Aug. 14, 1934. R. M. KALB 1,970,316

VACUUM TUBE CIRCUITS

Filed Jan. 13, 1932

FIG. 1

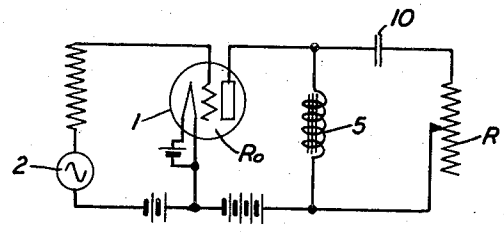
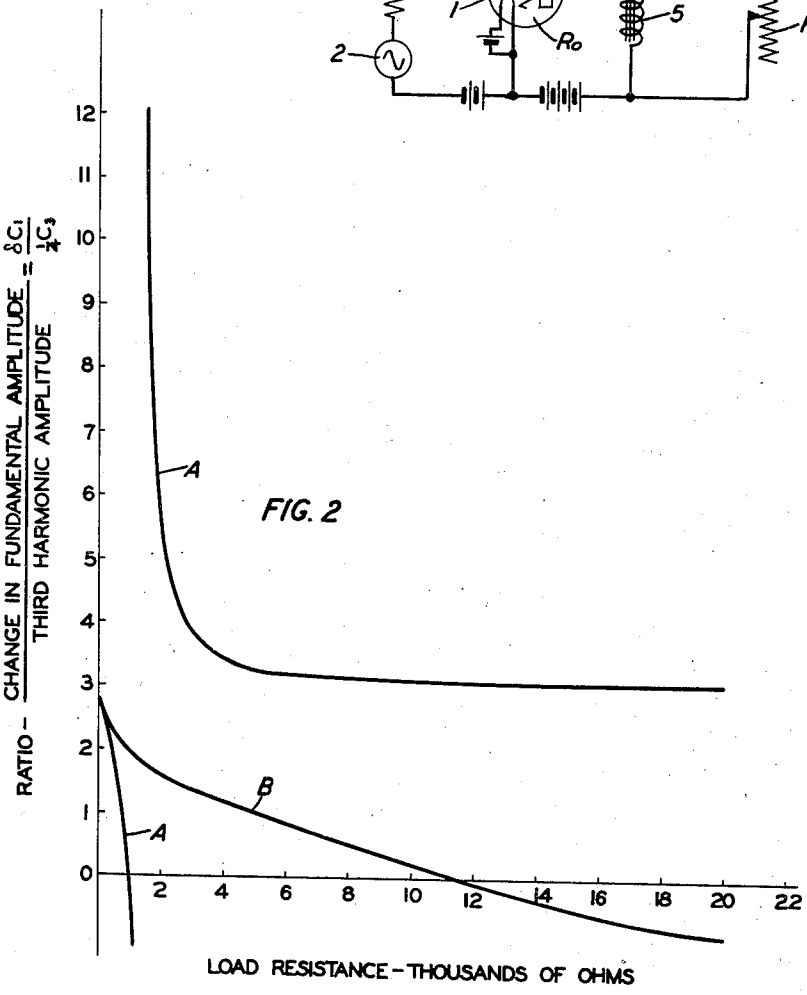


FIG. 2



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VACUUM TUBE CIRCUITS

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2 Claims. (Cl. 179—171)

This invention relates to wave translating systems, for example, signaling systems employing electric space discharge amplifying devices.

An object of the invention is to improve the grade of transmission in such systems.

It is also an object of the invention to control gain in such systems, for instance to operate vacuum tubes in such systems so that change of gain with transmission level is controlled as desired. For example, it may be desired that there be substantially no change of gain with transmission level.

As explained in a paper by E. Peterson "Impedance of a Non-Linear Circuit Element" Transactions of the American Institute of Electrical Engineers, Vol XLVI, pages 528 to 534, 1927, effects of harmonics generated in a vacuum tube, called by him loading and reaction effects, may change the amplitude of the fundamental wave. The harmonics shift the operating region on the (non-linear) plate voltage-plate current characteristic of the tube so that the plate impedance in the tube to the fundamental is changed, and moreover, the harmonics require energy dissipation or storage and dissipation at harmonic frequencies, which appears as impedance reaction at the fundamental frequency.

