

Oct. 19, 1937.

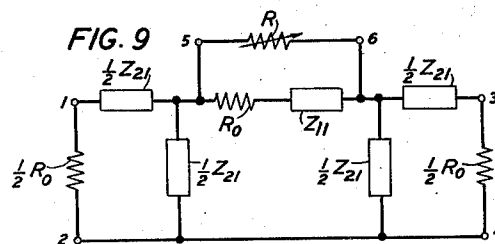
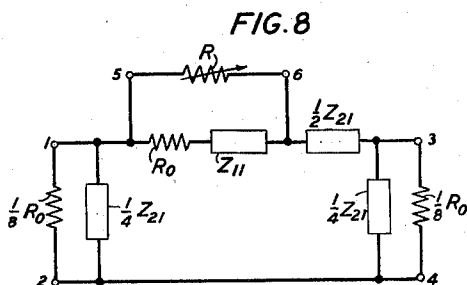
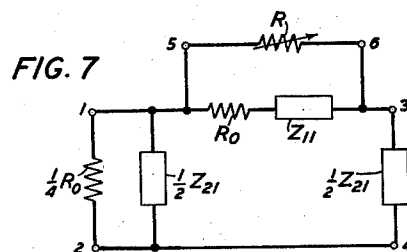
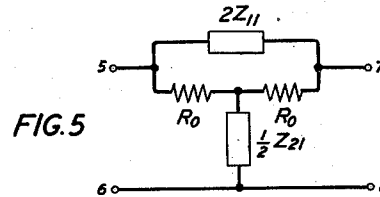
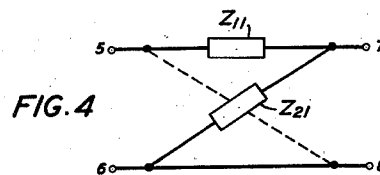
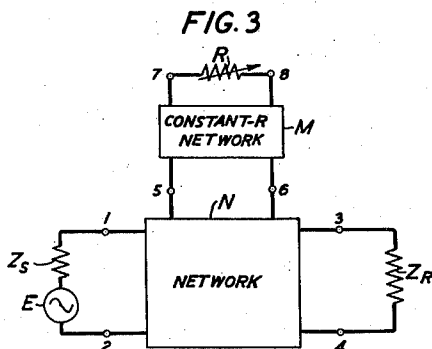
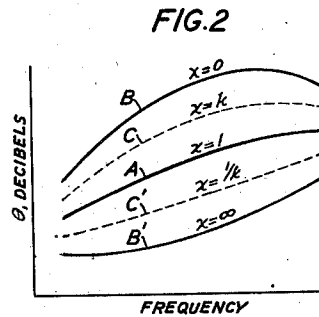
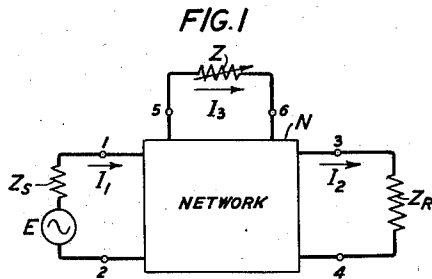
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2,096,027

ATTENUATION EQUALIZER

Filed Jan. 30, 1936

4 Sheets-Sheet 1



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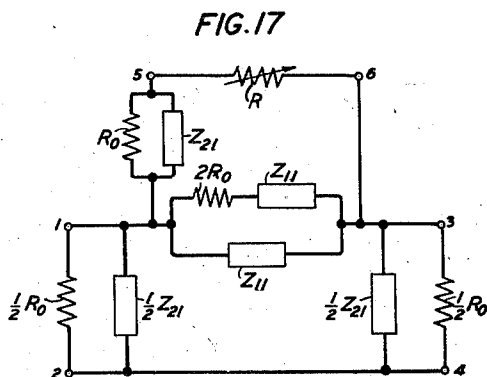
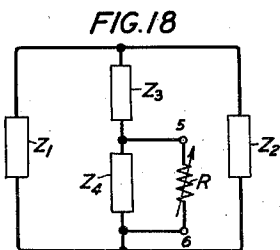
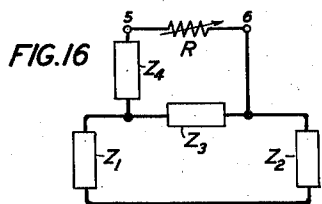
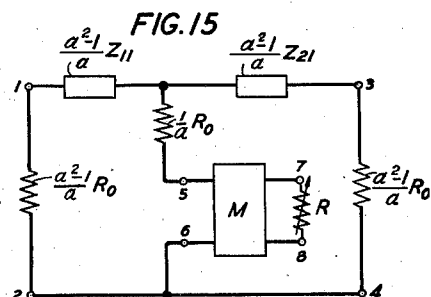
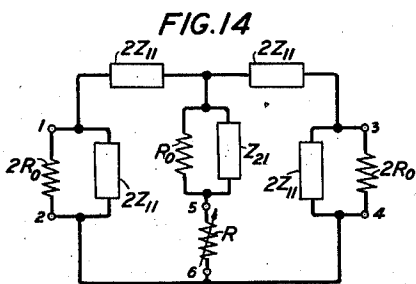
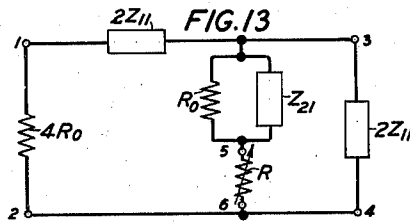
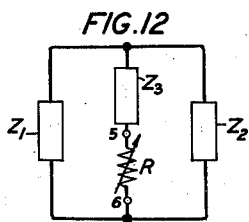
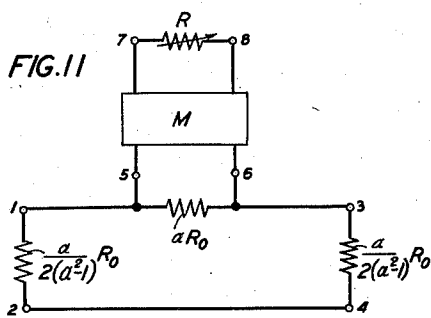
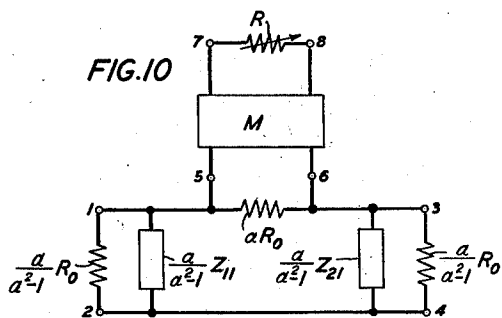
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2,096,027

