

Sept. 28, 1937.

S. DARLINGTON

2,094,062

ELECTROMECHANICAL IMPEDANCE

Filed Oct. 2, 1935

2 Sheets-Sheet 1

FIG. 1

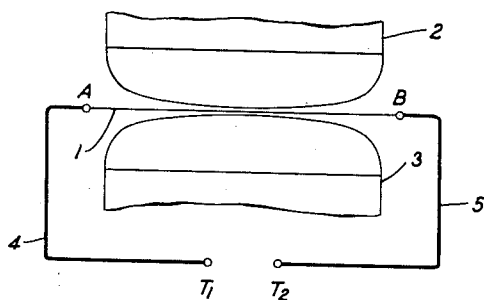


FIG. 2

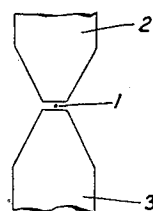


FIG. 3

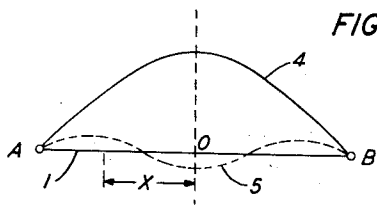
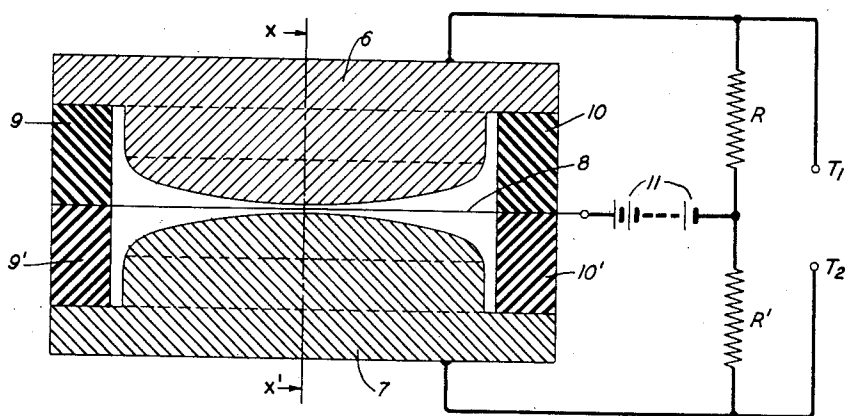


FIG. 4



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FIG. 5

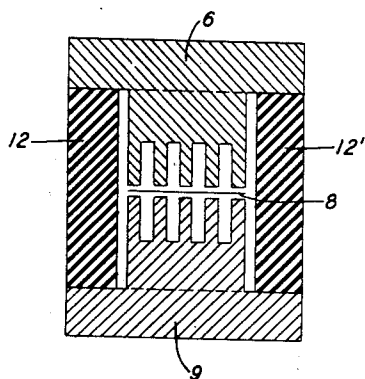


FIG. 6

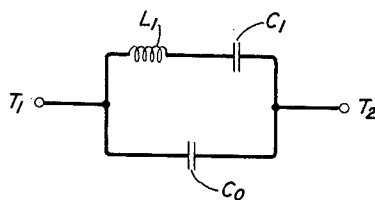


FIG. 7

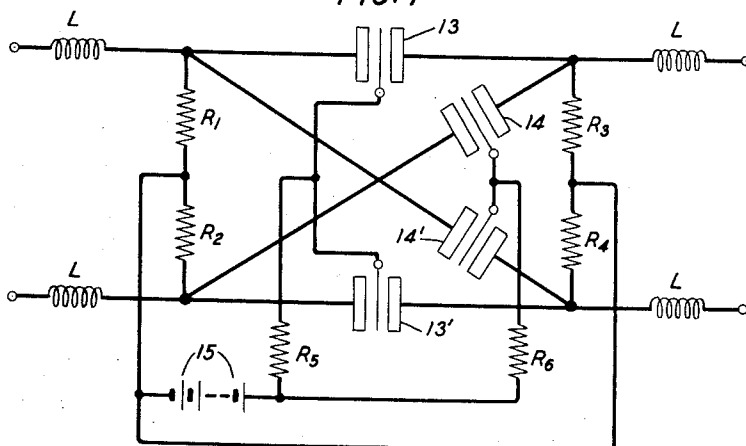
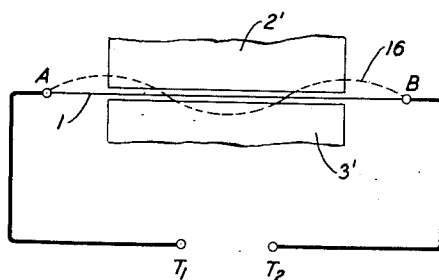


FIG. 8



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2,094,062

ELECTROMECHANICAL IMPEDANCE

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Application October 2, 1935, Serial No. 43,154

9 Claims. (Cl. 179—1)

This invention relates to electromechanical impedances and more particularly to impedance devices which exhibit resonance characteristics due to the resonant vibration of a mechanical element.

