May 21, 1935. H. W. BODE 2,002,216

WAVE FILTER

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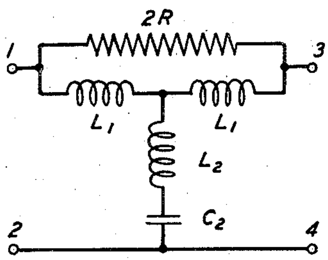
FIG. 1



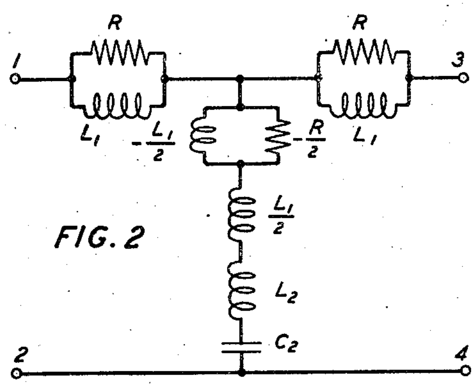


FIG. 2

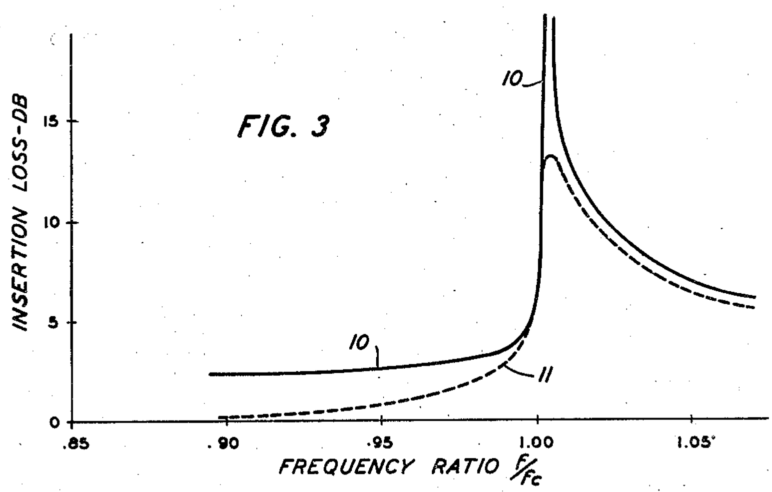
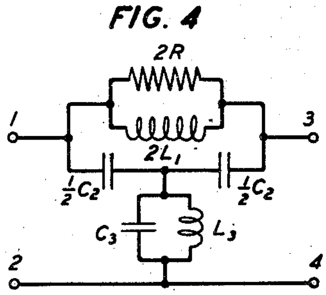


