

March 24, 1936.

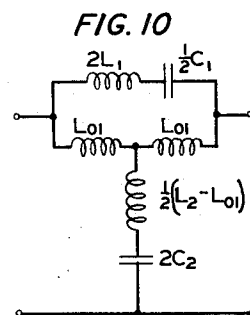
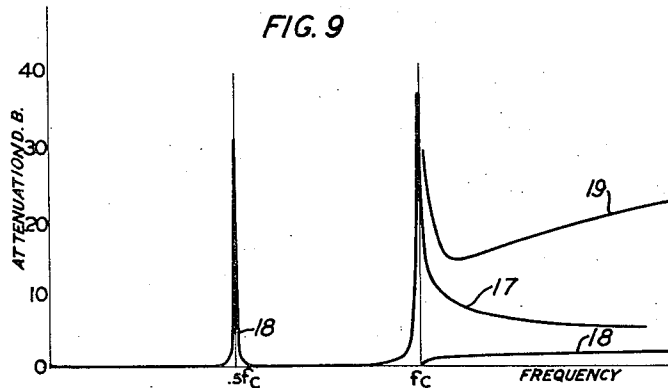
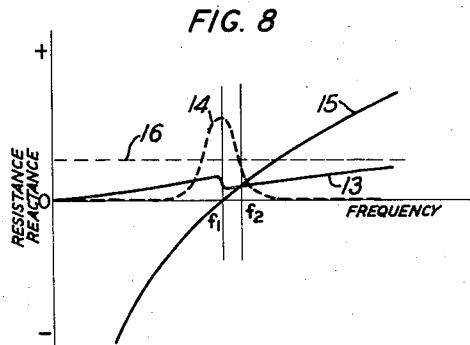
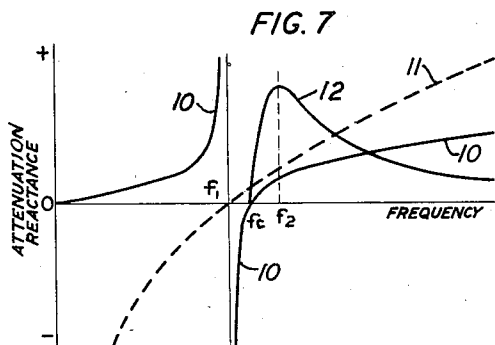
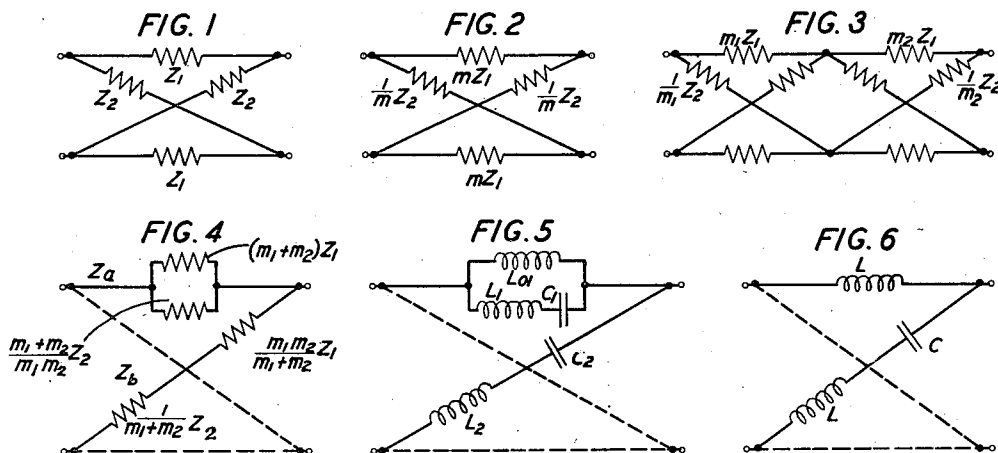
H. W. BODE

