April 17, 1934. E. L. NORTON 1,954,943

WAVE TRANSMISSION NETWORK

Filed July 29, 1932

FIG. 1

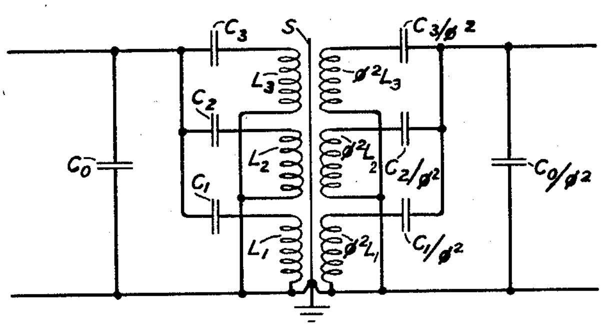


FIG. 2

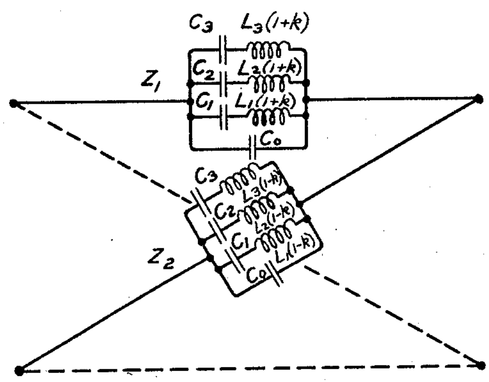


FIG. 3

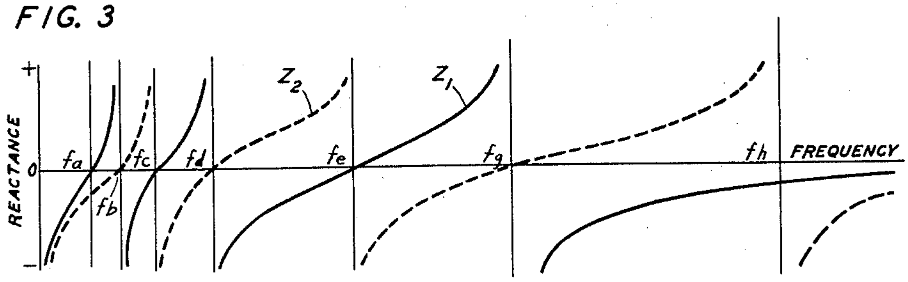
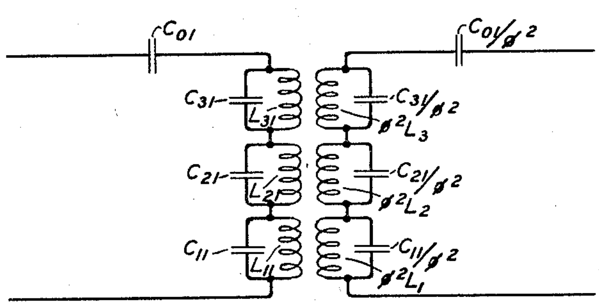


FIG. 4



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WAVE TRANSMISSION NETWORK

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9 Claims. (Cl. 178—44)

This invention relates to tuned transformers and more particularly to multiple transformer systems for the transmission of a broad band of frequencies.

An object of the invention is to increase the transmission band of a multiple transformer system. Another object is to increase the uniformity of the transmission throughout the transmission range.

As a transformer departs from the ideal of two infinite inductances with perfect coupling, its range of efficient transmission becomes limited, at high frequencies due to the impedance of the leakage inductance and at low frequencies because of the low impedance of the windings as a whole. The transmission characteristic resembles the familiar resonance curve of a highly damped tuned circuit. By tuning the windings it is possible to give the transformer a definite band transmission characteristic, a maximum band width being obtained when the windings are symmetrically tuned by series and shunt capacities in combination. Under this condition, it is found that the ratio of the upper to the lower limiting frequency is determined wholly by the degree of coupling between the transformer windings.

