

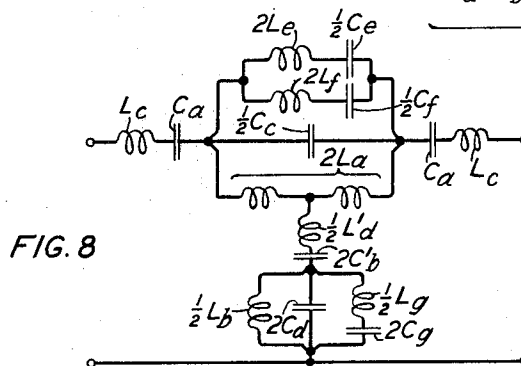
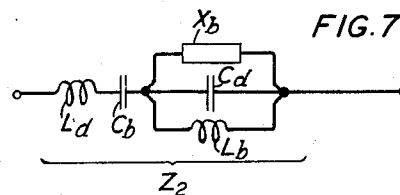
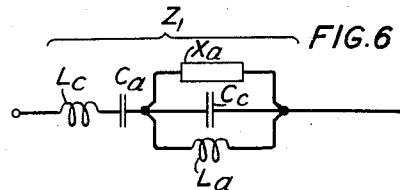
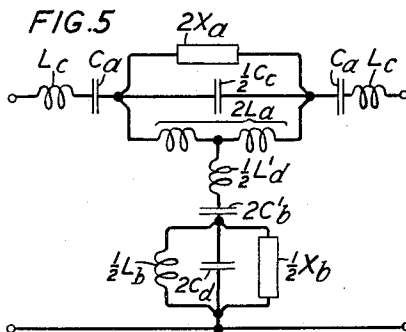
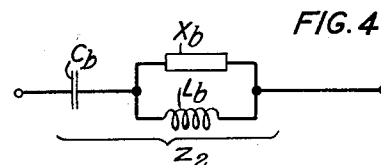
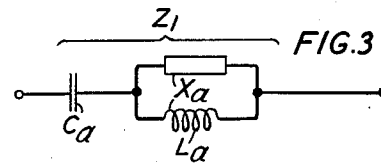
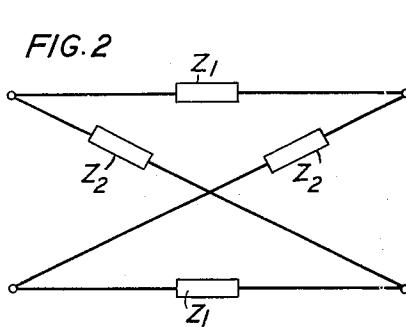
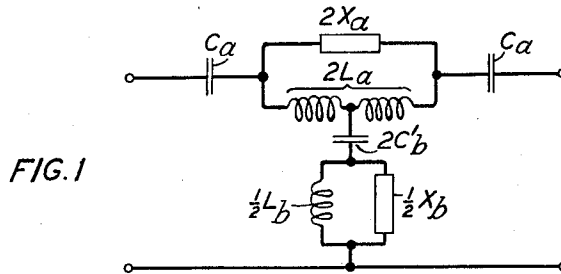
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H. W. BODE

1,944,209

TRANSMISSION NETWORK

Filed July 6, 1932



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1,944,209

TRANSMISSION NETWORK

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6 Claims. (Cl. 178-44)

This invention relates to wave transmission networks and more particularly to reactance networks, such as wave filters, of the unbalanced type in which one side of the circuit may be grounded.

It is well known that the symmetrical lattice type of network in its generalized form can be so designed as to have the same wave transmission properties as any other symmetrical four-terminal network. While this type of network therefore has the greatest possible diversity of transmission characteristics it is limited in its usefulness by the fact that it can not be readily adapted to unbalanced circuits and because of the duplication of impedance elements it involves.

The present invention provides a reactance network of the unbalanced type which has substantially the same degree of generality in respect to its transmission characteristics as a symmetrical lattice including only reactance elements.