2,035,258

WAVE FILTER

Filed March 24, 1933

2 Sheets-Sheet 1



INVENTOR
H. W. BODE

BY *G. H. Stevenson*
ATTORNEY

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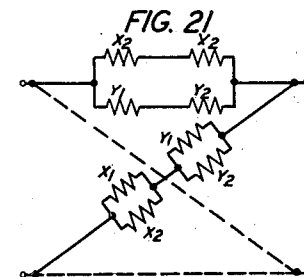
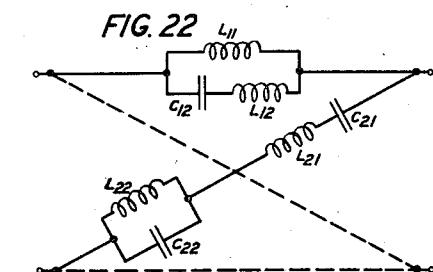
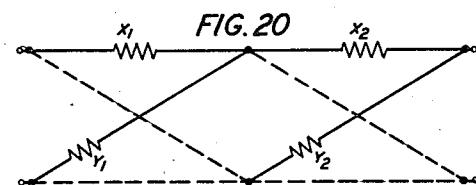
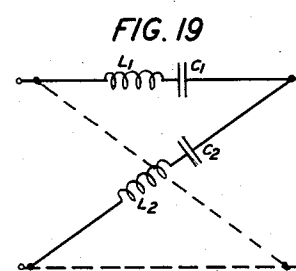
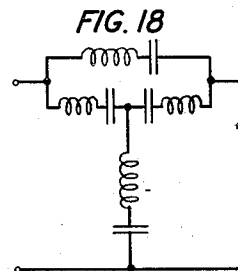
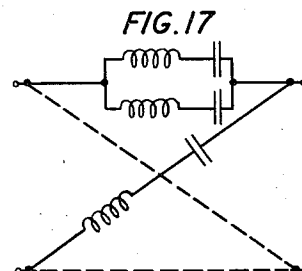
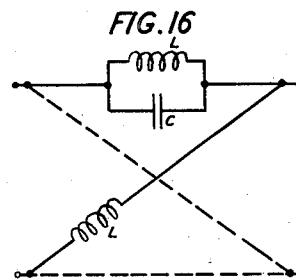
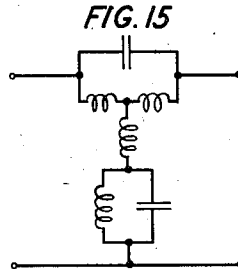
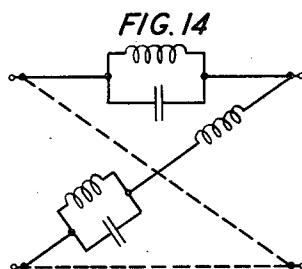
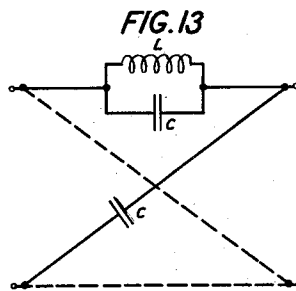
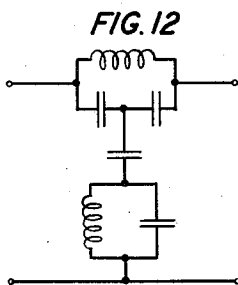
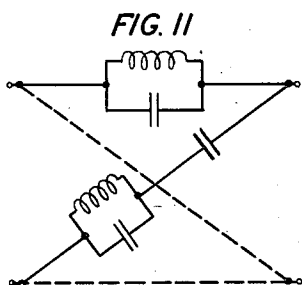
H. W. BODE

2,035,258

WAVE FILTER

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2 Sheets-Sheet 2



INVENTOR
H. W. BODE
BY *G. H. Stevenson*
ATTORNEY

UNITED STATES PATENT OFFICE

2,035,258

WAVE FILTER

Hendrik W. Bode, New York, N. Y., assignor to
Bell Telephone Laboratories, Incorporated, New
York, N. Y., a corporation of New York

Application March 24, 1933, Serial No. 662,412

9 Claims. (Cl. 178-44)

This invention relates to broad-band wave filters and has for its principal objects to improve the selectivity and to extend the range of transmission characteristics of such filters.

In accordance with the invention, broad-band wave filters are provided having peaks of infinite attenuation at frequencies close to or at the cut-off frequencies of the transmission band, or within the band itself. By an alternative form of the invention attenuation characteristics having a finite maximum of attenuation at any desired frequency outside the band are provided, this maximum being independent of the presence or absence of dissipation in the branches of the network.

The nature of the invention will be more clearly understood from the following detailed description and by reference to the appended drawings of which:

Figures 1, 2, 3 and 4 are schematics illustrative of the principles of the invention;

Fig. 5 shows one embodiment of the invention;

Fig. 6 is the prototype network corresponding to Fig. 5;

Figs. 7, 8, and 9 illustrate characteristics of the networks of Fig. 5;

Fig. 10 is a modified form of the network of Fig. 5;

Figs. 11 to 19 inclusive, show additional forms of the networks of the invention together with their prototypes;

Figs. 20 and 21 are explanatory of a theorem used in connection with further modifications of the invention, and

Fig. 22 illustrates a further modification based on this theorem.

