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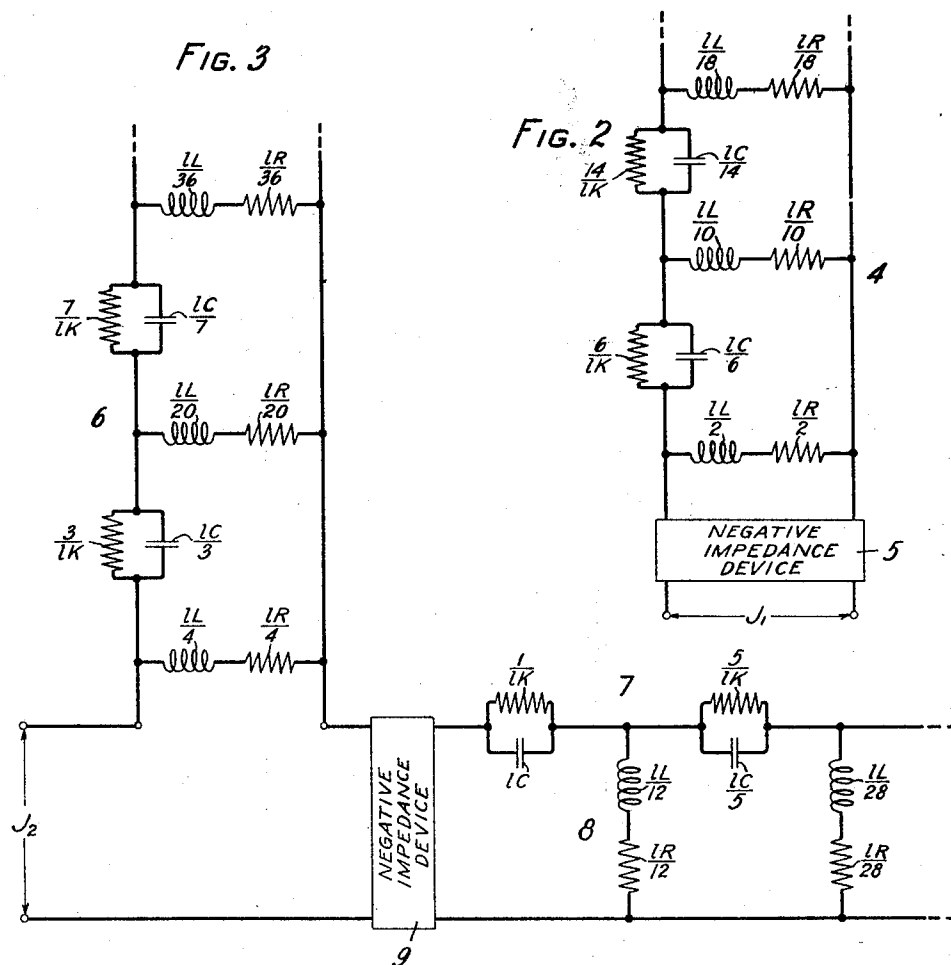
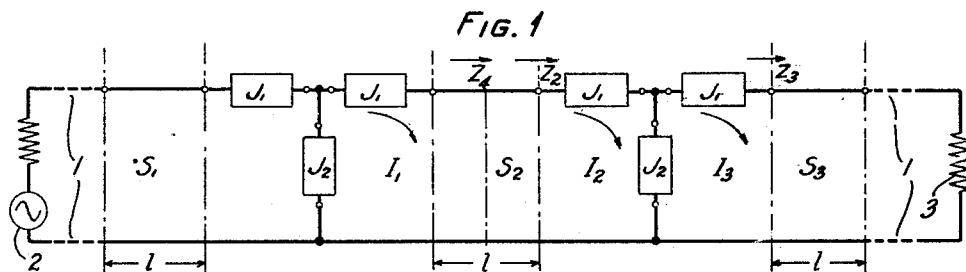
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DISTORTION CORRECTION IN TRANSMISSION SYSTEMS