It is well-known that mechanical vibratory elements can be constructed having much less damping than is usually found in resonant electric circuits and that for this reason they exhibit highly selective properties in the transmission of vibrations of different frequencies. In broad band wave filters this low energy dissipation contributes towards increased sharpness of cut-off at the edges of the transmission band and a higher efficiency of transmission throughout the band. However, it is difficult to construct mechanical vibrators, especially for high frequencies, which exhibit simple resonance characteristics, the reason for this being that the mass and the elasticity of the vibrating element are usually distributed in such a manner that its behavior resembles that of an extended transmission line. In consequence of this the vibrator usually exhibits a plurality of resonances at frequency intervals determined by the geometry of the structure.

In accordance with the present invention mechanical vibrating elements are incorporated in electric circuits in a particular manner hereinafter described whereby the effects of resonances other than the fundamental are substantially eliminated. The invention thus provides electromechanical devices which exhibit simple resonance characteristics and which are suitable for operation at relatively high frequencies.

In general an electromechanical impedance comprises an electric circuit to which a mechanical vibratory device is coupled either electromagnetically or electrostatically so that its motion induces an electromotive force in the circuit. The impedance of the combination with which the invention is concerned is that measured at the electrical terminals. A feature of the invention is the type of electromechanical coupling employed, this being so arranged that the driving forces applied to the vibrator are distributed along its length and are so graduated from point to point as to prevent vibrations in any other than the fundamental mode of the vibrator.

The nature of the invention will be more fully understood from the following detailed description and by reference to the attached drawings, of which

Figs. 1 and 2 illustrate one form of the invention;

Fig. 3 is a diagram explanatory of the operation of the device of Fig. 1;

Figs. 4 and 5 illustrate an alternative form of the invention;

Fig. 6 is a schematic representation of the device of Fig. 4;

Fig. 7 illustrates the application of the invention to wave filters, and

Fig. 8 shows a modified form of the invention.

In the embodiment of the invention illustrated diagrammatically in Fig. 1, 1 is a stretched wire extending between points A and B at which it is anchored to rigid supports, not shown, 2 and 3 are the poles of a magnet, in the space between which the wire 1 is located, 4 and 5 are conductors leading to electrical terminals T₁ and T₂. The magnet poles are tapered towards the air-gap, as shown in the end elevation in Fig. 2; to provide a concentrated magnetic field in the neighborhood of the wire. The pole faces instead of being parallel as is usual in devices of this type are closest together at the center and diverge symmetrically towards the ends. The purpose of this shaping of the pole faces is to provide a magnetic flux which varies in intensity along the wire in accordance with a sine law, the flux density having a maximum value at the midpoint of the wire and diminishing sinusoidally to a negligibly small value at the ends of the wire.

When a source of alternating electromotive force is connected to terminals T₁ and T₂ a current flows in the wire 1 which by interaction with the magnetic flux causes the wire to vibrate transversely in synchronism with the current variations and to generate a synchronous back electromotive force in the electric circuit. Due to its distributed mass and to the transverse elasticity arising from the tension, the wire has a series of resonances at harmonically related frequencies. In its fundamental mode of vibration all points in the wire move simultaneously in the same direction. At its higher resonances the wire exhibits evenly spaced nodal points, the motions on opposite sides of a nodal point being in opposite directions. Vibration in these higher frequency modes can be sustained only if energy can be supplied synchronously to the wire. Because of the sinusoidal distribution of the flux density in the device of the invention, energy is supplied to the wire only in its fundamental mode of motion and hence vibrations in the other possible modes are suppressed. The device thus exhibits a single resonance on the mechanical side which appears as a single anti-resonance in the electrical circuit.

The explanation of the suppression of the higher resonances will appear from a consideration of Fig. 3. In this figure the horizontal line AB represents the wire in its position at rest.

