May 14, 1935.

H. W. BODE

2,001,090

TRANSMISSION NETWORK

Filed Sept. 13, 1933

3 Sheets-Sheet 1

FIG. 1

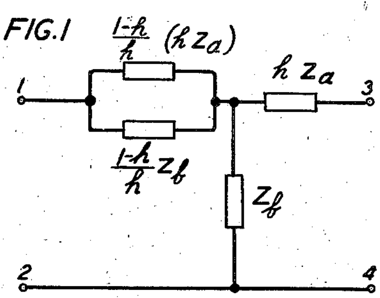


FIG. 2

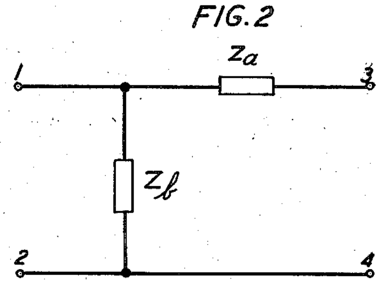


FIG. 3

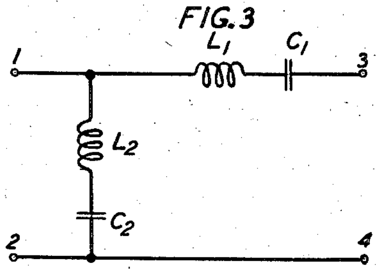


FIG. 4

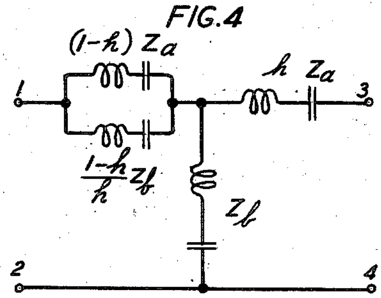


FIG. 5

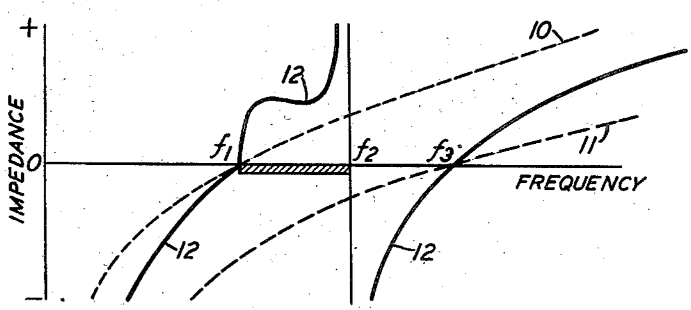
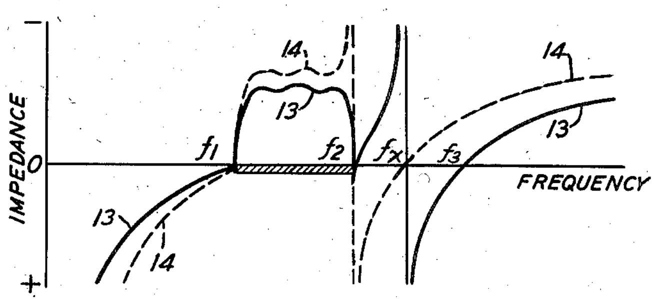