One method of introducing attenuation peaks into the transmission characteristic of a wave filter is described in U. S. Patent 1,538,964, issued May 26, 1925 to O. J. Zobel, which shows the so-called m -type filter and explains the manner of deriving it from elementary ladder type filters having no attenuation peaks. While it is possible to design filters of this type in which the peaks are located as close as may be desired to the cut-off frequencies, it has been found in practice that filters constructed in accordance with such designs do not exhibit the expected properties. The reason for this is that the normal dissipation in the filter elements, which in any case prevents the attenuation from rising to an infinite value at the peaks, becomes increasingly effective as the peaks are moved closer to the cut-off frequencies, tending to limit the attenuation to smaller and smaller values,

The filters of the present invention are in a sense related to those of the Zobel patent inasmuch that they may be arrived at from simpler prototype networks by the process of m -derivation, but are different in that the m -factors instead of being real numbers are complex quantities. In this way, it has been found possible not only to overcome the difficulty described above but to produce filters with additional novel characteristics.

When the m -derivation is applied to a symmetrical lattice network such as that of Fig. 1 which has line branches of impedance Z_1 and lattice branches of impedance Z_2 , the result is simply to multiply the line branch impedances by the factor m and to divide the lattice branches by the same factor. This is illustrated by the network of Fig. 2. The image impedance K_1 and the transfer constant θ_1 of the network of Fig. 1 are given by

$$K_1 = \sqrt{Z_1 Z_2} \quad (1)$$

and

$$\tanh \frac{\theta_1}{2} = \sqrt{\frac{Z_1}{Z_2}}$$

and the corresponding quantities K_2 and θ_2 for Fig. 2 are given by

$$K_2 = \sqrt{m Z_1 \frac{Z_2}{m}} = \sqrt{Z_1 Z_2} = K_1 \quad (2)$$

and

$$\tanh \frac{\theta_2}{2} = m \sqrt{\frac{Z_1}{Z_2}} = m \tanh \frac{\theta_1}{2}$$

The process of m -derivation has not changed the image impedance but has modified the transfer constant by the introduction of the factor m as a multiplier of the hyperbolic tangent of the half transfer constant.

When the factor m is positive and less than unity, the network of Fig. 2 can be transformed into a symmetrical T with physically realizable impedances, corresponding to the ladder networks of the Zobel patent. When m is greater than unity, this transformation can not be accomplished, except in a few special cases, although the m -derived lattice is evidently physically realizable.

If two m -derived lattices having the same image impedances but different values of m , namely m_1 and m_2 are connected in tandem as illustrated by Fig. 3, it may be shown that the combination is equivalent to the network of Fig. 4 which is also a symmetrical lattice having line branch impedances Z_a made up of impedances $(m_1 + m_2) Z_1$ and $Z_2(m_1 + m_2) / m_1 m_2$ in parallel, and lattice

branches Z_b composed of $Z_2/(m_1+m_2)$ and $Z_1 m_1 m_2/(m_1+m_2)$ in series. Only one line branch and one lattice branch are shown in this and subsequent figures, the other branches having identical impedances. The proof of the foregoing follows from a consideration of the transfer constants of the two networks. Let θ denote the transfer constant of the network of Fig. 4 and θ_1 and θ_2 the transfer constants of the component lattices of Fig. 3, then

$$\tanh \frac{\theta}{2} = \tanh \frac{\theta_1 + \theta_2}{2} = \frac{\tanh \frac{\theta_1}{2} + \tanh \frac{\theta_2}{2}}{1 + \tanh \frac{\theta_1}{2} \tanh \frac{\theta_2}{2}} \quad (3)$$

If now, the transfer constant of the $Z_1 Z_2$ prototype of the component lattices of Fig. 3 be designated by θ_0 , it follows that

$$\tanh \frac{\theta_1}{2} = m_1 \tanh \frac{\theta_0}{2}$$

and

$$\tanh \frac{\theta_2}{2} = m_2 \tanh \frac{\theta_0}{2}$$

and, hence, that

$$\tanh \frac{\theta}{2} = \frac{(m_1 + m_2) \tanh \frac{\theta_0}{2}}{1 + m_1 m_2 \tanh^2 \frac{\theta_0}{2}} \quad (4)$$