The ordinates of curve 4 measured from AB represent the flux density at the different points along the wire, the curve being sinusoidal. Dotted curve 5 outlines the form of the wire in its third harmonic mode of vibration, the displacements from the line AB likewise following a sinusoidal variation but of triple frequency.

Measuring along the wire from the mid-point O, the flux density B at a point distant x is given by

$$B = B_0 \cos \frac{\pi}{l} x \quad (1)$$

where B_0 is the flux density at the mid-point and l is the length of the wire. The velocity of vibration varies sinusoidally along the wire in the same manner as the displacement and its value V at the point x is given by

$$V = V_0 \cos \frac{3\pi}{l} x \quad (2)$$

where V_0 denotes the velocity at the mid-point for the assumed mode of vibration. The back electromotive force induced by the motion of an element dx of the wire at point x is given by

$$de = B_0 V_0 \cos \frac{\pi}{l} x \cos \frac{3\pi}{l} x dx \quad (3)$$

and the total back electromotive force, E_{b3} , is given by

$$E_{b3} = B_0 V_0 \int_{-l/2}^{l/2} \cos \frac{\pi}{l} x \cos \frac{3\pi}{l} x dx \quad (4)$$

Writing

$$\frac{\pi}{l} x = \theta$$

Equation 4 is readily transformed to

$$E_{b3} = B_0 V_0 \frac{l}{\pi} \int_{-\pi/2}^{\pi/2} \cos \theta \cos 3\theta d\theta \quad (5)$$

which has the value zero.

Since the motion in the assumed third harmonic mode of vibration generates zero back electromotive force, it follows from the principle of reciprocity that the wire when vibrating in this manner can receive no energy from the electric circuit and hence that vibrations of this type cannot be sustained. Consideration of other harmonic modes of vibration results in expressions similar to Equation 5 for the back electromotive force, the value in each case being zero. In the case of the fundamental mode of vibration the back electromotive force, denoted by E_{b1} , is given by

$$E_{b1} = \frac{B_0 V_1 l}{2} \quad (6)$$

V_1 being the mid-point velocity for this mode.

The character of the impedance of the vibrating wire as measured in the electric circuit may be ascertained from the differential equation for the motion of the wire. The driving force on an element of the wire dx at a point distant x from the center is equal to

$$B_0 i \cos \frac{\pi}{l} x dx$$

where i denotes the instantaneous value of the current in the wire. This is opposed by two reactions; one due to the transverse elasticity of the wire contributed by the tension, and one due

to the mass acceleration reaction of the elemental length dx . The first component has the value

$$-\tau \frac{d^2 y}{dx^2} dx \quad 5$$

where τ is the tension in the wire and y is the transverse displacement of the wire at the point x . The second component has the value

$$\rho \frac{d^2 y}{dt^2} dx \quad 10$$

where ρ is the linear density of the wire. The differential equation for the motion is therefore,

$$-\tau \frac{d^2 y}{dx^2} + \rho \frac{d^2 y}{dt^2} = B_0 i \cos \frac{\pi}{l} x \quad (7) \quad 15$$

the solution of which gives

$$y = -\frac{B_0 i}{\omega^2 \rho - \left(\frac{\pi}{l}\right)^2 \tau} \cos \frac{\pi}{l} x \quad (8) \quad 20$$

where ω denotes the pulsance of the current. It is to be noted that Equation 8 indicates a single resonance at a frequency corresponding to

$$\omega^2 = \frac{\tau}{\rho} \left(\frac{\pi}{l}\right)^2 \quad (9) \quad 25$$

The back electromotive force generated by the motion of the wire is given by

$$E_b = B_0 \int_{-l/2}^{l/2} \cos \frac{\pi}{l} x \frac{dy}{dt} dx \quad (10) \quad 30$$

$$= \frac{B_0^2 l i}{j 2 \left(\omega \rho - \frac{\pi^2 \tau}{\omega l^2} \right)} \quad (11) \quad 35$$

Except for a component equal to the resistance of the wire, which is simply added in series, the impedance is given by the ratio of the back electromotive force to the current and has the value

$$\left[\frac{B_0 l}{2} \right]^2 \frac{1}{j \omega \rho l - j \frac{\pi^2 \tau}{\omega l}} \quad 40$$

which corresponds to the impedance of a capacity of value

$$\frac{\rho l \left[\frac{2}{B_0 l} \right]^2}{2} \quad (12) \quad 45$$

shunted by an inductance equal to

$$\left[\frac{B_0 l}{2} \right]^2 \frac{2l}{\pi^2 \tau} \quad (13) \quad 50$$

When the several quantities are measured in c. g. s. units Formulae 12 and 13 give the capacity and inductance in absolute electrical units.