FIG. 6



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TRANSMISSION NETWORK

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FIG. 7

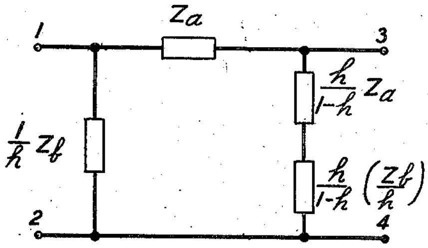

FIG. 8

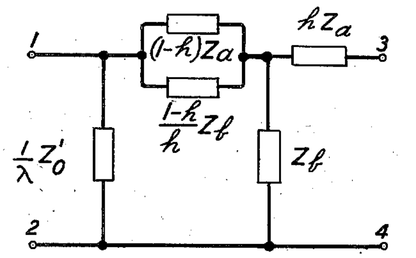

FIG. 9

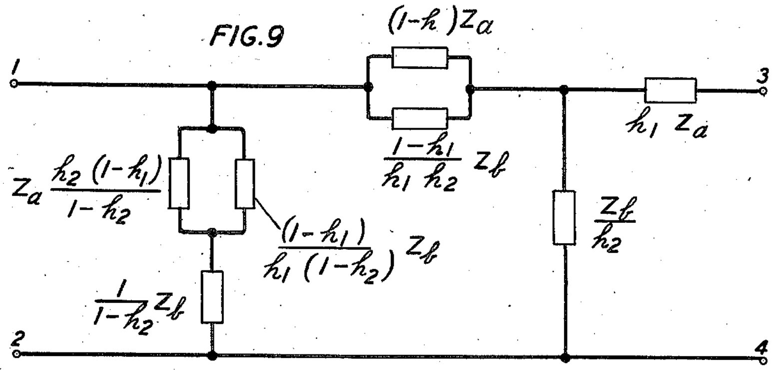

FIG. 10

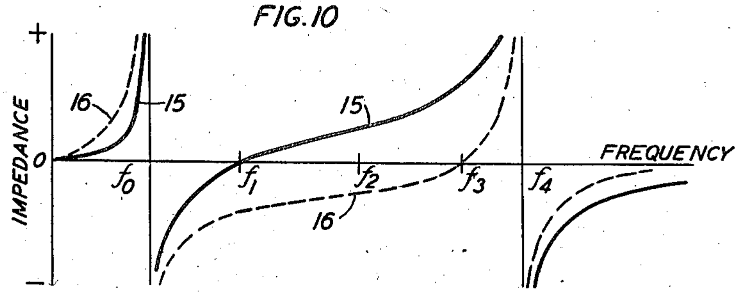

FIG. 11

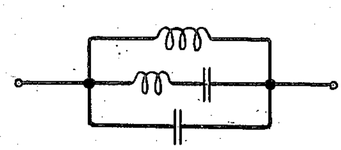

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FIG. 12

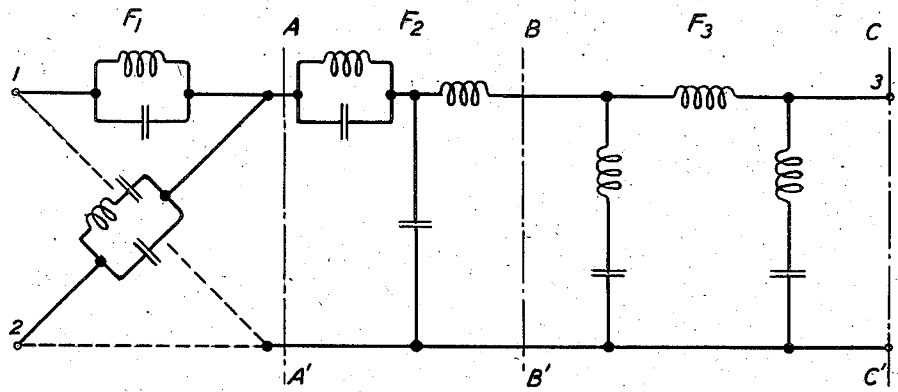


FIG. 13

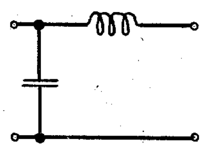


FIG. 14

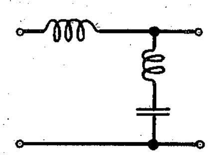


FIG. 15

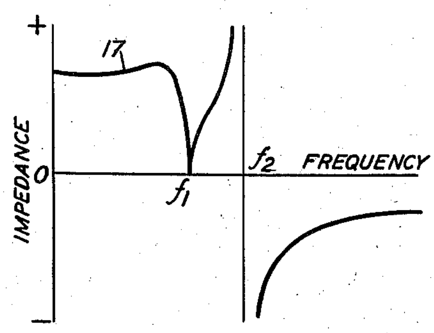
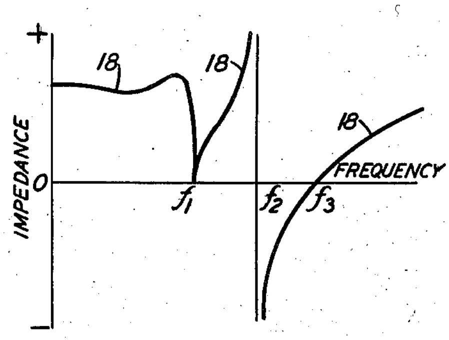


FIG. 16



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Patented May 14, 1935 2,001,090

UNITED STATES PATENT OFFICE

2,001,090

TRANSMISSION NETWORK

Hendrik W. Bode, New York, N. Y., assignor to Bell Telephone Laboratories, Incorporated, New York, N. Y., a corporation of New York

Application September 13, 1933, Serial No. 689,228

8 Claims. (Cl. 178—44)

This invention relates to wave filters and more particularly to wave filters of unsymmetrical structure having different image impedances at their input and output terminals.