In one specific aspect the invention is a system with prescribed control of effects of harmonics generated in vacuum tubes and appearing in the load circuits of the tubes upon the fundamental waves that produce the harmonics. For instance, the harmonics may be prevented from changing the fundamental amplitude. Such control or prevention may be accomplished, for example, by proportioning the load impedance of the vacuum tubes as disclosed hereinafter. It is possible by such prevention in accordance with the invention, to operate vacuum tubes so that there is substantially no change of gain with transmission level. Such operation is especially advantageous where a large number of tubes are connected in tandem. In such cases, uncontrolled changes of gain with transmission level may be large because of the cumulative action of the large number of repeaters, for example. The changes in gain not only cause distortion of signals, but may result in greater amplification at low levels than at high levels and consequently in decrease of signal-to-noise ratio.

Other objects and aspects of the invention will be apparent from the following description and claims.

Fig. 1 of the drawing shows a vacuum tube amplifier circuit embodying a form of the invention; and

Fig. 2 shows curves facilitating explanation of the invention.

In the system of Fig. 1, a vacuum tube 1 amplifies waves from source 2 and transmits them to a load resistance R. The usual cathode heating, grid biasing and plate voltage supply sources are shown. Current from the plate voltage supply source flows through a choke coil 5 and is prevented from flowing through resistance R by a stopping condenser 10.

The known fact that there exists a relation between the amplitude of the third harmonic and the change in fundamental amplitude in the output of a vacuum tube, caused by harmonics, suggests devising a method for determining either from the other. Once the relation is definitely determined for specified conditions, the change in fundamental can be calculated from the measured third harmonic, or vice versa, if the higher harmonics are small.

For zero external impedance the change in fundamental amplitude is thrice the amplitude of the third harmonic. This relation has been used quite generally. However, its applicability is restricted to one special case which is of limited practical importance, for at load impedances other than zero the ratio of fundamental increment to the amplitude of the third harmonic in general differs greatly from three, as brought out hereinafter. The difference is caused by the change, due to harmonics, of the external impedance to the fundamental, and by change in the diversion of energy from the fundamental to these harmonics. The latter effect (the diversion) is altered by a change in the impedance to any harmonic. As the harmonics are functions of the operating point and load impedance, the increment in the fundamental caused by them is in general a function of the same variables.

In taking account of the loading and reaction effects of harmonics, the effects of the variable amplification factor will also be included by following the methods used by Peterson and Evans in their paper on "Modulation in Vacuum Tubes Used as Amplifiers", published in Bell System Technical Journal, July, 1927, pp. 442–460. Their nomenclature will also be adopted. For an applied sinusoidal voltage of amplitude P the amplitude of the second harmonic is

$$\tfrac{1}{2}P^2C_2,$$

and of the third harmonic,

$$\tfrac{1}{4}P^3C_3,$$

as may be seen by expanding the second and third powers of $e=P\cos pt$ in terms of its harmonics and substituting the results for corresponding powers of e in Equation (5) of the paper just mentioned. (In these and following expressions the C's are functions of the impedances of the external plate circuit for the tube and of the parameters of the tube.) When this is done there

results for the first approximation to the change in the amplitude of the fundamental

$$P^3\delta C_1=\tfrac{3}{4}P^3C_3$$

This is seen to give the constant ratio three between fundamental increment and third harmonic amplitude. Closer approximation is obtained by taking account of the effects of harmonics by writing

$$v=-\Sigma Z_nJ_n$$

for the plate voltage and solving the result. In this expression, J_n is the nth harmonic of the plate current, and Z_n is the external plate impedance to J_n. In a large number of applications the direct current in the plate circuit is by-passed through a retard coil, such as 5, and only the alternating current flows through the output impedance such as R, via a condenser such as 10. The output impedance generally is high and of small phase angle, and may be represented by a resistance of value R, for all frequencies other than zero. The D. C. resistance of the retard coil is relatively low, and may be considered zero. With these simplifying assumptions a useful closer approximation to the fundamental can be found.

