

Oct. 6, 1936.

E. LAKATOS

2,056,281

WAVE FILTER

Filed June 1, 1935

3 Sheets-Sheet 1

FIG. 1

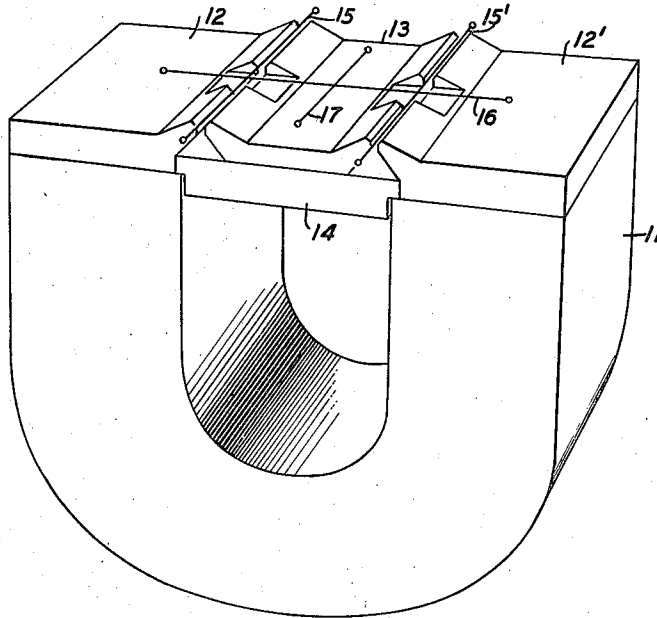


FIG. 2

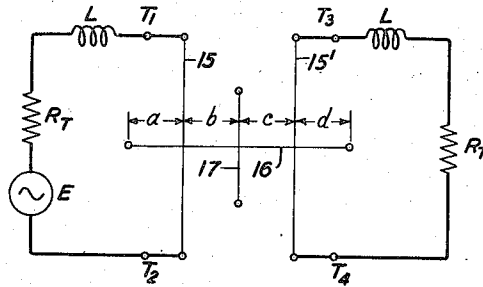
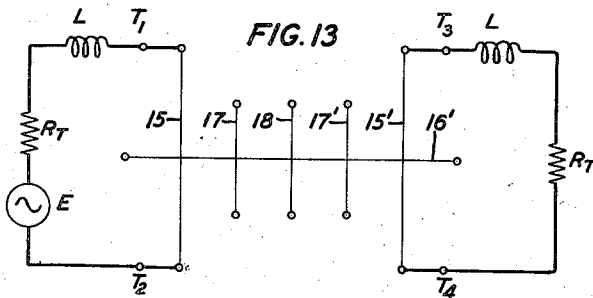


FIG. 13



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FIG.3

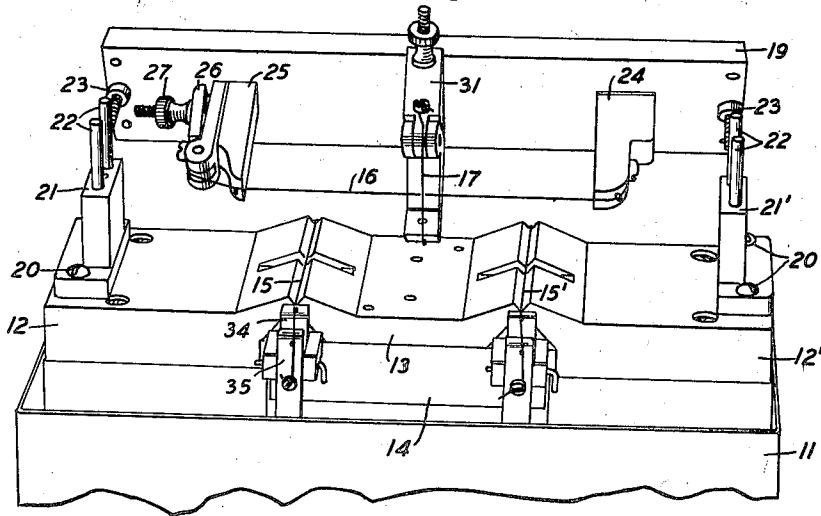


FIG. 5

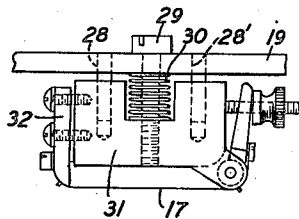


FIG. 4

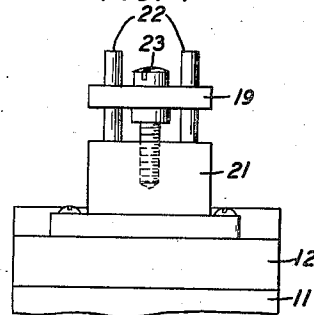
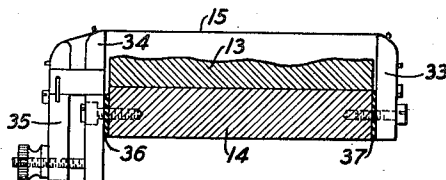