ATTENUATION EQUALIZER

Filed Jan. 30, 1936

4 Sheets-Sheet 2



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ATTENUATION EQUALIZER

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FIG. 19

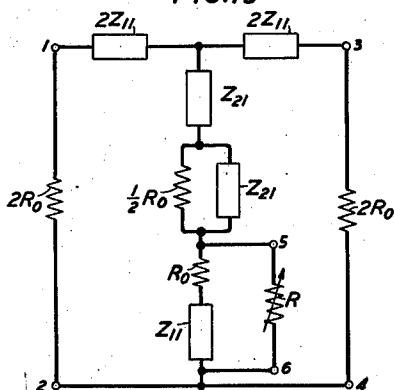


FIG. 20

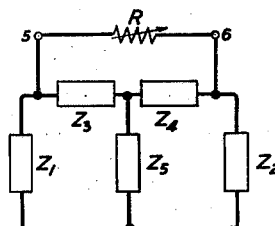


FIG. 21

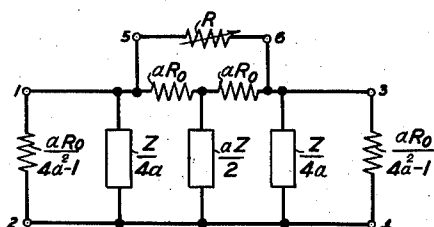


FIG. 22

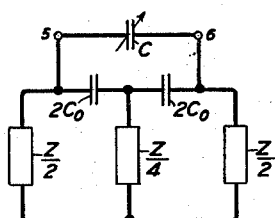


FIG. 23

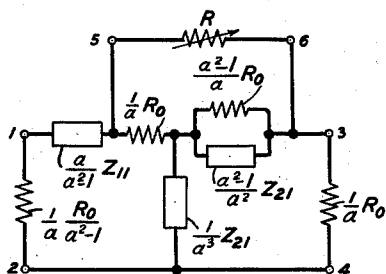
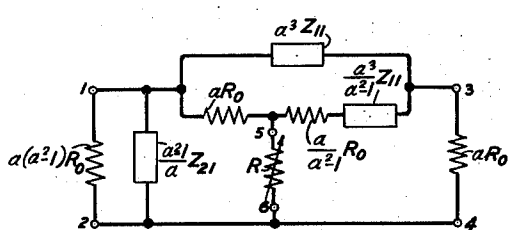


FIG. 24



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ATTENUATION EQUALIZER

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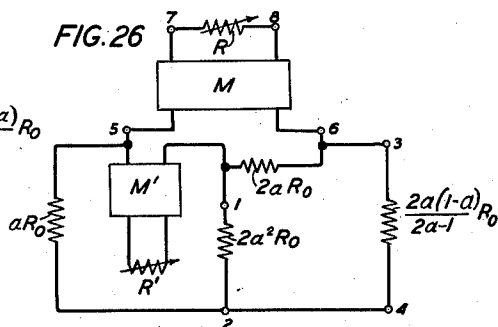
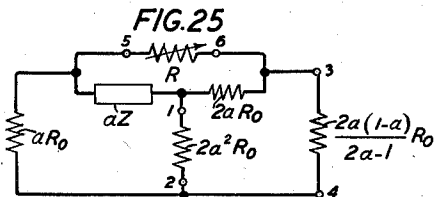


FIG. 27

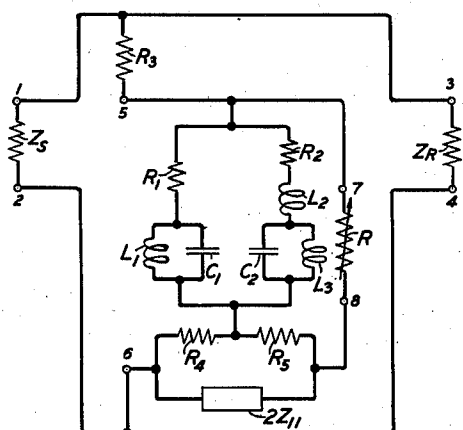


FIG. 28

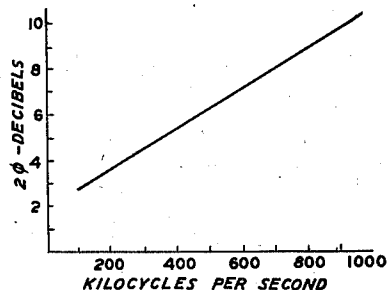
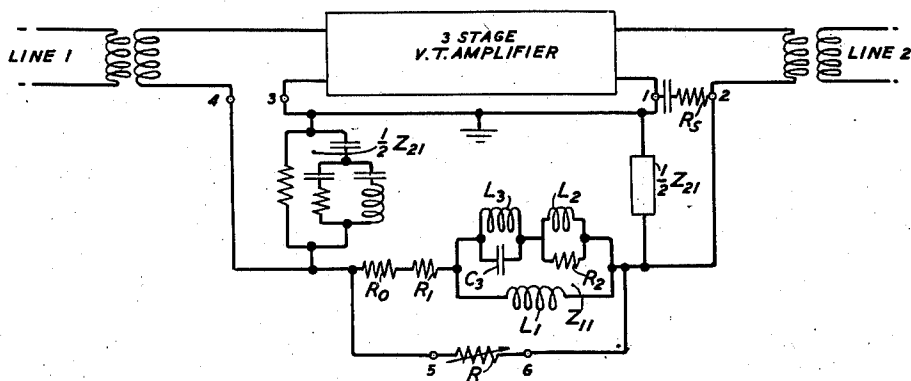


FIG. 29



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2,096,027

ATTENUATION EQUALIZER

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Application January 30, 1936, Serial No. 61,497

11 Claims. (Cl. 178-44)

This invention relates to attenuation equalizing networks and more particularly to adjustable equalizers for the compensation of transmission lines and networks the attenuation of which is subject to variation.

The maintenance of high quality transmission in long telephone circuits which include large numbers of repeaters imposes very severe requirements on the equalization of the attenuation distortion of the lines. Not only must the over-all equalization of the system be as nearly complete as possible, but the individual line sections must also be accurately equalized in order that the repeaters may always operate under optimum conditions and in order that intermediate portions of the line may be used separately. Since the errors of compensation of the individual line sections may be cumulative, large over-all distortions may result unless the accuracy of compensation of each section is very high.

Due to temperature variations and, in certain cases, to humidity changes, the attenuation in a line section tends to vary continually and in order that these variations may be continuously compensated with the required accuracy it is necessary that the compensating adjustment be of a simple character, preferably of a kind that lends itself readily to automatic control. Investigation has shown that the variations produced by departures of the temperature or of the humidity from normal values are functions of frequency and are of such character that they may be compensated with great accuracy by the addition to or subtraction from the normal line characteristic of some fraction of a determinable attenuation characteristic. In accordance with the invention adjustable equalizing devices are provided whereby compensations of the character described may be effected by the variation of a single adjustable impedance element and may be made with great accuracy for a wide variation of the line attenuation.

The equalizers of the invention comprise networks of a wide variety of forms all characterized by the inclusion of an adjustable impedance element, usually a variable resistance, the variation of which causes the addition or subtraction of fractional portions of a predetermined attenua-

tion characteristic. For a particular adjustment of the variable element, the network has a fixed normal characteristic from which the deviations for other adjustments are measured. In certain cases this fixed normal characteristic may be a constant attenuation and in other cases it may have a frequency variation related to the characteristic of the deviations by direct proportionality or in other explicit manner. Those networks in which the two characteristics are proportionally related find use in many practical instances, for example, in connections with lines in which temperatures changes produce proportionately related changes of attenuation. In such cases the network may be used for the equalization of both the normal line attenuation and the changes due to changing temperature. Other forms of the networks, in which the characteristics are not proportionally related, exhibit normal characteristics which are simple functions of frequency and are, therefore, readily combined with known types of equalizers to build up any desired reference characteristic.