The sinusoidal distribution of the flux density may be closely approximated by so shaping the pole faces that their separation is equal to

$$\sec \frac{\pi}{l} x \quad 60$$

x , as before, being measured from the mid-point of the wire or the air-gap. Due to the fringing of the magnetic field the flux density will not fall quite to zero at the ends of the air-gap, but by extending the wire at each end slightly beyond the magnet poles the fringing effect may be largely compensated and the flux density at the ends will be small enough to have negligible effect.

Another form of the invention, in which the electro-mechanical coupling is electrostatic instead of electromagnetic, is illustrated in Figs. 4 and 5, the former being a longitudinal section of the device and the latter a transverse section at the line X-X' in Fig. 4. In this device the

vibrating element is a thin metallic strip 8 which is held stretched in an air-gap between conducting electrodes 6 and 7. The strip 8 is clamped at its ends by insulating blocks 9, 9' and 10, 10' which also serve to support the electrodes and to maintain their proper separation at the air-gap. The details of the assembly are omitted for the sake of clarity, but these may be in accordance with familiar practice.

Electrodes 6 and 7 are polarized with respect to the vibrator 8 by battery 11 one terminal of which is connected directly to 8 and the other to electrodes 6 and 7 through protective high resistances R and R'. The operating terminals of the system T₁ and T₂ are connected to electrodes 6 and 7 respectively.

The device operates in the same manner as a balanced electrostatic telephone. Battery 11 establishes equal steady electric fields between vibrator 8 and electrodes 6 and 7 respectively, upon which are superimposed oppositely directed fields from alternating voltages applied to terminals T₁ and T₂. These superimposed voltages interacting with the steady polarizing fields produce synchronous vibrations of the strip 8 which in turn generate synchronous back electromotive forces in the electric circuit. Because of its uniformly distributed mass and transverse elasticity the strip 8 has a series of natural resonances at harmonically related frequencies just as in the case of the stretched wire of the device of Fig. 1. The higher frequency modes of vibration are prevented in this case also by so graduating the driving force along the length of the strip that energy is supplied thereto only in the fundamental mode of vibration. In this case the driving force at any point on the strip is proportional to the square of the electric field intensity at that point, the field being made up of two components, one due to the polarizing voltage and one due to the superimposed alternating voltage, both of which are distributed in the same manner. With a symmetrical arrangement of the electrodes and the circuits the forces on the two surfaces of the vibrator, due to each component separately, balance each other and the only forces tending to produce motion are those represented by the product of the two components. Since the polarizing voltage is steady the forces represented by the products are synchronous with the superimposed forces.

For suppression of the unwanted modes of vibration it is necessary that the driving force be distributed sinusoidally along the strip as in the case of the electromagnetic device of Fig. 1. However, since the graduation of the air-gaps in this case affects the distribution of the polarizing field intensity and the alternating field intensity equally, it is necessary to give the air-gaps a modified shape such that the separation of the electrodes from the vibrator is proportional to

$$\sqrt{\sec \pi x/l}$$

x being the distance from the middle of the vibrator and l being the length of the electrodes. Since each air-gap acts as a separate source of driving force the two should be symmetrical.

Due to the motion of the vibrator the air-gaps do not remain constant and in consequence the driving force does not retain its sinusoidal distribution for all positions of vibrator. This, however, has only a second order effect on the behavior of the system and does not prevent the substantial elimination of the higher modes of vibration. The fringing effect of the electric

field at the ends of the electrode may be largely compensated by extending the vibrator slightly beyond the electrodes as illustrated in Fig. 4.

Since very small air-gaps are desirable for efficient electromechanical coupling considerable damping of the vibration may arise from the viscosity of the air. This may be reduced greatly by cutting longitudinal slots in the faces of the electrodes as shown in Fig. 5 or, alternatively, the device may be completely enclosed by insulating side walls such as 12 and 12' in Fig. 5 and the air partially exhausted from the enclosure. The resistances R and R' in the polarizing circuits may be made sufficiently high to have negligible dissipative effect.