It has for its principal object to provide in such filters impedance characteristics of the most general possible type subject only to a reciprocal relationship between the impedances at the two ends. Another object is to provide for the control of the image impedances without affecting the value of the transfer constant. A further object is to provide an independent control of the image impedance in the case of filters of the series-shunt, or ladder, type.

In my earlier Patent No. 1,828,454 issued October 20, 1931, the control of the image impedance and the transfer constant of a symmetrical lattice type wave filter through the allocation of the resonance and anti-resonance frequencies of the branch impedances is described and the mutual independence of the controls of these parameters is pointed out. In that case, the branch resonances which lie within the transmission band control the transfer, or attenuation, constant alone and those lying outside the band control the image impedance alone. The transmission band of the symmetrical lattice is defined by the frequency range in which the line and the lattice branch reactances are of opposite sign and the attenuation range as that in which the reactances are of the same sign. This condition simplifies the impedance and transfer constant controls since it permits as many critical frequencies as may be desired to be introduced into either range independently of the other.

The symmetrical lattice filter provides a very wide range of image impedances and transfer constants but it is not completely general, that is, capable of producing all possible characteristics, for one reason amongst others, namely, that it is restricted to networks having the same image impedances at their input and output terminals. I have found by mathematical analysis that, so far as the transfer constant is concerned, the most general filter characteristic is the sum of the transfer constants of a symmetrical and an unsymmetrical network. The symmetrical portion can be built as a lattice, the desired transfer constant being obtained by proper allocation of the branch resonances within the transmission band, as explained in my above mentioned patent. The transfer constant of the unsymmetrical portion I have found to correspond to the sum of the transfer constants of known, physically realizable, ladder type half sections. No other types of transfer constant are necessary.

The most general filter can, therefore, be constructed by combining in tandem the lattice network representing the symmetrical portion and the several half sections representing the unsymmetrical portion providing the component structures can be so built that the resulting network will have matched image impedances at its internal junction points. This requires that the unsymmetrical half sections shall be capable of being constructed with at least as wide a range of image impedances as the symmetrical lattice.

It has been pointed out that the image impedance control in the case of the symmetrical lattice is simplified by reason of the fact that the reactances of the line and the lattice branches are everywhere of like sign in the attenuation ranges and are everywhere of opposite sign in the transmission band. In the case of the ladder type of filter, the problem of the image impedance control is complicated by the fact that the transmission and attenuation ranges do not have this simple demarcation. The reactances of the series and shunt branches are of opposite sign throughout the transmission band just as in the symmetrical lattice, but in the attenuation ranges the reactances are not everywhere of the same sign and, in certain cases may be of opposite sign at all frequencies. In all cases there exists a range adjacent one or both cut-off frequencies in which the reactances are of the opposite sign and, in the case of the elementary types of filters, for example, a low-pass filter in which the series branches are simple inductances and the shunt branches simple capacities this range may extend over the whole attenuation range.

If it is attempted to control the image impedance of a series-shunt type filter by introducing corresponding resonances or anti-resonances into the branch impedances in the portion of the attenuation range where the reactances are of opposite sign it will be found that this can not be done without changing the ratio of the branch reactances in this range and hence producing a change in the transfer constant of the network. Since the complete control of the image impedance requires that it be capable of assuming zero and infinite values in any frequency range outside the transmission band, the above restriction indicates that the ordinary types of series-shunt networks can provide only a limited range of impedance characteristics, for a given transfer constant.

The present invention provides networks having the same transfer constants as simple series-shunt half sections, but having image impedances which are quite general in that their zeros and poles can be placed anywhere in the attenuation ranges. This generality is obtained by building out the simple prototype with additional series and shunt branches having impedances proportional to the short circuit and open circuit impedances respectively of the network to which the branch is added. In this way the ratio of the short circuit and open circuit impedances, and hence the transfer constant, is maintained unchanged.

