}{dX}\frac{dX}{dt}$$

and using (21) we get

$$z_1 f'(t)=a_0\sqrt{\frac{y}{1-e^{-y}}}\frac{dz}{dy}\frac{dy}{dX} \tag{23}$$

Now

$$\frac{dz}{dy}$$

is found from (19):

$$\frac{dz}{dy}=\frac{1}{2}z_1\frac{1-e^{-y}+ye^{-y}}{\sqrt{y(1-e^{-y})y_1\left(1-e^{-y_1}\right)}}$$

This is substituted into (23), giving the differential equation

$$\frac{dy}{dX}=\frac{2f'(t)}{a_0}\frac{(1-e^{-y})\sqrt{y_1\left(1-e^{-y_1}\right)}}{1-e^{-y}+ye^{-y}}$$

Integrating from $X=0$ to $X=X$, $y=y_1$ to $y=y$:

$$X=\frac{a_0}{2\sqrt{y_1\left(1-e^{-y_1}\right)}}\int_{y_1}^{y}\frac{1+\frac{y}{e^y-1}}{f'(t)}dy \tag{24}$$

The appearance of a function of t in the integrand may be confusing, since integration is to be performed with respect to y. The way this is to be handled will be made clear in the specific cases now to be discussed.

Let the function $f(t)$ be that determined by Equation (4):

$$f(t)=\left(1+\frac{t}{t_1}\right)^m$$

Differentiating:

$$f'(t)=\frac{m}{t_1}\left(1+\frac{t}{t_1}\right)^{m-1}$$

But

$$1+\frac{t}{t_1}=[f(t)]^{\frac{1}{m}}$$

and from (19)

$$f(t)=\sqrt{\frac{y(1-e^{-y})}{y_1\left(1-e^{-y_1}\right)}}$$

Hence we have

$$f'(t)=\frac{m}{t_1}\left[\frac{y(1-e^{-y})}{y_1\left(1-e^{-y_1}\right)}\right]^{\frac{m-1}{2m}}$$

The possibility of expressing $f'(t)$ explicitly in terms of $f(t)$ was necessary to the solution of the present problem, the determination of a usable relation between X and y to arrive at the shape of the tapered sheath.

Finally, substituting this expression for $f'(t)$ into (24) we get the desired equation

$$\frac{2mX\left[y_1\left(1-e^{-y_1}\right)\right]^{\frac{1}{2m}}}{a_0 t_1}=\int_{y_1}^{y}\frac{1+\frac{y}{e^y-1}}{[y(1-e^{-y})]^{\frac{m-1}{2m}}}dy \tag{25}$$

This integral of Equation (25) can be expressed in series form, the first few terms of which will be given for certain values of m. These give sufficient accuracy for practical design purposes. If greater accuracy is desired, more terms must be computed. But it should be noted that whenever a series is difficult to compute one can always resort to graphical integration.

For the linear line ($m=1$, (25) becomes

$$\frac{2X\sqrt{y_1\left(1-e^{-y_1}\right)}}{a_0 t_1}=\int_{y_1}^{y}\left(1+\frac{y}{e^y-1}\right)dy \tag{26}$$

$$=2(y-y_1)-\frac{y^2-y_1^2}{4}+\frac{y^3-y_1^3}{36}-\frac{y^5-y_1^5}{3600}+\cdots$$

For the conical line ($m=2$) it is

$$\frac{4X\left[y_1\left(1-e^{-y_1}\right)\right]^{\frac{1}{4}}}{a_0 t_1}=\int_{y_1}^{y}\frac{1+\frac{y}{e^y-1}}{\left[y(1-e^{-y})\right]^{\frac{1}{4}}}dy \quad (27)$$

$$=4\left[y^{\frac{1}{2}}-y_1^{\frac{1}{2}}-\frac{1}{24}\left(y^{\frac{3}{2}}-y_1^{\frac{3}{2}}\right)+\frac{1}{640}\left(y^{\frac{5}{2}}-y_1^{\frac{5}{2}}\right)+\frac{5}{7168}\left(y^{\frac{7}{2}}-y_1^{\frac{7}{2}}\right)+\ldots\right]$$