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FIG. 5

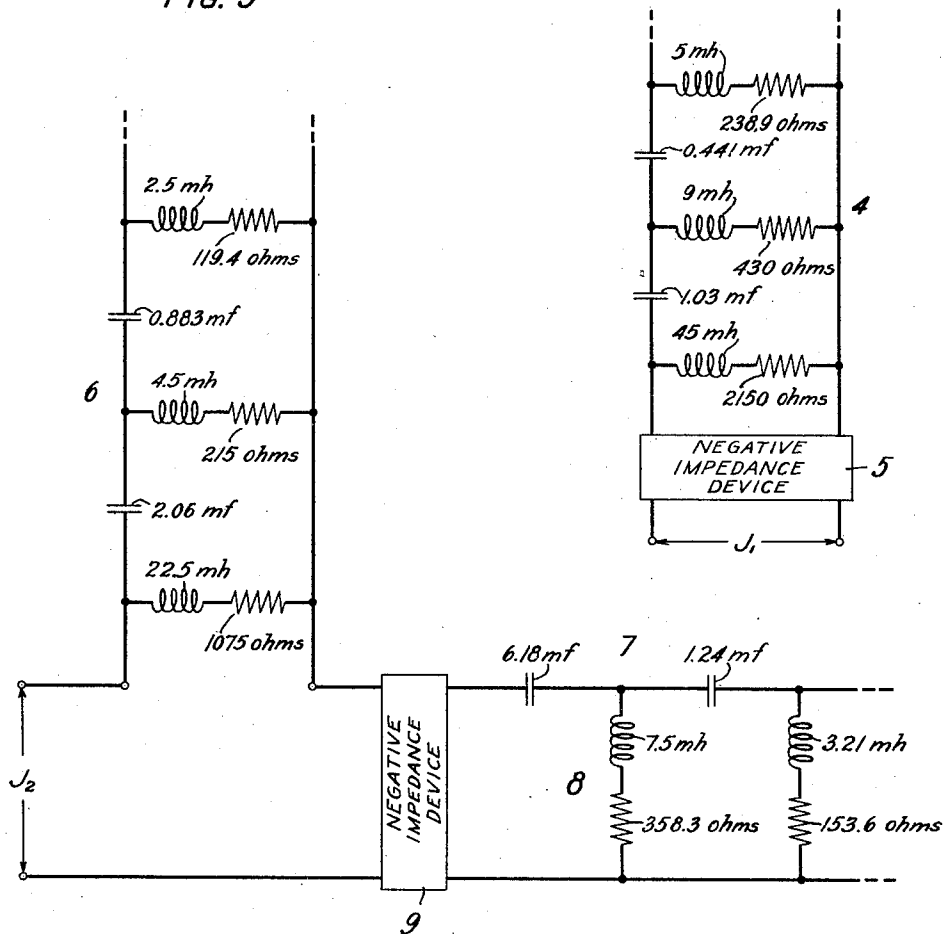


FIG. 4

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## UNITED STATES PATENT OFFICE

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## DISTORTION CORRECTION IN TRANSMISSION SYSTEMS

Application filed October 3, 1928. Serial No. 309,926.

This invention relates to systems for transmitting electrical wave energy, such as telephone transmission systems and the like, and particularly to correction of distortion in such systems.

From an ideal quality standpoint a transmission line should be designed so that the currents received thereover are exact copies of the corresponding currents entering the system at the transmitting end; in other words, the transmission system should be distortionless. When electrical waves, such as telephonic currents, are transmitted over conductors of relatively great length, the distortion produced in the transmitted waves due to the inherent properties of the conductors becomes of increasing importance. In long, high quality systems, such as are now being used for telephone transmission, program broadcasting, television, etc. the distortion produced by the line is often of such magnitude as to impair appreciably the quality of the received waves, and it thus reduces the commercial efficiency of such systems.

The changes produced in waves transmitted over transmission lines are, in general of three different kinds—first, attenuation of the transmitted waves by the line; second, variation in the degree of attenuation of waves of different frequencies by the line; and third, a relative phase change in the transmitted waves of different frequencies. These changes are due to characteristics of the line, such as its resistance and leakage and their improper proportioning.

The Pupin-Campbell and other methods of loading transmission systems have been devised with the object of reducing attenuation in the useful frequency range in such systems. The use of such loading methods in transmission systems has been supplemented in practice by the use of amplifiers, such as the telephone repeater, to compensate for reduction in the amplitude of the transmitted waves. The second kind of changes produced in transmitted waves, an unequal degree of attenuation in the transmitted waves of different frequencies, may be overcome by the use of attenuation equal-

izing networks at different points in the system, this expedient having been brought to a high state of development in practice. Phase corrective networks or phase compensators to supplement other types of correction, such as loading, have been used in connection with lines to correct phase distortion in the transmitted waves over a desired range of frequencies. Other networks have been devised and utilized in connection with transmission lines to combine the functions of phase distortion correction and attenuation equalization in transmitted waves of a desired range of frequencies. The latter networks have been either designed, in part at least, by empirical methods, or else, in their design, all factors contributing to distortion have not been taken into account. Their use, therefore, enables only an approximation of the ideal distortionless line to be obtained.

An object of the invention is to correct distortion in electric wave transmission.

Another and more specific object is to reduce the attenuation in and to minimize variations in the velocity of propagation of electric currents over transmission systems.