The nature of the invention will be more fully understood from the following detailed description and the accompanying drawings of which:

Fig. 1 represents in schematic form a general type of network in accordance with the invention;

Fig. 2 shows a prototype of the network of Fig. 1;

Figs. 3 and 4 illustrate respectively a simple band-pass prototype filter and a derived filter in accordance with the invention;

Figs. 5 and 6 show characteristics of the filters of Figs. 3 and 4;

Fig. 7 is a schematic of an alternative form of the network of Fig. 1;

Figs. 8 and 9 represent in schematic form additional general types of the networks of the invention;

Figs. 10 and 11 are explanatory of supplementary methods of impedance control;

Fig. 12 represents a composite filter in accordance with the invention;

Figs. 13 and 14 show prototypes of portions of the filter of Fig. 12; and

Figs. 15 and 16 illustrate the impedance characteristic of the filter of Fig. 12.

The principles of the invention will be readily understood from a consideration of the generalized networks of Figs. 1 and 2 which represent, respectively, one form of the invention and the prototype half section from which it is derived. The transmission parameters of the network of Fig. 1 and its prototype are most readily obtained from the open circuit and short circuit impedances measured at the input and at the output terminals. For the prototype network the open circuit impedance Z_{1o} and the short circuit impedance Z_{1c} at terminals 1, 2, are respectively

$$Z_{1o}=Z_b,$$

$$Z_{1c}=\frac{Z_aZ_b}{Z_a+Z_b}, \tag{1}$$

and the corresponding impedances Z_{2o} and Z_{2c} at terminals 3, 4, are

$$Z_{2o}=Z_a+Z_b \tag{2}$$

$$Z_{2c}=Z_a$$

The image impedances K_1 and K_2 at terminals 1, 2 and terminals 3, 4 respectively are given by

$$K_1=\sqrt{Z_{1o}Z_{1c}}=\sqrt{\frac{Z_aZ_b}{1+\frac{Z_a}{Z_b}}}=\sqrt{\frac{Z_aZ_b}{1+r}}, \tag{3}$$

and

$$K_2=\sqrt{Z_{2o}Z_{2c}}=\sqrt{Z_aZ_b(1+r)},$$

where r denotes the ratio Z_a/Z_b

The transfer constant Θ of the network is given by

$$\tanh\theta=\sqrt{\frac{Z_{1c}}{Z_{1o}}}=\sqrt{\frac{Z_{2c}}{Z_{2o}}}=\sqrt{\frac{r}{1+r}}. \tag{4}$$

The network of Fig. 1 is obtained from the prototype by changing the series impedance to hZ_a, h being a numerical multiplier, and adding a series branch on the other side of Z_b which is made up of the parallel combination of two impedances related to Z_b and hZ_a respectively by the common numerical factor $(1-h)/h$. That is, the added portion represents the short-circuit impedance of the portion to the right multiplied by the numerical factor $(1-h)/h$. It will be evident that the factor h must be positive and less than unity if the network of Fig. 1 is to be physically realizable. The open and closed circuit impedances Z'_{1o} and Z'_{2o} of this network measured at terminals 1, 2, are respectively by

$$Z'_{1o}=\frac{1-h}{h}\,\frac{hZ_aZ_b}{hZ_a+Z_b}+Z_b, \tag{5}$$

$$=\frac{Z_b(1+r)}{(1+hr)},$$

and

$$Z'_{1c}=\frac{1-h}{h}\cdot\frac{hZ_aZ_b}{hZ_a+Z_b}+\frac{hZ_aZ_b}{hZ_a+Z_b} \tag{6}$$

$$=\frac{Z_a}{1+hr}.$$

At the terminals 3, 4, the corresponding impedances Z'_{2o} and Z'_{2c} are respectively

$$Z'_{2o}=Z_b(1+hr),$$

and

$$Z'_{2c}=Z_a\frac{(1+hr)}{1+r}. \tag{7}$$

From these values the transfer constant, Θ', and the image impedances K'_1 and K'_2 at terminals 1, 2, and terminals 3, 4, respectively, are found to be

$$\tanh\theta'=\sqrt{\frac{r}{1+r}}=\tanh\theta \tag{8}$$

$$K'_1=\sqrt{\frac{Z_aZ_b(1+r)}{1+hr}}=\frac{K_2}{1+hr} \tag{9}$$

and

$$K'_2=(1+hr)\sqrt{\frac{Z_aZ_b}{1+r}}=K_1(1+hr) \tag{10}$$

It will be seen from Equations 8, 9 and 10 that the network has the same transfer constant as that of the prototype but that its image impedances are modified by the introduction of the factor $(1+hr)$, as a multiplier in one case and as a divisor in the other. A change in the image impedances has thus been effected without modification of the transfer or attenuation constant.

