

Oct. 20, 1931.

H. W. BODE

1,828,454

TRANSMISSION NETWORK

Filed July 3, 1930

5 Sheets-Sheet 1

Fig. 1.

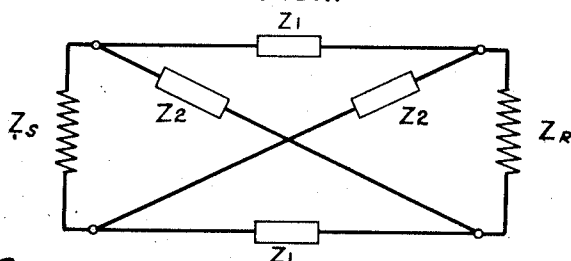


Fig. 2.

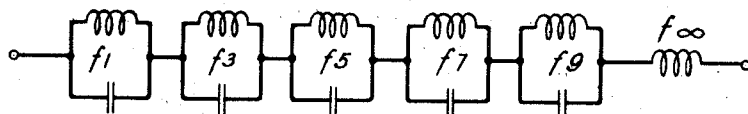


Fig. 3.

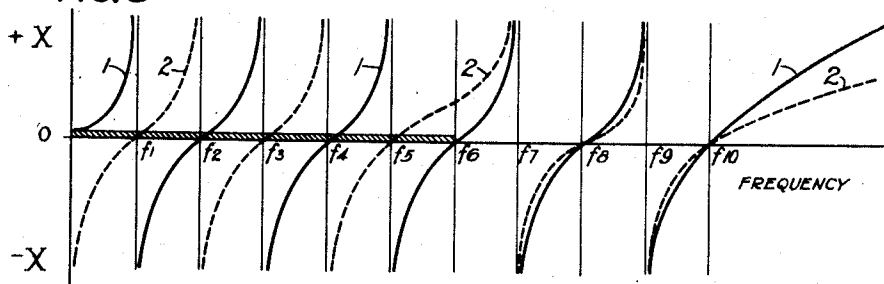


Fig. 4.

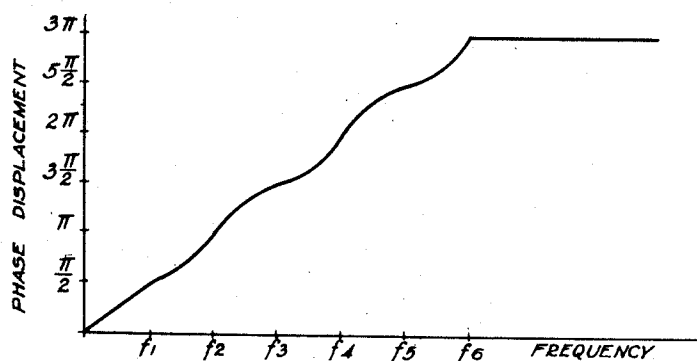
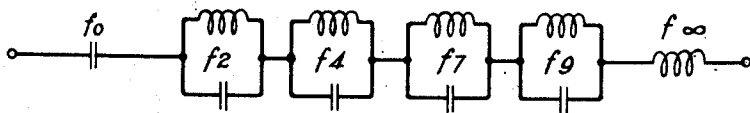


Fig. 5

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5 Sheets-Sheet 2

Fig. 6.

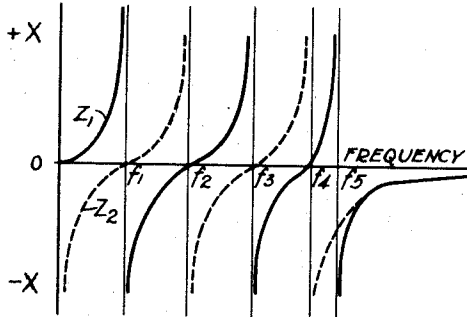


Fig. 7.

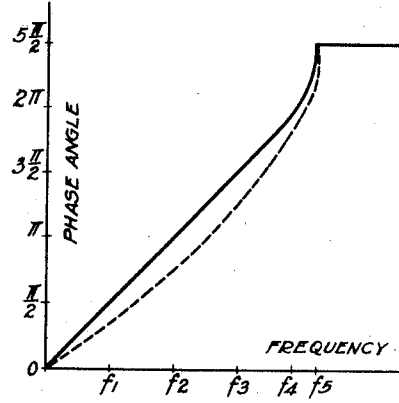


Fig. 8.