For the exponential line

$$\left(m=t_1=\infty,\frac{m}{t_1}=\alpha\right),$$

$$\frac{2\alpha X}{a_0}=\int_{y_1}^{y}\frac{1+\frac{y}{e^y-1}}{\sqrt{y(1-e^{-y})}}dy \quad (28)$$

or expressed in the series form

$$\frac{2\alpha X}{a_0}=2\log\frac{y}{y_1}-\frac{y^2-y_1^2}{96}+\frac{y^3-y_1^3}{288}+\ldots$$

II—Development of equations for transformer with tapered coil

Equations (25), (26), (27) and (28) refer to the case of a line with uniform coil and tapered sheath as in Fig. 1. The corresponding equations for a line with tapered coil and uniform sheath as in Fig. 4 are derived in a similar fashion. Let

r = radius of coil
r_0 = radius of sheath
N = number turns per unit length
r_1 = radius of coil at low impedance end
r_2 = radius of coil at high impedance end

$$a_0=\frac{c}{2\pi r_0 N} \quad (29)$$

$$c=3\times10^{10}$$

$y=2\log\frac{r_0}{r}$ *from which*

$$y_1=2\log\frac{r_0}{r_1},\; y_2=2\log\frac{r_0}{r_2} \quad (30)$$

Then

$$z=\sqrt{\frac{L}{C}}=2\pi rNc\,10^{-9}\sqrt{2\left(1-\frac{r^2}{r_0^2}\right)\log\frac{r_0}{r}} \quad (31)$$

$$a=\frac{1}{\sqrt{LC}}=\frac{c}{2\pi rN}\sqrt{\frac{2\log\frac{r_0}{r}}{1-\frac{r^2}{r_0^2}}} \quad (32)$$

or

$$z=\frac{c^2\times10^{-9}}{a_0}\sqrt{ye^{-y}(1-e^{-y})} \quad (33)$$

or

$$z=z_1\sqrt{\frac{(1-e^{-y})ye^{-y}}{\left(1-e^{-y_1}\right)y_1e^{-y_1}}} \quad (34)$$

$$a=a_0\sqrt{\frac{ye^{y}}{1-e^{-y}}} \quad (35)$$

$$\frac{dz}{dy}=\frac{z_1e^{-y}[e^{-y}y+1-e^{-y}-y(1-e^{-y})]}{2\sqrt{\left(1-e^{-y_1}\right)y_1e^{-y_1}(1-e^{-y})ye^{-y}}}$$

$$z_1f'(t)=\frac{dX}{dt}\frac{dz}{dy}\frac{dy}{dX}=a\frac{dz}{dy}\frac{dy}{dX}$$

$$=\frac{dy}{dX}\frac{a_0z_1}{2}\sqrt{\frac{ye^{y}}{1-e^{-y}}}\frac{e^{-y}[ye^{-y}+1-e^{-y}-y(1-e^{-y})]}{\sqrt{\left(1-e^{-y_1}\right)y_1e^{-y_1}(1-e^{-y})ye^{-y}}}$$

from which

$$X=\frac{a_0}{2}\int_{y_1}^{y}\frac{1+\frac{y}{e^y-1}-y}{f'(t)\sqrt{y_1\left(1-e^{-y_1}\right)e^{-y_1}}}dy$$

For

$$f(t)=\left(1+\frac{t}{t_1}\right)^m$$

$$f'(t)=\frac{m}{t_1}\left[1+\frac{t}{t_1}\right]^{m-1}=\frac{m}{t_1}[f(t)]^{\frac{m-1}{m}}$$

$$=\frac{m}{t_1}\left[\frac{(1-e^{-y}ye^{-y}}{\left(1-e^{-y_1}\right)y_1e^{-y_1}}\right]^{\frac{m-1}{2m}}$$

and finally

$$\frac{2Xm}{a_0t_1}\left[\left(1-e^{-y_1}\right)y_1e^{-y_1}\right]^{\frac{1}{2m}}=\int_{y_1}^{y}\frac{1+\frac{y}{e^y-1}-y}{\left[(1-e^{-y})ye^{-y}\right]^{\frac{m-1}{2m}}}dy \quad (36)$$

which corresponds to Equation (25).