The character of the change produced will be illustrated by the application of the transformation to the band-pass prototype filter shown in Fig. 3 of which the series impedance Z_a is made up of inductance L_1 and capacity C_1 in series and the shunt impedance Z_b comprises inductance L_2 and capacity C_2 in series. The filter obtained by the transformation described above is shown in Fig. 4, the added series branch comprising two simple resonant combinations connected in parallel.

The curves of Fig. 5 show the impedance-frequency characteristics of the filter of Fig. 3, dotted curves 10 and 11 representing the reactances of Z_a and Z_b respectively and curve 12 being the image impedance at terminals 1, 2. The transmission band extends from frequency f_1 at which impedance Z_a is resonant to frequency f_2 at which Z_a and Z_b are equal and of opposite sign. At frequency f_3 the shunt impedance is resonant and the image impedance has a zero value. This frequency is that of an attenuation peak produced by the shunt resonance. It will be observed that the two branch reactances have the same sign below f_1 and above f_3, but are of opposite sign in the attenuation range between the upper cut-off and the peak frequencies.

The values of the image impedances K_1 and K_2 are given by

$$K_1=\frac{1}{j\omega C_2}\frac{1}{\sqrt{1+C_1/C_2}}\sqrt{\frac{1-\omega^2/\omega_1^2}{1-\omega^2/\omega_2^2}}\left(1-\frac{\omega^2}{\omega_3^2}\right) \quad (11)$$

and

$$K_2=\frac{1}{j\omega C_1}\sqrt{1+\frac{C_1}{C_2}}\sqrt{(1-\omega^2/\omega_1^2)(1-\omega^2/\omega_2^2)} \quad (12)$$

where ω denotes 2π times frequency, ω_1 and ω_2 corresponds to the cut-off frequencies and ω_3 to the frequency f_3 of the attenuation peak. The upper cut-off frequency has the value given by

$$\omega_2^2=\frac{1}{L_1+L_2}\left(\frac{1}{C_1}+\frac{1}{C_2}\right) \quad (13)$$

corresponding to the condition that impedances Z_a and Z_b are equal and of opposite sign.

The frequency factor $1+hr$ introduced by the transformation has the value

$$1+hr=\frac{C_1+hC_2}{C_1}\frac{1-\omega^2/\omega_x^2}{1-\omega^2/\omega_3^2}, \quad (14)$$

where

$$\omega_x^2=\frac{1}{L_2+hL_1}\left(\frac{1}{C_2}+\frac{h}{C_1}\right), \quad (15)$$

and the image impedances of the transformed network have the values

$$K_1'=\frac{1}{j\omega C_1}\frac{\sqrt{1+C_1/C_2}\sqrt{(1-\omega^2/\omega_1^2)(1-\omega^2/\omega_2^2)(1-\omega^2/\omega_3^2)}}{\left(1+h\frac{C_2}{C_1}\right)(1-\omega^2/\omega_x^2)}, \quad (16)$$

and

$$K_2'=\frac{1}{j\omega C_2}\frac{(1+hC_2/C_1)}{\sqrt{1+C_1/C_2}}\sqrt{\frac{(1-\omega^2/\omega_1^2)}{(1-\omega^2/\omega_2^2)}}(1-\omega^2/\omega_x^2). \quad (17)$$

The frequency variations of the new image impedances are shown by the curves of Fig. 6 of which curve **13** represents the impedance K'_1 and dotted curve **14**, the impedance K'_2. In the transmission band where the impedances are real or resistive the effect of the new frequency factors is to produce an additional undulation on each characteristic thus making greater uniformity possible. The new frequency f_x at which K'_1 has a pole and K'_2 a zero lies in the range between the upper cut-off f_2 and the peak frequency f_3. That it must lie in this range follows from the value given by equation 15 and from the fact already shown that the factor h must be positive and less than unity in order that the transformed network can be physically constructed. If h is the minimum value, zero, the frequency f_x coincides with the peak frequency and if it has the maximum value, unity, its values become equal to that of the upper cut-off frequency as given by equation 13.