The change of the plate current is given by Peterson and Evans in their paper referred to above as

$$J=b_{10}v+b_{01}e+b_{20}v^2+b_{11}ve+b_{02}e^2+b_{30}v^3+b_{21}v^2e+b_{12}ve^2+b_{03}e^3+\ \ldots \quad (1)$$

Wherein the b_{mn} are parameters of the tube, and wherein

$$v=-R\sum_{n=1}^{\infty}J_n \quad (2)$$

according to the assumptions previously stated. When harmonics higher than the third are relatively negligible,

$$J=J_0+J_1+J_2+J_3, \quad (3)$$

and three successively closer approximations to v are

$$\begin{aligned} {}_1v&=v_1=-RJ_1\\ {}_2v&=v_1+v_2=-RJ_1-RJ_2\\ {}_3v&=v_1+v_2+v_3=-RJ_1-RJ_2-RJ_3. \end{aligned} \quad (4)$$

Equation (1) may be rewritten replacing v by ${}_3v$ in first order terms, by ${}_2v$ in second order terms, and by ${}_1v$ in third order terms. If the cross products and higher powers of J_2 and J_3 are negligible, the result is

$$\begin{aligned} {}_3J=&-b_{10}R(J_1+J_2+J_3)+b_{01}e\\ &+b_{20}R^2(J_1^2+2J_1J_2)-b_{11}R(J_1+J_2)e+b_{02}e^2\\ &-b_{30}R^3J_1^3+b_{21}R^2J_1^2e-b_{12}RJ_1e^2+b_{03}e^3, \end{aligned} \quad (5)$$

in which ${}_3J$ is identified with ${}_3v$. Taking only a second approximation by this process

$$\begin{aligned} {}_2J=&-b_{10}R(J_1+J_2)+b_{01}e\\ &+b_{20}R^2J_1^2-b_{11}RJ_1e+b_{02}e^2. \end{aligned} \quad (6)$$

A first approximation similarly gives

$${}_1J=-b_{10}RJ_1+b_{01}e. \quad (7)$$

By the trigonometric series (3) a first approximation to J is J_1 when J_0 is relatively small. Replacing ${}_1J$ in (7) by J_1 and solving gives

$$J_1=\frac{b_{01}e}{1+b_{10}R}=C_1e. \quad (8)$$

If the impressed voltage is a pure sinusoidal wave

$$e=P\cos pt, \quad (9)$$

$$J_1=C_1P\cos pt \quad (10)$$

$$J_1^2=\tfrac{1}{2}C_1^2P^2[1+\cos 2pt]. \quad (11)$$

Subtracting (8) from (6) leaves

$$J_0+J_2=-b_{10}RJ_2+b_{20}R^2J_1^2-b_{11}RJ_1e+b_{02}e^2, \quad (12)$$

which from (9), (10) and (11) becomes

$$J_0+(1+b_{10}R)J_2=\frac{\tfrac{1}{2}b_{20}b_{01}^2P^2R^2}{(1+b_{10}R)^2}[1+\cos 2pt]+\tfrac{1}{2}b_{02}P^2[1+\cos 2pt]-\frac{\tfrac{1}{2}b_{11}b_{01}P^2R}{(1+b_{10}R)}[1+\cos 2pt]. \quad (13)$$

As a consequence of the definition embodied in (3), the components J_2 and J_0 of this expression may be separated. They are as given below.

$$J_2=\frac{P^2}{2}\left(\frac{b_{20}b_{01}^2R^2}{(1+b_{10}R)^3}-\frac{b_{11}b_{01}R}{(1+b_{10}R)^2}+\frac{b_{02}}{1+b_{10}R}\right)\cos 2pt, \quad (14)$$

or, since, as shown by the above mentioned paper of Peterson and Evans, the quantity in brackets in equal to C_2,

$$J_2=\tfrac{1}{2}C_2P^2\cos 2pt. \quad (15)$$

$$J_0=\tfrac{1}{2}C_2P^2(1+b_{10}R). \quad (16)$$

J_3 can now be evaluated. Subtracting (6) from (5)