FIG. 6



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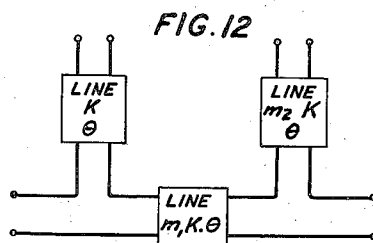
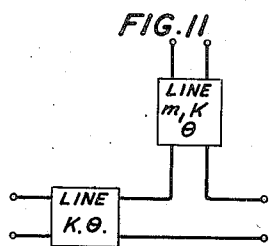
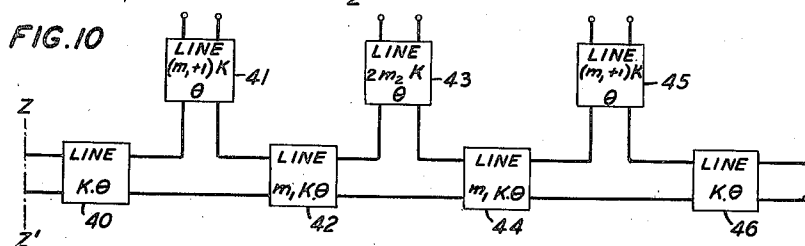
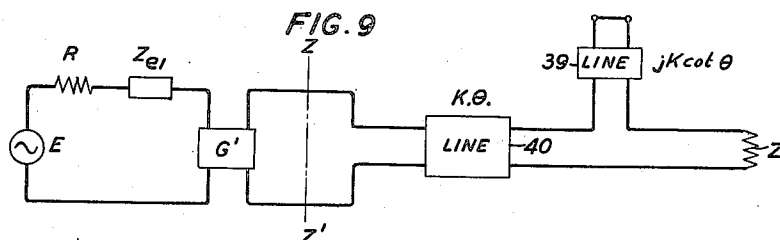
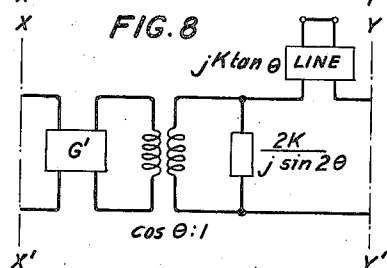
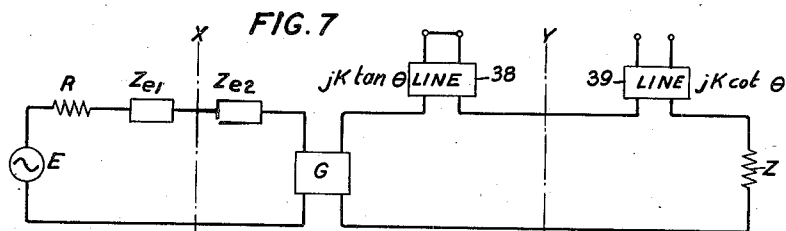
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3 Sheets-Sheet 3



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2,056,281

WAVE FILTER

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Application June 1, 1935, Serial No. 24,502

4 Claims. (Cl. 178-44)

This invention relates to wave filters and more particularly to wave filters using mechanical vibratory elements.

The principal object of the invention is to provide a mechanical vibratory structure which is capable of transmitting vibrations in a wide frequency range and at relatively high frequencies such as are used for carrier telephone transmission. Another object is to reduce the cost of band selective filters for high frequency transmission systems.

In the wave filters of the invention the mechanical vibratory system comprises a longitudinal stretched wire which acts as a main transmission line and a plurality of stretched wires coupled transversely thereto which act as loading impedances. The end transverse wires are disposed in the air-gaps of electromagnets and are included in separate electric circuits whereby they can function as driving means for converting electrical oscillations into mechanical vibrations or as receiving means for effecting the reverse transformation.

Particular features of the invention relate to the tuning of the wires whereby a continuous transmission band may be provided and to the proportioning of the mechanical impedances of the wires for the control of the width of the transmission band.