Another general form of the networks of the invention is illustrated in Fig. 7. This network is derived from the network of Fig. 2 by dividing the shunt impedance Z_b by the factor h and adding a shunt branch between terminals **3** and **4** consisting of $hZ_a/(1-h)$ and $Z_b/(1-h)$ in series. In this case the added shunt branch is equal to the open circuit impedance of the network of Fig. 2, after modificaton of Z_b, multiplied by the factor $h/(1-h)$. An analysis of the network in terms of its open circuit and short-circuit impedances shows that its image impedances and its transfer constant have the same values as given by equations 8, 9 and 10 for the network of Fig. 1

The derived structures described above have image impedances containing one additional frequency factor, that is, one impedance controlling factor. By continuing the process, additional frequency factors may be introduced one at a time without limit. The network, of course, becomes more complex with each subsequent step, but ordinarily it is not necessary for practical purposes to go further than the second step.

The general procedure in obtaining networks of higher order is illustrated by Figs. 8 and 9 of which Fig. 8 illustrates the first step in the derivation process and Fig. 9 the general form of the network resulting from two steps of derivation. Fig. 8 represents a network of the type shown in Fig. 1 built out by the addition of a shunt branch adjacent the series branch added by the first derivation. The added shunt branch has an impedance Z_{1o}'/λ, Z_{1o}' being the open circuit impedance of the first derived network measured at terminals **1**, **2**, and a numerical factor. The open circuit and short-circuit impedances Z_{1o}'' and Z_{1c}'' of the network thus formed have the respective values

$$Z_{1o}''=\frac{Z_{1o}'}{1+\lambda} \quad (18)$$

and

$$Z_{1c}''=\frac{Z_{1o}'Z_{1c}'}{\lambda Z_{1c}'+Z_{1o}'}, \quad (19)$$

from which the transfer constant θ'' is obtained as

$$\tanh\theta''=\sqrt{\frac{r(1+\lambda)}{1+r(1+\lambda)}}, \quad (20)$$

since

$$\frac{Z_{1c}'}{Z_{1o}'}=\frac{r}{1+r}.$$

This expression for the transfer constant is different from that for the prototype or for the first derived network as given by Equations 4 and 7, but the original value can be regained by giving Z_a and Z_b in the network new values such that their new ratio r' satisfies the equation

$$r'=\frac{r}{1+\lambda}, \quad (21)$$

since, obviously the substitution of r' in Equation 20 leads to the original transfer constant. This may most readily be accomplished by replacing each Z_b impedance by an impedance Z_b/h_2, such that

$$\lambda=\frac{1-h_2}{h_2}.$$

The resulting network is shown schematically in Fig. 9, the h parameter of the first derived network being designated h_1. The image impedances of the network thus obtained have the values

$$K_1''=\frac{\sqrt{Z_aZ_b}(1+h_2r)}{\sqrt{1+r(1+h_1h_2r)}},$$

and

$$K_2''=\sqrt{Z_aZ_b}\frac{\sqrt{1+r}(1+h_1h_2r)}{(1+h_2r)}, \quad (22)$$

K_1'' and K_2'' denoting the image impedances at terminals 1, 2, and 3, 4, respectively.

Comparison of Equations 22 with Equations 3 shows that the new image impedances correspond respectively with the image impedances K_1 and K_2 of the prototype network modified by the introduction of two different frequency factors, one appearing in the numerator and one in the denominator. Each subsequent step of derivation introduces an additional frequency factor, the new factors occurring alternately in the numerator and the denominator.

The procedure at each stage of derivation may be described as follows: Before adding a new series branch the Z_a impedances throughout the network already developed are modified by being reduced in proportion to the chosen value of h and the new branch is constructed to have an impedance equal to $(1-h)/h$ times the short-circuit impedance of the network to which it is added. Before adding a new shunt branch the Z_b impedances throughout the network are increased inversely in proportion to the chosen value of h and the new branch is made to have an impedance equal to $h/(1-h)$ times the open circuit impedance of the modified network. In the design development of a multiple derived network it is desirable that the added branches should not only have the values given above, but should retain the configurations of short-circuit and open-circuit impedances to which they are proportioned. When the network has been fully developed the individual branches may, if necessary, be simplified for constructional purposes by the application of well known theorems relating to equivalent impedances.

In the final networks each series branch retains its proportionality to the short-circuit impedance and each shunt branch retains its proportionality to the open-circuit impedance of the respective portions of the network to which they are added. Consequently, if starting with a single series impedance Z_a and a single shunt impedance Z_b, a network is built out by the addition of alternate series branches proportioned to the successive short-circuit impedances in accordance with proportionality factors h_1, h_2, $-h_n$ and shunt branches proportioned as to the successive open circuit impedances in accordance, with factors h_1', h_2' $-h_n'$, the resulting network will have the same transfer constant as a simple prototype consisting of a single series impedance equal to Z_a multiplied by the product h_1, h_2, $-h_n$ and a single short impedance equal to Z_b divided by the product h_1', h_2', $-h_n'$.