$$J_3=-b_{10}RJ_3+b_{20}R^2(2J_1J_2)-b_{11}RJ_2e-b_{30}R^3J_1^3+b_{21}R^2J_1^2e-b_{12}RJ_1e^2+b_{03}e^3. \quad (17)$$

$$(1+b_{10}R)J_3=2b_{20}R^2J_1J_2-b_{11}RJ_2e-b_{30}R^3J_1^3+b_{21}R^2J_1^2e-b_{12}RJ_1e^2+b_{03}e^3. \quad (18)$$

Solving (18) and substituting the relations (9), (10), (11) and (15) gives

$$\begin{aligned} J_3=&\frac{b_{20}R^2C_1C_2P^3\cos pt\cos 2pt}{1+b_{10}R}-\frac{\tfrac{1}{2}b_{11}RC_2P^3\cos pt\cos 2pt}{1+b_{10}R}-\\ &\frac{\tfrac{1}{2}b_{30}\overline{RC_1}^3P^3(\cos pt+\cos pt\cos 2pt)}{1+b_{10}R}+\frac{\tfrac{1}{2}b_{21}\overline{RC_1}^2P^3(\cos pt+\cos pt\cos 2pt)}{1+b_{10}R}-\\ &\frac{\tfrac{1}{2}b_{12}RC_1P^3(\cos pt+\cos pt\cos 2pt)}{1+b_{10}R}+\frac{\tfrac{3}{4}b_{03}P^3\cos pt+\tfrac{1}{4}b_{03}P^3\cos 3pt}{1+b_{10}R} \end{aligned} \quad (19)$$

$$=\frac{\tfrac{1}{2}b_{20}R^2C_1C_2-\tfrac{1}{4}b_{11}RC_2-\tfrac{3}{4}b_{30}\overline{RC_1}^3+\tfrac{3}{4}b_{21}\overline{RC_1}^2-\tfrac{3}{4}b_{12}RC_1+\tfrac{3}{4}b_{03}}{1+b_{10}R}P^3\cos pt+$$

$$\frac{\tfrac{1}{2}b_{20}R^2C_1C_2-\tfrac{1}{4}b_{11}RC_2-\tfrac{1}{4}b_{30}\overline{RC_1}^3+\tfrac{1}{4}b_{21}\overline{RC_1}^2-\tfrac{1}{4}b_{21}RC_1+\tfrac{1}{4}b_{03}}{1+b_{10}R}P^3\cos 3pt. \quad (20)$$

The coefficient of $P^3 \cos 3\,pt$ in Equation (20) is equal to

$$\tfrac{1}{4}C_3$$

by definition. The fundamental component of this equation is the increment of fundamental caused by the third harmonic, and will be written

$$\delta C_1 P^3 \cos pt.$$

Equation (2) then gives for the value of the output voltage.

$$v=-RC_1\left(1+P^2\frac{\delta C_1}{C_1}\right)P\cos pt-$$
$$\tfrac{1}{2}RC_2P^2\cos 2pt-\tfrac{1}{4}RC_3P^3\cos 3pt. \quad (21)$$

The relation between δC_1 and C_3 can be calculated from these results. Let

$$\xi=\frac{\tfrac{1}{2}RC_2(b_{20}RC_1-\tfrac{1}{2}b_{11})}{1+b_{10}R} \quad (22)$$

and

$$\zeta=\tfrac{1}{4}\frac{-b_{30}RC_1^3+b_{21}RC_1^2-b_{12}RC_1+b_{03}}{1+b_{10}R} \quad (23)$$

then

$$C_3=4(\xi+\zeta) \quad (24)$$

and

$$\delta C_1=\xi+3\zeta \quad (25)$$

The ratio of these quantities is

$$\frac{\delta C_1}{C_3}=\tfrac{1}{4}\frac{\xi+3\zeta}{\xi+\zeta}. \quad (26)$$

This expression is not tractable to numerical calculation as it contains second derivatives difficult to evaluate. Only first derivatives compose ξ, and only second derivatives compose ζ. It is therefore desirable to eliminate ζ from explicitly appearing in the equations connecting δC_1 and C_3, leaving only ξ. This is readily accomplished by subtracting (24) from (25) after so treating them as to obtain the same coefficient for ζ in both. The result is