FIG. 3

FIG. 4



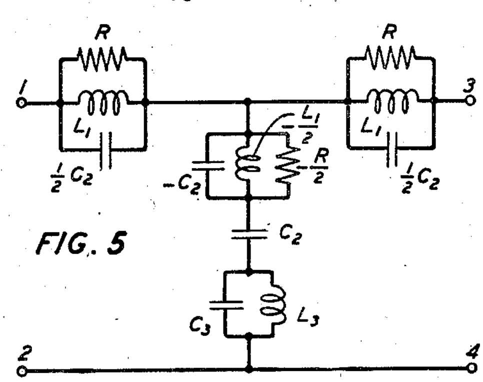


FIG. 5

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FIG. 6

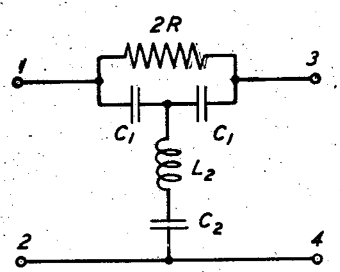


FIG. 7

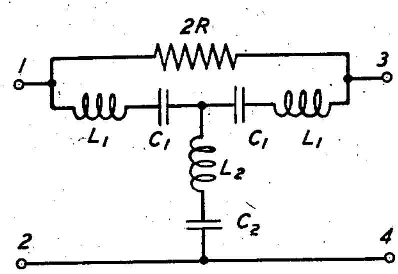


FIG. 8

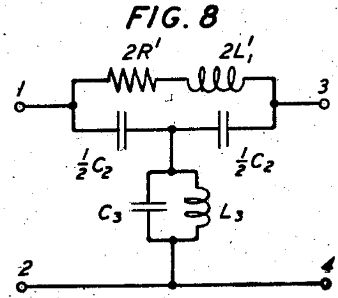


FIG. 9

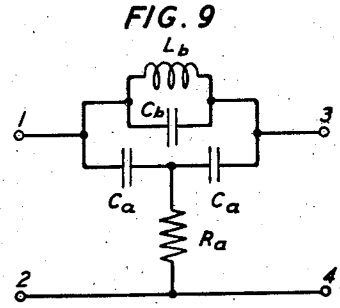
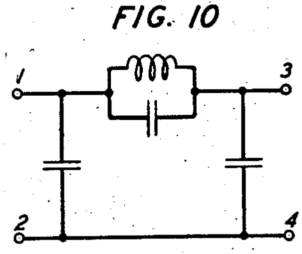


FIG. 10



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Patented May 21, 1935 2,002,216

UNITED STATES PATENT OFFICE

2,002,216

WAVE FILTER

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Application June 7, 1933, Serial No. 674,619

7 Claims. (Cl. 178—44)

This invention relates to broad-band wave filters and more particularly to filters having attenuation peaks close to the transmission band limits for the purpose of sharpening the cut-off.

The principal object of the invention is to increase the sharpness of selectivity of filters of the above mentioned type by providing a very high or substantially infinite attenuation at the attenuation peaks.

In broad-band filters of the ladder, or series-shunt type, the introduction of peaks into the attenuation characteristic is usually effected by the inclusion in the filter of one or more of the so-called M-derived sections described in U. S. Patent 1,538,964 issued May 26, 1925, to O. J. Zobel. With no dissipation in the filters elements, the attenuation at these peaks would be infinite but in practical filters the natural dissipation limits the attenuation to a finite value. As the peaks are brought closer to the transmission band limits for the purpose of sharpening the cut-off, the effect of the natural dissipation in diminishing the attenuation becomes more and more marked so that it becomes impracticable to provide peaks closer to the cut-off than about five per cent of the cut-off frequency.

In accordance with the present invention, the dissipation in a shunt resonant branch of a filter is compensated by means of a resistance bridged across adjacent series branches, thus permitting the shunt branch to introduce an infinite attenuation at its resonance frequency. The network thus formed retains the general characteristics of the original filter and can be connected in tandem with other sections without the introduction of noticeable reflection effects. Approximate neutralization of the shunt resistance extends over a range of frequencies on both sides of the peak and when the peak is located close to a cut-off frequency, the neutralization extends into the transmission band with a resultant diminution of the attenuation close to the cut-off.

By means of the invention, it has been found possible to produce infinite attenuation peaks at frequencies closer to the cut-off than one per cent of the cut-off frequency and at the same time to improve the uniformity of the transmission in the band up to points very close to the band limits.

The nature of the invention will be more fully understood from the following detailed description and by reference to the attached drawings, of which:

Fig. 1 shows in schematic form one embodiment of the invention;

Fig. 2 is a schematic explanatory of the network of Fig. 1;

Fig. 3 illustrates the characteristics of the network of Fig. 1;

Fig. 4 shows a modified form of the invention;

Fig. 5 is a schematic explanatory of the network of Fig. 4;

Figs. 6 and 7 illustrate additional networks embodying the invention;

Fig. 8 shows a modified form of the network of Fig. 4, and

Figs. 9 and 10 show respectively a further form of the invention and its prototype filter.