The device is equivalent to an electrical impedance of the type shown in Fig. 6 which comprises a capacity C₀ shunted by an inductance L₁ and capacity C₁ connected in series. The capacity C₀ is that between electrodes 6 and 7 with the vibrator at rest and at the mean potential of the two electrodes. Inductance L₁ and capacity C₁ are contributed by the motion of the vibrator and their resonance frequency is that of the fundamental mechanical resonance of the vibrator. The impedance Z of the device is given by

$$Z = -j \frac{(f^2 - f_1^2)}{2\pi f C_0 (f^2 - f_2^2)} \quad (14)$$

where f denotes frequency, f₁ is the fundamental resonance frequency of the vibrator, and f₂ is a higher frequency at which the combination is anti-resonant. The value of f₂ is given by

$$f_2^2 - f_1^2 = \frac{E^2}{24.48 \pi^2 \rho D_0^3} \quad (15)$$

where E is the polarizing voltage in absolute units, ρ is the surface density of the vibrating strip in grams per square centimeter, and D₀ is the minimum air-gap between the vibrator and the electrodes.

The resonant impedance characteristics of the devices of the invention make them useful as impedance elements for the construction of broad band wave filters in which they may be connected in substantially the same manner as other two-terminal impedances. In their application to broad band wave filters due regard must be given to the proportioning of the elements so that the resonances and the absolute magnitudes of the several elements cooperate to provide the desired selective characteristics. The principles of such cooperation are well-known and are explained, for example, in U. S. Patent 1,828,454, issued October 20, 1931, to H. W. Bode.

An example of the application of the electrostatic device of the invention in a lattice type wave filter is shown in Fig. 7. In this figure the line branches of the network include similar balanced electrostatic elements 13 and 13' of the type shown in Fig. 5 and the lattice branches include elements 14 and 14', also similar to each other, but resonant at frequencies different from the resonances of the line elements. Inductances L included in each of the four line conductors external to the lattice cooperate with the network impedances in controlling the width of the transmission band in a manner explained in an article by W. P. Mason entitled "Electrical wave filters employing quartz crystals as elements", Bell System Technical Journal vol. XIII No. 3, July 1934, pages 416 to 425. A network of high resistances R₁ to R₆ connects a polarizing battery 15 to the electrodes of the electromechanical elements, the resistances being so balanced as not

to disturb the distribution of currents in the filter branches and being of sufficiently high values to avoid excessive energy dissipation of the transmitted currents.

A further modified form of the invention, in which one particular harmonic mode of vibration is suppressed, is illustrated in Fig. 8. The device shown there is a modification of the electro-magnetic device of Fig. 1, the curved pole-pieces 2 and 3 being replaced by pole-pieces having parallel opposed faces but extending only over approximately two thirds of the length of the wire on each side of the mid-point. In this modification the third harmonic mode is suppressed. The reason for the suppression is readily seen from the dotted curve 16 which shows the wire in its third harmonic mode. The motion of the central loop of the wire generates a back electromotive force in one direction and the motion of the wire in its two outer loops generates back electromotive forces in the opposite direction. Since the field is of uniform intensity from the center of the one outer loop to the center of the other and substantially zero elsewhere, the sum of the back electromotive forces, is zero and the resultant electromagnetic coupling for this mode of motion is also zero.

What is claimed is:

1. An impedance element comprising a uniform stretched conducting wire, means providing a magnetic field perpendicular to said wire, a pair of electrical terminals, and an electrical path between said terminals including said wire, said field producing means being so proportioned as to provide a field of maximum intensity at the mid-point of said wire and diminishing sinusoidally to substantially zero at the ends of said wire.

2. An impedance element in accordance with claim 1 in which the field producing means is a magnet having opposed pole faces forming an air-gap in which the wire is free to vibrate, the separation of the pole faces being a minimum at the mid-point of the wire and increasing towards the ends of the wire substantially in accordance with the law

$$\omega = \omega_0 \sec \frac{\pi x}{l}$$

where ω denotes the width of the air-gap at a point distant x from the mid-point of the wire, ω_0 the width at the mid-point and l the length of the wire.

3. An impedance element comprising a rectangular stretched metallic foil, a conducting electrode having an extended face opposed to a surface of said foil, means producing a steady electric field between said electrode and said foil, a pair of electrical terminals, and an electrical path between said terminals including said electrode and said foil, the separation between said electrode and said foil being a minimum at the mid-point of the foil and increasing towards the ends of the foil substantially in accordance with the law

$$\omega = \omega_0 \sqrt{\sec \frac{\pi x}{l}}$$

where ω is the separation at a point distant x from the mid-point of the foil, ω_0 is the separation at the mid-point, and l is the length of the foil.