The network of the invention consists of a bridged-T network in which the series branches of the T are two equal inductances of finite value coupled together with unity coupling, in combination with a pair of equal impedances connected in series external to the bridged-T. The bridging arm and the shunt arm of the bridged-T are not restricted in character except to the extent that they should consist of reactance elements. The series impedances external to the bridged-T in general will consist only of simple capacitance elements but in certain cases may comprise simple combinations of inductance and capacitance in series.

The nature of the invention will be more fully understood by referring to the following detailed description and to the attached drawing, forming a part thereof, in which:

Figs. 1 and 5 show two general forms of the network of the invention;

Figs. 2, 3, 4, 6 and 7 are schematic diagrams to which reference is made in explaining the invention, and

Fig. 8 represents a specific embodiment of the invention in a wave filter.

The network of Fig. 1, which represents one general form of the invention, comprises a line impedance group made up of a generalized reactance $2X_a$ shunted by an inductance of value $2L_a$ and having two end impedances constituted

by condensers of capacity C_a , and a shunt impedance group consisting of a condenser of capacity $2C_b'$ in series with a parallel combination of an inductance of value

$$\frac{1}{2}L_b$$

and a second generalized reactance

$$\frac{1}{2}X_b.$$

The shunt group is connected between the midpoint of the inductance $2L_a$ and the other side of the network, the arrangement constituting a bridged-T system with external impedances added outside the bridge. The two halves of the coil $2L_a$ are coupled together with unity coupling, for example, by bifilar winding, so that the coil acts as a perfectly coupled transformer of finite inductance.

The network is equivalent in its transmission properties to a symmetrical lattice of the general type shown in Fig. 2, the branch impedances Z_1 and Z_2 of which have the form and values shown in Figs. 3 and 4 respectively. In Fig. 4 the capacity C_b is equal to the effective capacity of C_a and C_b' connected in series, or

$$C_b = \frac{C_a C_b'}{C_a + C_b'}$$

The propagation constant P and the characteristic impedance K of the lattice of Fig. 2, and hence, by virtue of the equivalence, of the network of Fig. 1, are given by the formulæ

$$\tanh(P + jn\pi) = \frac{2}{\sqrt{\frac{Z_1}{Z_2}} + \sqrt{\frac{Z_2}{Z_1}}} \quad (1)$$

and

$$K = \sqrt{Z_1 Z_2} \quad (2)$$

where n is an integer and j is the usual operator

$$\sqrt{-1}.$$

It is to be noticed that Equation (1) is not altered by the interchange of Z_1 and Z_2 except that a phase change of $n\pi$ radians may occur. Physically this means that the impedances Z_1 and Z_2 may be interchanged in the network of Fig. 2 without affecting the transmission properties

otherwise than by introducing a phase reversal of the output currents.

By known theorems relating to reactances, for which reference is made to the Bell System Technical Journal, vol. III, No. 2, April, 1924, A Reactance Theorem, by R. M. Foster, it may be shown that any combination of diverse reactance elements may be transformed to an equivalent combination of the type shown in Figs. 3 and 4 provided the capacities C_a or C_b are permitted to assume any value finite or infinite. Thus, a simple series resonant combination may be obtained by making the reactance X_a in Fig. 3 infinite and a simple parallel resonant combination may be obtained by making X_a a simple capacity and C_a infinitely great.

The steps to be taken in converting any reactive impedance into the form shown by Figs. 3 and 4, in accordance with the method described by Foster in the above mentioned article, are as follows: In Equation (1) of his article Foster gives the general expression for the reactance S , using his notation,

$$S = -iH \frac{(p_1^2 - p^2)(p_3^2 - p^2) \cdots (p_{2n-1}^2 - p^2)}{p(p_2^2 - p^2) \cdots (p_{2n}^2 - p^2)} \quad (3)$$

If, in accordance with his general method, the reactance is represented by a chain of anti-resonant loops connected in series, then the network will include a series condenser C of value

$$C = \frac{p_1^2 p_3^2 \cdots p_{2n-1}^2}{H p_1^2 p_3^2 \cdots p_{2n-1}^2} \quad (4)$$

It will be noted that in the special case where $p_1 = 0$, then C will be infinite, that is, absent from the chain.