While this limitation of the frequency range may not be serious in iron cored transformers designed for low frequencies, it becomes important in high frequency transformers in which for various reasons the use of magnetic cores may not be permissible. In air core transformers, it is found that coupling coefficients of greater than about 0.80 are impracticable and that, as the result of this, frequency ranges greater than 9 to 1 can not readily be obtained. To increase this range, combinations of differently tuned transformers have been used, but heretofore such combinations have been characterized by considerable irregularities of the transmission corresponding to the resonances of the individual transformers or else have required the use of an excessive number of transformers to keep the irregularities small.

In accordance with the present invention, the resonance frequencies of the several transformers of a multiple tuned transformer system are so coordinated with each other and with the coupling coefficients that the combination provides a maximum transmission range and a substantially uniform efficiency throughout this range.

The novel features of the invention and the principles of operation will be understood more clearly from the following detailed description and the accompanying drawing of which:

Fig. 1 shows a schematic arrangement of one form of the invention;

Figs. 2 and 3 are diagrams used in the explanation of the invention; and

Fig. 4 shows schematically another form of the invention.

The network of Fig. 1 comprises three transformers having parallel connected primary windings of inductances L_1, L_2, and L_3, and parallel connected secondary windings of inductances Φ^2L_1, Φ^2L_2, and Φ^2L_3, respectively. The transformers all have the same voltage transformation ratio, $1:\Phi$, and are so designed as to have the same coupling coefficient k between the windings. The primary windings are tuned by series capacities C_1, C_2, and C_3 and the secondary windings by corresponding series capacities C_1/Φ^2, C_2/Φ^2, and C_3/Φ^2. Additional tuning capacities C_0 and C_0/Φ are connected across the input and the output terminals respectively of the network.

In accordance with the invention, the elements of the network are proportioned to provide a continuous transmission band throughout a frequency range of maximum width. From one viewpoint each transformer by means of its series condensers and parts of the shunt capacities C_0 and C_0/Φ^2 is tuned to give the maximum band width consistent with its degree of coupling and the three transformers are coordinated to have their transmission bands contiguous. It is simpler, however, to consider the network as a unitary structure and to arrive at the design rules by the application of well known principles of wave filters.

Since the network is symmetrical except for a transformation ratio Φ, it may be shown to be equivalent to a symmetrical lattice network plus an ideal transformer of voltage ratio $1:\Phi$. For an explanation of the general method of derivation of the equivalent lattice, reference is made to an article by A. C. Bartlett on "An extension of a property of artificial line", Phil. Mag. (London) vol. 4, No. 24, November 1927. In the particular case illustrated the first step of the derivation is to make the network symmetrical by removing the transformation ratio of the several transformers and replacing this by an ideal transformer at the right hand of the network having a voltage transformation ratio $1:\Phi$, which is the same as the common ratio of the individual transformers. This procedure results in no change in the configuration of the network of Fig. 1, except for the addition of the ideal transformer, but makes the element values in the primary and secondary sides of the transformers respectively equal.

The individual transformers may now be replaced by their equivalent T-networks. Thus the transformer having winding inductance L_1 may be replaced by a T-network comprising equal series inductances $L_1(1-k)$ and a shunt inductance L_1k. This substitution brings the symmetrical portion of the network into a configuration to which the bisection principle disclosed in the above mentioned article may be directly applied.

The lattice corresponding to Fig. 1 is illustrated in Fig. 2. The line impedance Z_1 comprises three resonant circuits in parallel with a capacity C_0, the capacities of the resonant circuits being C_1, C_2, and C_3 and the inductances $L_1(1+k)$, $L_2(1+k)$ and $L_3(1+k)$. The lattice impedance is of similar form and differs only in the values of the inductances which are $L_1(1-k)$, $L_2(1-k)$ and $L_3(1-k)$ respectively.

