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E. L. NORTON

2,070,677

TRANSMISSION NETWORK

Filed June 9, 1934

FIG. 1

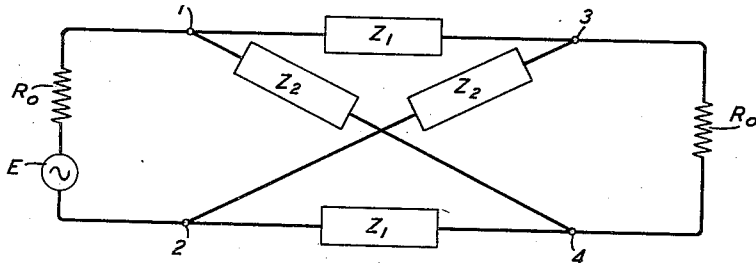


FIG. 2

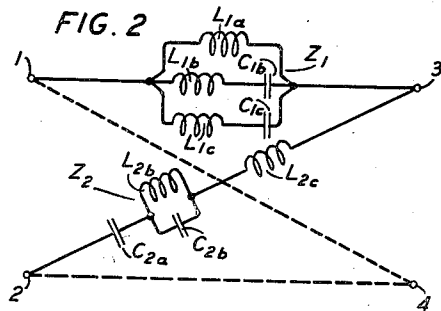


FIG. 3

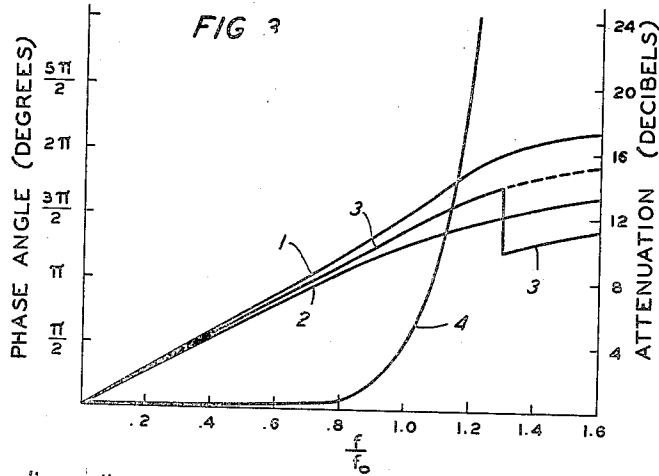
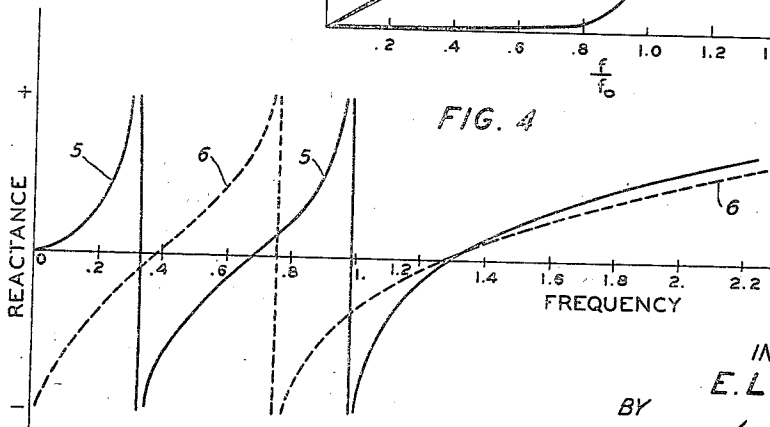


FIG. 4



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TRANSMISSION NETWORK

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3 Claims. (Cl. 178—44)

This invention relates to frequency selective networks and more particularly to the control of the phase characteristics of broad-band transmission systems. It has for its principal object the provision of a linear phase characteristic, not only in the transmission band of a band selective system, but also in the frequency ranges adjacent to the band wherein the attenuation is rapidly increasing. A further object is to provide for the control of the rate at which the attenuation increases at the edges of the transmission band while at the same time maintaining the linearity of the phase characteristic in these regions.

The control of the transmission characteristics of a reactive four-terminal network through the allocation of the resonance and anti-resonance frequencies of the branch impedance is described in United States Patent 1,828,454, to H. W. Bode, wherein particular frequency distributions are disclosed which provide linear variation of the phase characteristic throughout substantially the whole transmission band of the wave filter.