The equations corresponding to (26), (27) and (28) are then:

For the linear line:

$$\frac{2X}{a_0t_1}\sqrt{\left(1-e^{-y_1}\right)y_1e^{-y_1}}=\int_{y_1}^{y}\left(1+\frac{y}{e^y-1}-y\right)dy \quad (37)$$

$$=2(y-y_1)-\frac{3}{4}(y^2-y_1^2)+\frac{1}{36}(y^3-y_1^3)-\frac{1}{3600}(y^5-y_1^5)+\ldots$$

Conical line:

$$\frac{4X}{a_0t_1}\left[\left(1-e^{-y_1}\right)y_1e^{-y_1}\right]^{\frac{1}{4}}=\int_{y_1}^{y}\frac{1+\frac{y}{e_y-1}-y}{\left[(1-e^{-y})ye^{-y}\right]^{\frac{1}{4}}}dy \quad (38)$$

$$=4\left[y^{\frac{1}{2}}-y_1^{\frac{1}{2}}-\frac{y^{\frac{3}{2}}-y_1^{\frac{3}{2}}}{8}-\frac{23}{640}\left(y^{\frac{5}{2}}-y_1^{\frac{5}{2}}\right)+\ldots\right]$$

Exponential line:

$$\frac{2X\alpha}{a_0}=\int_{y_1}^{y}\frac{1+\frac{y}{e^y-1}-y}{\sqrt{(1-e^{y-})ye^{-y}}}dy \quad (39)$$

$$=2\log\frac{y}{y_1}-\frac{25}{96}(y^2-y_1^2)-\frac{7}{96}(y^3-y_1^3)-\ldots$$

III—Calculations for specific case involving tapered coil

It will now be shown how the design is to be carried out for the case where the coil is tapered and the sheath has a constant radius for which the equations are given in Section II. We shall assume a given problem in which the lowest frequency f_c which the tapered line is required to pass is 5.2×10^6 cycles per second, that the impedance z_1 at the small end of the tapered line should be 70 ohms and the impedance z_2 at the large end of the tapered line should be 700 ohms.

One must first decide upon the desired taper of the sheath whether it should be linear, conical or exponential. From the standpoint of electrical characteristics the exponential taper is preferable and will be chosen for the following discussion.

Certain preliminary steps are involved before one can directly find the parameters of the tapered line for the given values of z_1, z_2 and f_c.

Assuming we desire the exponential type of taper we must first find the value of αt_2, in order that this value may be used in Equation (12) for the insertion gain of the device.

From Equation (5)

$$\alpha t_2 = \log \frac{z_2}{z_1} = \log \frac{700}{70} = 2.303 \qquad (40)$$

It is now necessary to consider the insertion gain characteristic of the exponential tapered line and for this purpose a curve should be plotted with various assigned values of

$$\frac{2\omega}{\alpha}$$

as abscissae, and as ordinates the resulting insertion gains as given by Equation (12). For our assumed problem, the resulting curve will take the form shown in Fig. 7. It will now be necessary to choose some "cut-off" point for

$$\frac{2\omega}{\alpha}$$

beyond which the insertion gain is at a satisfactory high level, and assume that we choose this cut-off point as the point A on the curve of Fig. 7 for which the insertion gain is 4.5 decibels. It will be noted that for the point A the value of

$$\frac{2\omega}{\alpha}$$

is 2.43 for our specific example. Designate this value of

$$\frac{2\omega}{\alpha} \text{ as } \frac{2\omega_c}{\alpha} \text{ so that } \frac{2\omega_c}{\alpha} = 2.43$$

But $\omega_c = 2\pi f_c = 2\pi \times 5.2 \times 10^6$

Hence

$$\alpha = \frac{2\omega_c}{2.43} = \frac{4\pi \times 5.2 \times 10^6}{2.43} = 26.9 \times 10^6$$