In order that the lattice of Fig. 2, and therefore the network of Fig. 1 shall have a continuous transmission band, it is necessary that the reactances of the impedances Z_1 and Z_2 shall be of opposite sign at all frequencies throughout a continuous range. The frequency characteristics of the two reactances are illustrated by the curves of Fig. 3, the full line curves corresponding to impedance Z_1 and the dotted curves to impedance Z_2. The impedance Z_1 is resonant and anti-resonant alternately at a series of frequencies f_a, f_b, . . . f_g. Since impedance Z_2 is of the same form as Z_1, it will exhibit the same number of resonances and anti-resonances, but since the inductances of Z_2 are smaller than those of Z_1 the critical frequencies will be higher. To provide a continuous band the lowest resonance of Z_2 must coincide with the lowest anti-resonance frequency, f_b, of Z_1 and the remainng critical frequencies except the highest, f_h must coincide with f_c, f_d, f_e, etc. Under this condition, the band will extend from f_a the lowest resonant frequency of Z_1 to f_h the highest anti-resonant frequency of Z_2.

It is to be observed that the requirement of coincidence of the critical frequencies has to be met with two impedances which differ only in the values of the inductance elements and in which corresponding inductances are in the same ratio, namely

$$\frac{1-k}{1+k}.$$

This requirement serves to fix the relative values of the frequencies in the following manner. In accordance with the reactance theorem described by R. M. Foster in the Bell System Technical Journal, vol. III, April 1926, pages 259 to 267, the values of the impedances Z_1 and Z_2 at any frequency can be expressed in a factorial form in terms of the shunt capacity C_0 and the critical frequenceis. From these expressions the values of the capacities C_1, C_2 and C_3 can be obtained directly, also in terms of C_0 and the critical frequencies. In this way there are obtained two expressions for C_1, one from each impedance formula, and likewise two expressions for each of C_2 and C_3. These pairs of values, which are necessarily equal, give rise to equations involving the critical frequencies which suffice to determine their relative values.

The necessary relationship between the critical frequencies is found to be

$$\frac{f_a}{f_b}=\frac{f_b}{f_c}=\frac{f_c}{f_d}=\frac{f_d}{f_e}=\frac{f_e}{f_g}=\frac{f_g}{f_h}=\sqrt{\frac{1-k}{1+k}} \quad (1)$$

or, if f_1, f_2 and f_3 denote the resonance frequencies of the respective primary windings and their series capacities,

$$\frac{f_1}{f_2}=\frac{f_2}{f_3}=\frac{1-k}{1+k} \quad (2)$$

From Equation 1, it follows that the ratio of the upper cut-off frequency, f_h, to the lower cut-off frequency, f_a, is

$$\frac{f_h}{f_a}=\left[\frac{1+k}{1-k}\right]^3. \quad (3)$$

In general it may be shown by the same sort of analysis that for n transformers arranged in the manner of Fig. 1, the cut-off frequencies are in the ratio

$$\left[\frac{1+k}{1-k}\right]^n.$$

For the case of a single transformer, for example, the transformer L_1, Φ^2L_1, the limiting frequencies would be in the ratio

$$\frac{1+k}{1-k}$$

and the relationship between the tuning capacities would be

$$C_1=C_0\left[\frac{1+k}{1-k}-1\right].$$

Without the additional tuning condensers C_0 and C_0/Φ^2, the ratio of the limiting frequencies would be equal to

$$\sqrt{\frac{1+k}{1-k}}$$

which, with close coupling represents a very substantial reduction of the band width.

Each winding of the single transformer exhibits a resonance and an anti-resonance at a higher frequency, these frequencies, like the critical frequencies of the branches of the lattice of Fig. 2, being in the ratio

$$\sqrt{\frac{1-k}{1+k}}$$

For the frequency spacing indicated by Equations 1 and 2, the formulæ obtained for the capacity values in the three-transformer system of Fig. 1 are

$$\left.\begin{aligned}
C_1&=\frac{\left(\frac{1}{p}-1\right)\left(\frac{1}{p^3}-1\right)\left(\frac{1}{p^5}-1\right)}{\left(\frac{1}{p^2}-1\right)\left(\frac{1}{p^4}-1\right)}C_0;\\
C_2&=\frac{(p-1)\left(\frac{1}{p}-1\right)\left(\frac{1}{p^3}-1\right)}{(p^2-1)\left(\frac{1}{p^2}-1\right)}C_0;\\
C_3&=\frac{(p^3-1)(p-1)\left(\frac{1}{p}-1\right)}{(p^2-1)(p^4-1)}C_0;
\end{aligned}\right\} \quad (4)$$

where

$$p=\frac{1-k}{1+k}.$$

The inductance values follow from the resonance frequencies f_1, f_2, and f_3.