It is readily seen by comparing the products of the line and lattice impedances in each case, that each lattice of Fig. 3 and that of Fig. 4 has the same image impedance, namely,

$$K = \sqrt{Z_1 Z_2}$$

multiplying both sides of Equation 4 by K gives

$$K \tanh \frac{\theta}{2} = \frac{(m_1 + m_2) K \tanh \frac{\theta_0}{2}}{1 + m_1 m_2 K \tanh \frac{\theta_0}{2} \frac{1}{K} \tanh \frac{\theta_0}{2}} \quad (5)$$

From the general relationship between the image impedance and the transfer constant the quantity

$$K \tanh \frac{\theta}{2}$$

is identified as the line branch impedance of Fig. 3 and, since likewise,

$$K \tanh \frac{\theta_0}{2} = Z_1$$

and

$$\frac{1}{K} \tanh \frac{\theta_0}{2} = \frac{1}{Z_2}$$

Equation 5 becomes

$$Z_a = \frac{(m_1 + m_2) Z_1 Z_2}{Z_2 + m_1 m_2 Z_1} = \frac{(m_1 + m_2) Z_1 \left(\frac{m_1 + m_2}{m_1 m_2} \right) Z_2}{(m_1 + m_2) Z_1 + \frac{m_1 + m_2}{m_1 m_2} Z_2} \quad (6)$$

Equation 6 is readily identified as the parallel combination of the two impedances constituting the line branches in Fig. 4. The value of Z_b may be developed in the same way, using the relationship

$$\frac{1}{Z_b} = \frac{1}{K} \tanh \frac{\theta}{2}$$

In the general network of Fig. 4 the ratio of each parallel connected impedance in the line branches to the corresponding series impedance component in the lattice branches has the value

$$\frac{(m_1 + m_2)^2}{m_1 m_2}$$

which for real values of m has a minimum pos-

sible value of 4. So long therefore, as this ratio is greater than 4, the network may be broken down into two simpler lattices having real elements, but when the ratio is less than 4, this is no longer possible. In this case the m 's of the component lattices must be conjugate complex quantities in order that the above impedance ratio may be real. That is, the network may be regarded as the composition of two m -type lattices having conjugate complex m 's.

If m_1 and m_2 are conjugate complex quantities defined by

$$\begin{aligned} m_1 &= a + jb \\ m_2 &= a - jb \end{aligned} \quad (7)$$

the impedance multiplying factors in the network of Fig. 4 have the values

$$m_1 + m_2 = 2a \quad (8)$$

and

$$\frac{m_1 + m_2}{m_1 m_2} = \frac{2a}{a^2 + b^2}$$

which are positive real quantities so long as a is positive. The impedance ratio referred to above has the value

$$\frac{(m_1 + m_2)^2}{m_1 m_2} = \frac{4a^2}{a^2 + b^2}$$

which obviously, must be less than 4.

With this relationship in mind specific networks of the invention having the general schematic form of Fig. 4 are readily arrived at.

One example of the networks of the invention is shown in Fig. 5, the case illustrated being that of a low-pass filter. The simple filter structure from which this network may be derived in the above indicated manner is the symmetrical lattice shown in Fig. 6 which has line branches consisting of inductances L and lattice branches consisting of simple resonant combinations of inductance L in series with capacities C .

In the network of Fig. 5 each line branch consists of a parallel combination of an inductance L_{01} with a resonant circuit $L_1 C_1$ and each lattice branch consists of a simple series combination of an inductance L_2 and capacity C_2 . The values of the elements when the m -factors have the complex values of Equation 7 are as follows:

$$\left. \begin{aligned} L_{01} &= 2aL \\ L_1 &= \frac{2a}{a^2 + b^2} L \\ L_2 &= \frac{1 + a^2 + b^2}{2a} L \\ C_1 &= \frac{a^2 + b^2}{2a} C \\ C_2 &= 2aC \end{aligned} \right\} \quad (9)$$

Representative impedance and attenuation characteristics for this network are shown in Fig. 7 for the case in which the coefficients a and b have the values 0.36 and 0.217 respectively. Curve 10 shows the variation with frequency of the line branch reactance and curve 11 the variation of the lattice branch reactance. The band cut-off occurs at the frequency f_c at which the combination $L_1 C_1$ is resonant. The lattice branch $L_2 C_2$ resonates at a lower frequency f_1 at which the line branch is anti-resonant.

It will be noticed that in the range above the cut-off frequencies the two reactances become most nearly equal at a frequency f_2 not far removed from the cut-off. At this frequency the lattice, regarded as a bridge, is most nearly balanced and the current transmitted through it is therefore a minimum. This is illustrated by

curve 12 of the figure which shows, to an arbitrary scale, the frequency variation of the attenuation constant of the network. Instead of rising to a sharp peak, as in the m -derived filters of Zobel Patent 1,538,964, the attenuation increases to a finite maximum determined by the ratio of the reactances.