Up to the present point the calculations apply to either a tapered sheath or a tapered coil. If we wish a tapered coil and uniform sheath we have by definition from Equation (30) the following expressions for

y_1 (the value of y at the low impedance end of the line) and

y_2 (the value of y at the high impedance end of the line):

$$y_1 = 2 \log \frac{r_0}{r_1}$$

$$y_2 = 2 \log \frac{r_0}{r_2}$$

where r_0 is the radius of the sheath, r_1 the radius of the small end of the coil and r_2 the radius of the large end of the coil. As an expression involving N, the number of turns per centimeter of the winding surrounded by the sheath, we also have by definition from Equation (29),

$$a_0 = \frac{c}{2\pi r_0 N}$$

One is free to specify any one of the quantities y_1, y_2 or a_0, after which the others may be uniquely determined. But it is preferable to assign a definite value to y_2 for it can be shown from Equation (31) that the largest value which can be used for

$$\frac{r_0}{r}$$

is 2.06 and that for a larger value of

$$\frac{r_0}{r}$$

the impedance of the tapered line will not vary monotonically from one end to the other. Furthermore if we took

$$\frac{r_0}{r} = 2.06$$

the shape of the coil form would have an infinite slope at the high impedance end which would be undesirable in practice. Therefore we have the requirement that

$$\frac{r_0}{r}$$

must be less than 2.06 and since

$$y_2 = 2 \log \frac{r_0}{r_2}$$

this means that y_2 should be less than 1.44 in the case of the tapered coil-uniform sheath line. At the same time it must be noted that the smaller y_2 is chosen the smaller will be y_1 and if y_1 is made very small the separation between the sheath and the coil at the low impedance end will also be very small. A study of these factors indicates that $y_2=1$ is a reasonable value.

Assuming this value of $y_2=1$, we now solve for the values of y_1 and a_0. The value of y_1 is to be obtained from Equation (34) letting $z=z_2$ and $y=y_2$ in that equation from which

$$\sqrt{e^{-y_1} y_1 (1-e^{y_1})} = \frac{z_1}{z_2} \sqrt{e^{-y_2} y_2 (1-e^{y_2})}$$

This is a transcendental equation in y_1 and its exact solution cannot be obtained algebraically. Although a graphical solution is generally advisable, we can use the first few terms in the power series for an approximate formula for y_1 in terms of y_2 and

$$\frac{z_1}{z_2}$$

providing

$$\frac{z_2}{z_1}$$

is 10 or greater so that

$$y_1 \doteq \frac{z_1}{z_2} \sqrt{y_2 e^{-y_2}\left(1-e^{-y_2}\right)} + \frac{3}{4}\left[\frac{z_1}{z_2}\sqrt{y_2 e^{-y_2}\left(1-e^{-y_2}\right)}\right]^2$$

For $y_2=1$ and

$$\frac{z_1}{z_2} = 0.1$$

the radical

$$\sqrt{y_2 e^{-y_2}\left(1-e^{-y_2}\right)}$$

is equal to 0.483. Thus, from the above equation

$$y_1 = 0.050$$

Now from Equation (33) letting $z=z_2$ and $y=y_2$

$$a_0 = \frac{c^2 \times 10^{-9}}{z_2} \sqrt{y_2 e^{-y_2}\left(1-e^{-y_2}\right)} = $$

$$\frac{9 \times 10^{20} \times 10^{-9} \times 0.483}{700} = 6.20 \times 10^8$$

By definition,

$$a_0 = \frac{c}{2\pi N r_0}$$

or

$$r_0 N = \frac{3 \times 10^{10} \times 10^{-8}}{2\pi \times 6.20} = 7.71$$

We can now choose N and r_0 in any way as long as their product equals 7.71. If, for example, we make $r_0=1$ inch, then N becomes 7.71 turns per inch, a reasonable value.

The length L of the tapered coil is that value of X where $y=y_2$ and hence from Equation (39) after dividing both sides by

$$\frac{2\alpha}{a_0}$$

$$L=\frac{a_0}{\alpha}\left[\log\frac{y_2}{y_1}-\frac{25}{192}(y_2^2-y_1^2)-\frac{7}{192}(y_2^3-y_1^3)\right]=$$

65.3 cm.=25.7 inches

The value of X corresponding to each value of y chosen between $y_1=0.050$ and $y_2=1$ can be computed in the same way. For example, ten values of y intermediate these two values may be chosen.