Filters of a type similar to those of my invention, but differing therefrom in respect of the arrangements provided for controlling the transmission band width, are disclosed in the copending application of R. B. Blackman, Serial No. 17,431, filed April 20, 1935. In the filters of the Blackman application control of the band width is effected by the use of loading wires coupled directly to the stretched wires which convert the electrical currents into mechanical vibrations. In the filters of my invention the use of special impedance transforming filter sections permits the control of the band width to be effected by means of loading wires located remote from the converting wires. As a result of this the mechanical construction of the filters is simplified.

The nature of the invention will be more fully understood from the following detailed description and the attached drawings, of which

Fig. 1 is a diagrammatic representation of one embodiment of the invention;

Fig. 2 shows schematically the arrangement of the electrical circuits and the mechanical vibratory system of the system of Fig. 1;

Figs. 3, 4, 5 and 6 illustrate the mechanical con-

struction and details thereof of the system of Fig. 1;

Figs. 7, 8, 9, 10, 11 and 12 are schematic diagrams illustrative of the principles of the invention; and

Fig. 13 is a schematic representation of a modified form of the invention.

Description of physical structure

Referring to the drawings, a schematic arrangement of the magnetic and mechanical vibratory systems of one embodiment of the invention is shown in Fig. 1, in which, for the sake of clearness, much of the detail structure has been omitted. The magnetic system comprises a permanent magnet 11 provided with three pole-pieces 12, 12', and 13 which are arranged to provide two separated air-gaps in the form of narrow slits parallel to each other. The vibratory system consists of a pair of stretched actuating wires 15 and 15' disposed in the respective air-gaps of the magnetic system, a stretched coupling wire 16 lying perpendicular to the actuating wires and contacting therewith at their mid-points, and a loading wire 17 disposed transversely of wire 16 and in contact therewith at its mid-point. The supports for the stretched wires are not shown in Fig. 1, but are illustrated in Figs. 3, 4, 5 and 6, the details of which will be described later. Pole-pieces 12 and 12' may be secured by screws directly to the magnet. Pole-piece 13 is supported on a plate 14 of non-magnetic metal which in turn is rigidly secured to the magnet. The several pole-pieces are shaped to provide mechanical clearance for all of the wires of the vibratory system and are notched in the manner indicated in the drawings to provide clearance for the coupling wire 16.

Referring to Fig. 3, which shows the details of a practical construction partially disassembled for clarity, the coupling wire 16 and the loading wire 17 are carried on a removable plate 19 which is arranged to be supported from the magnetic structure by brackets 21 and 21' and guide pins 22. An end view of the structure with the plate 19 in position is shown in Fig. 4. Vertical adjustment is provided for by means of collared screws 23, one at each end, which engage the plate and are threaded into brackets 21 and 21'. By this means an adequate mechanical contact between the driving wires and the coupling wire 16 can be assured. The coupling wire 16 is supported from plate 19 by bracket members 24 and 25 the latter of which includes a tensioning arrangement consisting of a hinged lever arm 26 to which

the end of the wire is attached and an adjusting screw and nut 27.

The loading wire 17 is mounted on tensioning holder 31 which is adjustably supported from plate 19. Details of this holder and of the supporting arrangements are shown in Fig. 5. At one end of the holder an anchor plate 32 is attached to which the stretched wire is secured. At the other end an adjustable tensioning lever is provided similar to that used for tensioning the coupling wire 16. The holder is carried on plate 19 by guide pins 28 and 28' which are rigidly attached to the plate and engage in close fitting holes in the holder. Vertical adjustment is provided by means of screw 29 which is threaded into the holder and passes through a clearance hole in plate 19. Spring 30 which encircles screw 29 and abuts against the holder and the plate serves to keep the holder in its adjusted position.

The driving wires 15 and 15' are supported directly from the magnetic structure and are provided with adjustable tensioning means similar to those for the other wires. Details of the mounting are shown in Fig. 6, only the essential parts of the magnetic structure being indicated. The wire 15 is attached at one end to an anchor plate 33 mounted on plate 14 and at the other to a tensioning support comprising bridge piece 34 and tensioning lever 35. Anchor plate 33 and bridge piece 34 are insulated from plate 14 by plates 36 and 37 of insulating material and are attached to plate 14 by suitably insulated screws. This is necessary since the wires 15 and 15' carry the input and output electrical currents. While the other wires of the system do not carry electrical currents it is preferable that they should be insulated in similar manner to the driving wires, at least at one end of each wire. The symmetry of the wire structure prevents the transmission of current from the one driving wire to the other through the coupling wire 16.

Theory of operation and design

Before discussing the detailed theory of the invention, the operation of the filter will be described briefly with reference to Fig. 2. In this figure, which is a schematic of the stretched wire system and its connections to the cooperating electrical circuits, the filter is shown connected between electrical input terminals T₁ and T₂ and output terminals T₃ and T₄ to which are connected resistive terminal impedances R_r. In series with one of the terminal resistances is included a wave source of voltage E.