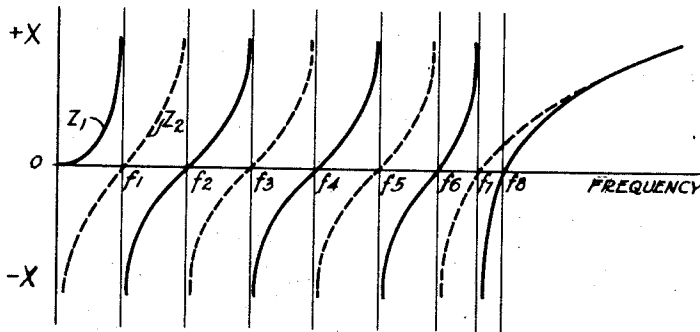
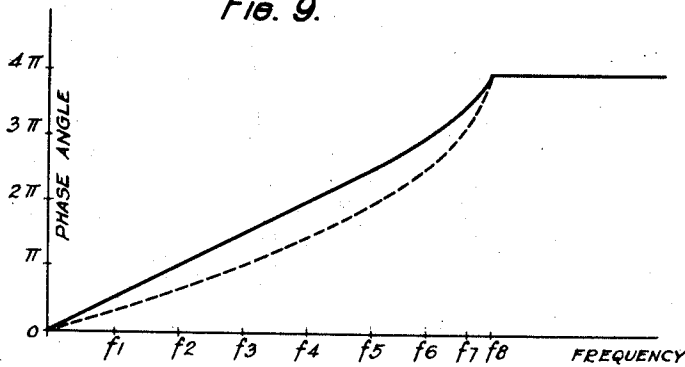


Fig. 9.



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TRANSMISSION NETWORK

Filed July 3, 1930

5 Sheets-Sheet 3

Fig. 10.

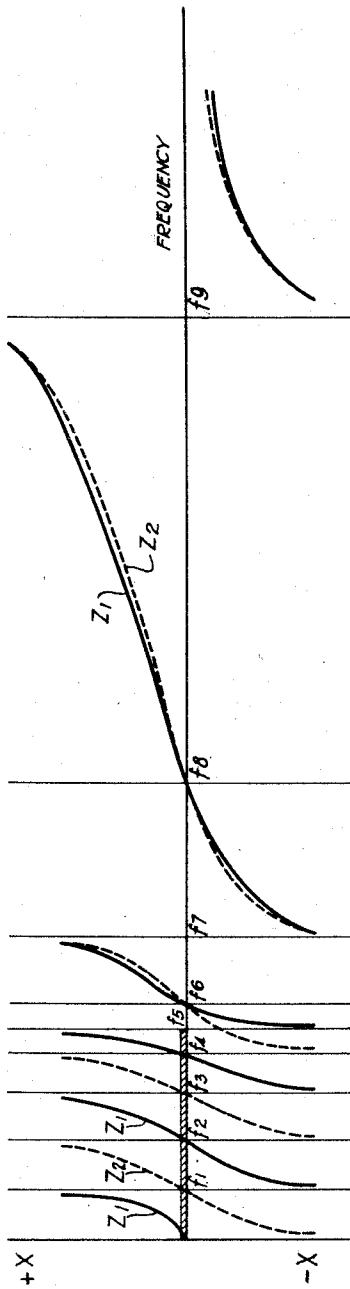
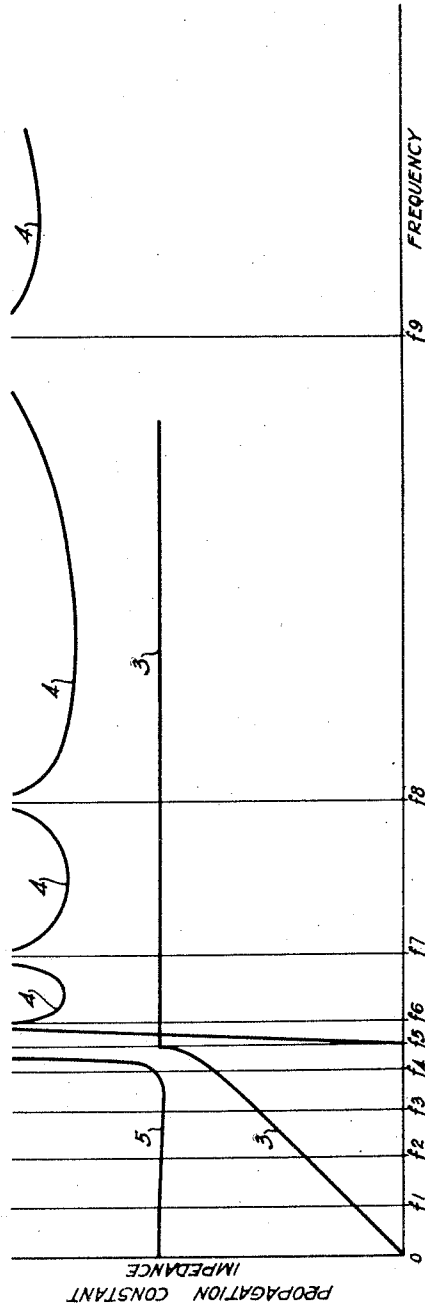


Fig. 11.