The shape of the form which supports the tapered coil may be found by plotting

$$\frac{r}{r_0}$$

against

$$\frac{X}{L}$$

the value of

$$\frac{r}{r_0}$$

being computed for each chosen value of y by the Formula (30)

$$\frac{r_0}{r}=e^{\frac{y}{2}}$$

For these assigned values of y between

$y_1=0.050$ and $y_2=1$

we can find not only the ratio

$$\frac{r_0}{r}$$

but also the distance that ratio holds from the small impedance end of the line by means of Equation (39) and hence can plot

$$\frac{r_0}{r}$$

against

$$\frac{X}{L}$$

as has been done for our example in Fig. 8. The curve of Fig. 8, therefore, gives the coil shape for our specific example.

The radii of the two ends of the coil can be readily determined since $r_0=1$ inch and since r_1 is the value of r where $y=y_1=0.050$ and r_2 is the value of r where $y=y_2=1$. That is

$$r_1=\frac{r_0}{e^{\frac{y_1}{2}}}=\frac{1}{e^{\frac{0.050}{2}}}=0.975\text{ inch}$$

and

$$r_2=\frac{r_0}{e^{\frac{y_2}{2}}}=\frac{1}{e^{\frac{1}{2}}}=0.606\text{ inch}$$

That is, for our typical example the tapered coil will be 25.7 inches long, will have a radius of 0.975 inch at its low impedance end, will have a radius of 0.606 inch at its high impedance end and will have the contour prescribed by the curve of Fig. 8. This coil will, of course, be surrounded by a sheath of constant radius of 1 inch. This construction, it will be recalled, is for coupling a line of 70 ohm impedance to a line of 700 ohm impedance with a lower cut-off frequency of 5.2×10^6 cycles per second, and as previously stated the amount of insertion gain in decibels realized by the employment of this device is shown by Fig. 7.

IV—Calculations for specific case involving tapered sheath

It will now be shown how the design is to be carried out for the case where the sheath is tapered and the coil is of constant radius, the equations for which are given in Section I. We shall assume as in the other example in Section III that z_1 the low impedance is 70 ohms, z_2 the high impedance is 700 ohms, and the cut-off frequency f_c is 5.2×10^6 cycles per second. The notations employed are similar to those employed for the tapered coil except that in the present case r is the variable radius of the sheath and r_0 is the radius of the coil.

The expression for the value of the insertion gain for the tapered sheath device is the same as that used for the tapered coil, namely, Equation (12), so that the curve of insertion gain plotted against

$$\frac{2\omega}{\alpha}$$

will be that of Fig. 7 and we will utilize the same cut-off point where

$$\frac{2\omega_c}{\alpha}=2.43$$

from which $\alpha=26.9\times10^6$ as in previous example.

Again let $y_2=1$ as in previous case, although it may be pointed out that the restriction that y_2 for the tapered coil line should be less than 2.06 does not apply to the tapered sheath construction. We have by definition from Equation (17)

$$y_1=2\log\frac{r_1}{r_0}$$

$$y_2=2\log\frac{r_2}{r_0}$$

$$a_0=\frac{c}{2\pi r_0N}$$

where r_1 is the radius of small impedance end of sheath; r_2 is radius of large impedance end of sheath; r_0 is radius of coil; and N is the number of turns of coil per unit length.

Equation (19) gives an expression for y_1 in terms of the known quantities y_2, z_1 and z_2 or

$$\sqrt{y_1\left(1-e^{-y_1}\right)}=\frac{z_1}{z_2}\sqrt{y_2\left(1-e^{-y_2}\right)}$$

and as in the case of the tapered coil we will use the first few terms of the power series of e^{-y_1} and obtain an approximate solution for y_1, namely

$$y_1=\frac{z_1}{z_2}\sqrt{y_2\left(1-e^{-y_2}\right)}+\frac{1}{4}\left[\frac{z_1}{z_2}\sqrt{y_2\left(1-e^{-y_2}\right)}\right]^2$$

although a graphical solution of Equation (19) may be employed if desired.