Current from the source E causes the driving wire 15 to vibrate transversely to the direction of the magnetic field in the air-gap in which the wire is situated. The vibrations of wire 15 are transmitted to wire 15' through coupling wire 16. Vibrations of wire 15' in its magnetic field induce corresponding electromotive forces which cause currents to flow in the output circuit. The band-pass characteristic is obtained by proper dimensioning and tuning of the driving wires 15 and 15' and coupling wire 16 and by proper proportioning of the load due to transverse wire 17.

The manner in which the several wires must be tuned and proportioned to provide a single broad-band transmission characteristic will be understood from the following analysis.

It will be observed that the transducing means, namely stretched wires 15 and 15', whereby conversion of the energy from electrical to mechanical vibrations is effected, are flexible elements

the different points of which partake of different motions. The reaction in the electrical system due to the motion, and the resulting mechanical force at the middle point where coupling to wire 16 is effected will therefore have somewhat complex characters. For this reason it is desirable to examine first the theory of the stretched wire transducer.

The differential equations for the transverse motion of the driving wire differ from those of an ordinary stretched spring or wire for the reason that each elemental length of the wire besides introducing a mass acceleration reaction is subject to a mechanical force due to the interaction of the current flowing in it with the magnetic field. This mechanical force is the same for each element of the wire.

Let

l = length of the wire in centimeters,
 ρ = linear density of the wire in grams per centimeter,
 τ = tension in the wire in dynes,
 β = magnetic flux density in the air-gap in c. g. s. units, and
 I = current in the wire in c. g. s. units.

An elemental portion of the wire of length dx at a distance x from the mid-point will move under the action of two forces, one a force βIdx due to the current and the other a mechanical force p equal to the difference of the transverse components of the tension τ at the two ends of the element, that is, to the decrement $-dp$ of the transverse mechanical force. These forces are opposed by the mass-acceleration reaction of the element, giving rise to the relationship

$$\beta Idx - dp = \rho dx \frac{d^2y}{dt^2} \quad (1)$$

where y denotes the transverse displacement. Assuming, the motion to be sinusoidal and of pulsance ω , Equation 1 may be rewritten as

$$\frac{dp}{dx} = \beta I - j\omega\rho y \quad (2)$$

where y denotes the transverse velocity.

Due to the tension τ in the wire each elemental length has a transverse stiffness equal to

$$\frac{\tau}{dx}$$

The change in the lateral displacement from one end of the element to the other due to the transverse force p is therefore given by

$$dy = -p \frac{dx}{\tau}$$

and the change of the transverse velocity by

$$d\dot{y} = -\frac{dx}{\tau} \frac{dp}{dt} \quad (3)$$

Assuming p to vary sinusoidally with pulsance ω , this equation becomes

$$\frac{d\dot{y}}{dx} = -j\frac{\omega}{\tau} p \quad (3)$$

From Equations 2 and 3 is obtained

$$\frac{d^2\dot{y}}{dx^2} + \frac{\omega^2\rho}{\tau}\dot{y} + j\frac{\omega\beta I}{\tau} = 0 \quad (4)$$

which expresses the motion of the wire. In deriving Equations 3 and 4, it is assumed that the wire has a flexural stiffness, due to its dimensions and material, which is negligibly small in com-

parison with the flexural stiffness due to the tension. I have found that this assumption is justified in practice and that the effects of the flexural stiffness of the wire itself are negligible except at frequencies well removed from the operating frequencies of the filters of the invention.

Equation 4 may be solved for each half of the wire to give the total mechanical reaction at the mid-point due to that half. Since each half will contribute the same reaction as the other half the reaction of the whole wire is simply twice that of either half. Measuring x from the mid-point of the wire the velocity $\dot{y}$ at the point x is given by

$$\dot{y} = \dot{y}_0 \frac{\sin \frac{\pi}{2} \frac{\omega}{\omega_0} \left(1 - \frac{2x}{l}\right)}{\sin \frac{\pi}{2} \frac{\omega}{\omega_0}} - 2\beta l \frac{\sin \frac{\pi}{4} \frac{2x}{l} \frac{\omega}{\omega_0} \sin \frac{\pi}{4} \frac{\omega}{\omega_0} \left(1 - \frac{2x}{l}\right)}{\cos \frac{\pi}{4} \frac{\omega}{\omega_0}} \quad (5)$$

where $\dot{y}_0$ is the velocity at the mid-point and

$$\omega_0 = \frac{\pi}{l} \sqrt{\frac{T}{\rho}} = 2\pi f_0 \quad (6)$$

f_0 being the fundamental resonance frequency of the whole wire.