In accordance with the invention exact design formulæ have been developed for impedance structures of the T or equivalent types of structures which, when combined with a transmission line, will, without altering the characteristic impedance of the line, enable distortionless transmission to be obtained thereover for waves of all frequencies, and networks have been designed in accordance with these formulæ to accomplish this result.

In a preferred embodiment of the invention, substantially distortionless transmission of waves of all frequencies over a line may be obtained by combining with the line one or more structures of the T type having transmission characteristics simulating those of the line or a given section thereof, but of effectively opposite sign and impedance characteristics simulating those of the line and of the same sign. Specifically, each arm of the T structure comprises one or more two-terminal ladder type networks of a plurality

of sections each having similar impedance elements but varying in value from section to section in accordance with the terms of a continued fraction formed by the expansion of the proper impedance functions given by the design formulae to a desired approximation. Devices which will affectively change the positive impedance of a network to a negative impedance of equal value are connected in tandem with several of these networks to give a negative impedance when required in accordance with the design formulae.

The objects and advantages of the invention will be better understood from the following detailed description when read in connection with the accompanying drawings in which

Fig. 1 shows a diagram of a transmission system embodying the invention;

Figs. 2 and 3 show diagrammatically the construction of the various elements of the distortion correcting structures utilized in the system of Fig. 1; and

Figs. 4 and 5 show the actual values which the elements of the correcting structures of Fig. 1 should have in order to compensate for the distortion in an actual section of telephone cable of given constants.

In Fig. 1 is shown a transmission line for transmitting alternating current waves generated by the source 2 to a load circuit represented by the resistance 3. The line 1 comprises a plurality of smooth line sections  $S_1, S_2, S_3 \dots$  of length  $l$  and a 4-terminal structure of the T type inserted between each two adjacent line sections. Each of the T structures comprises two series arms each of impedance  $J_1$ , represented in the figure by the boxes so labeled, and a shunt arm of impedance  $J_2$ , represented in the figure by a box so labeled. These impedance arms are so designed that each structure will compensate completely for the attenuation and phase distortion in transmitted waves of all frequencies produced by one section of the line of length  $l$ .

In determining mathematically the proper values for the impedances  $J_1$  and  $J_2$ , the following notation will be used:

Let  $Z$  = the characteristic impedance of the line 1 before the T-type impedance networks are inserted therein;

$Z_2$  = the impedance measured across the input terminals of one of the T-type networks;

$Z_3$  = the impedance measured across the output terminals of the same T-type network;

$Z_4$  = the impedance measured at the middle of a line section;

$I_1$  = the current flowing into a particular line section;

$I_2$  = the current flowing out of this line section into the T network, and

$I_3$  = the current flowing out of the T network into the next succeeding section of line;

$l$  = the length of each line section;

$\Gamma$  = the propagation constant of the complex line 1 including the smooth sections  $S_1, S_2 \dots$  without the inserted T networks;

$\alpha$  = the overall attenuation produced by the complex line 1 in waves transmitted thereover;

$\beta j\omega$  = the overall phase shift produced by the complex line 1 in the waves transmitted thereover, where  $\omega = 2\pi$  times the frequency of the transmitted waves;

$L$  = the effective inductance of a section of line 1 for unit length;

$C$  = the effective capacity of a section of line 1 for unit length;

$R$  = the effective resistance of a section of line 1 for unit length;

$K$  = the effective leakance of a section of line 1 for unit length;

If the line 1 is to produce a specified attenuation and a specified phase shift in the transmitted waves of all frequencies, and is also to have a specified impedance, the compensating T-type networks inserted therein should satisfy the following requirements.

The first requirement is

$$\frac{I_3}{I_1} = e^{-\alpha - \beta j\omega} \quad (1)$$

The second requirement is that the line shall have a specified impedance  $Z_4$  at a given point, this point being taken for convenience as the middle of each line section.

The second requirement implies that  $Z_4$  shall be a desired function of frequency.