The reactance S' of the network with C removed is given by the expression

$$S' = S - \frac{1}{i p C} = -iH \frac{(p_1^2 - p^2)(p_3^2 - p^2) \cdots (p_{2n-1}^2 - p^2)}{p(p_2^2 - p^2)(p_4^2 - p^2) \cdots (p_{2n}^2 - p^2)} - \frac{1}{C} \frac{(p_2^2 - p^2)(p_4^2 - p^2) \cdots (p_{2n}^2 - p^2)}{p(p_2^2 - p^2)(p_4^2 - p^2) \cdots (p_{2n}^2 - p^2)} \quad (5)$$

The numerator of the expression on the right hand side of Equation (5) is a polynominal in p^2 whose roots can be obtained by standard mathematical processes. If the roots are designated $p_a^2, p_b^2, \dots$, then the numerator can be rewritten as

$$-H' p^2 (p_a^2 - p^2)(p_b^2 - p^2) \cdots (p_j^2 - p^2),$$

where H' is some new constant. The reactance S' then becomes

$$S' = iH' p \frac{(p_a^2 - p^2)(p_b^2 - p^2) \cdots (p_j^2 - p^2)}{(p_2^2 - p^2)(p_4^2 - p^2) \cdots (p_{2n}^2 - p^2)} \quad (6)$$

From the mode of derivation this is the reactance of a physical network, and it can be built by Foster's methods as a set of resonant branches connected in parallel, including a parallel inductance L given by

$$L = H' \frac{p_a^2 p_b^2 \cdots p_j^2}{p_2^2 p_4^2 \cdots p_{2n}^2} \quad (7)$$

The inductance L of Equation (7) is the inductance L_a or L_b of Figs. 3 and 4, while the remainder of the parallel resonant branches are represented by the reactance X_a or X_b of those figures.

If, now, a reactance network is designed as a lattice to meet a given set of requirements, for example, in accordance with the principles set forth in my earlier Patent 1,828,454 issued October 20, 1931, it follows that such a lattice can always be transformed into an unbalanced net-

work of the type of Fig. 1, the line capacities C_a having any value finite or infinite. The procedure in the conversion is first of all to transform the impedances Z_1 and Z_2 to the form of Figs. 3 and 4. Usually it is simplest in the initial design to determine Z_1 and Z_2 as chains of anti-resonant circuits or as parallel connected systems of series resonant circuits, in accordance with the rules given in the aforementioned article by Foster. The transformation may be carried out by analytical methods or by a step-by-step application of the elementary substitutions and transformation formulae given in the Bell System Technical Journal, vol. II, No. 1, January, 1923, pages 45 and 46. Whatever the initial form of these impedances may be the determination of the capacities C_a and C_b is always simple, these being the total effective capacities of the impedances at zero frequency.

After converting the impedances to the desired form, the one having the larger series capacity is used as the basis of the design of the line impedance group of the network of Fig. 1. The remainder of the conversion follows naturally from the impedance relationships indicated by the figures. The choice of either one or the other of the impedances for the line group of Fig. 1 does not affect the transmission properties of the resulting network otherwise than by introducing a phase reversal of the output currents, as already described.

In certain cases it will be found that one or possibly both of the impedances Z_1 and Z_2 have no series capacity, that is, the series capacities are infinite and have zero reactance. In such cases the resulting unbalanced network reduces to one of known type.

Another general form of the invention is shown

in Fig. 5, which differs from the network of Fig. 1 in having a condenser of capacity

$$\frac{1}{2} C_c$$

added in shunt with the inductance $2L_a$, an additional inductance L_c in series with each end impedance C_a , an inductance

$$\frac{1}{2} L_{d'}$$

added in series with the capacity $2C_b'$ and a condenser $2C_a$ shunting the inductance

$$\frac{1}{2} L_b.$$

The inductance $L_{d'}$ is equal to L_d minus L_c . The network is equivalent in its transmission properties to a symmetrical lattice having the branch impedances Z_1 and Z_2 shown in Figs. 6 and 7. An advantage of the network shown in Fig. 5 is that the unavoidable inter-winding capacity of the inductance $2L_a$ effective between its outer terminals can be compensated for by a corresponding reduction in the value of the shunting capacity