The high degree of accuracy and the increased range of compensation characteristic of the networks of the invention depend to a large extent upon certain broad relationships, explained hereinafter in detail, which obtain among the impedances of the network. By means of the invention equalization to within 0.1 or 0.2 decibels can be obtained for a total range of variation as great as 35 decibels and for smaller ranges of variation much closer compensation can be effected.

In one modification of the invention, the variable characteristic of the networks may be changed without affecting the fixed or normal characteristic. This feature permits a wide range of relationships between the fixed and the variable characteristics to be obtained and facilitates the design and adaptation of the networks to any particular set of conditions.

Other features of the invention will appear from the following detailed description and by reference to the attached drawings, of which Fig. 1 shows the general schematic arrangement of the networks of the invention;

Fig. 2 illustrates the operation of the invention;

Fig. 3 shows the schematic arrangement of a modified form of the invention;

Figs. 4 and 5 are explanatory of Fig. 3;

Figs. 6 to 26, inclusive, illustrate various particular networks in accordance with the invention;

Fig. 27 shows the detail design of a particular network of the invention;

Fig. 28 shows the line characteristic compensated by the network of Fig. 27; and

Fig. 29 shows the detail arrangement and the external connections of another network of the invention.

Preliminary mathematical theory

The diagram of Fig. 1 represents a general schematic form to which the networks of the invention may be reduced. The system comprises a six terminal network N, the form of which is so far undetermined. Terminals 1, 2, and 3, 4, are the normal input and output terminals of the network leading respectively to a wave source of impedance Z_s and voltage E and to a load of impedance Z_R , while terminals 5, 6, are the terminals to which the variable impedance Z is connected. Such a system can be specified in the ordinary manner by a set of mesh equations, in writing which it will be assumed that the first mesh current, I_1 , passes through the generator, the second I_2 , passes through the load, and the third, I_3 , passes through the variable impedance and, further, that the meshes may be so chosen that the impedances Z_s , Z_R and Z , appear only as self impedances. This leads to the following set of equations:

$$\begin{aligned} I_1 Z_{11} + I_2 Z_{12} + I_3 Z_{13} \dots + I_n Z_{1n} &= E \\ I_1 Z_{21} + I_2 Z_{22} + I_3 Z_{23} \dots + I_n Z_{2n} &= 0 \\ I_1 Z_{31} + I_2 Z_{32} + I_3 (Z_{33} + Z) \dots + I_n Z_{3n} &= 0 \dots \\ I_1 Z_{n1} + I_2 Z_{n2} + I_3 Z_{n3} \dots + I_n Z_{nn} &= 0 \end{aligned} \quad (1)$$

in which the impedance Z_{33} represents the self impedance in the third mesh exclusive of the variable impedance Z , and in which the impedances such as Z_{11} , Z_{22} and Z_{nn} represent the total self impedances of the respective meshes and impedances such as Z_{12} , Z_{13} and Z_{1n} , represent mutual impedances.

The transmission loss in the system depends upon the value of the current I_2 flowing through the load. The solution of Equations (1) for I_2 gives

$$I_2 = -E \frac{\Delta_{1233}}{\Delta} \frac{Z + \frac{\Delta_{12}}{\Delta_{1233}}}{1 + Z \frac{\Delta_{33}}{\Delta}} \quad (2)$$

where Δ is the determinant of the coefficients of the currents in Equations (1) when the impedance Z is equal to zero, Δ_{12} is the minor of Δ obtained by suppressing the first row and the second column in Δ , Δ_{33} is the minor when the third row and third column are suppressed, and Δ_{1233} the minor when the first and third rows and the second and third columns are suppressed.

The quantities Δ , Δ_{12} , etc. in Equation (2) depend only on the fixed parts of the network N and the generator and load impedances, which are also fixed. The ratios Δ_{1233}/Δ , $\Delta_{12}/\Delta_{1233}$, and Δ_{33}/Δ therefore represent functions of frequency which can be fixed independently of the value of the variable impedance. By choosing them in various ways the loss characteristic can be prescribed for three different values of the variable impedance, upon which the loss characteristics for other values become determined.

The relationship imposed on these quantities by the invention is the following:

$$\frac{\Delta}{\Delta_{33}} \frac{\Delta_{12}}{\Delta_{1233}} = Z_0^2 \quad (3)$$

where Z_0 is a chosen reference value of the variable impedance Z . By means of this relationship Equation (2) may be transformed to

$$\frac{I_2 Z_0}{E} = -Z_0^2 \frac{\Delta_{1233}}{\Delta} \frac{\frac{Z}{Z_0} + Z_0 \frac{\Delta_{33}}{\Delta}}{1 + \left(\frac{Z}{Z_0}\right) \left(Z_0 \frac{\Delta_{33}}{\Delta}\right)} \quad (4)$$

which may be written in the form

$$e^{-\theta} = e^{-\theta_0} \frac{x + e^\phi}{1 + x e^\phi} \quad (5)$$

in which the various quantities are identified as follows:

$$e^{-\theta} = \frac{I_2 Z_0}{E} \quad (6)$$

$$e^{-\theta_0} = -Z_0^2 \frac{\Delta_{1233}}{\Delta} \quad (7)$$

$$e^\phi = Z_0 \frac{\Delta_{33}}{\Delta} \quad (8)$$

and

$$x = \frac{Z}{Z_0} \quad (9)$$

Equation (6) defines $e^{-\theta}$ as the ratio of the current I_2 to a reference current equal to E/Z_0 . Ordinarily the total insertion loss of a network is defined in terms of the ratio of the output current, or voltage, when the network is present to the value when the network is absent. The quantity $e^{-\theta}$ is related to this ratio by the factor $Z_0/(Z_s + Z_R)$ and the total insertion loss is therefore equal to

$$\theta + \log_e \frac{Z_0}{Z_s + Z_R} \quad (10)$$

the additional term being readily computed in any given case. In most cases the ratio

$$Z_0/(Z_s + Z_R) \quad (11)$$

may be made a constant real quantity so that θ , defined as above, differs from the actual insertion loss only by an additive constant.

Equation (7) gives the value of θ when Z is equal to Z_0 and therefore defines a normal or reference attenuation characteristic of the system.

Equation (8) defines the variable insertion loss, or variable attenuation, characteristic of the network. The quantity ϕ represents the total amount that is added to or subtracted from θ_0 as the impedance Z is varied from Z_0 to zero or from Z_0 to infinity. By virtue of the relationship imposed by Equation (3) the amount is the same in each case. This may be shown readily by giving the quantity x the successive values zero, unity and infinity.

In most cases it is preferable that the impedance Z_0 should be a real, constant quantity, that is, a pure resistance. In many cases the impedance may be simply a variable resistance, but in others it may be a complex impedance which for its reference adjustment has a pure resistance value. Examples of both types will be described later.