The location of the frequency of maximum attenuation with respect to the cut-off depends on the value of the quantity $a^2 + b^2$. If this quantity is very small relatively to unity the maximum will occur very close to the cut-off frequency and will move away from the cut-off frequency as the value is increased towards unity. For values greater than unity the attenuation maximum is absent from the characteristic. The height of the attenuation maximum may be increased as much as desired by making the coefficient b small with respect to a .

The characteristics discussed above are those which would be obtained with dissipationless impedances. If the effect of dissipation is taken into account, it is possible, by proportioning the complex m 's in accordance with the dissipation to obtain additional characteristics of unique types. This will be illustrated by the consideration of how the network of Fig. 5 may be designed to have an infinite attenuation peak at its cut-off frequency.

If the coils in the lattice of Fig. 6 have the same dissipation factor, defined as the ratio of reactance to resistance, then the line and the lattice impedances, denoted by Z_1 and Z_2 respectively, may be expressed as

$$Z_1 = j\omega L \left(1 + \frac{1}{jQ} \right) \quad (10)$$

and

$$Z_2 = j\omega L \left[\left(1 - \frac{\omega_c^2}{\omega^2} \right) + \frac{1}{jQ} \right]$$

where Q is the dissipation factor, ω denotes 2 π times frequency, and ω_c corresponds to the cut-off frequency.

The transfer constant θ is given by

$$\tanh^2 \frac{\theta}{2} = \frac{Z_1}{Z_2} = \frac{1 + Q^2 \left(1 - \frac{\omega_c^2}{\omega^2} \right) + jQ \frac{\omega_c^2}{\omega^2}}{1 + Q^2 \left(1 - \frac{\omega_c^2}{\omega^2} \right)^2} \quad (11)$$

which, at the cut-off frequency, becomes

$$\tanh^2 \frac{\theta}{2} \Big|_{\omega=\omega_c} = 1 + jQ \quad (12)$$

$$\omega = \omega_c$$

If a network m -derived from Fig. 6 could be constructed with its m factor of the value

$$m = \frac{1}{\sqrt{1 + jQ}} \quad (13)$$

The transfer constant, θ_m , of that network would have the value defined by

$$\tanh \frac{\theta_m}{2} = \frac{1}{\sqrt{1 + jQ}} \tanh \frac{\theta}{2} \quad (14)$$

which would be unity at the cut-off frequency, corresponding to infinite attenuation at that point.

While such a network can not be constructed, the network equivalent to its combination with the conjugate m -derived network is physically possible as shown above, and, since the attenuation of the combination network is the sum of those of the components, the infinite peak at the cut-off will appear in the combined characteristic. Moreover, I have found that the attenua-

tions of the two components are of opposite sign at most frequencies, that is, the conjugate component introduces a gain, so that when they are combined, the resultant attenuation is large only in the neighborhood of the peak.

The requisite values of the m factors and the impedance coefficients of the combination network are readily computed for the special case under consideration. The square root of the complex quantity $1 + jQ$ is given by

$$\sqrt{1 + jQ} = \sqrt{\frac{1}{2}\sqrt{1 + Q^2} + \frac{1}{2}} + j\sqrt{\frac{1}{2}\sqrt{1 + Q^2} - \frac{1}{2}}$$

from which the values of the conjugate m 's may be determined as

$$m_1 = \frac{\sqrt{\frac{1}{2}\sqrt{1 + Q^2} + \frac{1}{2}} - j\sqrt{\frac{1}{2}\sqrt{1 + Q^2} - \frac{1}{2}}}{\sqrt{1 + Q^2}} \quad (15)$$

and

$$m_2 = \frac{\sqrt{\frac{1}{2}\sqrt{1 + Q^2} + \frac{1}{2}} + j\sqrt{\frac{1}{2}\sqrt{1 + Q^2} - \frac{1}{2}}}{\sqrt{1 + Q^2}}$$

The impedance coefficients have the following values

$$m_1 + m_2 = 2a = \frac{2\sqrt{\frac{1}{2}\sqrt{1 + Q^2} + \frac{1}{2}}}{\sqrt{1 + Q^2}} \quad (16)$$

and

$$\frac{m_1 + m_2}{m_1 m_2} = \frac{2a}{a^2 + b^2} = 2\sqrt{\frac{1}{2}\sqrt{1 + Q^2} + \frac{1}{2}}$$

the value of Q being, of course, that corresponding to the cut-off frequency.

