Feb. 16, 1937. T. M. ODARENKO 2,070,678

COAXIAL CONDUCTOR SYSTEM

Filed June 11, 1935

FIG. 1

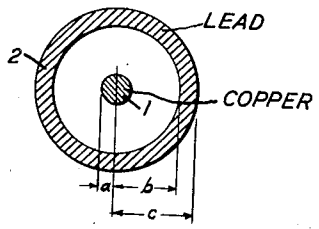


FIG. 2

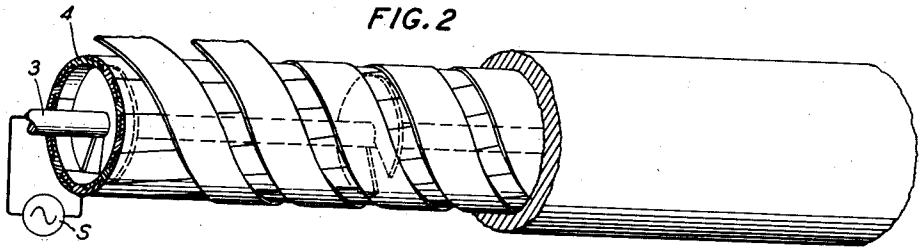


FIG. 3

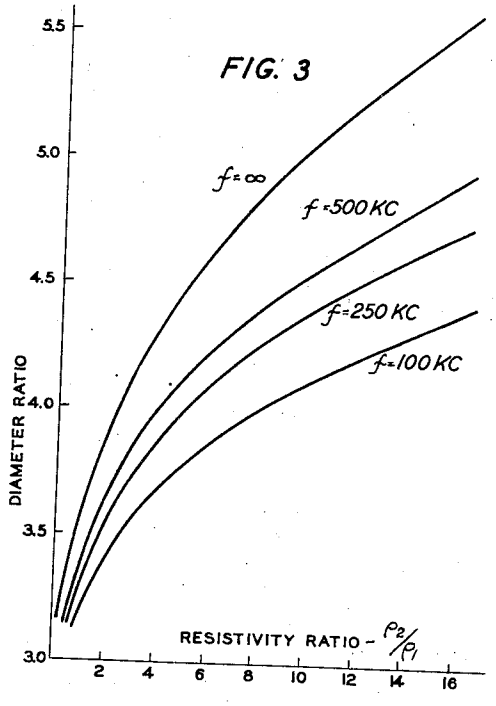
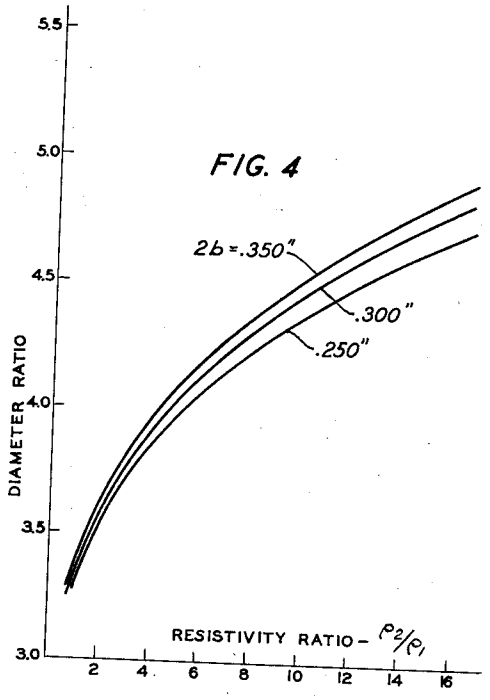


FIG. 4



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COAXIAL CONDUCTOR SYSTEM

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7 Claims. (Cl. 178—44)

This invention relates to electrical transmission systems and more particularly to coaxial conductor systems adapted to transmit radio frequency signals.

An object of the present invention is to increase the efficiency of transmission of a coaxial conductor line of fixed outer diameter.

A more specific object of the invention is to obtain a proportioning of the diameters of a pair of coaxial conductors that is the optimum with respect to signal attenuation.