The characteristics with which the above mentioned patent deals are those of the network itself without reference to the impedances between which it is connected, namely, the characteristic impedance and the propagation constant. When such networks are connected to operate between fixed terminal impedances, which in practice are usually fixed resistances, the overall transmission characteristics of the system include not only the propagation constant of the network itself, but also the effects of wave reflection at the junctions to the terminal impedances. These latter effects may be negligibly small throughout the greater part of the band when the characteristic impedance is a substantially constant resistance, but in the frequency ranges adjacent to the band limits, the wave reflection exercises a strong modifying influence.

In the networks of the invention, the reflection effects for a preassigned value of the terminal resistances are coordinated with the propagation constant in such a manner that the total phase characteristic of the system represented by the network and its connected terminal impedance is linear throughout the whole of the transmission band and part of the attenuation ranges adjacent thereto. This is accomplished by allocating the resonance and anti-resonance frequencies of the branch impedances of the networks in accordance with a novel arrangement as hereinafter described, one characteristic of the arrangement being that the coincidence of the resonance and anti-resonance frequencies of the different network branches, which has heretofore been regarded as necessary for the pro-

vision of a continuous transmission band, is not necessarily present.

Of the attached drawing:

Fig. 1 represents a general type of network in which the invention may be embodied;

Fig. 2 shows the schematic structure of a particular network of the invention; and

Figs. 3 and 4 are curves showing characteristics of the networks of the invention and illustrating the design procedure.

Referring to Fig. 1, which shows a symmetrical lattice network having branch impedances Z_1 and Z_2 connected between load resistances R_2 one of which includes a wave source of voltage E , the insertion loss θ due to the lattice will be given by

$$e^{\theta} = \frac{I_1}{I_2} \quad (1)$$

where I_1 is the current through the receiving-end resistance with the network absent and I_2 is the corresponding current with the network in the circuit. Expressing I_1 and I_2 in terms of E and the impedances and substituting in the above equation, the insertion loss may be written as

$$e^{\theta} = \frac{\left(1 + \frac{Z_1}{R_0}\right)\left(1 + \frac{Z_2}{R_0}\right)}{\frac{Z_2}{R_0} - \frac{Z_1}{R_0}} \quad (2)$$

Assuming that the impedances are pure reactances, and writing

$$\begin{aligned} Z_1 &= jX_1 \\ Z_2 &= jX_2 \end{aligned}$$

Equation 2 may be transformed to

$$e^{\theta} = \frac{\left(1 + \frac{jX_1}{R_0}\right)\left(1 + \frac{jX_2}{R_0}\right)}{\frac{jX_2}{R_0} - \frac{jX_1}{R_0}} \quad (3)$$

Now let X_1 and X_2 be defined as follows:

$$\begin{aligned} \frac{X_1}{R_0} &= \tan y_1 \\ -\frac{R_0}{X_2} &= \tan y_2 \end{aligned} \quad (4)$$

then

$$e^{\theta} = \frac{\left(1 + j \tan y_1\right)\left(1 + \frac{1}{j \tan y_2}\right)}{\frac{1}{j \tan y_2} - j \tan y_1} = \frac{1 - \tan y_1 \tan y_2}{1 + \tan y_1 \tan y_2} + j \frac{\tan y_1 + \tan y_2}{1 + \tan y_1 \tan y_2} \quad (5)$$

If we write $\theta = A + jB$, where A is an attenuation factor and B is the phase angle, then

$$e^{\theta} = e^{A+jB} = e^A \cos B + je^A \sin B$$

Substituting in the preceding equation and equating real and imaginary terms we will have

$$e^A \cos B = \frac{1 - \tan y_1 \tan y_2}{1 + \tan y_1 \tan y_2} \quad (6)$$

$$e^A \sin B = \frac{\tan y_1 + \tan y_2}{1 + \tan y_1 \tan y_2}, \quad (7)$$

from which it can be readily deduced that

$$B = y_1 + y_2 \quad (8)$$

and

$$e^{-A} = \cos (y_1 - y_2) \quad (9)$$

It is thus seen that the phase angle B is equal to the sum of the angles y_1 and y_2 , which are determined by the reactance arms X_1 and X_2 according to Equations 1 and 2; and that the attenuation A is determined by the difference between these angles. When y_1 and y_2 are nearly equal the attenuation of the network is small and when their difference approaches 90 degrees the attenuation approaches infinity. It is to be noted that in the foregoing discussion no assumption has been made as to the coincidence of the resonance and anti-resonance frequencies of the reactances X_1 and X_2 as is usually done in the design of filters of this type.

The application of the foregoing principles to the design of networks of the invention will now be described. To obtain linearity of the phase characteristic through a given frequency range it is necessary to provide reactive impedances Z_1 and Z_2 such that the sum of the angles y_1 and y_2 , defined above, increases linearly with frequency through the range. If this linearity is to be accompanied by broad band selective properties it is further necessary that the impedances be such as to make the difference between y_1 and y_2 small throughout the transmission band and approximately 90 degrees elsewhere. A graphic method of design may be used which is described below in connection with the design of a low pass filter.