If the value of Q be assumed equal to 50, which is representative of air core inductances of commercial type, the various factors have the following numerical values:

$$\begin{aligned} m_1 &= .101 - j.099 \\ m_2 &= .101 + j.099 \\ 2a &= .202 \\ \frac{2a}{a^2 + b^2} &= 10.1 \end{aligned} \quad (17)$$

The substitution of these numerical values in Equations 9 gives the values of the elements of Fig. 5 in terms of the L and C of the prototype for the case under consideration. These are:

$$\begin{aligned} L_{01} &= .202L \\ L_1 &= 10.1L \\ L_2 &= 5.05L \\ C_1 &= .099C \\ C_2 &= .202C \end{aligned} \quad (18)$$

From a physical standpoint, the manner in which the peak at the cut-off is produced may best be appreciated by a consideration of the frequency variation of the line and lattice impedances. This is illustrated by the curves of Fig. 8 in which curve 13 represents the variation of the line branch reactance, curve 14 the line branch effective resistance and curve 15 and straight line 16 represent respectively the reactance and resistance of the lattice branch. For the sake of clarity, the resistance curves are drawn to a greater ordinate scale than the reactance curves.

It is to be observed that anti-resonance and resonance effects in the line branch, which in the absence of dissipation would make curve 13 similar to curve 10 of Fig. 9, appear only as a slight irregularity at the frequency f_1 . This is due to the fact that the inductance L_{01} is relatively small and the shunting impedance $L_1 C_1$ is a resonant circuit of great stiffness, the combined reactance

being substantially that of inductance L_{01} . A second result of the proportions of this combination is that the effective resistance at the anti-resonance frequency is quite small, its value being approximately equal to the resistance of the shunting circuit L_1C_1 . At the cut-off frequency the resistance falls to half this value and is equal to the resistance of the lattice branch. For the case under consideration, the balance of the two impedances at the cut-off may be described broadly as follows. The reactance of the lattice arm is balanced by the low reactance of the shunting inductance L_{01} in the line branch while the resistance of the lattice arm is balanced substantially by the resistance of the resonant circuit L_1C_1 in the line branch, this resistance, by virtue of the parallel combination in the line branch appearing only in the neighborhood of the desired frequency.

The attenuation characteristic of the computed network is illustrated by curve 17 of Fig. 9 which shows the attenuation in decibels plotted as a function of frequency, the curve being approximately to scale. The cut-off is extremely sharp and above the suppression peak there is a reasonably high attenuation which can be supplemented if desired by additional filter sectors of the ordinary type.

It is evident from Equation 11 that the quantity

$$\tanh \frac{\theta}{2}$$

is complex at all frequencies and, hence that, by suitably choosing the complex m factors, the suppression peak may be located anywhere in the frequency range including the transmission band. The procedure in computing the m factors will be the same as outlined above, the value of Q in each case being that corresponding to the frequency for which suppression is desired. The form of the resulting network, of course, remains unchanged.

The attenuation characteristic of a network like that of Fig. 5, but having a suppression peak in the middle of the band is illustrated by curves 18 of Fig. 9. For this case the value of Q was again assumed to be 50, giving for the values of the m factor

$$m_1 = .022 + j1.733$$

and

$$m_2 = .022 - j1.733$$

It will be noticed that the suppression peak is extremely sharp and falls very quickly to zero at frequencies quite close to the peak. The attenuation above the cut-off is also very small.

Numerical values of the actual inductances and capacities for the two cases illustrated are given in the table below, assuming a cut-off frequency of 3000 cycles per second and a characteristic impedance at zero frequency of 600 ohms.

Element	Value	
	Peak at cut-off	Peak at mid band
L_{01}	6.36	1.54, millihenries
L_1	322.00	.51, millihenries
L_2	161.00	2630, millihenries
C_1	.0087,	5.45, microfarads
C_2	.0178,	.00425, microfarads
Q	50	50

The general form of the networks of the invention, as illustrated by Fig. 4, is that of a symmetrical lattice and, therefore, best adapted for use

in circuits having both sides balanced to ground. However, in many cases, and particularly in the suppression networks of the invention, unbalanced equivalent networks in the form of symmetrical bridged-T's can be obtained by well-known transformation of the symmetrical lattice and these unbalanced equivalents can be used in circuits having one side grounded. Usually the possibility of this transformation can be ascertained by inspection of the developed lattice network, the requirement being that the lattice arm should contain an impedance in series corresponding to one of the parallel connected impedances in the line arm but of greater magnitude. In the suppression networks of the invention, the values of the m factors are such that this is practically always the case.