The network shown schematically in Fig. 1 represents a low-pass filter section in accordance with the invention. It comprises a symmetrical T- network made up of two equal series inductances designated L_1 and a shunt branch consisting of an inductance L_2 in series with a capacity C_2 together with a relatively high resistance 2R bridging the two series inductances. The T-network is a mid-series terminated filter section of the so-called M-derived type described in the above mentioned U. S. patent to Zobel, the shunt branch being proportioned to resonate at approximately the frequency of the attenuation peak.

At the resonance frequency the impedance of the shunt branch is very low and would be zero except for the effect of dissipation, represented mainly by the resistance of the inductance L_2. In many cases, the resistance of this coil may be quite low, giving a very high peak of attenuation, but when it is attempted to bring the peak very close to the cut-off of the filter the inductance L_2 must be made very large with the result that its resistance is high and the shunting effect of the branch at resonance is diminished.

Design formulæ for most of the important types of M-derived filter sections are given in an article by A. J. Zobel on the Theory and Design of Uniform and Composite Electric Wave Filters, Bell System Technical Journal, vol. II, No. 1, January 1923. For a low-pass filter of the type corresponding to the T-network of Fig. 1 having series inductances L_0^1 and shunt inductance and capacity L_{02} and C_{02} respectively, the element values are given by

$$L_{01}=m\frac{K_0}{2\pi f_c},$$
$$L_{02}=\frac{1-m^2}{2m}\frac{K_0}{2\pi f_c},$$
$$C_{02}=\frac{m}{\pi f_c K_0}, \qquad (1)$$
$$m=\sqrt{1-\frac{f_c^2}{f_\infty^2}},$$

where f_c denotes the filter cut-off frequency, f_∞ the frequency of the attenuation peak, and K_0 the zero frequency value of the characteristic impedance.

The ratio of the inductances is given by

$$\frac{L_{02}}{L_{01}}=\frac{1-m^2}{2m^2}=\frac{f_c^2}{2(f_\infty^2-f_c^2)}, \qquad (2)$$

which shows that the shunt inductance becomes very large relatively to the series inductance as the peak frequency is brought very close to the cut-off.

The elements of the network of Fig. 1 will have values approximately the same as those given by Equations (1), but, due to the presence of the bridging resistance, there will be some modification which must be taken into account in order that the cut-off frequency and the peak frequency may be accurately placed.

The determination of the proper value of the bridging resistance and the necessary modification of the element values will be readily understood from a consideration of the network shown in Fig. 2 which is the T equivalent of the network of Fig. 1. The shunt branch of this network includes, in addition to the original elements L_2 and C_2, an inductance $L_{1/2}$ and a negative impedance consisting of an inductance $-L_{1/2}$ and a resistance $-R/2$. The series branches each consist of an inductance L shunted by a resistance R. The network of Fig. 2 is not intended to represent a physically realizable equivalent of the network of the invention shown in Fig. 1, the negative elements being, in general impossible to construct except as rough approximations. It is, however, a mathematical equivalent which serves to illustrate in a simple and graphic manner the relationships which should obtain amongst the elements of the network of Fig. 1. The equivalence of the two networks may be shown by deriving the lattice equivalent network for each by means of the bisection theorem of A. C. Bartlett, described in an article entitled "An Extension of a Property of Artificial Lines", Phil. Mag. (London), vol. 4, No. 24, Nov. 1927.

It is evident that the negative impedance appearing in the shunt branch may be proportioned to have an effective resistance which just neutralizes the total positive resistance of the branch and if this neutralization is made to occur at the frequency for which the total reactance is also zero, the attenuation of the network will be infinite at this frequency.

When the attenuation peak is located very close to the cut-off the series inductance will in general be so small that its resistance may be neglected. On this assumption, the impedance, Z_1 of the parallel combination of L_1 and R is given by

$$Z_1=R\frac{\omega^2L_1^2}{R^2+\omega^2L_1^2}+j\omega L_1\frac{R^2}{R^2+\omega^2L_1^2}, \qquad (3)$$

where ω denotes 2π times frequency.