It has been shown heretofore that the attenuation of a coaxial conductor transmission line at a predetermined frequency can be made a minimum by observing a critical ratio between the internal diameter of the outer conductor and the external diameter of the central conductor. The critical ratio varies over a fairly wide range depending on the absolute values of conductor diameter and frequency and on other factors peculiar to the particular type of conductor under consideration. It is important to know in each particular case what the critical or optimum diameter ratio is. Others have demonstrated that at high radio frequencies the optimum ratio for a coaxial line comprising solid or integral central and return conductors of the same material is 3.59. In J. M. West Patent 1,970,350, August 14, 1934, it is shown that the optimum ratio is a variable that approaches a limiting asymptotic value, 3.59 in one particular case, as the frequency and/or absolute diameter is increased.

Applicant has discovered that the optimum diameter ratio departs from the values ascertained by West if inner and outer conductors are not of the same material, or if the physical composition or structure of the conductors is such that the effective resistivity of one is different than that of the other. Formulae and curves are given from which may be determined the optimum diameter ratio for coaxial lines falling within this category.

The nature of the present invention will be set forth more fully in the following detailed description, in the course of which reference will be made to the accompanying drawing.

Fig. 1 of the drawing is a cross-sectional view of a coaxial pair;

Fig. 2 represents a coaxial conductor system in accordance with the invention; and

Figs. 3 and 4 show graphically the relation between optimum diameter ratio and the ratio of the effective resistivity of the outer conductor to the effective resistivity of the inner conductor, frequency and absolute diameters being the respective parameters.

Referring to Fig. 1, there is represented in cross-section a typical coaxial pair in accordance with the present invention comprising a central conductor 1 of copper and a tubular outer conductor 2 of lead. The existence of an optimum diameter ratio can be demonstrated and its value for particular cases determined mathematically, as follows:

The attenuation α of a coaxial conductor transmission line may be expressed as

$$\alpha=\frac{R}{2}\sqrt{\frac{C}{L}}+\frac{G}{2}\sqrt{\frac{L}{C}} \tag{1}$$

where R is the series resistance, C is the capacitance, L is the inductance, and G is the leakance, per unit length of line, all quantities being in C. G. S. units.

An expression for C, suitable for present purposes, is to be found in "A Treatise on the Theory of Alternating Currents" by Russell, second edition, University Press, 1914, at page 155;

$$C=\frac{\lambda}{2\log_\epsilon\frac{b}{a}} \tag{2}$$

or

$$C=\frac{K_1}{\log_\epsilon\frac{b}{a}} \tag{3}$$

where λ is the effective dielectric coefficient of the insulating medium, a is the radius of the central conductor, b is the internal radius of the outer conductor, and k_1 is $\frac{\lambda}{2}$.

Suitable expressions for R and L may be derived from Equations (98) and (99), respectively, of Russell (ibid, page 221), as follows:

Let

$p=\frac{2b}{2a}$, the diameter ratio

f=frequency in cycles per second

ρ_1=resistivity of the material of the inner conductor in C. G. S. units

ρ_2=resistivity of the material of the outer conductor in C. G. S. units

μ=permeability of the conducting materials

$\omega=2\pi f$

$$m_1=2\pi\sqrt{\frac{2\mu f}{\rho_1}}$$

$$m_2=2\pi\sqrt{\frac{2\mu f}{\rho_2}}$$

$$A=\frac{1}{b}\sqrt{\mu\rho_1 f}$$

$$B=\frac{\rho_1}{4\pi b^2}$$

$$E=\frac{\sqrt{\mu\rho_1}}{2\pi\sqrt{f}}\frac{1}{b}=A\frac{1}{2\pi f}=\frac{A}{\omega}$$