Returning to Equation (3), certain physical significances of the determinant ratios will be investigated. It will be observed that each ratio has the dimensions of an impedance, since each is the ratio of a determinant of impedances to one of the next lower order. From the familiar

mathematical solutions of Equations (1) it will be seen that the ratio Δ/Δ_{33} represents the impedance of the system looking in at terminals 5 and 6, the terminal impedances Z_S and Z_R being included. Hence

$$\frac{\Delta}{\Delta_{33}} = Z_{56} \quad (10)$$

where Z_{56} denotes the network impedance at terminals 5 and 6 as defined above.

From Equation (2), the current I_{20} in the second mesh, when the variable impedance Z has zero value, is given by

$$I_{20} = -E \frac{\Delta_{12}}{\Delta} \quad (11)$$

and the current $I_{2\infty}$, when Z is infinite, is

$$I_{2\infty} = -E \frac{\Delta_{1233}}{\Delta_{33}} \quad (12)$$

Hence, defining the transfer impedances Z_{20} and $Z_{2\infty}$ as respectively equal to E/I_{20} and $E/I_{2\infty}$ it follows that

$$\frac{Z_{2\infty}}{Z_{20}} = \frac{\Delta_{33}}{\Delta_{1233}} \frac{\Delta_{12}}{\Delta} \quad (13)$$

From this equation and Equation (10) is obtained

$$\frac{\Delta_{12}}{\Delta_{1233}} = Z_{56}^2 \frac{Z_{2\infty}}{Z_{20}} \quad (14)$$

which shows that the determinant ratio $\Delta_{12}/\Delta_{1233}$ is equal to the impedance Z_{56} multiplied by the ratio of the two transfer impedances $Z_{2\infty}$ and Z_{20} . Equation (3) may therefore be rewritten in the form

$$Z_{56}^2 \frac{Z_{2\infty}}{Z_{20}} = Z_0^2 \quad (15)$$

thereby expressing the condition imposed upon the networks of the invention in terms of the terminal impedances of the system. In the form of Equation (3) the statement of the condition becomes somewhat ambiguous when applied to two-mesh networks since, in that case, the minors Δ_{1233} and Δ_{33} cannot be readily identified. This ambiguity is completely removed in Equation (15) which is applicable alike to two-mesh and multi-mesh networks.

By means of the impedances defined in the foregoing Equations (7) and (8), the significant characteristics of the network may be transformed into expressions of simpler character which in many cases prove more convenient for design purposes. Thus, for the normal characteristic, there is obtained

$$e_0^\theta = \frac{Z_{20}}{Z_0} e^\phi = \frac{Z_{2\infty}}{Z_0} e^{-\phi} = \frac{Z_{20}}{Z_{56}} = \frac{\sqrt{Z_{2\infty} Z_{20}}}{Z_0} \quad (16)$$

and, for the variable characteristic

$$e^\phi = \frac{Z_0}{Z_{56}} = \frac{Z_{56}}{Z_0} \frac{Z_{2\infty}}{Z_{20}} \quad (17)$$

It has already been noted that the extreme departures from the normal characteristic of the network are given by ϕ , as defined in Equation (8), these departures occurring when x is zero and when x is infinite. The departure characteristics for intermediate values of x will now be investigated. For any pair of reciprocally related values of x , namely x_1 and $1/x_1$, Equation (5) takes the forms

$$e^{-\theta} \Big|_{x_1} = e^{-\theta_0} \frac{x_1 + e^\phi}{1 + x_1 e^\phi} \quad (18)$$

and

$$e^{-\theta} \Big|_{1/x_1} = e^{-\theta_0} \frac{1 + x_1 e^\phi}{x_1 + e^\phi} \quad (19)$$

which show that the corresponding current

changes are reciprocally related and the changes in the insertion loss are equal and of opposite sign. The intermediate deviation characteristics thus occur in pairs which are symmetrically related to the normal characteristic. It is evident, however, from Equations (18) and (19) that they will not be exactly proportional to the extreme deviation characteristics ϕ and $-\phi$ and it is therefore of interest to determine the extent of the departure from proportionality and the effect of this upon the accuracy of compensation. I have found that θ , as given by Equation (5), may be expanded into a rapidly converging series in terms of powers of ϕ , giving

$$\theta = \theta_0 + \gamma\phi + C_3\phi^3 + C_5\phi^5 + C_7\phi^7 + \dots \quad (20)$$

where

$$\gamma = \frac{x-1}{x+1}$$

$$C_3 = \frac{\gamma}{12}(1-\gamma^2) \quad (21)$$

$$C_5 = \frac{\gamma}{240}(1-\gamma^2)(2-3\gamma^2)$$

$$C_7 = -\frac{\gamma}{6720}(1-\gamma^2)\left(\frac{17}{3} - 20\gamma^2 + 15\gamma^4\right) \quad (22)$$

and where θ , θ_0 and ϕ are expressed in nepers, one neper being equal to 3.686 decibels.

The first two terms of Equation (20) represent the sum of the normal characteristic of the system and a fractional part of the variable characteristic, ϕ , determined by the particular setting of the variable impedance Z . These terms evidently correspond to an ideal regulator in which all of the intermediate deviation characteristics are proportional to the extreme deviation characteristic. The remaining terms represent the departure from this ideal but, since the series is very rapidly convergent, only the third and fourth terms are of any practical interest.

Consider the third term first. The maximum value of the coefficient C_3 is 0.032 and occurs when γ has the value 0.578, for which the values of x are 0.266 and 3.76. If it be assumed that ϕ is a real quantity, the maximum change in the attenuation represented by the third term will be $0.32\phi^3$ which will be equal to 1 decibel when ϕ is 1.53 nepers or 13.3 decibels. Since this error may be distributed equally on each side of a desired characteristic and since the range of adjustment is 2ϕ , the resulting error is limited to 0.5 decibel for a total range of 26.6 decibels.

The assumption made in the foregoing that ϕ is a real quantity does not hold in practice. Actually ϕ is nearly always a complex quantity such as $A+jB$ and the real part of its cube, which measures the error due to the third term, has the value $(A^3 - 3AB^2)$. The error due to the third term will therefore be considerably less than that indicated above and in certain cases may be made equal to zero by proportioning the network so that A^2 is equal to $3B^2$. If this is done for the frequency at which ϕ is a maximum the error elsewhere will be negligible.

A further reduction of the error arises from the fact that the third and fourth terms of equation have coefficients of opposite sign and of such relative magnitudes that when the third term is large enough to be of importance the fourth term is large enough to cancel a substantial part of it. As a net result it has been found possible to effect attenuation compensation to an accuracy of 0.1 to 0.2 decibel when the total range of adjustment is as great as 35 decibels, or 4 nepers.

Characteristics typical of the networks of the

invention are illustrated by the curves of Fig. 2 in which the variations of θ are plotted as ordinates against frequency as abscissae. Curve A represents θ_0 , the normal characteristic obtained when x is unity. Curves B and B' represent respectively $\theta_0 + \varphi$ and $\theta_0 - \varphi$, the extreme variations obtained when x is zero and when x is infinite. The difference between curves B and A or between curves A and B' is equal to the variable characteristic φ . Curves C and C' represent a related pair of intermediate characteristics for values of x equal to k and $1/k$ respectively. These curves, by virtue of the impedance relationship imposed on the system, are symmetrical with respect to θ_0 and their departures from θ_0 are very closely proportional to φ at all frequencies.