The following relations exist between the currents in and impedances of line 1

$$\frac{Z_2}{Z} = \frac{Z_4 \cosh \frac{\Gamma l}{2} - Z \sinh \frac{\Gamma l}{2}}{Z \cosh \frac{\Gamma l}{2} - Z_4 \sinh \frac{\Gamma l}{2}} \quad (2)$$

$$\frac{Z_3}{Z} = \frac{Z_4 \cosh \frac{\Gamma l}{2} + Z \sinh \frac{\Gamma l}{2}}{Z \cosh \frac{\Gamma l}{2} + Z_4 \sinh \frac{\Gamma l}{2}} \quad (3)$$

$$\frac{Z_4}{Z} = \frac{Z_2 \cosh \frac{\Gamma l}{2} + Z \sinh \frac{\Gamma l}{2}}{Z \cosh \frac{\Gamma l}{2} + Z_2 \sinh \frac{\Gamma l}{2}} \quad (4)$$

$$Z_2 = J_1 + \frac{1}{\frac{1}{J_2} + \frac{1}{J_1 + Z_3}} \quad (5)$$

$$\frac{I_3}{I_1} = \frac{J_2}{J_1 + J_2 + Z_3} \left\{ \frac{Z \cosh \frac{\Gamma l}{2} - Z_4 \sinh \frac{\Gamma l}{2}}{Z \cosh \frac{\Gamma l}{2} + Z_4 \sinh \frac{\Gamma l}{2}} \right\} e^{-\alpha - \beta j \omega} \quad (6)$$

Solving these equations, it is found that

$$\frac{J_1}{Z} = \frac{\left( Z \sinh \frac{\Gamma l}{2} - Z_4 \cosh \frac{\Gamma l}{2} \right) + \left( Z \sinh \frac{\Gamma l}{2} + Z_4 \cosh \frac{\Gamma l}{2} \right) e^{-\alpha - \beta j \omega}}{\left( Z_4 \sinh \frac{\Gamma l}{2} - Z \cosh \frac{\Gamma l}{2} \right) - \left( Z_4 \sinh \frac{\Gamma l}{2} + Z \cosh \frac{\Gamma l}{2} \right) e^{-\alpha - \beta j \omega}} \quad (7)$$

$$\frac{J_2}{Z} = \frac{2ZZ_4}{\left( Z_4 \sinh \frac{\Gamma l}{2} - Z \cosh \frac{\Gamma l}{2} \right) e^{\alpha + \beta j \omega} - \left( Z_4 \sinh \frac{\Gamma l}{2} + Z \cosh \frac{\Gamma l}{2} \right) e^{-\alpha - \beta j \omega}} \quad (8)$$

In the above expressions,  $\alpha$ ,  $\beta$  and  $Z_4$  may be any desired functions of frequency.

From the mathematical standpoint, then, the line 1 may be made to have any desired impedance and propagation characteristics provided  $J_1$  and  $J_2$  as expressed by Equations (7) and (8) respectively can be realized physically.

Let it be assumed that the impedance  $Z_4$  at the middle of each line section is identical with the characteristic impedance of the line 1. This is a particularly desirable choice, because for this condition, each of the designed  $\mathbf{T}$  structures will have an effective impedance which is exactly equal to that of the line at all junction points, so that there can be no echoes or reflection losses other than those due to improperly matched terminal devices.

For distortionless overall transmission in a given section of line 1 the current  $I_3$  flowing

line 1 from section to section will be distortionless.

The following continued-fraction expansion for  $\tanh x$  is well known:

$$\tanh x = \frac{1}{\frac{x}{1} + \frac{3}{\frac{x}{1} + \frac{5}{\frac{x}{1} + 1}}} \quad (11)$$

and since

$$-\operatorname{csch} 2x = \frac{1}{2} \tanh x - \frac{1}{2} \coth x \quad (12)$$

it appears, therefore, that the hyperbolic cosecant  $2x$  is capable of the following representation:

$$-\operatorname{csch} 2x = \frac{1}{\frac{2}{x} + \frac{3}{\frac{2}{2x} + \frac{5}{\frac{2}{2x} + \frac{7}{\frac{2}{2x} + 1}}}} - \frac{1}{2x} - \frac{1}{\frac{3}{x} + \frac{5}{\frac{3}{2x} + 1}} \quad (13)$$

out of an adjacent one of the  $\mathbf{T}$  structures should be identical with the current  $I_1$  flowing into the preceding section of line, in other words, the overall attenuation  $\alpha$  and the overall phase distortion  $\beta$  for that line section should be zero at all frequencies. Then, if the values  $\alpha = \text{zero}$ ,  $\beta = \text{zero}$ , and  $Z_4 = Z$ , are substituted in Equations (7) and (8) respectively, it is found that for the conditions imposed

$$J_1 = -Z \tanh \frac{\Gamma l}{2} \quad (9)$$

$$J_2 = -Z \operatorname{csch} \Gamma l \quad (10)$$

The following two equations are also well known

$$Z^2 = \frac{Lj\omega + R}{Cj\omega + K} \quad (14)$$

$$\Gamma^2 = (Lj\omega + R)(Cj\omega + K) \quad (15)$$

If in Equations (9) and (10) for  $J_1$  and  $J_2$ , respectively, the hyperbolic tangent and hyperbolic cosecant, are replaced by their equivalent continued-fraction expansions, and  $Z$  and  $\Gamma$  are replaced by their values as given in Equations (14) and (15), the follow-