The dissipation in the shunt branch is substantially wholly due to the resistance of the inductance coil L_2 which will be designated r_2. It follows from Equation (3) and from the composition of the shunt branch that the neutralization of this resistance requires

$$\frac{R}{2}\frac{\omega_\infty^2L_1^2}{R^2+\omega_\infty^2L_1^2}=r_2, \qquad (4)$$

where ω_∞ is the value of ω corresponding to the peak frequency. The requirement that the reactance shall also be zero at this frequency gives

$$\omega_\infty\left[L_2+\frac{L_1}{2}-\frac{L_1}{2}\frac{R^2}{R^2+\omega^2_\infty L_1^2}\right]-\frac{1}{\omega_\infty C_2}=0. \qquad (5)$$

In addition the effective inductances of the series and shunt branches should have respectively the values L_{o1} and L_{o2} as given by Equations (1) at the peak frequency, and the shunt capacity should have the value C_{o2}. This will ensure the proper location of the cut-off frequency with respect to the peak, particularly when the two are close together, and also that the filter will have substantially the same general transmission characteristics as the M-derived section. These requirements, expressed mathematically, are

$$L_1\frac{R^2}{R^2+\omega_\infty^2L_1^2}=L_{o1} \qquad (6)$$

and

$$L_2+\frac{L_1}{2}\frac{\omega_\infty^2L_1^2}{R^2+\omega_\infty^2L_1^2}=L_{o2} \qquad (7)$$

From Equations (4) to (7), inclusive, the required values of L_1, L_2, and R are found to be the following:

$$L_1=L_{o1}\left[1+\frac{4r_2^2}{\omega_\infty^2L_{o1}^2}\right],$$

$$L_2=L_{o2}-L_1\frac{r_2}{R}, \qquad (8)$$

and

$$2R=\frac{\omega_\infty^2L_{o1}^2}{r_2}+4r_2.$$

The following example will illustrate the application of the design formulæ given above. The design data are as follows: Cut-off frequency 2500 cycles per second; characteristic impedance at zero frequency 800 ohms; attenuation peak to be located at a frequency one half of one per cent above the cut-off frequency. Inductance coils having a ratio of reactance to resistance of 200 will be assumed.

From equations, the values of L_{01}, C_{02}, and L_{02} are found to be

L_{01}=.0051 henry
L_{02}=.252 henry
C_{02}=$.0159\times10^{-6}$ farads

The resistance of the inductance L_{02} is 19.8 ohms at the frequency of infinite attenuation. Since the final value of inductance L_2 is very close to L_{02}, the value of resistance r_2 may be taken at 19.8 ohms. The values of 2R, L_1 and L_2 are then found by means of Equations 8 to be

2R=405 ohms,
L_1=.00627 henry,
L_2=.251 henry.

The capacity C_2 is the same as C_{02}.

The attenuation of this network is illustrated by curve 10 of Fig. 3 which shows the attenuation loss in decibels plotted as ordinates against the frequency ratio f/f_c as abscissæ. For purpose of comparison dotted curve 11 shows the loss of an ordinary M-derived filter section having the same cut-off and attenuation peak frequencies. Curve 10 shows a greater loss at frequencies below the cut-off, this being due to the presence of the bridging resistance, but a markedly sharper cut-off and a much higher attenuation at the peak are also indicated.

A modified form of the invention having substantially the same transmission characteristics as the network of Fig. 1, but having series and shunt inductances of more nearly the same order of magnitude is shown in Fig. 4, the equivalent T-network being shown in Fig. 5. In this network the shunt capacity of Fig. 1 has been divided into two equal capacities $C_2/2$ and these have been made the series branches of the T-network. The two series inductances of Fig. 1 have been combined and their resultant inductance $2L_1$ placed in parallel with the resistance 2R in the bridging branch. The shunt inductance L_2 has been replaced by a smaller inductance L_3 shunted by a capacity C_3.