The bridged-T equivalent of the network of Fig. 5 is shown in Fig. 10, the element values being designated in terms of those of Fig. 5.

The prototype lattice, Fig. 6, from which the networks of Figs. 5 and 10 are derived is equivalent to a simple T-network having series inductances L and a shunt capacity $2C$, that is, to a mid-series terminated low-pass filter section of the so-called constant- k type. It is obvious that other forms of the networks of the invention may be arrived at using other simple networks as prototypes. Two additional forms are illustrated by the lattice of Fig. 11 and the bridged-T of Fig. 12, both of which are derived in the manner described above from the prototype lattice of Fig. 13 which corresponds to a ladder type mid-shunt terminated low-pass filter section, having a series inductance $2L$ and shunt capacities C at each end.

Further examples of the networks of the invention are shown in Figs. 14 and 15 which are derived from the mid-shunt high-pass filter of Fig. 16 and by Figs. 17 and 18 which are band-pass structures derived from the elementary band-pass filter of Fig. 19. The inductance and capacity values of the complex networks may be determined directly from the designated coefficients in the prototypes by the application of the relationships indicated by Figs. 3 and 4, and Equations 8. In each case the complex network has the same image impedance as the prototype, but has a transfer constant modified in accordance with the values of the complex m 's. The procedure in the design of these networks to produce peaks of infinite attenuation at any desired frequencies is the same as in the case of the example already discussed. The steps comprise, first, developing an expression for the hyperbolic tangent of the half transfer constant for the dissipative case, and, second, determining the particular complex value of this quantity at the chosen frequency, the inverse of which gives the first of the required conjugate complex m -factors.

It will be noticed from the numerical values given for the low-pass filter case that the impedance elements may have greatly disparate values. For example, in the case where the peak is made to occur at half the cut-off frequency, the largest inductance in the network is about five thousand times as great as the smallest. While this does not render the design impracticable, it is frequently desirable for manufacturing purposes to have the element values more nearly of the same order of value, as this permits the same type of coil construction to be used throughout. This is accomplished in a modified form of the invention in which the network is

constructed as the equivalent of the combination of the prototype network together with the two complex- m derivatives thereof. The principle of the combination is illustrated by Figs. 20 and 21 of which Fig. 21 shows the single symmetrical lattice which is the equivalent of the tandem connection of the two lattices of Fig. 20 when these have the same image impedances.

The two lattices of Fig. 20 have line impedances X_1 and X_2 respectively and lattice impedances Y_1 and Y_2 respectively, these being related so that $X_1 Y_1 = X_2 Y_2$ to make the image impedances equal. The equivalent lattice of Fig. 21 has for its line branches the parallel combination of $(X_1 + X_2)$ with $(Y_1 + Y_2)$ and for its lattice branches the series combination of these impedances in parallel. Using this relationship, it is possible to construct a single lattice equivalent to the combination of the general network of Fig. 4 with its prototype Fig. 1, or, as a specific example, the combination of the complex network of Fig. 5 with its prototype Fig. 6.

The branches of the network obtained by the direct application of this equivalence relationship to Figs. 5 and 6 will be of a rather complex character but by the application of the equivalent transformations described by O. J. Zobel in an article on "Theory and design of uniform and composite electric wave filters," Bell System Technical Journal Vol. II, No. 1, January, 1923, pages 45 and 46, the network may be reduced to the relatively simple form shown in Fig. 22. In terms of the a and b coefficients of the complex m -factors as given by Equations 7, the elements of the network of Fig. 22 have the following values:

$$\left. \begin{aligned} L_{11} &= (1 + 2a)L \\ L_{12} &= \left(\frac{1 + 2a}{2a} \right) L \\ L_{21} &= L \\ L_{22} &= \frac{2a(a^2 + b^2)}{(1 + 2a)^2} L \\ C_{12} &= \frac{2a[(1 + a)^2 + b^2]}{(1 + 2a)^2} \\ C_{21} &= (1 + 2a)C \\ C_{22} &= \frac{(1 + 2a)[(1 + a)^2 + b^2]}{2a(a^2 + b^2)} \end{aligned} \right\} \quad (19)$$

In this network the disparity of the element values is greatly reduced, for example, in the case of the low-pass filter having a peak at the mid-band frequency the ratio of the largest to the smallest inductance is reduced from about 5000 to about 164. The impedance transformations referred to may, evidently, be applied in various ways giving other forms of the final network and in certain cases one or others of these possible modifications may give better uniformity of the element values. The best form in any particular case may usually be arrived at by trial.