ing algebraic expressions for  $-J_1$  and  $-J_2$  will be obtained:

$$-J_1 = \frac{1}{\frac{l}{2}(Lj\omega + R)} + \frac{1}{\frac{l}{6}(Cj\omega + K)} + \frac{1}{\frac{l}{10}(Lj\omega + R)} + \frac{1}{\frac{l}{14}(Cj\omega + K)} + \frac{1}{\frac{l}{18}(Lj\omega + R)} + \dots \quad (16)$$

$$-J_2 = \frac{-J_1}{2} + \frac{1}{l(Cj\omega + K)} + \frac{1}{\frac{l}{3 \times 4}(Lj\omega + R)} + \frac{1}{\frac{l}{5 \times 4}(Cj\omega + K)} + \frac{1}{\frac{l}{7 \times 4}(Lj\omega + R)} + \dots \quad (17)$$

To obtain  $J_1$  it is therefore only necessary to provide a structure which is effectively the negative of a structure which will satisfy the continued fraction expansion (16). The former structure is physically realized, as shown in Fig. 2 by a ladder network 4 of the singly infinite type, and in tandem therewith a device 5 which will effectively change the positive impedance of the network 4 to a negative impedance of equal value.

The device 5 may be of any suitable type, for example, the feed-back amplifier described and claimed in the patent to R. C. Mathes, No. 1,779,382, issued October 21, 1930, which will effectively give between two terminals thereof for a wide range of frequencies, the negative of a resistance, inductance, capacity or complex combination thereof utilized as the feed-back impedance. If the structure illustrated in Fig. 1 of the said Mathes patent is used as the negative impedance device 5 in the structure of Fig. 2 of this application, the terminals  $J_1$  of the latter structure would be connected to the terminals 16 and 17 of the former structure, and one pair of terminals of network 4 in said latter structure would be connected to the terminals of box 14 in the circuit of said former structure so that the network 4 acts as the feed-back impedance  $Z$  in said circuit.

The shunt elements of network 4 have admittances corresponding to alternate terms of Equation (16), and the series elements have impedances corresponding to the other terms of Equation (16). For example, the first shunt element of network 4 has an ad-

mittance corresponding to the first term of the continued fraction of Equation (16).

$$\frac{1}{\frac{l}{2}(Lj\omega + R)},$$

and its impedance must therefore be

$$\frac{lL}{2}j\omega + \frac{lR}{2}.$$

This element may be obtained, as indicated in Fig. 2, by an inductance of value

$$\frac{lL}{2}$$

in series with a resistance of value

$$\frac{lR}{2}.$$

Similarly, the second shunt element has an admittance corresponding to the third term of the continued fraction Equation (16),

$$\frac{1}{\frac{l}{10}(Lj\omega + R)},$$

and may be represented therefore (as indicated in Fig. 2) by an inductance of value

$$\frac{lL}{10}$$

in series with a resistance of value

$$\frac{lR}{10}.$$

The third shunt element corresponding to the fifth term of the continued fraction Equa-

tion (16), may be represented by an inductance of value

$$\frac{LL}{18}$$

in series with a resistance of value

$$\frac{LR}{18}$$

The first series element of structure 4 has an impedance corresponding to the second term of the continued fraction expansion Equation (16), and may be represented by a capacity of value

$$\frac{LC}{6}$$

in parallel with a resistance of value

$$\frac{6}{LK}$$

Similarly the second series element of network 4 has an impedance corresponding to the fourth term of the continued fraction Equation (16) and may be represented by a capacity of value

$$\frac{LC}{14}$$

in parallel with a resistance of value

$$\frac{14}{LK}$$

The succeeding shunt and series elements of the singly infinite ladder network 4 may be determined in similar manner from the terms of Equation (16) until the desired approximation of  $-J_1$  is obtained.

To obtain  $J_2$  it is only necessary to provide a structure which is effectively the negative of a structure which will satisfy Equation (17). This structure is physically realized by the complex network of Fig. 3 which comprises a positive part 6 and a negative part 7 in series. The positive part 6 is a network having one-half the impedance of the network 4 which satisfies Equation (16), and is realized, as indicated in Fig. 3, by a singly infinite ladder network which is identical with the network 4 of Fig. 2 except that each impedance element therein has half the value of the corresponding element of network 4. Thus, the first shunt element of network 6 comprises an inductance of value

$$\frac{LL}{4}$$

in series with a resistance of value

$$\frac{LR}{4}$$

and the first series element of network 6 comprises a capacity of value

$$\frac{LC}{3}$$

in parallel with a resistance of value

$$\frac{3}{LK'}$$

and the other elements are similarly obtained.