These formulæ serve to fix the relative values of the elements; to fix their absolute values it is necessary to assign a value to C_0. If the transformer is intended to operate between a resistance R on the primary side and a resistance Φ^2R on the secondary, the requirement for impedance matching furnishes also the proper value of C_0. The characteristic impedance K of the lattice of Fig. 2 is given by

$$K=\frac{1}{2\pi f C_0}\sqrt{\frac{f^2-f_a^2}{f_h^2-f^2}}, \quad (5)$$

which at the mean frequency of the band has the value K_m given by

$$K_m=\frac{1}{2\pi f_h C_0} \quad (6)$$

The characteristic impedance remains fairly close to this value throughout the greater portion of the band and satisfactory freedom from reflection effects is obtained when the terminal resistance is matched to K_m. The impedance matching condition gives for the value of C_0,

$$C_0=\frac{1}{2\pi f_h R} \quad (7)$$

In the foregoing it is assumed that the transformer windings with their respective series capacities represent simple resonant circuits. In practice this will not be the case due to the presence of self capacity in the coil windings wihch introduces an anti-resonance at a frequency above the resonance of the series combination. The effect of such self capacity can be compensated by slight changes in the coil inductances and in the values of the series tuning condensers together with a reduction of the shunt capacities C_0 and C_0/Φ^2. For example, if it is found that the coil used for inductance L_1 in Fig. 1 has a self capacity C_x, then instead of using the values L_1 and C_1 as computed from the foregoing formulæ the coil should have an inductance L_1 given by

$$L_1'=\frac{1}{n^2}L_1 \quad (8)$$

and the series capacity a value C_1' given by

$$C_1'=nC_1 \quad (9)$$

where

$$n=\frac{1+\sqrt{1+4\frac{C_x}{C_1}}}{2}\doteq 1+\frac{C_x}{C_1}.$$

To complete the compensation, the shunt capacity C_0 should be diminished by the amount

$$\frac{1}{n}C_x.$$

Similar corrections would, of course, be made in the other branches of the circuit.

Other disturbing effects may arise from capacity between the primary and secondary windings of the transformers. These may be eliminated by the use of a grounded electrostatic shield between the windings as indicated by S in Fig. 1 and by grounding one end of each winding.

Another form of the network of the invention is illustrated in Fig. 4. In this form the transformer windings are connected in series and are individually tuned by shunt condensers. In series with each set of tuned windings are included additional tuning capacities corresponding to the additional shunt capacities C_0 and C_0/Φ^2 of Fig. 1.

The primary winding inductances and their individual tuning capacities are designated L_{11}, L_{21}, L_{31} and C_{11}, C_{21}, C_{31} respectively, and the additional primary capacity C_{01}. The values of the secondary elements are related to the primary values by the impedance transformation ratio Φ^2 as in Fig. 1.

As in Fig. 1 each transformer may be regarded as tuned to give the maximum band width consistent with the degree of coupling. The tuning is effected in each case by a combination of shunt and series capacities as in the case of Fig. 1 but the positions of the shunt and series capacities are interchanged. The several series capacities combine to give the single capacities C_{01} and C_{01}/Φ^2. In addition the resonances of the different transformers are so adjusted as to make the several transmission bands contiguous.

Design formulæ for the element values may be obtained by a procedure similar to that outlined in connection with Fig. 1, the requirement for continuity of the transmission band being that

$$\frac{f_1}{f_2}=\frac{f_2}{f_3}=\frac{1-k}{1+k} \quad (10)$$

where k is the common coefficient of coupling of the several transformers and f_1, f_2 and f_3 are the anti-resonance frequencies of $L_{11}C_{11}$, $L_{21}C_{21}$ and $L_{31}C_{31}$, respectively.