In Fig. 3 the curves represent the variations of the network constants with frequency, the abscissae being proportional to the ratio f/f_0 where f denotes frequency and f_0 is a frequency of the character of a cut-off frequency marking the limit of the transmission band. Curve 3 represents a desired phase characteristic, the ordinates being taken for convenience as equal to the angle $B/2$. Curve 4 represents the desired attenuation characteristic, the ordinates being in decibels attenuation, corresponding to the scale on the right hand side of the figure. It is to be noted that the desired phase characteristic is substantially linear up to a frequency equal to $1.3f_0$ and that the attenuation is less than 4 decibels at all frequencies below f_0 but rises sharply to infinity at the frequency $1.3f_0$.

The design now proceeds by drawing in the curves representing the angles y_1 and y_2 using the above mentioned principles as a guide. In the first place it is clear that y_1 and y_2 must be evenly spaced about curve 3 since their sum is to be equal to twice the ordinates of curve 3. In the second place, the difference between the ordinates of the two curves is determined by the desired attenuation loss in accordance with the formula

$$db = 20 \log_{10} \frac{1}{\cos (y_1 - y_2)} \quad (10)$$

where db is the attenuation loss in decibels. Proceeding in this way curves 1 and 2 may be drawn representing the angles y_1 and y_2 respectively. For example, at the frequency f_0 where the at-

tenuation is 4 decibels, the difference of the angles y_1 and y_2 is 51 degrees or $.284 \pi$ radians.

Having determined the curves representing y_1 and y_2 the next step is to determine the reactances corresponding to these characteristics in accordance with Equation 4. From these equations it is seen that the reactance X_1 will be zero at zero frequency and at those frequencies for which the ordinates of curve 1 are even multiples of $\pi/2$ and will be infinite at frequencies for which the ordinates are odd multiples of $\pi/2$. In the case of reactance X_2 an inverse relationship holds. For the case illustrated in Fig. 3, the critical frequencies of reactance X_1 occur at zero, $0.335f_0$, $0.68f_0$, $.99f_0$, and $1.3f_0$ and those of X_2 at zero, $0.38f_0$, $0.75f_0$, and $1.3f_0$. The next critical frequency in each case is assumed to be infinity, that is, the reactances have no further resonances or anti-resonances at finite frequencies and the curves of y_1 and y_2 flatten out as the frequency increases. The frequency characteristic of the reactances are shown in Fig. 4, curve 5 corresponding to X_1 and curve 6 to reactance X_2 . The lack of coincidence of the critical frequencies is clearly shown.

It will be observed that the increasing slope of curve 1 in Fig. 3 as the frequency increases entails a progressive diminution of the intervals between critical frequencies of the reactance X_1 and that the diminishing slope of curve 2 entails a progressive increase in the intervals between the critical frequencies of reactance X_2 . This is illustrated in Fig. 4 by the increasing lack of coincidence between the resonances, or zeros, of curve 6, and the anti-resonances, or poles, of curve 5. In the range of low attenuation the zeros of X_2 and the poles of X_1 are nearly coincident, but separate gradually as the attenuation increases until in the range of high attenuation the zeros of the two reactances substantially coincide. In the example illustrated each reactance has only one critical frequency in the attenuating range, but as many as desired may be added. Since the curves 1 and 2 must continue to be separated by approximately $\pi/2$ radians to maintain high attenuation, the location of the added critical frequencies must be such as to maintain the curves substantially parallel.

The physical character of reactance X_1 is illustrated by the impedance Z_1 of Fig. 2 and that of X_2 by the impedance Z_2 of the same figure. The former consists of two resonant circuits and a simple inductance all connected in parallel and the latter of a capacity and an inductance and an anti-resonant loop all in series.

It is well known that, theoretically, the lack of coincidence of the critical frequencies should produce a multi-band system. Thus the network of Fig. 2, with the critical frequencies indicated above, should have two attenuation bands below f_0 in the ranges adjacent the first anti-resonance and the first resonance of X_1 where the reactances X_1 and X_2 are of like sign. However, the amount of attenuation in those bands is quite small and by virtue of the reflection effects at the terminal impedances is made unnoticeable. All of the effects are summed up in Equations 8 and 9 which show that the attenuation is small so long as the difference of y_1 and y_2 is small.