Assuming that $2b$, the inner diameter of the outer conductor, is 0.200" or more and that the frequency f is 200 kilocycles per second or higher,

then the members $(ma)^2$ and $(ma)^3$ in Russell's Equations (98) and (99) may be neglected and the last expressions in these two equations, involving hyperbolic and trigonometric functions may be regarded as unity. Russell's equations may then be written as follows:

$$R=\frac{\rho_1 m_1}{2\pi a}\left[\frac{1}{\sqrt{2}}+\frac{1}{2m_1 a}\right]+\frac{\rho_2 m_2}{2\sqrt{2}\pi b} \tag{4}$$

and

$$L=2\mu \log_\epsilon \frac{b}{a}+\frac{\sqrt{2}\mu}{m_1 a}+\frac{\sqrt{2}\mu}{m_2 b} \tag{5}$$

or rewriting these equations in applicant's notation

$$R=A\left[p+\sqrt{\frac{\rho_2}{\rho_1}}\right]+Bp^2 \tag{6}$$

and

$$L=2\log_\epsilon p+E\left[p+\sqrt{\frac{\rho_2}{\rho_1}}\right] \tag{7}$$

Substituting the expressions for C, R and L as given by Equations (3), (6) and (7), respectively, into Equation (1), we have

$$\alpha=\frac{\sqrt{k_1}}{2}\frac{A\left(1+\frac{G}{\omega C}\right)\left(p+\sqrt{\frac{\rho_2}{\rho_1}}\right)+Bp^2+2\frac{G}{C}\log_\epsilon p}{\sqrt{\log_\epsilon p\left[2\log_\epsilon p+E\left(p+\sqrt{\frac{\rho_2}{\rho_1}}\right)\right]}} \tag{8}$$

To determine the value of diameter ratio p for which the attenuation α is a minimum, the derivative

$$\frac{da}{dp}$$

is equated to zero:

$$\log_\epsilon^2 p\left[A\left(2+\frac{G}{\omega C}\right)+4Bp\right]+\log_\epsilon p\left\{\left[2BEp+\frac{A^2}{2\omega}\left(1+\frac{G}{\omega C}\right)-\frac{A}{p}\left(2+\frac{G}{\omega C}\right)\right]\left(p+\sqrt{\frac{\rho_2}{\rho_1}}\right)-\frac{1}{2}BEp^2-2Bp\right\}-\frac{1}{2}\left\{A\left(1+\frac{G}{\omega C}\right)\left(p+\sqrt{\frac{\rho_2}{\rho_1}}\right)^2+Bp^2\left(p+\sqrt{\frac{\rho_2}{\rho_1}}\right)\right\}\frac{E}{p}=0 \tag{9}$$

This is a transcendental equation for p and can be solved only graphically. Solving it for $\log_\epsilon p$:

$$\log_\epsilon p=\frac{-\left\{\left[2BEp+\frac{1}{2\omega}A^2\left(1+\frac{G}{\omega C}\right)-\frac{A}{p}\left(2+\frac{G}{\omega C}\right)\right]\left(p+\sqrt{\frac{\rho_2}{\rho_1}}\right)-\frac{1}{2}BEp^2-2Bp\right\}}{2\left[A\left(2+\frac{G}{\omega C}\right)+4Bp\right]}+\frac{\sqrt{\left\{\left[2BEp+\frac{1}{2\omega}A^2\left(1+\frac{G}{\omega C}\right)-\frac{A}{p}\left(2+\frac{G}{\omega C}\right)\right]\left(p+\sqrt{\frac{\rho_2}{\rho_1}}\right)-\frac{1}{2}BEp^2-2Bp\right\}^2+2\left[A\left(2+\frac{G}{\omega C}\right)+4Bp\right]\left\{A\left(1+\frac{G}{\omega C}\right)\left(p+\sqrt{\frac{\rho_2}{\rho_1}}\right)^2+Bp^2\left(p+\sqrt{\frac{\rho_2}{\rho_1}}\right)\right\}\frac{E}{p}}}{2\left[A\left(2+\frac{G}{\omega C}\right)+4Bp\right]} \tag{10}$$

This is the fundamental equation for optimum values of the diameter ratio.

The limiting value which the diameter ratio approaches asymptotically with increase of frequency may be determined by equating B and E in Equation (10) to zero, in which case the following equation must be satisfied:

$$\log_\epsilon p=1+\frac{1}{p}\sqrt{\frac{\rho_2}{\rho_1}} \tag{11}$$

The uppermost curve of Fig. 3 shows how in this limiting case the optimum diameter ratio p varies with the effective resistivity ratio

$$\frac{\rho_2}{\rho_1}$$

This curve is independent of absolute diameters.