Now for $y_2=1$ and

$$\frac{z_1}{z_2}=0.1$$

we obtain from the above formula

$y_1=0.081$

Also from Equation (18) we obtain

$$a_0=\frac{c^2\times10^{-9}}{z_2}\sqrt{y_2\left(1-e^{-y_2}\right)}$$

from which

$a_0=1.023\times10^9$

But by definition Equation (17)

$$a_0=\frac{c}{2\pi r_0 N}$$

or

$$\frac{c}{2\pi r_0 N}=1.023\times 10^9$$

or

$$r_0 N=4.68.$$

If we take r_0 (the radius of the coil) as equal to 1 inch then $N=4.68$, that is, the coil will have 4.68 turns per inch.

For the length of the coil we have from Equation (28) (where L is the value of X where $y=y_2$) after dividing both sides of the equation by

$$\frac{2\alpha}{a_0}$$

$$L=\frac{a_0}{\alpha}\left[\log\frac{y^2}{y_1}-\frac{y_2^2-y_1^2}{192}\right]$$

and by substituting the known values,

$$L=95.5\text{ cm.}=37.6\text{ inches.}$$

The shape of the sheath is computed by solving Equation (28) for X for several values of y between $y_1=0.081$ and $y_2=1$ and then calculating the ratio

$$\frac{r}{r_0}$$

for each of these values of y by the Formula (17)

$$\frac{r}{r_0}=e^{\frac{y}{2}}$$

Finally a curve is plotted of

$$\frac{X}{L}$$

versus

$$\frac{r}{r_0}$$

to obtain the shape of the sheath and such a curve is shown in Fig. 10.

If we wish the values of r_1, the radius of the sheath at the low impedance end, and r_2, the radius of the sheath at the high impedance end, we have from Equation (17), knowing that $r_0=1$ inch,

$$r_1=e^{\frac{y_1}{2}}=e^{\frac{0.081}{2}}=1.041\text{ inches}$$

and

$$r_2=e^{\frac{y_2}{2}}=e^{\frac{1}{2}}=1.649\text{ inches}$$

Therefore, when employing a tapered sheath construction for coupling a 70 ohm impedance line to a 700 ohm impedance line and with a lower cut-off frequency of 5.2×10^6 cycles per second, the coil will have a radius of 1 inch, will be 37.6 inches long, will have 4.68 turns per inch and this coil will be surrounded by a sheath having the shape shown by the curve of Fig. 10 with a radius of 1.041 inches at the low impedance end and a radius of 1.649 inches at the high impedance end.

We will now refer more in detail to the tapered line constructions shown in the drawings.

In Fig. 1 the impedance transformer is of the tapered sheath type the parameters of which are given generally in Section I above, and for a specific example in Section IV above. The coil 11 is of constant radius being wound on a suitable cylinder 12 of insulating material. As explained under Section IV for use between impedances of 70 and 700 ohms and with a lower cut-off frequency of 5.2×10^6 the coil 11 would be 37.6 inches long with a constant radius of one inch. Surrounding the winding 11 is a circumferentially complete cylindrical sheath 13 of conducting material such as copper, the shape of the sheath being in conformity with the curve of Fig. 9 previously described. The sheath 13 may be suitably supported about the coil 11 by means of spaced washers 14 of insulating material. This tapered line of Fig. 1 is shown in cross-section in Fig. 2.

It has been assumed that the low impedance end of the tapered line is to be connected to a line having an impedance of 70 ohms for the frequency range to be transmitted while the high impedance end is to be connected to a line having an impedance of 700 ohms. The manner of connection is shown in Fig. 3 where line 15 having an impedance of 70 ohms has one terminal connected to the terminal 16 of coil 11 and has its other terminal connected to the surrounding sheath 13, while line 18 having an impedance of 700 ohms has one terminal connected to terminal 17 of coil 11 and has its other terminal connected to sheath 13. The insertion gain achieved by the use of this impedance transformer for the transmission of current from the frequency source 19 to line 18 is slightly more than 4.5 decibels as shown by Fig. 7 and this substantially constant insertion gain applies for a frequency range of ten to one or greater.