In the foregoing the nature of the variable impedance Z has not been restricted. In many cases it may consist of a simple variable resistance in which case the quantities x and y in the preceding equations are always real quantities. One general form of the invention in which the variable impedance is not a simple resistance is shown schematically in Fig. 3. In this form the variable impedance consists of a symmetrical 4-terminal constant resistance network M connected at one end to terminals 5 and 6 and at the other end to a variable resistance R. This network may be of the symmetrical lattice type shown in Fig. 4 or the bridged-T type shown in Fig. 5, wherein Z_{11} and Z_{21} are impedances of general type subject to the requirement.

$$Z_{11}Z_{21} = R_0^2 \quad (21)$$

An extensive discussion of the properties of these networks is given in an article by O. J. Zobel entitled Distortion correction in electrical circuits with constant resistance recurrent networks, Bell System Technical Journal, vol. VII, No. 3, July 1928. Other suitable forms of networks are also disclosed in this article.

Throughout the discussion which follows and in the illustrations of the various embodiments of the invention, the notations Z_{11} and Z_{21} will be used to designate generalized impedances which are inversely related in accordance with Equation (21).

Because of the relationship expressed in Equation (21) the network M has image impedances at both pairs of terminals equal to R_0 . If its transfer constant be denoted by Ψ , then the impedance seen at terminals 5 and 6 when it is terminated by the resistance R is the variable impedance Z and has the value

$$Z = R_0 \frac{R + R_0 \tanh \Psi}{R_0 + R \tanh \Psi} \quad (22)$$

Substituting this value for Z in Equations (4) and (5) and using x to denote the ratio R/R_0 leads to the expression

$$e^{-\theta} = e^{-\theta_0} \frac{x + \frac{e^{\varphi} + \tanh \Psi}{1 + e^{\varphi} \tanh \Psi}}{1 + x \frac{e^{\varphi} + \tanh \Psi}{1 + e^{\varphi} \tanh \Psi}} \quad (23)$$

which may be written in a form corresponding to that of Equation (5), namely,

$$e^{-\theta} = e^{-\theta_0} \frac{x + e^{\varphi'}}{1 + x e^{\varphi'}} \quad (24)$$

where φ' is a new variable characteristic given by

$$e^{\varphi'} = \frac{e^{\varphi} + \tanh \Psi}{1 + e^{\varphi} \tanh \Psi} \quad (25)$$

It is to be noted that the introduction of the

network M has not changed the normal characteristic θ_0 of the system, but has made it possible to control the variable characteristic independently by changing the transfer constant Ψ of the introduced network. Independent control of the normal characteristic θ_0 may be obtained, if desired, by the introduction of an appropriate constant resistance network between the network N and either the source impedance Z_s or the load impedance Z_R . This is, of course, merely equivalent to adding an external equalizer.

The quantities $\tanh \Psi$ and e^{φ} appearing in Equation (25) are both impedance ratios which for any given network are ascertainable in terms of the network branch impedances by ordinary processes of circuit analysis. When combined as in Equation (25) the resulting expression is also an impedance ratio and is in form suitable for design purposes. This equation and others preceding it may be subject to various mathematical transformations giving forms which may in certain instances be of greater convenience. However, for most purposes the forms given have been found to be adequate.

Network structures

While the invention may find embodiment in a very large number of individual networks, the more useful types resolve themselves into a limited number of schematic forms, a number of which are illustrated in the drawings.

Fig. 6 shows schematically one general type in which the whole network, including the source and load impedances, takes the form of a Π -network with the variable impedance bridging the series branch. The variable impedance is a variable resistance R, the end branches of the network comprise generalized two-terminal impedances Z_1 and Z_2 , and the series branch a third generalized impedance Z_3 . These may consist of complex assemblages of resistance and reactance elements, the impedances Z_1 and Z_2 including the source and load impedances as branches thereof. The relationship imposed by Equations (3) and (15) may be expressed in terms of the impedances themselves as follows:

The impedance of the network measured at terminals 5 and 6 is given by

$$Z_{56} = \frac{Z_3 (Z_1 + Z_2)}{Z_1 + Z_2 + Z_3} \quad (26)$$

The ratio of the transfer impedances $Z_{2\infty}$ and Z_{20} is given by

$$\frac{Z_{2\infty}}{Z_{20}} = \frac{Z_1 + Z_2 + Z_3}{Z_1 + Z_2} \quad (27)$$

the factor expressing the ratios of the source and load currents to the total currents in impedances Z_1 and Z_2 being the same for both of the transfer impedances and canceling out in the ratio. The design condition given by Equation (15) therefore becomes

$$(Z_3) \frac{Z_3 (Z_1 + Z_2)}{Z_1 + Z_2 + Z_3} = R_0^2 \quad (28)$$

in which the two factors on the left side can be identified as the impedances measured at terminals 5 and 6 with the interconnection between Z_1 and Z_2 open and closed respectively at a point such as X. The determination of suitable forms for the branch impedances thus reduces to finding an L-type network having a constant resistance image impedance.

From Equation (17) the variable characteristic is found to be

$$e^{\varphi} = \frac{Z_3}{R_0} \quad (29)$$

which is general for this type of network. The normal characteristic involves the relationship of the source and load impedances to the network as a whole and may be computed in any particular case. In the examples that follow the expressions for the total insertion loss will be given, this being the characteristic of greatest practical interest. Its value is equal to the normal characteristic multiplied by $Z_0/(Z_S + Z_R)$.

Fig. 7 shows one example of the general type illustrated in Fig. 6. In this network the impedance Z_3 is made up of a resistance R_0 in series with an impedance Z_{11} . The shunt branch corresponding to Z_1 is the parallel combination of a resistance $R_0/4$, which represents the source impedance, and an impedance $Z_{21}/2$. The branch corresponding to Z_2 comprises the single impedance $Z_{21}/2$. The variable characteristic is given by

$$e^{\varphi} = 1 + \frac{Z_{11}}{R_0} \quad (30)$$

The normal insertion loss, denoted by Γ and in this case defining the ratio of the source electromotive force to the voltage across terminals 3, 4, is given by

$$e^{\Gamma} = e^{2\varphi} \quad (31)$$

This form of equalizer is useful for insertion between stages of a vacuum tube amplifier, the voltage at the output terminals being impressed upon the input terminals of the later stage.

A modification of Fig. 7 is shown in Fig. 8 in which the impedance in Fig. 7 corresponding to Z_1 is divided into two equal portions one of which is placed at each end. Resistance branches are thus provided for both the source and the load impedances. The variable insertion loss is the same as for Fig. 7 and the normal insertion loss is given by

$$e^{\Gamma} = \frac{Z_{11} + 2R_0}{Z_{11}} e^{2\varphi} = \frac{e^{\varphi} + 1}{e^{\varphi} - 1} e^{2\varphi} \quad (32)$$

Fig. 9 shows a symmetrical network of the general type of Fig. 6. As in the two preceding examples, the series branch consists of a resistance R_0 in series with an impedance Z_{11} . The shunt branches are alike, each comprising an impedance $Z_{21}/2$, inversely related to Z_{11} in accordance with Equation 21, in parallel with the series combination of $R_0/2$ and $Z_{21}/2$. The variable loss and the normal insertion loss are the same as for the network of Fig. 8.