This network is a true low-pass filter having only a single attenuating range and a single transmission range if the combination L_3C_3 is proportioned to be anti-resonant at the same frequency as the combination of L_1, and $C_2/2$ in parallel. However, since in the cases primarily of interest, when the attenuation peak and the cut-off are located very close together, the latter anti-resonance is at so high a frequency as to be well out out the useful frequency range, it is not of importance that the two anti-resonances should be made to coincide.

From a comparison of the equivalent, T-networks shown in Figs. 2 and 5 it will be seen that in the shunt branch of the latter the inductance $L_1/2$ is absent and in each of the branches the parallel combinations of L_1 and R are shunted by the capacity C_2. Since the combination of L_1 and $C_2/2$ in parallel is anti-resonant at a very high frequency the capacity may be ignored at the frequencies of interest. Moreover, since the inductance $L_1/2$ in the case of Fig. 2 is a very small part of the total shunt inductance its omission does not noticeably effect the network. Correction factors may, of course, be used in the design formulæ to take account of these modifications.

The combination L_3 and C_3 should be so proportioned that the effective inductance at the peak frequency is approximately the same as the shunt inductance L_2 of Fig. 1. More accurately, it should be proportioned to make the effective inductance of the shunt branch of the equivalent T-network equal to the inductance L_{o2} as given by Equation (1) at the peak frequency. At frequencies below anti-resonance the reactance of a parallel connected inductance and capacity, such as L_3 and C_3, is of an inductive character and is greater than the reactance of the inductance alone. At any given frequency in this range the combination may, therefore, have a relatively large effective inductance although the inductance element itself may be relatively small. The above requirement of making the effective inductance of the combination L_3 C_3 equal to the inductance L_{o2} at the peak frequency may, therefore, be met using a relatively small value for the inductance L_3.

The general principle of neutralizing the dissipation in a shunt resonant circuit by means of a series bridging resistance may be applied to high-pass and band-pass filters as well as to low-pass filters. Fig. 6 shows a high-pass filter in accordance with the invention comprising a T-network having series capacities C_1, a shunt branch consisting of an inductance L_2 and a capacity C_2 in series, together with a bridging resistance 2R. A band-pass filter is shown in Fig. 7 which differs from the filters of Figs. 1 and 6 only in that the series branches of the T consist of resonant combinations L_1 and C_1 instead of simple inductances or capacities.

The T-networks in each of these filters approximate to known types of M-derived filters described in the article by Zobel hereinbefore mentioned. The modification factors for the element values to take account of the bridging resistance are determined by the same general procedure as outlined above.

A second modification of the network of Fig. 1 is illustrated by Fig. 8. The network is the same as that of Fig. 4 except that the parallel inductance and resistance $2L_1$ and 2R in the bridging branch of Fig. 4 are replaced by an inductance and resistance $2L_1'$ and $2R'$ in series. The elements $2L_1'$ and $2R'$ are so proportioned that the impedance of their series combination at the frequency of infinite attenuation is the same as that of the bridging branch of Fig. 4. The transmission characteristic of the network near the cut-off and the peak of infinite attenuation, then, is substantially the same as that of the structure of Fig. 4. In the network of Fig. 8, however, the line current flows through the resistance $2R'$ at low frequencies so that the network gives some transmission loss in this range. This transmission loss is, in fact, nearly the same as that at the cut-off. The transmission loss of the structure of Fig. 8 is thus substantially distortionless throughout the entire transmission band, which is a great advantage in many practical applications.

The process can also be applied to the mid-shunt M-derived type filter sections described in the above mentioned paper by Zobel. For example, Fig. 10 shows a mid-shunt M-derived low-pass section. In the absence of dissipation the anti-resonant network in the series branch has infinite impedance, and gives infinite attenuation, at its frequency of anti-resonance. When dissipation is considered, however, the impedance of the series branch is finite at all frequencies and only a finite peak of attenuation is secured.