The mechanical reaction at the mid-point due to the half wire is obtained from Equation 5 by means of Equation 3. Denoting this reaction by p_0 , Equation 3 gives

$$p_0 = j \frac{\tau}{\omega} \left[\frac{d\dot{y}}{dx} \right]_{x=0} \quad (7)$$

Performing the differentiation indicated and substituting for ω_0 , where it appears in the resulting coefficients, the value given in Equation 6, the mid-point reaction is found to be

$$p_0 = -j\sqrt{\rho\tau} \left[\dot{y}_0 \cot \frac{\pi\omega}{2\omega_0} + \frac{\beta l}{j\omega\rho} \tan \frac{\pi\omega}{4\omega_0} \right] \quad (8)$$

Assuming that a driving force F is applied to the wire at its mid-point and that a mechanical load of impedance Z is attached at the driving point, the equation for the resultant motion of the mid-point is

$$F = \dot{y}_0 Z + 2p_0 \quad (9)$$

or

$$F = -l \frac{2\beta l \omega_0}{\pi \omega} \tan \frac{\pi \omega}{4 \omega_0} + \dot{y}_0 \left[Z + j\sqrt{\rho\tau} \left(\tan \frac{\pi \omega}{4 \omega_0} - \cot \frac{\pi \omega}{4 \omega_0} \right) \right] \quad (10)$$

On the electrical side the electromotive force, at the wire terminals, is given by

$$E = RI + 2\beta \int_0^l \dot{y} dx \quad (11)$$

which, when the integration is performed becomes

$$E = \left[R + \frac{\beta^2 l}{j\omega\rho} \left(1 - \frac{4\omega_0}{\pi\omega} \tan \frac{\pi\omega}{4\omega_0} \right) \right] \dot{y}_0 + \frac{2\beta l \omega_0}{\pi\omega} \tan \frac{\pi\omega}{4\omega_0} \dot{y}_0 \quad (12)$$

where R is the electrical resistance of the wire. Equations 10 and 12 have the form

$$F = A\dot{y}_0 - GI$$

and

$$E = G\dot{y}_0 + BI$$

where A and B represent mechanical and elec-

trical impedances, respectively, and G is the force factor of the transducer. The force factor has the value

$$G = \frac{\beta l \tan \theta}{2} \quad (13)$$

where

$$\theta = \frac{\pi\omega}{4\omega_0} = \frac{\omega l}{4} \sqrt{\frac{\rho}{\tau}} \quad (14)$$

Since the phase constant of the whole wire is equal to

$$\omega l \sqrt{\rho/\tau}$$

the angle θ represents the phase constant of one-quarter of the wire. The electrical impedance B has the value

$$B = R + \frac{\beta^2 l}{j\omega\rho} \left(1 - \frac{\tan \theta}{\theta} \right)$$

which, since

$$\tan \theta = \frac{\sin 2\theta}{2(1 + \tan^2 \theta)} \quad (15)$$

may be written as

$$B = R + \frac{1}{j\omega} \cdot \frac{\beta^2 l^2}{4\rho l} \left(1 - \frac{\sin 2\theta}{2\theta} \right) + j \frac{\beta^2 l}{\omega\rho} \frac{\sin 2\theta}{2\theta} \tan^2 \theta \quad (15)$$

or

$$B = R + Z_{e1} + Z_{e2} \quad (16)$$

The mechanical impedance A consists of three parts, first the load impedance Z , second, a component

$$j\sqrt{\rho\tau} \tan \theta \quad (17)$$

which is the impedance of a short-circuited uniform line of characteristic impedance

$$K = \sqrt{\rho\tau} \quad (18)$$

and of length corresponding to a phase angle θ . The third component represents the impedance of a similar line open-circuited at its remote end.

The complete system of the transducer is shown schematically in Fig. 7. The mechanical portion, which is shown in accordance with equivalent electrical conventions, includes the load impedance Z and impedances $jK \tan \theta$ and $-jK \cot \theta$. These are represented as sections of uniform lines of impedance K and phase constant θ with their remote terminals respectively short-circuited and open-circuited.