The other type of tapered line which has been computed has a tapered coil with a surrounding sheath of constant radius as described in Sections II and III. Such a tapered line is shown in Fig. 4 where the winding 20 is wound on a tapered cylindrical form 21 of insulating material tapered in accordance with the curve of Fig. 8 for the example specifically calculated in order to give the proper taper to the winding. The winding as shown more clearly in Fig. 6 may comprise ribbon shaped wire fitting into a helical groove in the external surface of the cylinder 21. Surrounding the winding 20 is a concentric sheath 23 of conducting material such as copper suitably supported by one or more intermediate insulating washers 24. The core 21 at the high impedance end of the winding may have a collar 25 which supports the sheath at that end while at the low impedance end of the transformer the sheath 23 may be in contact with core 21 at a point beyond the termination of the winding 20. The tapered line of Fig. 4 may be employed in the transmission circuit of Fig. 3 in the same manner as the tapered line of Fig. 1, the low impedance end of winding 20 being connected to line 15 and the high impedance end of winding 20 being connected to line 18.

Although certain specific embodiments of this invention have been described for illustrative purposes it is to be understood that the invention is not limited thereto as other embodiments can be readily realized commensurate with the scope of the appended claims.

What is claimed is:

1. In combination, two circuits each having a fixed, predetermined impedance, and means connecting said circuits in energy transfer relation comprising an impedance transformer, said impedance transformer comprising two elements, first, a long cylindrical coil and, second, a concentric metallic sheath surrounding said coil, at least one of said elements being tapered in radius over substantially its entire length, the input and output impedances of said transformer being equal to the respective impedances of said two circuits.

2. In combination, two circuits each having a fixed, predetermined impedance, and means connecting said circuits in energy transfer relation comprising an impedance transformer, said impedance transformer comprising two elements, first, a long cylindrical coil, and second, a concentric metallic sheath surrounding said coil, one of said elements being tapered in radius over substantially its entire length, the other of said elements being of substantially constant radius over its entire length, the input and output impedances of said transformer being equal to the respective impedances of said two circuits.

3. In combination, two circuits each having a fixed, predetermined impedance, and means connecting said circuits in energy transfer relation comprising an impedance transformer, said impedance transformer comprising a long single layer solenoidal winding of substantially constant radius throughout its length and a metallic sheath of substantially circular cross-section surrounding said winding, said sheath being tapered in radius over substantially its entire length, the input and output impedances of said transformer being equal to the respective impedances of said two circuits.

4. In combination, two circuits each having a fixed, predetermined impedance, and means connecting said circuits in energy transfer relation comprising an impedance transformer, said impedance transformer comprising a long single layer solenoidal winding and a metallic sheath of substantially circular cross-section surrounding said winding, said sheath being of substantially constant radius throughout its length, the radius of said winding being tapered over substantially its entire length, the input and output impedances of said transformer being equal to the respective impedances of said two circuits.

5. An impedance transformer comprising a long cylindrical coil and a concentric metallic sheath surrounding said coil, said coil being tapered in radius over substantially its entire length, said sheath being of substantially constant radius, the ratio of sheath radius to coil radius at the high impedance end of said transformer being less than 2.06.

6. A radio frequency impedance matching device whose terminals are adapted to face different impedances comprising two elements, first, a solenoidal winding, and second, a conducting sheath enclosing said winding, one set of terminals for said device comprising one end of said winding and the adjacent end of said sheath, the other set of terminals for said device comprising the other end of said winding and the adjacent end of said sheath, said elements having such parameters that the impedance of said device tapers substantially exponentially as the time of wave transmission along its length, whereby its terminals present impedances having values which are substantially equal to the impedances to be faced.

7. A radio frequency impedance matching device whose terminals are adapted to face different impedances comprising two elements, first, a long cylindrical shaped winding, and second, a conducting sheath enclosing said winding, the radius of at least one of said elements varying substantially continuously along its length, one set of terminals for said device comprising one end of said winding and the adjacent end of said sheath, the other set of terminals for said device comprising the other end of said winding and the adjacent end of said sheath, said elements having such parameters that the impedance of said device tapers approximately exponentially as the time of wave transmission along its length whereby its terminals present impedances having values which are substantially equal to the impedances to be faced.

8. A radio frequency impedance matching device in accordance with claim 7 in which said winding is tapered in radius along its length and said sheath is of substantially constant radius.

9. A radio frequency impedance matching device in accordance with claim 7 in which said sheath is tapered substantially continuously along its length and said winding is of substantially constant radius.