For finite values of frequency, Equation (10) must be solved graphically for each specific combination of the variables.

In all of the numerical examples herein given, a typical value of the ratio

$$\frac{G}{C}$$

has been assumed; specifically, it has been assumed in these examples that

$$\frac{G}{C}=0.0045f \tag{12}$$

This expression is accurate for a coaxial pair in which the separator comprises slotted hard rubber washers one-sixteenth inch thick spaced approximately three-quarters of an inch apart. The optimum diameter ratio is not a very sensitive function of

$$\frac{G}{C}$$

in a well designed coaxial pair having low dielectric losses, hence Equation (12) may be applied with sufficient accuracy to a variety of similar structures. Generally, of course, the appropriate value of

$$\frac{G}{C}$$

must be employed in each graphical solution of Equation (10). In Fig. 3 are shown the results of the solution for one specific case, viz., a coaxial pair in which the internal diameter ($2b$) of the outer conductor is 0.250''. Optimum diameter ratio is plotted against the resistivity ratio

$$\frac{\rho_2}{\rho_1}$$

for three finite frequencies, 100, 250 and 500 kilocycles per second, and also for infinite frequency.

For specific example, consider a coaxial pair comprising a central conductor of copper and a tubular outer conductor of lead, the latter being of one-quarter inch internal diameter. Assume the resistivity of the lead is twelve times as great as that of the copper, and assume further that the frequency involved is 250 kilocycles per second. Reference to Fig. 3 shows that the optimum diameter ratio is 4.52.

The optimum diameter ratio for other cases where the resistivity ratio is 12 is shown in the following schedule:

$2b$ (inches)	f (kilocycles/sec.)				
	100	250	500	1000	∞
0.250	4.24	4.54	4.74	4.88	5.24
0.300	-------	4.60	4.79	4.92	5.24
0.350	-------	4.64	4.83	4.96	5.24

Fig. 4 shows optimum diameter ratio as a function of resistivity ratio for a coaxial conductor system in which the frequency of interest is 250 kilocycles per second, the three curves applying where the internal diameter (2b) of the outer conductor is 0.250", 0.300" and 0.350", respectively. A similar family of curves may be ploted from Equation (10) for other values of frequency.

Although the present invention has been thus far discussed with reference to coaxial pairs in which the difference in effective resistivity arises from the use of different materials for the inner and outer conductors, the invention is not thus restricted in its application. The invention may be applied also to coaxial pairs in which the difference in effective resistivity is due, wholly or in part, to structural differences.

If, for example, the outer conductor is not a solid tube but a composite structure made up of a plurality of interengaging profiled strips, the optimum diameter ratio can still be ascertained by proper use of the equations and curves set forth herein. In this case, it is only necessary to substitute for the resistivity ratio

$$\frac{\rho_2}{\rho_1}$$

a modified or effective resistivity ratio

$$X^2\frac{\rho_2}{\rho_1}$$

where X^2 represents the ratio of the effective resistance of the composite outer conductor to the resistance of a solid tubular outer conductor that is in all other respects the same. The factor X can be computed directly or from line attenuation measurements. In the appended claims where a difference in the effective resistivities of the two conductors is specified, it is contemplated that this difference may arise from differences of intrinsic resistivity, of structure, or of both, and that in any case the difference is significant with respect to the optimum diameter ratio.

As a concrete illustration of the application of the present invention to a coaxial conductor line comprising a composite outer conductor, consider the system represented schematically in Fig. 2. In this system, S represents a signal source, for example, the terminal circuits of a multiplex carrier telephone or television system operating over a frequency range of from 100 to 1000 kilocycles per second. The coaxial conductor line comprises a central copper wire 3 and a composite tubular copper outer conductor 4 made up of a plurality of overlapping profiled strips assembled with moderate lay. The construction of the line is described more fully in United States Patent No. 2,018,477, issued on October 22, 1935 to J. F. Wentz.