In accordance with the invention, the effects of dissipation can be balanced out by a low resistance in the shunt branch of a bridged-T network whose series and bridging impedances are respectively the shunt and series impedances of the original filter section. The configuration is shown by Fig. 9. It can be derived from the structure of Fig. 1 by interchanging the bridging and shunt impedances of that figure and then replacing each impedance of the structure by its inverse network. Thus, for example, C_a of Fig. 9 is the inverse network of L_1 in Fig. 1, the anti-resonant circuit constituted by L_b and C_b in parallel in Fig. 9 is the inverse network of the series resonant circuit L_2, C_2 of Fig. 1, etc. The values of the elements of Fig. 9 can thus be obtained from the formulæ previously derived for the elements of Fig. 1 and these inverse relationships, which can be expressed as

$$\frac{L_2}{C_b}=\frac{L_b}{C_2}=\frac{L_1}{C_a}=2R_aR=K_o^2$$

where K_o is, as before, the characteristic impedance of the network at zero frequency. It will be understood that the same process of interchanging the shunt and bridging impedances of the bridged-T and then replacing each branch by its inverse network can also be applied to all of the other structures herein described.

What is claimed is:

1. A wave filter network comprising three impedance branches disposed in series-shunt relation between a pair of input terminals and a pair of output terminals, said branch impedances consisting of reactance elements proportioned with respect to each other to provide a broad transmission band, and one of said branches including an inductance coil and a capacity adapted by their resonance characteristic to produce an attenuation peak at a frequency close to the edge of said transmission band, and a resistance included in a fourth branch of the network proportioned to compensate the resistance of said inductance coil at the frequency of the attenuation peak whereby the attenuation is made substantially infinite at the peak frequency.

2. A wave filter section comprising a plurality of reactance elements arranged as a T-network and proportioned with respect to each other to provide a transmission band and an attenuation peak at a frequency close to a cut-off frequency, and a resistance bridged across the series branches of said network, said resistance being proportioned to compensate the energy dissipation in the reactance elements of the shunt branch of said network at the frequency of the attenuation peak whereby the attenuation at the peak frequency is made substantially infinite.

3. A wave filter section comprising two similar series impedances and a resonant shunt impedance connected to form a symmetrical T-network between a pair of input terminals and a pair of output terminals, and a resistance bridging said two series impedances, the reactances of said impedances and the resonance frequency of said shunt impedance being proportioned with respect to each other to provide a transmission band and an attenuation peak close to the band, and said bridging resistance being proportioned to compensate the energy dissipation in said shunt impedance at the resonance frequency thereof whereby the attenuation at the peak frequency is made substantially infinite.

4. A wave filter section in accordance with claim 3 in which the shunt impedance is proportioned to resonate at a frequency differing from a cut-off frequency of the filter by less than one per cent.

5. A wave filter section in accordance with claim 3 in which the series branches of the T-network are constituted by equal inductances and the shunt branch by an inductance and a capacity in series, whereby a low-pass transmission band is defined, and in which the shunt branch is proportioned to resonate at a frequency less than one per cent above the cut-off frequency of the band.

6. A wave filter section in accordance with claim 3 in which the series branches of the T-network are constituted by equal capacities and the shunt branch by an inductance and a capacity in series, whereby a high-pass transmission band is defined, and in which the shunt branch is resonant at a frequency less than one per cent below the cut-off frequency of the band.

7. A wave filter section comprising two equal capacities and a shunt impedance constituted by an inductance and a capacity in parallel arranged as a symmetrical T-network between a pair of input terminals and a pair of output terminals, an inductance connected to bridge said equal capacities, said inductance and the elements of said T-network being proportioned to provide a low-pass transmission band and an attenuation peak at a frequency close to the cut-off of said band, and a resistance connected in parallel with said bridging inductance, said resistance being proportioned to compensate the energy dissipation of said shunt impedance at the frequency of the attenuation peak.

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