As in the case of the first derived network, the additional frequency factors introduced by successive derivations correspond to zeros and poles of the image impedances located in the frequency range in which the impedances Z_a and Z_b of the prototype are of opposite signs. By means of the transformations alone it is, therefore, not possible to obtain the most completely general image impedance since this requires that the zeros and poles of the image impedance can be located anywhere in the attenuation ranges. This problem can however, be solved by a preliminary modification of the prototype network before making the transformation. The required modification consists in substituting for the impedances Z_a and Z_b related impedances which have the same ratio but which are simultaneously resonant or anti-resonant at additional frequencies lying in the ranges where Z_a and Z_b have the same sign. The procedure will be readily understood by the consideration of an illustrative example based on the prototype network of Fig. 3. In that network the branch impedances have the values

$$Z_a=\frac{1}{j\omega C_1}(1-\omega^2/\omega_1^2),$$

and

$$Z_b=\frac{1}{j\omega C_2}(1-\omega^2/\omega_3^2), \tag{23}$$

their ratio r being given by

$$r=\frac{C_2}{C_1}\frac{(1-\omega^2/\omega_3^2)}{(1-\omega^2/\omega_1^2)}. \tag{24}$$

If for Z_a an impedance Z_a be substituted having the value

$$Z_a'=j\omega L_1'\frac{(1-\omega^2/\omega_1^2)}{(1-\omega^2/\omega_0^2)(1-\omega^2/\omega_4^2)} \tag{25}$$

and for Z_b an impedance

$$Z_b'=j\omega L_2'\frac{(1-\omega^2/\omega_3^2)}{(1-\omega^2/\omega_0^2)(1-\omega^2/\omega_4^2)} \tag{26}$$

The ratio of these impedances will be the same as the ratio of Z_a to Z_b provided only that

$$\frac{L_1'}{L_2'}=\frac{C_2}{C_1}. \tag{27}$$

The transfer constant of the modified network thus remains unchanged, but the image impedances, which depend upon the product of the branch impedances, are modified by the introduction of the new frequency factors.

In order that the impedances Z_a' and Z_b' may be physically realizable it is necessary that the frequencies corresponding to ω_0 and ω_4 be respectively lower than f_1 the resonance frequency of Z_a and higher than f_3 the resonance frequency of Z_b. The variations of the impedances Z_a' and Z_b' with frequency are shown respectively by curves 15 and 16 of Fig. 10 and one form that the modified branches may take is illustrated by Fig. 11. The values of the branch elements may be determined from the resonance and anti-resonance frequencies in the manner described by R. M. Foster in an article entitled "A Reactance Theorem", Bell System Technical Journal, vol. III No. 2, April 1924.

The application of the transformation of Figs. 1 and 8 to the modified prototype networks introduces additional impedance controlling factors while retaining those introduced by the modification of the prototype. The two processes, therefore, supplement each other and together provide for complete control of the image impedances.

The derivation processes described in the foregoing, may be applied to prototype networks of any type, but their greatest utility lies in their application to wave filter networks. It has already been pointed out that the most general transfer constant of a wave filter can be represented as the sum of the transfer constants of a symmetrical portion, which may be constructed as a symmetrical lattice network in accordance with my earlier Patent 1,828,454 issued October 20, 1931, and of a number of simple series-shunt type half sections. The image impedance of the symmetrical lattice may be made to have any possible characteristic by the appropriate allocation of the branch impedance resonances and anti-resonances, but in order that the completely general transfer constant may be obtained it is necessary that the additional half sections be capable of assuming impedances which match each other and that of the symmetrical lattice when connected in tandem. The complete control of the image impedances of unsymmetrical half sections by means of the processes described above permits the required impedance matching to be accomplished and so makes possible the construction of wave filters of completely general characteristics.

An example of a composite filter employing the networks of the invention is shown in Fig. 12. This is a low-pass filter network comprising a symmetrical lattice F_1; and two unsymmetrical networks F_2 and F_3 represented respectively by the portion of the network between dotted AA' and BB' and the portion between BB' and CC'. The network F_2 is an h-derived network corresponding to the elementary prototype half section of Fig. 13, the term h-derived network being used herein to define a network related to a prototype in accordance with the principles of the invention as described above. The network F_3 is an h-derived network corresponding to the prototype half section shown in Fig. 14 which is an elementary low-pass half section of the type having an infinite value of the transfer constant at some frequency in the attenuation range.