$$\delta C_1=\tfrac{3}{4}C_3-2\xi. \quad (27)$$

The ratio of the change in amplitude of the fundamental to the amplitude of the third harmonic can now be found from (27) and (21). It is

$$\frac{\delta C_1}{\tfrac{1}{4}C_3}=3-8\frac{\xi}{C_3}. \quad (28)$$

When C_3 is known this relation is readily calculable, requiring only first derivatives.

Determination of the change in fundamental for two tubes in cascade by a series of approximations similar to those used for a single tube would be very complicated. Inasmuch as some method applicable to two tubes is useful, notably in repeater circuits, the following expedient may be adopted.

The output impedances of the two tubes are designated by R_1 and R_2, respectively, for all frequencies except direct current, which meets zero impedance. Let the coefficients for the first tube be C_η, and for the second tube, D_η. The output voltage, e, of the combination without regard to the increment in fundamental or the short-circuited direct current is derived as follows:

If a voltage with an alternating component e be applied to the first tube, the alternating component v of the output voltage will be given by the series

$$v=-R_1C_1e-R_1C_2e^2-R_1C_3e^3-\ldots$$

wherein the C's are coefficients determined by characteristics of the first tube. The voltage v is the alternating component of the voltage applied to the grid of the second tube, producing an output voltage ϵ determined by the relation

$$\epsilon=-R_2D_1v-R_2D_2v^2-R_2D_3v^3-\ldots$$

where the D's are coefficients determined by the characteristics of the second tube. Substituting for v its value in terms of e I have

$$\epsilon=R_1R_2C_1D_1e+R_1R_2C_2D_1e^2+R_1R_2C_3D_1e^3+\ldots$$
$$-R_1^2R_2C_1^2D_2e^2-2R_1^2R_2C_1C_2D_2e^3-\ldots$$
$$+R_1^3R_2C_1^3D_3e^3+\ldots$$
$$\epsilon=(-R_1C_1)(-R_2D_2)\left[e+\left(\frac{C_2}{C_1}-R_1C_1\frac{D_2}{D_1}\right)e^2+\right.$$
$$\left.\left(\frac{C_3}{C_1}-2R_1C_2\frac{D_2}{D_1}+\overline{R_1C_1^2}\frac{D_3}{D_1}\right)e^3\right]. \quad (29)$$

To transform this equation into one which takes account of the change in fundamental it is necessary to replace throughout

$$C_1 \text{ by } C_1\left(1+P^2\frac{\delta C_1}{C_1}\right)$$

and

$$D_1 \text{ by } D_1\left(1+P^2\frac{\delta D_1}{D_1}\right).$$

As

$$P^2\frac{\delta C_1}{C_1}<1$$

and

$$P_2\frac{\delta D_1}{D_1}<1,$$

the binomial power series expansion may be used to simplify the result and cross products of the small terms may be dropped. The value of e given by (9) is then substituted in the result, and all direct current products dropped, since they are necessarily zero. There remains

$$\epsilon=R_1R_2C_1D_1\left[1+P^2\left(\tfrac{3}{4}\left(\frac{C_3}{C_1}-2R_1C_2\frac{D_2}{D_1}+\overline{R_1C_1^2}\frac{D_3}{D_1}\right)+\left(1-\tfrac{3}{4}\left[2R_1C_2\frac{D_2}{D_1}-3\overline{R_1C_1^2}\frac{D_3}{D_1}\right]\right)\frac{\delta C_1}{C_1}+\right.\right.$$
$$\left.\left.\left(1+\tfrac{3}{4}\frac{C_3}{C_1}\right)\frac{\delta D_1}{D_1}\right)\right]P\cos pt+\tfrac{1}{2}R_1R_2C_1D_1\left[\frac{C_2}{C_1}\left(1+P^2\frac{\delta D_1}{D_1}\right)-R_1C_1\frac{D_2}{D_1}\left(1+2P^2\frac{\delta C_1}{C_1}\right)\right]P^2\cos 2pt+$$
$$\tfrac{1}{4}R_1R_2C_1D_1\left[\frac{C_3}{C_1}\left(1+P^2\frac{\delta D_1}{D_1}\right)-2R_1C_2\frac{D_2}{D_1}\left(1+P^2\frac{\delta C_1}{C_1}\right)+\overline{R_1C_1^2}\frac{D_3}{D_1}\left(1+3P^2\frac{\delta C_1}{C_1}\right)\right]P^3\cos 3pt. \quad (30)$$