4. An electrical circuit element comprising an electrical path extending between a pair of terminals, a mechanical vibratory element included in said path, said vibratory element comprising

an extended longitudinal member having distributed mass and elasticity whereby it is inherently capable of transverse vibration in a fundamental mode and also in different secondary modes at different frequencies, means establishing a steady field normal to the longitudinal axis of said member which coacts with the vibratory motion thereof to produce an oscillatory electromotive force in said path, said field establishing means including polar members shaped to distribute the field along the length of said vibratory member and to provide different field intensities at different points along the length thereof, the gradation of the field intensity thereby produced being proportioned to make the net resultant electromotive force generated in said electrical path substantially zero for vibration of said vibratory member in at least the two secondary modes adjacent the fundamental mode.

5. An electrical circuit element comprising an electrical path extending between a pair of terminals, a mechanical vibratory element included in said path, said vibratory element comprising an extended longitudinal member having distributed mass and elasticity whereby it is inherently capable of transverse vibration in a fundamental mode and also in different secondary modes at different frequencies, means establishing a steady field normal to the longitudinal axis of said member which coacts with electrical oscillations in said path to produce oscillatory mechanical forces upon said member, said field producing means including polar members shaped to distribute the field along the length of said vibratory member and to provide different field intensities at different points along the length thereof, the gradation of the field intensity thereby produced being proportioned to make the net oscillatory mechanical force tending to set the vibratory member into motion in its secondary modes substantially zero for at least the two secondary modes adjacent the fundamental mode.

6. An electrical circuit element comprising an electrical path extending between a pair of terminals, a mechanical vibratory element included in said path, said vibratory element comprising a longitudinally extended member having uniform longitudinal distribution of mass and elasticity whereby it is inherently capable of transverse vibration in a fundamental mode and in different secondary modes at harmonically related frequencies, means establishing a steady field normal to the longitudinal axis of said member which coacts with electrical oscillations in said path to produce oscillatory mechanical forces upon said member, said field producing means including polar members shaped to distribute the field along the length of said vibratory element and to provide different field intensities at different points along the length thereof, the gradation of the field intensity thereby produced being proportioned to make the resultant oscillatory mechanical force tending to set the vibratory element into motion in its secondary modes substantially zero for at least the two secondary modes adjacent the fundamental mode.

7. An electrical circuit element comprising an electrical path extending between two terminals, a mechanical vibratory element included in said path, said vibratory element comprising a stretched longitudinal member having uniform mass per unit length, means establishing a steady field normal to the longitudinal axis of said

member which coacts with electrical oscillations in said path to produce mechanical forces upon said member, said field establishing means including polar members shaped to distribute the field along the length of said vibratory member and to provide different field intensities at different points along the length thereof, the gradation of the field intensity thereby produced being proportioned to make the resultant oscillatory mechanical force tending to set the vibratory element in motion in its inherent harmonic modes of vibration substantially zero for at least the first two said harmonic modes.

8. An electrical circuit element comprising an electrical path extending between two terminals, a mechanical vibratory element included in said path, said vibratory element comprising a longitudinal member having uniform mass per unit length and uniformly distributed transverse elasticity, whereby it is inherently capable of transverse vibration in a fundamental mode and in different secondary modes at harmonically related frequencies, means establishing a steady field normal to the longitudinal axis of said member which coacts with oscillations in said electrical path to produce oscillatory mechanical forces acting upon said member, said field producing means including polar members shaped

to distribute the field along the length of said vibratory element and to provide different field intensities at different points along the length thereof, the gradation of the field intensity thereby produced being proportioned to provide a mechanical force on said vibratory member which is a maximum at the midpoint thereof and diminishes sinusoidally to substantially zero at the ends thereof.

9. An electrical circuit element comprising an electrical path extending between two terminals, a mechanical vibratory element included in said path, said vibratory element comprising a stretched longitudinal member having uniform mass per unit length, means establishing a steady field normal to the longitudinal axis of said member and extending along the length thereof which coacts with electrical oscillations in said path to produce mechanical forces acting upon said member, said field producing means including polar members shaped to provide a continuous gradation of the field intensity along the length of said vibratory member such that the mechanical force on said vibratory member is a maximum at the midpoint thereof and diminishes sinusoidally to substantially zero at the ends thereof.

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