Assume that by calculation or by electrical measurement it is found that the structure of the outer conductor is responsible for an increase in line attenuation of five per cent over the attenuation of a line having a corresponding solid tubular outer conductor. By computation the value of X will be found to be 1.4, and $X^2=1.4^2=1.96$, assuming, as may safely be done, that the change in attenuation is due practically solely to a change in the resistance of the outer conductor. If both conductors are of the same material, or more accurately, if the materials comprising the respective conductors are of the same intrinsic resistivity, then the resistivity ratio to be used is 1.96. If the materials are of different intrinsic resistivities, then the resistivity ratio to be used is

$$1.96\frac{\rho_2}{\rho_1}$$

Assuming the proper resistivity ratio for the particular case to be 1.96 the optimum diameter ratio p as determined from Equation (10) or from the curves of Figs. 3 and 4, is as shown in the following schedule:

2b (inches)	f (kilocycles/sec.)				
	100	250	500	1000	∞
0.250	3.4	3.55	3.64	3.71	3.9
0.300	-------	3.60	3.68	3.74	3.9
0.350	-------	3.64	3.71	3.76	3.9

Various other specific types of coaxial pairs to which the present invention is applicable suggest themselves. To mention only a few, the inner conductor may be of high conductivity copper and the outer conductor of low conductivity copper or to reduce attenuation-temperature variations the inner conductor may be of an aluminum-copper alloy.

The frequency f to be selected as the basis for determining the optimum diameter ratio depends upon the system under consideration. In the wide-band signaling system represented in Fig. 2 the highest frequencies are subject to the greatest attenuation and may tend to approach too closely the noise level. Hence, in determining the optimum diameter ratio it is the highest frequency of the signaling band or a frequency in the vicinity thereof that is used as the basis for the design.

It is to be understood that the present invention is not limited to the specific embodiments disclosed herein for purposes of illustration, but comprehends all embodiments that come within the scope and spirit of the appended claims.

What is claimed is:

1. A coaxial conductor transmission line comprising inner and outer conductors of different effective resistivities, the outer of said conductors being less than an inch in diameter, and the ratio of the internal diameter of said outer conductor to the external diameter of said inner conductor being optimum with respect to the attenuation of an ultra-audible wave having a frequency of the order of a megacycle or less and lying near the top of the frequency range transmitted over said system.

2. A coaxial conductor transmission line comprising inner and outer conductors of different effective resistivities and means for transmitting over said line a band of ultra-audible waves, the ratio of the internal diameter of said outer conductor to the external diameter of said inner conductor being such that the attenuation of said line at a frequency near the top of said band is a minimum, said frequency being so low that said ratio of diameters is a function of frequency.

3. A coaxial conductor transmission line comprising inner and outer conductors connected one as a return for the other, the effective resistivity of said outer conductor being greater than the effective resistivity of said inner conductor and the ratio of the internal diameter of said outer conductor to the external diameter of said inner conductor being optimum with re-

spect to the attenuation of waves lying near the top of the frequency band transmitted, said optimum ratio being at least ten per cent smaller than the optimum ratio for waves of infinitely high frequency.

4. A transmission line in accordance with the claim next preceding in which the intrinsic resistivity of the material comprising said outer conductor is greater than that of the material comprising said inner conductor.

5. A transmission line in accordance with claim 3 in which the difference in effective resistivities is due primarily to the composite nature of said outer conductor.

6. A coaxial conductor transmission line comprising a central conductor and a composite tubular return conductor having a greater effective resistance than an integral tubular conductor of the same material and dimensions, and means for transmitting over said line a wide band of ultra-audible signals, the ratio of the internal diameter of said return conductor to the external diameter of said central conductor being the optimum for low attenuation of waves lying near the top of said wide band, said optimum ratio being a sensibly dependent function of frequency.

7. A transmission line comprising a central conductor and a coaxial tubular return conductor separated therefrom by a dielectric that is chiefly gaseous, said return conductor having a smooth composite inner conducting surface and an effective resistivity greater than that of said central conductor, and means for transmitting over said line waves of a wide range of frequencies extending up to a high frequency well above audibility, the ratio of the internal diameter of said return conductor to the external diameter of said central conductor being optimum for low attenuation of waves lying near the top of said wide range of frequencies, the diameter of said return conductor and the highest frequency in said range being such that the optimum diameter ratio is a function of either or both.

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