The negative portion 7 of the structure of Fig. 3 comprises a singly infinite ladder type network 8 the shunt elements of which have admittances corresponding to alternate terms of the second part of Equation (17) (the first part of Equation (17) being the first term,

$$\frac{-J_1}{2})$$

and the series elements of which have impedances corresponding to the other terms of the said second expansion in Equation (17), and in tandem therewith a device 9, similar to the device 5 in Fig. 2, which will effectively change the positive impedance of network 8 to a negative impedance of equal value.

For example, the first series element of network 8 has an impedance corresponding to the second term of Equation (17),

$$\frac{1}{L(Cj\omega + K)}$$

and is represented by a capacity of value  $LC$  in parallel with a resistance of value

$$\frac{1}{LK'}$$

Likewise, the second series element of network 8 has an impedance corresponding to the fourth term of Equation (17), and is represented by a capacity of value

$$\frac{LC}{5}$$

shunted by a resistance of value

$$\frac{5}{LK'}$$

The first shunt element of network 8 has an admittance which corresponds to the third term of Equation (17),

$$\frac{1}{\frac{l}{12}(Lj\omega + R)}$$

and is represented by an inductance of value

$$\frac{LL}{12}$$

in series with a resistance of impedance

$$\frac{LR}{12}$$

Likewise, the second shunt element of network 8 has an admittance corresponding to the fifth term of Equation (17), and may be represented by an inductance of value

$$\frac{L}{28}$$

in series with a resistance of value

$$\frac{LR}{28}$$

Succeeding shunt and series elements for networks 6 and 8 are determined in similar manner from Equation (17) until the desired approximation of  $J_2$  is obtained.

The following numerical example will give a conception of the magnitudes of the inductances, capacities, and resistances which would be needed in the arms of a  $T$  structure such as has been described to compensate completely for attenuation and phase distortion in an actual section of line of known constants. A 100 mile section of 16-gauge cable has been used for illustration purposes. The cable has a capacity  $C=0.062$  mf. per mile, a resistance  $R=43$  ohms per mile, an inductance  $L=0.9$  mh. per mile and a leakance  $K$  assumed to be negligibly small. The values of the various elements making up the structures  $J_1$  for this case are shown in Fig. 4. Referring to this figure it will be noted that the first shunt element of the  $J_1$  structure comprises an inductance.

$$\frac{L}{2}$$

which is equivalent to 45 mh., and a resistance

$$\frac{LR}{2}$$

which is equivalent to 2,150 ohms, the second shunt element comprises an inductance of 9 mh. in series with a resistance of 430 ohms, and the third shunt element comprises an inductance of 5 mh. and a resistance of 238.9 ohms. In the case of the series elements of the  $J_1$  structure the capacity

$$\frac{LC}{6} = 1.03 \text{ mf.}$$

and the resistance

$$\frac{6}{LK} = \infty.$$

Therefore, no resistance in shunt to the capacity need be used. The second series element comprises a capacity of 0.441 mf., and so on.

The values of the various elements in the network  $J_2$  of each  $T$  structure for the line of the particular constants given are shown in Fig. 5. Referring to that figure, it will be noted that in the positive portion 6 thereof the first shunt element comprises an in-

ductance of 22.5 mh. in series with a resistance of 1075 ohms, the second shunt element an inductance of 4.5 mh. in series with the resistance of 215 ohms, etc. The first series element in the network 6 is a capacity of 2.06 mf. and the second series element a capacity of 0.883 mf.

In the network 8 in the negative portion 7 of the structure  $J_2$  in Fig. 4, the first series element comprises a capacity of 6.18 mf. and the second series element a capacity of 1.24 mf., while the first shunt element thereof comprises an inductance of 7.5 mh. in series with a resistance of 358.3 ohms and the second shunt element an inductance of 3.1 mh. in series with a resistance of 153.6 ohms.

It will be noted that the largest inductance used in either the  $J_1$  or the  $J_2$  parts of the  $T$  structure has an inductance of only 45 mh. and the largest capacity in these networks has a value of 6.18 mf. It is apparent therefore that the  $T$  structures of the invention can be easily constructed to compensate for the distortion in 16-gauge cable for as great a length as 100 miles. For shorter lengths of cable, of course, the network elements would be even smaller.