A more convenient equivalent system is shown schematically in Fig. 9. This equivalent is arrived at from that of Fig. 7 by a transformation of the portion between the dotted lines xx' and yy' . The first step in the transformation is indicated by the schematic of Fig. 8. The force factor G is replaced by the equivalent combination of a new force factor G' of value

$$G' = \frac{\beta l \sin \theta}{2} \quad (19)$$

and an ideal transformer having a transformation ratio $\cos \theta:1$. The electrical impedance Z_{e2} is replaced by an equivalent shunt impedance on the mechanical side of value

$$\frac{G^2}{Z_{e2}} = j \sin 2\theta \quad (20)$$

The combination of the ideal transformer with this shunt impedance and the series impedance $jK \tan \theta$ is equivalent to a section of uniform line

of characteristic impedance K and phase angle θ included in the circuit as indicated at 40 in Fig. 9.

The electrical impedance Z_{e1} is of the character of a capacity, the value of which is proportional to the mass of the driving wire but variable with frequency in accordance with the inverse of the factor $(1 - \sin 2\theta/2\theta)$. The magnitude of the impedance is small and may be neglected in most instances or, if desired, may be compensated by means of an inductance of appropriate value.

The line elements appearing in Fig. 9 have each a phase constant θ and hence correspond in length to one-quarter of the driving wire.

The complete mechanical portion of the filter may be considered to include all the elements to the right of the vertical line ZZ' in Fig. 9, the load impedance Z being that due to the other wires of the system including wire 15' of the second transducer. At its right-hand end the filter terminates in an electric circuit similar to that at the left of Fig. 9, to which it is coupled by the force factor G' .

In the filters of this invention the coupling wires and the loading wires have characteristic impedance which are different from each other and from that of the transducer wires. The requisite values of the characteristic impedances are obtained by proper proportioning of the linear densities of the wires and by proper adjustment of the tensions.

A schematic of the complete mechanical portion of the filter is shown in Fig. 10 in which the several elements are represented as uniform transmission lines in accordance with electrical conventions. The system comprises a series of uniform line sections 40, 42, 44 and 46, connected in tandem, and intermediate series impedances 41, 43, and 45, consisting of uniform line sections open circuited at their remote ends. All of the line elements have the same phase constant θ . That is, all have the same wave-length at any given frequency although their actual physical lengths may differ depending upon the linear densities of the individual wires and the tensions therein.

The end sections 40 and 46, which enter the system from the transducer wires, have characteristic impedances K . Intermediate sections 42 and 44, which correspond to the portions of the coupling wire designated b and c respectively in Fig. 2, have characteristic impedances m_1K , m_1 being a numerical factor greater than unity. Line element 43, which corresponds to the two halves of wire 17 has a characteristic impedance $2m_2K$, each half of the wire contributing an impedance m_2K . Line element 41 represents the combination of the impedance $-jK \cot \theta$ contributed by transducer wire 15, designated 39 in Fig. 7 with the impedance contributed by the end section a of the coupling wire. The total characteristic impedance of this combination is $(m_1 + 1)K$. Line element 45 represents a similar combination corresponding to the impedances contributed by transducer wire 15' and end section d of the coupling wire.

The filter may be regarded as being composed of a series of sections of two different types having the schematic arrangements shown in Figs. 11 and 12. The section shown in Fig. 11 consists of a line element of characteristic impedance K and phase constant θ forming a transmission path and an open-circuit line element of impedance m_1K and phase constant θ . Two of these sections reversed with respect to each other form the end

portions of the filter, the line elements $K\theta$ being contributed by the transducer wires and the elements $m_1K\theta$ being contributed by the end portions of the coupling wires. The section shown in Fig. 12 has three elements, first a series impedance constituted by an open-circuit line of impedance K , second a transmission path constituted by a line of impedance m_1K and third a series impedance constituted by an open-circuit line of impedance m_2K , all three line elements having phase constant θ . Two of these sections reversed with respect to each other form the central portion of the filter. The line elements $K\theta$ in these sections are contributed by the transducer wires and correspond to element 39 of Fig. 7. The other elements are contributed by the coupling wire 16 and loading wire 19 as already described.

Although the filter sections are not symmetrical in structure, the image impedances at their opposite terminals have similar frequency characteristics, differing only in magnitude and bearing a fixed ratio. They correspond, therefore, in their properties to symmetrical sections in combination with ideal transformers having transformation ratios different from unity. By properly proportioning the elements the image impedances of the different sections may be matched so that the sections may be coupled in cascade without giving rise to reflection effects at the junctions.