$$\frac{1}{2} C_c.$$

In this way the theoretical transmission properties of the equivalent lattice, given by Equations (1) and (2), may be realized very closely in the generalized bridged-T network. In order for the network to be physically realizable it is necessary

that the impedance Z_1 , used as the basis of the design of the line impedance group of the network of Fig. 5, shall have the larger series capacity C_a and the smaller series inductance L_c , as compared with the corresponding elements C_b and L_d of impedance Z_2 .

The transmission properties of the network of the invention depend, as stated above, upon the characteristics of the impedances Z_1 and Z_2 . For example, if the reactance $2X_a$ of Fig. 5 consists of two series resonant branches connected in parallel and the reactance

$$\frac{1}{2}X_b$$

is a simple series resonant branch, then the network shown in Fig. 8 will result. In this figure the reactance $2X_a$ is constituted by the parallel connected resonant circuits $2L_c$,

$$\frac{1}{2}C_e$$

and $2L_f$,

$$\frac{1}{2}C_f$$

and the reactance

$$\frac{1}{2}X_b$$

by the single resonant circuit

$$\frac{1}{2}L_g$$

$2C_g$. The other parts of the network correspond to Fig. 5. The branches of the equivalent lattice structure are readily found as indicated above. To design the structure as a band-pass filter, the method described in my aforementioned Patent No. 1,828,454 may be followed for the determination of the values of the impedance elements of the equivalent lattice. Conversion to the bridged-T structure may then be accomplished as explained above.

What is claimed is:

1. A wave transmission network comprising a bridged-T network and a pair of equal reactances connected in series therewith, one on each side thereof, the series arms of said bridged-T being constituted by a pair of equal finite inductances inductively coupled, and the shunt arm and the bridging arm of said bridged-T being lumped impedances of different characteristics.

2. A wave transmission network comprising a bridged-T network and a pair of equal reactances connected in series therewith, one on each side

thereof, the series arms of said bridged-T being constituted by a pair of equal finite inductances coupled series aiding with unity coupling factor, and the shunt arm and the bridging arm of said bridged-T comprising reactive impedances of different characteristics.

3. A wave filter comprising a bridged-T network and a pair of equal reactances connected in series therewith, one on each side thereof, the series arms of said bridged-T being constituted by a pair of equal finite inductances coupled series aiding with unity coupling factor, and the shunt arm and the bridging arm of said bridged-T comprising reactive impedances of different characteristics, said equal reactances being so proportioned with respect to said reactive impedances that said filter transmits with negligible attenuation currents of all frequencies lying between the upper limiting frequency and the lower limiting frequency of the range of frequencies to be transmitted, while approximately extinguishing currents of neighboring frequencies lying outside of said limiting frequencies.

4. A transmission network comprising a bridged-T network and two equal capacitances connected in series with said bridged-T, one on each side, the series arms of said bridged-T consisting of a pair of equal finite inductances coupled series aiding, and the shunt arm and the bridging arm of said bridged-T comprising reactive impedances of different characteristics.

5. A wave transmission network comprising a bridged-T network and a pair of equal reactances connected in series therewith, one on each side thereof, each of said reactances comprising a capacitance and an inductance connected in series relation, the series arms of said bridged-T being constituted by a pair of equal finite inductances coupled series aiding with substantially unity coupling factor, and the shunt arm and the bridging arm of said bridged-T comprising reactive impedances of different characteristics.

6. A wave transmission network comprising a bridged-T network and a pair of equal reactances connected in series therewith, one on each side thereof, each of said reactances comprising a capacitance and an inductance connected in series relation, the series arms of said bridged-T consisting of a pair of finite inductances inductively coupled, the bridging arm of said bridged-T comprising a reactive impedance in parallel with a capacitance, the value of said last-mentioned capacitance being adjusted to compensate for the intertwining capacitance of said pair of coupled inductances.

HENDRIK W. BODE.