From the above it is apparent that transmission lines can be constructed in accordance with the methods of the invention to give distortionless transmission without any reflection losses whatever, and this over a sufficiently wide range of frequencies as to permit the use of the very wide frequency bands desirable in high quality telephone circuits, picture transmission, television, etc., by inserting  $T$  networks such as have been described above at comparatively rare intervals along the line. The form of these compensating networks is determined solely by the requirement that the propagation shall be distortionless, and is not in any way influenced by a particular choice for the value of the impedance  $Z_0$ . Distortionless transmission will always be obtained with such a built-up line, no matter what type of terminal apparatus is used, provided this terminal apparatus is inserted at the mid-point of a line section.

It should be noted that each of the impedance elements used in the  $T$  structures of the invention which has been illustrated and described contributes both to attenuation distortion correction and phase distortion correction, this being an important distinguishing feature from the structures of the prior art in which attenuation distortion correction or phase distortion correction is the sole function of separate elements therein.

Although in the specific embodiment of the invention illustrated and described, the correction structures have been of the  $T$ -type, it is to be understood that the invention is not so limited, but that well known equivalents of  $T$  networks may be substituted therefor in which case the impedance arms of the equivalent



lent networks may be constructed of continued fraction networks and negative impedance devices designed in accordance with the principles of the invention as described above.

Although the invention as described has been principally directed to the obtaining of complete compensation for attenuation and phase distortion, it is apparent that it may be utilized for obtaining any desired degree of approximation of this condition. Sufficiently good results may be obtained in some cases with a small number of series and shunt elements in the arms of the T or equivalent structures.

Although the invention has been described in connection with a particular transmission system comprising a plurality of uniform smooth sections of line and compensating structures spaced at equal intervals along the transmission line, it is not so limited, but the system may comprise a smooth or a loaded line in combination with a single distortion compensating structure, or a plurality of distortion compensating structures spaced at unequal intervals along the line or any other combination within the scope of the appended claims.

What is claimed:

1. In combination in a wave transmission system, a transmission line, and means for neutralizing amplitude and phase distortion in waves of all frequencies transmitted thereover, said means comprising an electrical network inserted in said line in the path of said waves and having a plurality of impedance arms connected with said line in different ways, said network having substantially the same characteristic impedance as said line and a propagation constant which is substantially the negative of that of said line.

2. In a wave transmission system, a transmission line, and means associated therewith for neutralizing amplitude and phase distortion in waves of all frequencies transmitted thereover, said means comprising an electrical structure inserted in said line in the path of said waves and having a series impedance arm and a shunt impedance arm, said structure as a whole having a characteristic impedance which is substantially the same as that of said line and a propagation constant which is substantially the negative of that of said line.

3. In combination in a wave transmission system, a transmission line, and means for neutralizing amplitude and phase distortion in waves of all frequencies transmitted over said line, said means comprising an electrical structure of the T type having a shunt arm and two series arms inserted in said line and having a characteristic impedance which is substantially the same as that of said line and a propagation constant which is substantially the opposite of that of said line.

4. A combination in accordance with that

of claim 3 in which at least one of the arms of said T-type structure includes an impedance network of the singly infinite ladder type, said network being connected effectively in series with said one arm.

5. A combination in accordance with that of claim 3 and in which at least one of the arms of said T-type structure includes an impedance network of the singly infinite type in tandem with a device which will effectively change the positive impedance of said network to a negative impedance of equal value, said network being connected effectively in series with said one arm.

6. A combination in accordance with that of claim 3 in which at least one of the arms of said T-type structure includes an impedance network of the singly infinite ladder type comprising a plurality of sections each having similar impedance elements the values of which vary from section to section in accordance with the terms of a continued fraction expansion of an impedance function representing the impedance of a corresponding arm of the equivalent T network of said line.

7. A combination in accordance with that of claim 3 in which at least one of the arms of said T-type structure includes an impedance network of the singly infinite ladder type of a plurality of sections each having impedance elements the values of which vary from section to section in accordance with the terms of a continued fraction expansion of an impedance function representing the impedance of a corresponding arm of the equivalent T network of said line, and a device which will effectively change the resultant positive impedance of said network to a negative impedance of equal value, said negative impedance being connected effectively in series with said arm.

8. In combination in a wave transmission system, a transmission line, and means for correcting phase and amplitude distortion in waves of all frequencies transmitted over said line, said means comprising a T type electrical structure having one arm in shunt with said line of impedance substantially equal to  $-Z \operatorname{csch} \Gamma l$  and two arms in series with said line each of impedance substantially equal to

$$-Z \tanh \frac{\Gamma l}{2},$$

where Z is the characteristic impedance and  $\Gamma$  the propagation constant of said transmission line without said T structure and  $l$  is the length of said line.