The series condensers C_{01} and C_{01}/Φ^2 in the circuit of Fig. 2, and the corresponding shunt condensers in Fig. 1 have the principal function of extending the band width to the maximum attainable. Continuity of the band can be obtained without these condensers, but the band width is reduced in proportion to the factor

$$\sqrt{\frac{1+k}{1-k}}$$

where k has a value of about 0.8 the effect of omitting the additional condensers will be to reduce the band width to about ⅓ of the maximum.

What is claimed is:

1. A broad-band transmission network comprising a plurality of transformers having their primary windings connected between a common pair of input terminals and their secondary windings connected between a common pair of output terminals, said transformers having substantially the same transformation ratios and having their windings closely coupled with substantially the same degrees of coupling, capacitative means for tuning the windings of said transformers individually, the primary and the secondary of each transformer being tuned to the same frequency, and the resonance frequencies of the several transformers having different values arranged substantially in accordance with an expanding geometric series the ratio of the successive terms of which is proportioned with respect to the common degree of coupling of the transformers to provide a single continuous transmission band.

2. A transmission network comprising a plurality of transformers having their primary and their secondary windings connected respectively to common input terminals and common output terminals, said transformers having substantially the same degrees of coupling between their windings and the same transformation ratios, capacitive tuning means for tuning the windings of said transformers individually, the primary and the secondary of each transformer being tuned to the same frequency, and the resonance frequencies of the several transformers being proportioned successively in the ratio

$$\frac{1+k}{1-k},$$

where k denotes the coupling coefficient, whereby a continuous transmission band is provided.

3. A transmission network comprising a pair of transformers having primary and secondary windings, said transformers having the same degree of coupling between their windings and the same transformation ratio, common input terminals for said primary windings and common output terminals for said secondary windings, capacitive tuning means for tuning said windings individually, the primary and the secondary of each transformer being tuned to the same frequency and the two transformers being tuned respective to frequencies in the ratio

$$\frac{1+k}{1-k}$$

where k denotes coefficient of coupling between the windings of the transformers.

4. A transmission network comprising a plurality of transformers having their primary and secondary windings connected in parallel between common input terminals and common output terminals respectively, said transformers having the same degree of coupling between their windings and the same transformation ratios, condensers in series with each of the windings of said transformers adapted to tune the windings of each transformer to the same frequency, the several transformers being tuned to different frequencies substantially in accordance with an expanding geometric series of ratio

$$\frac{1+k}{1-k}$$

where k denotes the coupling coefficient of the windings.

5. A network according to claim 4 including additional means for extending the transmission range comprising capacities connected between the network input terminals and output terminals respectively, said capacities having values in the ratio of the square of the transformation ratio.

6. A transmission network comprising a plurality of transformers having their primary and secondary windings connected in series between common input and common output terminals respectively, said transformers having the same degree of coupling between their windings and the same transformation ratios, capacities in shunt to each winding of said transformers adapted to tune the windings of each transformer to the same frequency, the several transformers being tuned to different frequencies substantially in accordance with an expanding geometric series of ratio

$$\frac{1+k}{1-k},$$

where k is the coupling coefficient of the windings.

7. A network in accordance with claim 6 including additional means for extending the transmission range comprising a capacity connected in series with all of the primary windings and a capacity similarly connected to the secondary windings, said capacities having values in the ratio of the square of the transformation ratio.

8. A transformer having a primary winding and a secondary winding, the windings of said transformer being closely coupled with a degree of coupling less than unity, series and shunt condensers connected to each of said windings whereby each winding is tuned to resonance at equal frequencies and to anti-resonance at equal frequencies, said series and shunt condensers being proportioned with respect to each other and to the degree of coupling between the windings to provide a single continuous transmission band.

9. A transformer having a primary winding and a secondary winding, series and shunt condensers connected to each of said windings whereby each winding is tuned to resonance at equal frequencies and to anti-resonance at equal frequencies, said series and shunt condensers being proportioned with respect to each other to make the ratio of said resonance frequency to said anti-resonance frequency equal to

$$\sqrt{\frac{1-k}{1+k}},$$

where k is the coefficient of coupling between the windings.

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