In each of the foregoing examples the variable impedance is a simple resistance and the two characteristics Γ and φ are explicitly related. By inserting constant resistance networks in front of the variable resistance in the manner indicated by Figs. 3, 4, and 5, further modified forms are obtained in which the variable characteristic may be independently controlled. The insertion of such networks does not change the normal insertion loss characteristics but modifies the variable characteristic in accordance with Equation (25).

Fig. 10 shows a network of the general type of Fig. 6 in which control of the variable characteristic is dependent entirely on the use of an inserted network. The series branch consists of a simple resistance aR_0 , a being a numerical mul-

tiplier greater than unity. The shunt branch corresponding to Z_1 of Fig. 6 comprises a resistance $aR_0/(a^2 - 1)$ in parallel with an impedance $aZ_{11}/(a^2 - 1)$ and that corresponding to Z_2 comprises a similar resistance in parallel with an impedance $aZ_{21}/(a^2 - 1)$. The sum of the impedances of the two combinations is a constant resistance. The variable defined by Equation (17) is given by

$$e^{\varphi} = a \quad (33)$$

and does not vary with frequency. However, by the insertion of the constant resistance network M in front of the variable resistance, the deviation characteristic becomes equal to φ' , as defined by Equation (25) and will have a frequency variation determined by the characteristic of the inserted network. This network should be of the general types illustrated in Figs. 4 and 5 and should have an image impedance equal to R_0 . The normal insertion loss of the network is given by

$$e^{\Gamma} = \left(1 + \frac{Z_{11} + Z_{21}}{2R_0}\right) e^{\varphi} \quad (34)$$

If the inversely related impedances are omitted, the network reduces to the simple form shown in Fig. 11, the normal insertion loss of which is a constant.

A second class of networks corresponds to the general schematic form shown in Fig. 12. The impedance Z_3 in this type appears as a shunt branch intermediate the end impedances Z_1 and Z_2 and the variable resistance R is connected in series with Z_3 . The relationship imposed by Equations (3) and (15) becomes, in this case,

$$Z_3 \left(Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} \right) = Z_3 \cdot Z_{30} = R_0^2 \quad (35)$$

from which is obtained, by means of Equation 17

$$e^{\varphi} = \frac{Z_3}{R_0} \quad (36)$$

An example of this general type is shown in Fig. 13 in which the impedance Z_3 is the parallel combination of a resistance R_0 and an impedance Z_{21} , impedance Z_1 comprises a resistance $4R_0$ in series with impedance $2Z_{11}$ and impedance Z_2 consists of the single impedance Z_{11} . It is to be observed that the network represents an inversion of the network of Fig. 7, in consequence of which the characteristics of the two networks are closely related. The variable characteristic is given by

$$e^{-\varphi} = 1 + \frac{Z_{11}}{R_0} \quad (37)$$

which is the inverse of that of Fig. 7, and the normal insertion loss is given by

$$e^{\Gamma} = \frac{2R_0}{Z_{11}} e^{-2\varphi} \quad (38)$$

which, as in the case of Fig. 7, expresses the ratio of the source voltage to the output voltage.

A second example of this class is shown in Fig. 14, this network being an inversion of that of Fig. 9. For the impedance values indicated in the figure the variable loss has the value given by Equation (37) and the normal insertion loss is given by

$$e^{\Gamma} = \frac{2R_0 + Z_{11}}{Z_{11}} e^{-2\varphi} \quad (39)$$

A third example, corresponding to an inversion

of Fig. 10, is shown in Fig. 15. For the impedance values indicated the loss characteristics are

$$e^{-\varphi} = a$$

and

$$e^{\Gamma} = \left(1 + \frac{Z_{11} + Z_{21}}{2R_0}\right) e^{-\varphi} \quad (40)$$

As in the case of the network of Fig. 10, the variable characteristic is modified by the insertion of a constant resistance network M in front of the variable resistance, the modified value having a frequency variation controlled by the transfer constant of the network.

The type of network illustrated in Fig. 16 is similar to that of Fig. 6 except that an additional impedance Z_4 is included in series with the variable resistance R. The structural requirement for this type is that

$$(Z_3 + Z_4)Z_{56} = R_0^2 \quad (41)$$

under which condition the variable loss has the value

$$e^{\varphi} = \frac{Z_3 + Z_4}{R_0} \quad (42)$$

The design requirement of Equation (41) is most readily met by so constructing the network that Z_1 and Z_2 are together equal to Z_4 . An example of a network of this type is shown in Fig. 17 in which Z_3 consists of the parallel combination of an impedance Z_{11} with a like impedance in series with a resistance $2R_0$. For the impedance values indicated in the figure the variable loss is given by

$$e^{\varphi} = \frac{1}{2} \left[1 + \frac{Z_{11}}{R} + \frac{1}{1 + \frac{Z_{11}}{R}} \right] \quad (43)$$

and the normal insertion loss by

$$e^{\Gamma} = \left[1 + \frac{Z_{11}}{R} \right]^2 \quad (44)$$

The inverse networks corresponding to those of Figs. 16 and 17 are shown in Figs. 18 and 19 respectively. In these networks the impedance Z_3 appears in a shunt branch and the additional impedance Z_4 is connected in parallel with the variable resistance. For the general type, Fig. 18, the design requirement is

$$\frac{Z_3 Z_4}{Z_3 + Z_4} Z_{56} = R_0^2 \quad (45)$$

under which condition the variable loss is

$$e^{-\varphi} = R_0 \left(\frac{1}{Z_3} + \frac{1}{Z_4} \right) \quad (46)$$

In the particular network shown in Fig. 19 the variable loss and the normal insertion loss, for the impedance values indicated, become

$$e^{-\varphi} = \frac{1}{2} \left[1 + \frac{Z_{11}}{R_0} + \frac{1}{1 + \frac{Z_{11}}{R_0}} \right] \quad (47)$$

and

$$e^{\Gamma} = \left[1 + \frac{Z_{11}}{R_0} \right]^2$$

Another class of networks in accordance with the invention is illustrated in Fig. 20 in which the network takes the form of a Wheatstone bridge with the variable impedance included in one diagonal branch. The four sides of the bridge are constituted by impedances Z_1 , Z_2 , Z_3 and Z_4 and a fifth impedance Z_5 forms the other diagonal

branch. The structural requirement imposed by Equation (3) takes the form

$$\left(Z_3 + Z_4 + \frac{Z_3 Z_4}{Z_5} \right) Z_{56} = R_0^2 \quad (48)$$

which by imposing further structural relations upon the bridge network reduces to simpler forms in particular cases.