The increment to the fundamental consists of those terms involving P cos pt which have the common multiplier, P^2. This increment comprises three additive terms, the first due to modulation of the complex wave applied to the grid of the second tube, the second due to production by the

first tube of a third harmonic, and the last due to production by the second tube of a third harmonic. The change in fundamental due to any of these contributory causes, or to their combined effect, can be calculated by means of Equation (30).

It is unnecessary in these calculations to know the third order coefficient provided its ratio to the fundamental is known. This statement applies equally well to a single tube or a two-stage arrangement.

In Fig. 2 curve A and curve B show the ratio of the change in amplitude of the fundamental to the amplitude of the third harmonic at various load resistances for Western Electric Company tubes of types 104–D and 101–D, respectively, (which are typical low-μ tubes), with a grid bias of —22 volts for the 104–D tube and —9 volts for the 101–D tube, a filament current of 1.2 amperes for the 104–D tube and 1.1 amperes for the 101–D tube, and a plate voltage of 120 volts for each tube. These ratios were calculated by Equation (28), using computed values of C_3 checked experimentally. This coefficient for the 104–D tube changes sign within the limits of load resistance used in the calculations, and the ratio becomes infinite for the load resistance at which this change occurs. At this point the increment in fundamental amplitude is $-2\xi P^3$. For higher usual values of load resistance the ratio approximates three, the value already mentioned as being applicable when the load impedance is zero to harmonic frequencies. When the plate impedance is matched by the load the ratio is between four and five. The ratio for the 101–D tube departs rapidly from three as the load resistance is increased, and is negative for the higher values considered. It is near unity when the impedances of plate and load are matched.

When the third order coefficient has the same sign as the fundamental the increment to the fundamental is an increase. When the signs differ the fundamental is decreased by the effects of the harmonics. For the 104–D tube the fundamental amplitude is increased (as distinguished from decreased) for all but very low load resistances. For the 101–D tube the fundamental is increased for values of load resistance above twice the plate resistance, under the operating conditions assumed. For each of these tubes a value of load resistance can be found such that the fundamental amplitude does not change because of harmonics. This resistance may be found from the curve, and is that value for which the ratio is zero.

Where increment of fundamental caused by harmonics is to be used for obtaining transmission volume expansion or compression, for example, the sign of the increment becomes of interest. The sign can be found from the curves by computing C_3 to ascertain whether the sign of C_3 is like or opposite that of the fundamental, or the sign of the fundamental increment can be found by direct measurement of the fundamental increment.

From the foregoing discussion it is seen that the ratio of change in fundamental amplitude to third harmonic, is not a constant, but a function of the load resistance. As the coefficients used vary with plate voltage and grid potential, this ratio is also a function of these. It may be evaluated in any case from measured or calculated values of the third order coefficient, and values of ξ, calculated from curves of μ and R_0 as functions of plate voltage.

What is claimed is:

1. An amplifying system comprising a space discharge device having an input circuit and an output circuit, a source of waves to be amplified associated with said input circuit, and a load associated with said output circuit having its impedance proportioned substantially to a critical value with respect to said discharge device at which value the increment of fundamental waves that is due to harmonics is made substantially zero.

2. In a space discharge circuit, a space discharge device having electrodes, an input and an output circuit associated with said electrodes, an impedance in said output circuit, means for applying potentials to said electrodes, the values of said impedance and said potentials being proportioned so that the change of fundamental products that is due to harmonics is a minimum.

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