The transfer constant of the combination network of Fig. 22 will, of course, be the sum of the transfer constants of the component sections and the inclusion of the prototype section will have the effect of increasing the attenuation at frequencies beyond the cut-off. For the case in which the suppression peak is located at the cut-off the attenuation above the cut-off is shown by curve 19 of Fig. 9.

What is claimed is:

1. A symmetrical four-terminal transmission network comprising a path between one input terminal and one output terminal, a plurality

of reactance elements included in said path constituting an impedance characterized by at least one resonance and one anti-resonance at different frequencies, a second path between the same input terminal and the other output terminal, a plurality of reactance elements included in said second path constituting an impedance having a resonance at substantially the same frequency as the frequency of the said anti-resonance of the impedance in the first mentioned path, the reactances of the elements of said impedances being proportioned with respect to each other to provide a transmission band between preassigned frequencies and being further proportioned with respect to the dissipation factors of the elements whereby the effective resistance and the reactance of the first path are simultaneously balanced by the effective resistance and the reactance, respectively, of the said second path at a frequency close to the frequency of the said anti-resonance of the first path and whereby a peak of substantially infinite attenuation is provided at the frequency of balance.

2. A network in accordance with claim 1 in which the said two paths constitute respectively a line branch and a lattice branch of a symmetrical lattice network.

3. A network in accordance with claim 1 in which the said path including parallel connected impedances constitutes a line path between an input and an output terminal and in which the other path includes a portion connected from the mid-point of one of the parallel connected impedances to the other input and output terminals in common.

4. A network in accordance with claim 1 in which the impedances are proportioned to provide substantially infinite attenuation at one of the cut off frequencies of the transmission band determined by the reactances of the elements constituting the network branches.

5. A network in accordance with claim 1 in which the impedances are proportioned to provide substantially infinite attenuation at a frequency within the transmission band determined by the reactances of the elements constituting the network branches.

6. A wave transmission network comprising two pairs of impedance branches connected to form a symmetrical lattice between a pair of input terminals and a pair of output terminals, the line branches of said lattice each comprising two parallel connected reactive impedances of different frequency characteristics and the lattice branches each comprising two series connected impedances similar respectively to the impedances of the line branches, the reactances of said impedances being proportioned with respect to each other to provide a transmission band between preassigned frequencies, and the parallel connected reactances of the line branches being related in magnitude to the respectively similar series connected reactances of the lattice branches by a numerical factor less than four.

7. A symmetrical four-terminal transmission network comprising a path between one input terminal and one output terminal including an anti-resonant circuit of low impedance at frequencies other than that of anti-resonance, a second path between the same input terminal and the other output terminal including a series resonant circuit of high impedance at frequencies other than that of resonance, the resonance frequency of said second circuit being the same as the anti-resonance frequency of said first

circuit, and the reactances and the effective resistances due to dissipation in the reactance elements being proportioned to balance each other at a frequency substantially equal to the said anti-resonance frequency whereby the transmission of currents of this frequency is suppressed.

8. A symmetrical four-terminal transmission network comprising a path between one input terminal and one output terminal including two reactive impedances of different frequency characteristics connected in parallel, the parallel combination of said impedances having an anti-resonance at a preassigned frequency, and a second path between the same input terminal and the other output terminal including a reactive impedance having a resonance at the said anti-resonance frequency of the parallel connected impedances and being proportioned with respect to said parallel combination to provide a broad transmission band, said parallel connected impedances being also proportioned so that their joint reactance and their effective resistance due to dissipation in the reactance elements balance the reactance and effective resistance of the said second path at a frequency sub-

stantially equal to the said anti-resonance frequency, whereby the transmission of currents of this frequency is suppressed.

9. A lattice type broad band wave filter comprising two equal line branches of impedance Z_a and two equal lattice branches of impedance Z_b and having the same characteristic impedance as that of a prototype lattice filter having line branches of impedance Z_1 and lattice branches of impedance Z_2 , the impedances Z_1 and Z_2 being constituted by reactance elements proportioned to provide a transmission band between preassigned frequencies, the impedances Z_a being equal in value to the impedance of the parallel connection of $(m_1 + m_2)Z_1$ and

$$\frac{m_1 + m_2}{m_1 m_2} Z_2$$

and the impedances Z_b being equal in value to the series connection of impedances

$$\frac{m_1 m_2}{m_1 + m_2} Z_1 \text{ and } \frac{1}{m_1 + m_2} Z_2$$

where m_1 and m_2 are conjugate complex numbers.

HENDRIK W. BODE.