The relationships of the impedances of the several line elements necessary for the matching of the image impedances are determined as follows:

Consider first the section shown in Fig. 11. The image impedance at the left-hand terminals is most readily computed from the product of the open-circuit and short-circuit impedances at these terminals. The open-circuit impedance Z_o is simply that of the line $K\theta$ open circuited at its remote end, and is given by

$$Z_o = -jk \cot \theta$$

The closed circuit impedance Z_c is the impedance of the line $K\theta$ terminated at its remote end by an impedance constituted by the open-circuit line $m_1K\theta$. The expression for Z_c is obtained readily from a formula given in Johnson's textbook on Transmission Circuits for Telephonic Communication, first edition, page 137, Equation 46. Its value is found to be

$$Z_c = jK \frac{\tan \theta - m_1 \cot \theta}{1 + m_1}$$

The image impedance, denoted by W_1 , is equal to the square root of the product of Z_o and Z_c and has the value

$$W_1 = K \sqrt{\frac{1 - m_1 \cot^2 \theta}{1 + m_1}} \quad (17)$$

Similarly the image impedance, W_1' , at the right-hand terminals is found to be

$$W_1' = K \sqrt{(1 + m_1)(1 - m_1 \cot^2 \theta)} \quad (18)$$

The frequency factor $(1 - m_1 \cot^2 \theta)$ is the same in both expressions indicating similar frequency variations. The magnitudes of the impedances are in the fixed ratio

$$\frac{W_1'}{W_1} = 1 + m_1 \quad (19)$$

In a like manner the image impedances W_2 and W_2' at the left and right terminals respectively

of the section shown in Fig. 12 may be computed. Their values are found to be

$$W_2 = K \sqrt{\frac{m_1^2(1+m_1)}{m_1+m_2}} \sqrt{1 - \frac{m_1+m_2+m_1m_2}{m_1^2} \cot^2 \theta} \quad (20)$$

and

$$W_2' = K \sqrt{\frac{m_1^2(m_1+m_2)}{m_1+1}} \sqrt{1 - \frac{m_1+m_2+m_1m_2}{m_1^2} \cot^2 \theta} \quad (21)$$

In this case also the frequency factors are the same and the magnitudes have a fixed ratio given by

$$\frac{W_2'}{W_2} = \frac{m_1+m_2}{m_1+1} \quad (22)$$

In order that the two types of section may be joined together, as in Fig. 10, without reflection it is necessary that the m 's be so chosen as to make W_2 and W_1' equal. To make the frequency factors equal requires that

$$m_1 = \frac{m_1+m_2+m_1m_2}{m_1^2} \quad (23)$$

and to make the magnitudes equal requires that

$$1+m_1 = \frac{m_1^2(1+m_1)}{m_1+m_2} \quad (24)$$

Both relationships are satisfied when

$$m_2 = m_1(m_1-1) \quad (25)$$

Equation 25 gives the relationship between the characteristic impedance of loading wire 17 and that of coupling wire 16. In the limiting case where the coupling wire has the same characteristic impedance as the transducer wires, that is $m_1=1$, m_2 becomes zero and no loading wire is required.

Referring to Equations 17 and 18 it will be seen that the image impedance of the filter has a real value when $m_1 \cot^2 \theta$ is less than unity. Since the image impedance is resistive in the transmission bands the band limits are therefore given by

$$\tan^2 \theta = m_1 \quad (26)$$

The filter has an indefinite number of transmission bands all of uniform width and centered about the frequencies for which $\cot \theta$ is zero or θ is an odd multiple of $\pi/2$. From Equation 14 giving the value of θ it follows that the mid-frequencies of the successive bands are $2f_0$, $6f_0$, $10f_0$,—and so on. The lowest frequency band is the only one of interest, the mid-band frequency in this case being twice the fundamental resonance frequency of the driving wires.

Equation 26 indicates that the width of the band is dependent on the value of m_1 and decreases as m_1 increases. That is, the band is made narrower by increasing the characteristic impedance of the coupling wire. The widest band is obtained when m_1 is equal to unity, the band limits in this case being f_0 and $3f_0$. Under this condition the loading wire 17 is absent.

While the mechanical portion of the system has an indefinite number of bands only the lowest frequency band appears in the over-all electromechanical system. The elimination of the higher frequency bands is due to the frequency characteristic of the electromechanical transducer. Referring to Fig. 9, it will be seen that the mechanical portion is coupled to the electrical cir-

cuits by force factors G' , the value of which is given by Equation 15 and is variable with frequency. At the mid-frequency of the first band the phase constant θ has the value $\pi/2$, the force factor having the value G_1' given by

$$G_1' = \frac{\beta l}{\pi} \quad (27)$$

and at the mid-frequencies of the successively higher bands being respectively $1/3$, $1/5$, $1/7$, etc. as great. Since the efficiency of transmission is proportional to the square of the force factor, the loss in the higher frequency bands is very large.