The image impedances of the symmetrical filter F_1, which due to the symmetry of this portion are both alike are represented by curve 17 of Fig. 15. The characteristic has one pole at frequency f_2 above the cut-off frequency f_1, this providing a single impedance controlling factor. The image impedance at the other end of the network is represented by curve 18 of Fig. 16 and is characterized by a pole at frequency f_2 and a zero at a higher frequency f_3. The zero in this case corresponds to the frequency of the attenuation peak of the prototype half section of Fig. 14.

The difference in the two image impedances of the composite filter arises from the fact that, in the case of reactive networks, if the transfer constant is given only one image impedance can be chosen arbitrarily, the second image impedance being determined as soon as the transfer constant and the first image impedance are chosen.

What is claimed is:

1. The method of building out a series shunt transmission network comprising a series impedance Z_a and a shunt impedance Z_b to produce a new network having the same transfer constant as the original network, but having different image impedances therefrom, which comprises alternate steps of adding series branches having impedances equal to $(1-h)/h$ times the short-circuit impedances of the networks to which the said branches are added, h being a positive numeric less than unity, and shunt branches having impedances equal to $h/(1-h)$ times the open circuit impedances of the networks to which the said shunt branches are added, the Z^a impedances throughout the network at each stage being diminished in accordance with the factor h before a new series branch is added and the Z^b impedances being increased in accordance with the factor $1/h$ before a new shunt branch is added.

2. The network according to claim 1 in which the value of the factor h is changed with each step of adding a new branch.

3. A four-terminal network having the same transfer constant as a prototype network consisting of a single series impedance Z_a and a single shunt impedance Z_b, comprising a series branch having an impedance k_1Z_a, a shunt branch having an impedance Z_b/k_2 and a built out portion comprising a plurality of successively added series and shunt branches each series branch having an impedance equal to

$$\frac{1-h}{h}$$

times the short-circuit impedance of the portion of the network to which it is added and each shunt branch having an impedance equal to $h'/(1-h')$ the open-circuit impedance of the portion of the network to which it is added, the quantities h and h' being positive numerics less than unity which have values h_1, h_2, $-h_n$, respectively for the successively added series branches and h_1', $h_2'-h_n$ for the successively added shunt branches, and the factors k_1 and k_2 having respectively the values h_1, h_2, $-h_n$, and h_1', h_2', $-h_n'$.

4. A four-terminal transmission network comprising a portion consisting of a single series impedance and a single shunt impedance and a built out portion consisting of a plurality of successively added series and shunt branches each series branch of said built out portion having an impedance proportional to the short-circuit impedance of the portion of the network to which it is added and each added shunt branch having an impedance proportional to the open-circuit impedance of the portion of the network to which it is added.

5. A four-terminal transmission network having the same transfer constant as a prototype network comprising a single series impedance Z_a and a single shunt impedance Z_b, comprising a series branch having an impedance hZ_a, a shunt branch of impedance Z_b and a second series branch having an impedance $(1-h)/h$ times the impedance of said first mentioned series branch and said shunt branch in parallel, where h is a positive numeric less than unity.

6. A four-terminal transmission network having the same transfer constant as a prototype network comprising a single series impedance Z_a and a single shunt impedance Z_b, comprising a shunt branch having an impedance Z_b/h, a series branch having an impedance Z_a and a second shunt branch having an impedance equal to $h/(1-h)$ times the impedance of said first mentioned shunt branch and said series branch in series.

7. A composite wave filter comprising a reactance network proportioned to transmit currents of frequencies in a pre-assigned band and to attenuate currents of other frequencies, and having a pre-assigned image impedance which has the same value at both the input and the output terminals of the network, and connected in tandem therewith, an h-derived network having the same transfer constant as a prototype half-section wave filter adapted to transmit the same band of frequencies as said first mentioned network, but having image impedances which differ from that of said first mentioned network, said h-derived network having an image impedance which matches that of said first mentioned network at the junction point.

8. A composite wave filter comprising a plurality of h-derived networks connected in tandem, said h-derived networks having the same transfer constants as respective prototype half-section wave filters comprising single series branches and single shunt branches, but having image impedances differing from those of the respective prototypes by the inclusion of additional resonance frequencies in the attenuating ranges, and the image impedances of said h-derived networks being further proportioned to provide impedance matching at the junction points of the several networks.

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