Having ascertained the critical frequencies of the reactances by the graphic method outlined above the values of the several inductances and capacities can be ascertained by the method de-

scribed by R. M. Foster in an article entitled "A Reactance Theorem", Bell System Technical Journal, Vol. III, No. 2, April 1924. Before the formulae given in this article can be applied, however, it is necessary that one element in each impedance be determined. This can be done most conveniently by considering the slopes of the y_1 and y_2 characteristics at zero frequency, which may be made each equal to the slope of the curve 3, that is, to half the slope of the desired linear phase characteristic.

From Equation 4 the slopes of the y_1 and y_2 characteristics are found to be

$$\frac{dy_1}{df} = \frac{1}{1 + \frac{(X_1)^2 R_0}{(R_0)^2}} \frac{1}{R_0} \frac{dX_1}{df} \quad (11)$$

and

$$\frac{dy_2}{df} = -\frac{R_0}{1 + \frac{(R_0)^2}{(X_2)^2}} \frac{d}{df} \frac{1}{X_2} \quad (12)$$

which at zero frequency become

$$\left. \frac{dy_1}{df} \right|_{f=0} = \frac{1}{R_0} \left. \frac{dX_1}{df} \right|_{f=0} \quad (13)$$

and

$$\left. \frac{dy_2}{df} \right|_{f=0} = -R_0 \left. \frac{d}{df} \frac{1}{X_2} \right|_{f=0} \quad (14)$$

At zero frequency the reactance of Z_1 is evidently wholly that of the simple shunt inductance L_{1a} and the reactance of Z_2 is wholly determined by the series capacity C_{2a} . Hence if S_0 be the desired linear slope of the resultant phase characteristics, Equations 13 and 14 become

$$\frac{1}{2} S_0 = \frac{2\pi L_{1a}}{R_0} \quad (15)$$

and

$$\frac{1}{2} S_0 = 2\pi R_0 C_{2a} \quad (16)$$

which suffice to determine the elements L_{1a} and C_{2a} .

It will be observed that the phase characteristic illustrated by curve 3 of Fig. 3 undergoes a sudden shift of $\pi/2$ at the frequency $1.3f_0$, representing a phase shift of π , or a current reversal, in the over-all characteristic. A discontinuity of this type will occur whenever the attenuation passes through an infinite value, as at $1.3f_0$, but it is not of great consequence since the current that is reversed is infinitesimally small. Linearity of the phase characteristic is maintained up to the first attenuation peak above the cut-off frequency and, if reversals be ignored, can be maintained as far as may be desired by increasing the complexity of impedances Z_1 and Z_2 so that they have additional critical frequencies above the cut-off.

The invention has been described in connection with its embodiment in a broad band selective

network, that is, a network characterized by a steep transition between the transmission and the attenuation ranges. It is obvious from the discussion given that the design principles may be applied in other connections, for example, in the design of a network giving a gradually rising or gradually falling attenuation coupled with phase linearity and useful as an attenuation equalizer.

What is claimed is:

1. A four-terminal broad-band wave filter comprising a symmetrical network of reactances, said network having characterizing reactances X_1 and X_2 which by their frequency variations and magnitudes determine the transmission properties of the network, equal resistive terminal impedances of resistance R_0 connected to the input and the output terminals of said filter, said characterizing reactances each having a plurality of critical frequencies corresponding to resonances and anti-resonances, said critical frequencies being so spaced in the frequency scale that the sum of the angles y_1 and y_2 defined by

$$y_1 = \tan^{-1} \frac{X_1}{R_0}$$

and

$$y_2 = -\tan^{-1} \frac{R_0}{X_2}$$

varies substantially linearly with frequency throughout the transmission band of the filter and throughout part of the attenuation range adjacent the band.

2. A wave filter in accordance with claim 1 in which the critical frequencies and magnitudes of the reactances X_1 and X_2 are so proportioned that the angles y_1 and y_2 are zero at zero frequency, are substantially equal in a low frequency range, and diverge in a higher frequency range to a difference equal to $\pi/2$ radians, their sum being proportional to the frequency throughout both ranges, whereby the filter has a low-pass characteristic and a linear phase shift throughout and beyond the transmission range.

3. A wave filter in accordance with claim 1 of the low-pass type in which the critical frequencies of the reactances X_1 and X_2 are approximately uniformly spaced in the lower portion of the transmission band whereby the angles y_1 and y_2 are approximately equal and proportional to frequency and in which at higher frequencies the intervals of the critical frequencies of X_1 diminish while the intervals of the corresponding frequencies of X_2 increase so that over a preassigned frequency range the difference of the angles y_1 and y_2 increases gradually to an approximately constant value of $\pi/2$ radians, the average value of the two angles retaining the same proportionality to frequency as in the lower range.

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