The impedance Z_{el} , which is added to the electrical circuit by the transducer has the value, given by Equation 15,

$$Z_{el} = \frac{1}{j\omega} \frac{\beta^2 l^2}{4} \frac{4}{\rho l} \left(1 - \frac{\sin 2\theta}{\theta}\right) \quad (28)$$

which at the mid-frequency of the lowest transmission band becomes

$$Z_{el} = \frac{1}{j\omega} \frac{\beta^2 l^2}{m} \quad (29)$$

where m is the mass of the transducer wire. This is equivalent to the impedance of a capacity equal to $m/\beta^2 l^2$. The effect of this impedance may be substantially neutralized throughout the band by the addition of a series inductance proportioned to resonate with the capacity at the mid-band frequency. The value of the neutralizing inductance is found from Equation 6 and 21 to be

$$L = \frac{\beta^2 l^2}{4\pi^2} \frac{1}{\pi} \quad (30)$$

Fig. 2 shows inductances L inserted in the input and output electrical circuits in series with the driving wires.

In the construction of the filter it is preferable to use aluminum alloy such as duralumin for the driving wires 15 and 15'. Such materials have high tensile strength and, in addition to having low density, have relatively high electrical conductivity. Because of the low density, the length of the wire for a given resonance frequency and tension will be greater than for other materials and hence will permit greater values of the force factor. The other wires may be of the same material, but in many cases steel piano wire may be preferred because of its greater tensile strength. In the case of the loading wires, the greater density of steel permits the relatively high characteristic impedances to be obtained without resorting to excessively large mechanical tensions.

If all of the wires are of the same material and diameter and are subject to the same mechanical tension, wires 15, 15' and 16 will be of the same length and wire 17 half as long. The four sections a , b , c , and d of wire 16 will all be equal in length. These relationships are shown in Fig. 2. The fundamental resonance frequencies of the wires will be inversely proportional to their lengths. Loading wire 17 will resonate at the mid-frequency of the first transmission band or twice the resonance frequency f_0 of the driving wires. Coupling wire 16 will resonate at the same frequency as the driving wire.

When different materials are used for the several wires the required relationships of the phase constants and characteristic impedances can be maintained by appropriate adjustments of the lengths and diameters of the wires and of the mechanical tensions. The relationships of the fundamental resonance frequencies will remain

unchanged. The lengths and diameters of the wires should be so chosen that the required resonances are obtained with mechanical tensions suitable for the material used.

- 5 The filter described in the foregoing includes only a single loading wire and has a total of four sections. The filter may be extended by the insertion of additional sections of the type shown in Fig. 12, but since these are of unsymmetrical structure it is necessary to insert them in pairs, the two sections of each pair being reversed with respect to each other to form a symmetrical double section. A filter with one additional pair of sections is shown schematically in Fig. 13.
- 10 This comprises transducer wires 15 and 15' as in Fig. 2, a coupling wire 16' corresponding to that of Fig. 2, but lengthened by two sections of phase angle θ , two loading wires 17 and 17' corresponding to loading wire 17 of Fig. 2, and a third
- 20 loading wire 18 of the same characteristic impedance K as the transducer wires but only half as long.

What is claimed is:

- 25 1. An electromechanical wave filter comprising a longitudinal stretched wire, a plurality of transverse stretched wires coupled at their mid-points to said longitudinal wire at points along the length thereof dividing said longitudinal wire into equal portions, magnetic means providing magnetic
- 30 fields perpendicular to the two transverse wires nearest the ends of said longitudinal wire, and separate electrical circuits connected to the ends of said two end wires respectively, said transverse wires having characteristic impedances different

from that of said longitudinal wire, and the lengths of the several said wires and the relative values of their characteristic impedances being proportioned to provide a transmission band between two preassigned frequencies.

- 5 2. A wave filter in accordance with claim 1 in which the transverse wires are three in number, the transverse wires adjacent the ends of the coupling wire having characteristic impedances of value K , the central transverse wire having a characteristic impedance $m(m-1)K$, and the longitudinal wire having a characteristic impedance, mK , m being a numerical factor greater than unity.

- 15 3. A wave filter in accordance with claim 1 in which the transverse wires are three in number, the transverse wires adjacent the ends of the coupling wire having characteristic impedances of value K and an over-all phase constant 4θ , the central transverse wire having a characteristic impedance $m(m-1)K$ and a phase constant 2θ , and the longitudinal wire having a characteristic impedance mK and a phase constant 4θ , m being a numerical factor greater than unity.

- 25 4. A wave filter in accordance with claim 1 in which the longitudinal wire has a characteristic impedance mK and the transverse wires have alternately characteristic impedances K and $m(m-1)K$, m being a numerical factor greater than unity, the end wires having characteristic impedances of value K .

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