9. A combination in accordance with that of claim 3 and in which each of the series arms of said T structure comprises a ladder network of the continued fraction type hav-

ing a characteristic impedance of substantially

$$Z \tanh \frac{\Gamma l}{2}$$

in tandem with a device which will effectively change its positive impedance to a negative impedance of equal value, and in which the shunt arm of said T-structure comprises a ladder network of the continued fraction type having an impedance which is substantially equal to

$$\frac{Z}{2} \coth \frac{\Gamma l}{2},$$

in tandem with a device which will effectively change its positive impedance to a negative impedance of equal value, and in series therewith a ladder type network of the continued fraction type having a characteristic impedance which is substantially equal to

$$\frac{Z}{2} \tanh \frac{\Gamma l}{2},$$

where Z is the characteristic impedance and  $\Gamma$  the propagation constant of said transmission line without said T structure and  $l$  is the length of said line.

10. In combination in a wave transmission system, a transmission line, and means for correcting phase and amplitude distortion in waves of all frequencies transmitted thereover, said means comprising a T-type electrical structure having one arm in shunt with said line of impedance substantially equal to  $-Z \operatorname{csch} \Gamma l$  and two arms in series with said line and each of impedance substantially equal to

$$-Z \tanh \frac{\Gamma l}{2},$$

where  $\Gamma$  is the propagation constant and Z the characteristic impedance of said transmission line without said T structure and  $l$  is the length of said line, each of said series arms comprising a two-terminal ladder network of a plurality of sections, each having similar impedance elements the values of which vary in value from section to section in accordance with the terms of a continued fraction formed by the expansion of

$$Z \tanh \frac{\Gamma l}{2}$$

to the desired degree of approximation in tandem with a device which will effectively change the net positive impedance of said ladder network to a negative impedance of equal value.

11. The combination with a transmission line, of a T-type structure designed to substantially compensate for phase and amplitude distortion in waves of all frequencies transmitted over said line, said T-type structure having two arms in series with said line,

each of characteristic impedance substantially equal to

$$-Z \tanh \frac{\Gamma l}{2}$$

and an arm in shunt with said line of impedance substantially equal to  $-Z \operatorname{csch} \Gamma l$ , where Z is the characteristic impedance and  $\Gamma$  the propagation constant for said transmission line without said T structure and  $l$  is the length of said line, said shunt arm comprising a two-terminal ladder network of a plurality of sections, each having similar impedance elements which vary in value from section to section in accordance with the terms of a continued fraction expansion of

$$\frac{Z}{2} \tanh \frac{\Gamma l}{2}$$

to the desired degree of approximation in series with a two-terminal ladder network of a plurality of sections, each having similar impedance elements which vary in value from section to section in accordance with the terms of a continued fraction formed by the expansion of

$$\frac{Z}{2} \coth \frac{\Gamma l}{2}$$

to the desired degree of approximation, and in tandem with the last-mentioned network a device which will effectively change its net positive impedance to a negative impedance of equal value.

12. The combination of claim 10, and in which each section of the ladder type network in each series arm comprises a parallel combination of capacity and resistance in series with the terminals of said network and a series combination of resistance and inductance in shunt with said terminals.

13. The combination of claim 11, and in which each section of one of the ladder type networks in said shunt arm comprises a parallel combination of resistance and capacity in series with the terminals of said network and a series combination of resistance and inductance in shunt with said terminals.

14. A combination in accordance with that of claim 8 and in which said shunt arm of said T structure comprises two multi-section ladder type networks, each network having similar impedance elements which vary in value from section to section in accordance with the terms of a continued fraction formed by the expansion of different functions, said different functions being

$$\frac{1}{2} \tanh \frac{\Gamma l}{2}$$

and

$$-\frac{1}{2} \coth \frac{\Gamma l}{2},$$

respectively, and each of said series arms comprises a ladder type network having a

plurality of sections, each having similar impedance elements which vary in value from section to section in accordance with the terms of a continued fraction formed by the expansion of

$$-Z \tanh \frac{\Gamma l}{2}$$

the number of sections of the networks in the series and shunt arms depending on the degree of approximation to complete distortion correction desired.

15. The combination with a line for transmitting waves of a plurality of frequencies, of a structure for correcting for the distortion in phase and amplitude produced in said waves in transmission over said line, said structure being of the T-type having two arms in series with said line and one arm in shunt therewith, each of said series arms and said shunt arm including a network of impedance elements, each of said impedance elements contributing both to the phase distortion correction and to the amplitude distortion correction in the transmitted waves.

16. The combination of claim 8 in which said network in each of the arms of said structure is of the singly infinite ladder type.

In witness whereof, I hereunto subscribe my name this 2nd day of October, 1928.

THORNTON C. FRY.

In witness whereof, I hereunto subscribe my name this 2nd day of October, 1928.

DIRK J. STRUIK.