In the particular example of this type shown in Fig. 21 the impedances corresponding to Z_3 and Z_4 are equal and are simple resistances of value aR_0 , a being a numerical factor greater than one half. The impedances corresponding to Z_1 and Z_2 are likewise equal, each consisting of a resistance in parallel with an unrestricted impedance. Because of the equality of the branch impedances the bridge is balanced so that the impedance corresponding to Z_5 does not enter into the value of the impedance Z_{56} . By virtue of this simplification it becomes possible to proportion the several branches so that the impedance Z_5 as determined from Equation (48) is physically realizable.

For the values indicated in the figure, the impedances Z_{56} and Z_5 become

$$Z_{56} = \frac{R_0 Z}{2a(R_0 + Z)} \quad (49)$$

and

$$Z_5 = \frac{aZ}{2} \quad (50)$$

The variable loss and the normal insertion loss, from Equations (16) and (17), are

$$e^{\varphi} = 2a \left(1 + \frac{R_0}{Z} \right) \quad (51)$$

and

$$e^{\Gamma} = 2a \left[1 + \frac{2a}{2a+1} \frac{R_0}{Z} \right] \left[1 + \frac{2a}{2a-1} \frac{R_0}{Z} \right] \quad (52)$$

A modified form of the network of Fig. 21 is shown in Fig. 22 in which the variable resistance is replaced by a variable capacity the reference value of which may be denoted by C_0 , defining the impedance Z_0 of the fundamental equations. The series bridge arms are fixed capacities of value $2C_0$, their impedances being equal to $Z_0/2$. The element values correspond to the case for which the multiplier a appearing in Fig. 21 has the value one half. For greater values of the multiplier the end branches would include capacities added in shunt corresponding to the end resistances of Fig. 21.

The variable loss is given by

$$e^{\varphi} = \left(1 + \frac{1}{j\omega C_0 Z} \right) \quad (53)$$

and the normal insertion loss by

$$e^{\Gamma} = \left(1 + \frac{1}{j2\omega C_0 Z} \right) \quad (54)$$

assuming the end impedances to be those of the source and load. The impedance Z may include a shunt or a series resistance which, in the case of the end branches, may be constituted by the source and the load impedances. In that case an appropriate modification of the expression for the insertion loss would be required. An obvious modification of this network would be obtained by substituting inductances for the capacities $2C_0$ and the variable capacity. More complex impedances might also be used but these would require the use of several coordinated variable elements.

Fig. 23 shows another form of network of the balanced bridge type. As compared with Fig. 20,

the bridge arm impedances Z_2 and Z_3 comprise equal resistances of value R_0/a , where a is greater than unity. Impedance Z_1 comprises the series combination of a resistance $R_0/a(a^2-1)$ and an impedance $aZ_{11}/(a^2-1)$ and impedance Z_4 consists of a parallel combination inversely related to Z_4 so that their product has the value $(R_0/a)^2$.

The impedance Z_{56} thus becomes a constant resistance having the value

$$Z_{56} = \frac{R_0}{a}$$

and, for the values of the bridge arms indicated, the value of Z_5 as required by Equation (48), becomes

$$Z_5 = \frac{Z_{21}}{a^2}$$

The variable loss and the normal insertion loss are given by

$$e^r = a$$

and

$$e^r = a \left[1 + \frac{Z_{11}}{R_0} \right]^2 \quad (53)$$

The inversely related network to Fig. 23 is shown in Fig. 24, the variable resistance in this case appearing in the other diagonal branch of the bridge and the several impedances being inversions of the corresponding impedances in the network of Fig. 24. The variable loss is the inverse of that of the preceding network and the normal insertion loss is the same as given by Equation (53). In these networks the variable loss is constant with frequency, but by the inclusion of a constant resistance frequency selective network in front of the variable resistance a desired frequency characteristic may be obtained.

The network of Fig. 25 is also of the general class of Fig. 21 but is characterized by the use of a single frequency selective impedance. As distinguished from the other networks of this class, one of the terminal branches is constituted by the Z_5 branch. The bridge formed by the branches Z_1 , Z_2 , Z_3 and Z_4 is not balanced, but because of the fact that all but one of the branches are resistive and by virtue of the disposition of the source and load branches the fundamental requirement of Equation (3) is satisfied when the resistances have the following values

$$Z_1 = aR_0, Z_2 = \frac{2a(1-a)}{2a-1}R_0$$

$$Z_4 = 2aR_0, \text{ and } Z_5 = 2a^2R_0 \quad (54)$$

where a is a numerical multiplier having any value between one half and unity. For these proportions the variable loss and the normal insertion loss have the values

$$e^r = a \left(1 + \frac{R_0}{Z_1 + R_0} \right)$$

and

$$e^r = \frac{2a}{2a^2 - 2a + 1}$$

the frequency selective impedance having the value aZ . The normal insertion loss in this case is constant.

The impedance aZ in Fig. 25 may be constituted by a constant resistance selective network of either of the types indicated in Figs. 4 and 5, terminated by a resistance. This modification is illustrated in Fig. 26 which shows the impedance constituted by such a network, denoted by M' , terminated by an adjustable resistance R' . By a proper design of the network characteristic and

choice of the resistance R' a desired variable loss characteristic can be obtained. By making the resistance R' adjustable the total value of the variable loss can be varied through a family of proportionally related characteristic curves. This arrangement is convenient for providing an auxiliary control of the variable loss, as for example in adapting the network to lines of different types or lengths. Further modification of the variable loss characteristic can be obtained, as in the preceding examples, by the inclusion of an appropriate constant resistance network in front of the variable resistance R . The characteristics of the two networks may be coordinated for a desired over-all characteristic.

An actual example of a network of the type of Figs. 12 and 15, designed for the compensation of the temperature variations of the attenuation in a concentric conductor cable, is shown in Fig. 27. This is of the simplified type in which the impedances Z_1 , Z_2 and Z_3 of Fig. 12 are resistances, giving a constant normal insertion loss, and in which the frequency characteristic of the variable loss is determined by a network inserted in front of the variable resistance. The range of the variable characteristic for a temperature variation of 125 degrees F. is shown by the curve of Fig. 28 which represents the variable characteristic of a line section of 10 miles. The controlling network is a bridged-T structure of the type shown in Fig. 5. The system as a whole is designed for operation between resistive impedances of 10400 ohms, the value of the multiplier a appearing in Fig. 15 being chosen as two. The values of the several elements are as follows:

$$\begin{aligned} Z_S &= Z_R = 10400 \text{ ohms} \\ R_3 &= 1730 \text{ ohms} \\ R_4 &= R_5 = 3460 \text{ ohms} \\ R_1 &= 23200 \text{ ohms} \\ R_2 &= 3547 \text{ ohms} \\ L_1 &= 3.92 \text{ millihenries} \\ L_2 &= 3.5 \text{ millihenries} \\ L_3 &= .523 \text{ millihenry} \\ C_1 &= 2 \text{ to } 35 \text{ micromicrofarads} \\ C_2 &= 20 \text{ to } 80 \text{ micromicrofarads} \end{aligned}$$

The capacities C_1 and C_2 have normal values of 20 and 50 micromicrofarads respectively but are made variable for final adjustments of the network. The impedance $2Z_{11}$ is an inverse to that of the shunt branch of the bridged-T.