10. In combination, a pair of radio frequency electrical circuits having different impedances, an impedance matching device whose terminals are adapted to connect together said pair of circuits, each set of terminals of said device presenting an impedance substantially equal to the impedance of the electric circuit which it faces, said device comprising a long cylindrical coil and a concentric conducting sheath surrounding said coil, one set of terminals of said device comprising one end of said winding and the adjacent end of said sheath, the other set of terminals of said device comprising the other end of said winding and the adjacent end of said sheath, said device having an impedance which tapers continuously along its length substantially in accordance with Equation (4).

11. An impedance transformer comprising a long cylindrical shaped winding and a concentric metallic tapered sheath surrounding said winding in which the parameters of the transformer are substantially defined by the formula

$$X=\frac{a_0 t_1}{2m\left[y_1\left(1-e^{-y_1}\right)\right]^{\frac{1}{2m}}}\int_{y_1}^{y}\frac{1+\frac{y}{e^{y}-1}}{\left[y\left(1-e^{-y}\right)\right]^{\frac{m-1}{2m}}}dy$$

where X=axial length of winding

$a_0=\frac{c}{2\pi r_0 N}$

$c=3\times10^{10}$

r_0=radius of winding

N=number of turns of winding per unit length

$y=2\log\frac{r}{r_0}$

$y_1=2\log\frac{r_1}{r_0}$

r=radius of sheath at any point along its length

r_1=radius of sheath at low impedance end

t_1 is the value obtained from the expression

$$t_1=\frac{t_2}{\left[\frac{z_2}{z_1}\right]^{\frac{1}{m}}-1}$$

t_2=time required for a wave to travel the length of the winding

z_1=impedance at low impedance end

z_2=impedance at high impedance end

m=a constant preferably one of the following values: 1, 2, infinity

e=base of natural logarithms

12. An impedance transformer comprising a long cylindrical shaped winding whose radius is tapered along its axis and a concentric metallic sheath surrounding said winding in which the

parameters of the transformer are substantially defined by the formula

$$X=\frac{a_0t_1}{2m\left[\left(1-e^{-y_1}\right)y_1e^{-y_1}\right]^{\frac{1}{2m}}}\int_{y_1}^{y}\frac{1+\frac{y}{e^y-1}-y}{\left[(1-e^{-y})ye^{-y}\right]^{\frac{m-1}{2m}}}dy$$

where X=axial length of winding

$a_0=\frac{c}{2\pi r_0N}$

$c=3\times10^{10}$

r_1=radius of coil at low impedance end

N=number of turns of coil per unit length

t_1=value of expression

$$t_1=\frac{t_2}{\left[\frac{z_2}{z_1}\right]^{\frac{1}{m}}-1}$$

t_2=time required for a wave to travel the length of the winding

z_2=impedance at high impedance end

z_1=impedance at low impedance end

r_0=radius of sheath

$y=2\log\frac{r_0}{r}$

$y_1-2\log\frac{r_0}{r_1}$

r=radius of winding at any point along its axis

m=a constant preferably one of the following values: 1, 2, infinity

e=base of natural logarithms

13. In combination, two electrical circuits having different impedances and an impedance matching device interconnecting them for the transfer of radio frequency waves occupying a wide range of radio frequencies such that the highest frequency of said range is at least several times the lowest frequency, said device comprising a long cylindrical coil and a concentric metallic sheath surrounding said coil, at least one of said elements being tapered in radius over substantially its entire length in such manner that the impedance varies with the time of transmission of said waves in accordance with Equation (4).

14. In combination, two electrical circuits having different impedances and an impedance matching transmission line interconnecting them for the transfer of radio frequency waves comprising a long cylindrical coil and a concentric conducting sheath surrounding said coil, the impedance of said line varying gradually along said line with the time of transmission in such manner that the insertion gain is substantially uniform over a frequency band the highest frequency of which is at least several times the lowest.

15. A combination in accordance with claim 14 in which said sheath is of substantially constant radius throughout its length and the ratio of sheath radius to coil radius at the high impedance end is less than 2.06.

16. A combination in accordance with claim 14 in which said line is tapered in impedance in accordance with Equation (4), where m lies between 2 and infinity.

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