Another example of an actual network designed for the same purpose as that of Fig. 27 is shown in Fig. 29. In this figure the equalizer network forms the β -circuit of a stabilized feed-back amplifier of the type described by H. S. Black in an article entitled Stabilized Feed-Back Amplifiers, Bell System Technical Journal Vol. XIII, January 1934. The figure shows a three-stage vacuum tube repeater amplifier coupled to two line sections through transformers. The input terminals 1 and 2 of the equalizer are connected to a resistance R_s included in the amplifier output circuit in series with a blocking condenser, and the output terminals 3 and 4 are connected in series with the amplifier input circuit. The feed-back through the equalizer is strongly degenerative with the result that, as shown in the above mentioned article by Black, the over-all voltage amplification of the repeater corresponds to the insertion loss of the β -circuit. By making the insertion loss of the equalizer correspond, in its normal characteristic and in its variations, directly to the line characteristics complete compensation is obtained. In the ordinary method of compensation,

where the equalizer is connected in tandem with the line, inverse characteristics are required whereas in the arrangement shown in Fig. 29 the characteristic of the compensating network is the same as that of the line.

The equalizer is designed for operating from a source impedance of 500 ohms which is constituted by the resistance R_s in parallel with the internal impedance of the amplifier. The latter impedance is usually so great that the input impedance of the network may be regarded as being constituted by R_s alone. The structures of the impedance Z_{11} and of the inverse impedance Z_{21} are indicated in the drawings. For the impedance Z_{11} the elements have the following values

$$\begin{aligned} R_1 &= 900 \text{ ohms} \\ R_2 &= 1910 \text{ ohms} \\ L_1 &= 2.23 \text{ millihenries} \\ L_2 &= 12.0 \text{ millihenries} \\ L_3 &= 2.54 \text{ millihenries} \\ C_3 &= 10 \times 10^{-6} \text{ microfarads} \end{aligned}$$

The reference resistance R_0 is 2000 ohms.

By reference to Equations (30) and (31) it will be seen that the normal insertion loss and the variable loss characteristic for this type of equalizer have the same frequency variations and are proportionally related in magnitude. In certain types of lines, notably concentric conductor lines in which the inner conductor is supported from the outer by spaced insulators of glass, porcelain or rubber, the same relationship has been found to exist between the normal attenuation at a chosen reference temperature and the departures therefrom at other temperatures. The compensating arrangement shown in Fig. 29 is therefore suitable for the complete attenuation equalization of such a line. Its normal insertion loss may be proportioned to compensate the normal attenuation of the line at the chosen reference temperature and the variations due to temperature changes may be compensated by adjustment of the variable resistance R .

In the case of other types of lines in which the normal attenuation and the departures therefrom due to temperature changes are not proportionally related, the variable characteristic of the equalizer may be appropriately modified by the inclusion of a suitable constant resistance selective network in front of the variable resistance in the manner shown in Fig. 3. Other forms of equalizing networks may also be used for this purpose, for example, the networks shown in Figs. 10, 15, 23 and 24, in which the normal and the variable characteristics are mutually independent, an appropriate constant resistance network being included in front of the variable resistance in each case to provide the desired frequency variation of the variable characteristic.

For automatic control of the equalization in response to temperature changes in the line, the adjustment of the variable resistance in the equalizer may be made mechanically in accordance with variations of a pilot current transmitted over the line in the manner disclosed in U. S. Patent 1,743,132 issued January 14, 1930 to E. I. Green. Alternatively, the variable resistance may be thermally controlled as shown in U. S. Patent to E. I. Green, 1,918,390, July 19, 1933. The variable resistance need not be calibrated in any special manner, but need only be capable of adjustment through a sufficiently large range of values, above and below its normal value, to insure an adequate range of the variable characteristic.

What is claimed is:

1. An adjustable attenuation equalizer for wave transmission lines and the like comprising a fixed impedance network having three pairs of terminals, a wave source impedance connected to one of said pairs of terminals, a load impedance connected to a second pair of terminals, and an adjustable impedance connected to the third pair of terminals, said network and its connected source and load impedances being proportioned in accordance with the relationship

$$(Z_{56})^2 \frac{Z_{2\infty}}{Z_{20}} = Z_0^2$$

where Z_{56} denotes the impedance of the system measured at the said third pair of terminals, $Z_{2\infty}$ and Z_{20} are the transfer impedances from said source impedance to said load when the third pair of terminals are open-circuited and short-circuited respectively, and Z_0 is a chosen reference value of the said adjustable impedance.

2. An adjustable attenuation equalizer for wave transmission lines and the like, comprising a fixed impedance network having three pairs of terminals, a wave source impedance connected to one of said pairs of terminals, a load impedance connected to a second of said pairs of terminals, and an adjustable impedance connected to the third pair of terminals, said network and its connected source and load impedances being proportioned to have a preassigned normal insertion loss between the source and the load for an assigned normal value of the said adjustable impedance and being proportioned in accordance with the relationship

$$(Z_{56})^2 \frac{Z_{2\infty}}{Z_{20}} = Z_0^2$$

where Z_{56} denotes the impedance of the system measured at the said third pair of terminals, $Z_{2\infty}$ and Z_{20} are the transfer impedances from the said source impedance to said load when the third pair of terminals are open-circuited and short-circuited respectively, and Z_0 is the said normal value of the adjustable impedance, whereby variations of the adjustable impedance from its normal value result in the addition to the normal insertion loss characteristic of the system of fractional parts of a frequency variable loss characteristic.

3. An attenuation equalizer in accordance with claim 2 in which the adjustable impedance is a variable resistor.

4. An attenuation equalizer in accordance with claim 2 in which the adjustable impedance comprises a four-terminal frequency selective network having constant resistance image impedances at its input and output terminals and a variable resistor connected to its output terminals.

5. An attenuation equalizer in accordance with claim 2 in which the adjustable impedance is a variable reactance element.

6. An attenuation equalizer in accordance with claim 2 in which the adjustable impedance is a variable capacity.

7. An attenuation equalizer in accordance with claim 2 in which the said fixed network together with the connected impedances constitute a Wheatstone bridge network, the said adjustable impedance forming a diagonal branch of the bridge and the source and load impedances being included in adjacent bridge arms.

8. An attenuation equalizer in accordance with claim 2 in which the said fixed network together with the said source and load impedances constitute a Π -type network, the source and load impedances being included in the shunt branches thereof, and in which the said adjustable im-

pedance is bridged across the series branch of the network.

9. An attenuation equalizer in accordance with claim 2 in which the said fixed network together
5 with the said connected impedances constitute a network comprising three parallel connected impedance branches, the source and load impedances being included respectively in two of said
10 branches and the said adjustable impedance being connected in series in the third of said branches.

10. An attenuation equalizer in accordance with claim 2 in which the said normal insertion loss and the said frequency variable loss are related in direct proportion at all frequencies.

11. In an attenuation equalizer in accordance
5 with claim 2, means independent of said adjustable impedance for varying the maximum value of the said frequency variable loss characteristic produced by the variation of said adjustable im-
10 pedance.

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