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1,828,454

TRANSMISSION NETWORK

Filed July 3, 1930

5 Sheets-Sheet 4

FIG. 12.

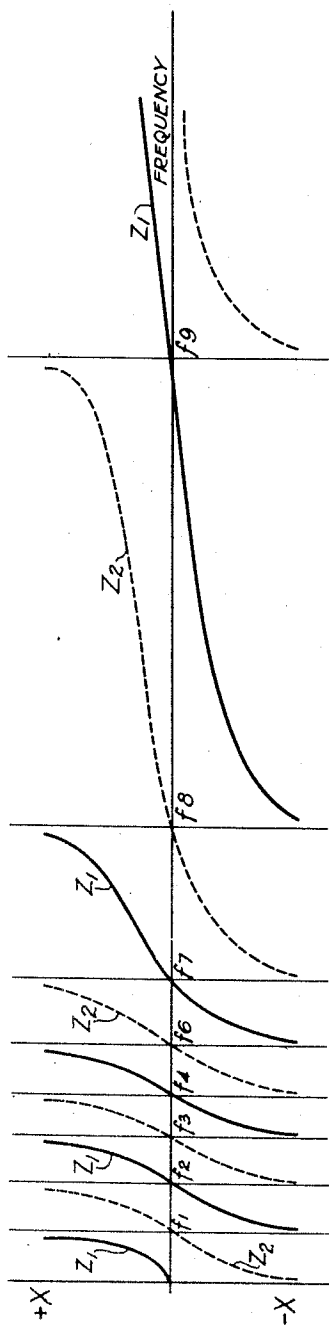
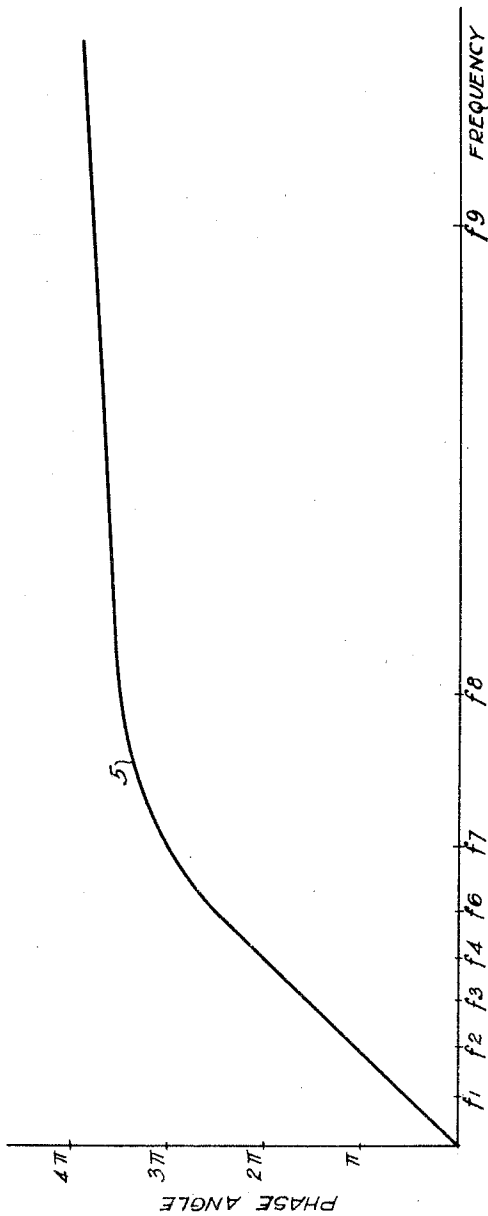


FIG. 13.



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1,828,454

TRANSMISSION NETWORK

Filed July 3, 1930

5 Sheets-Sheet 5

Fig. 14

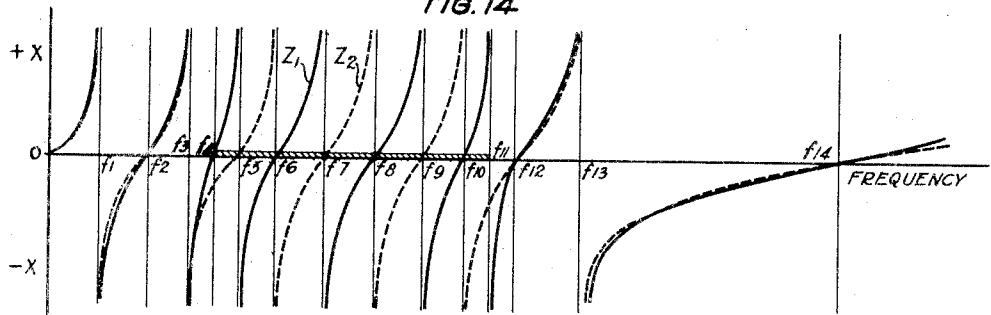


Fig. 15.

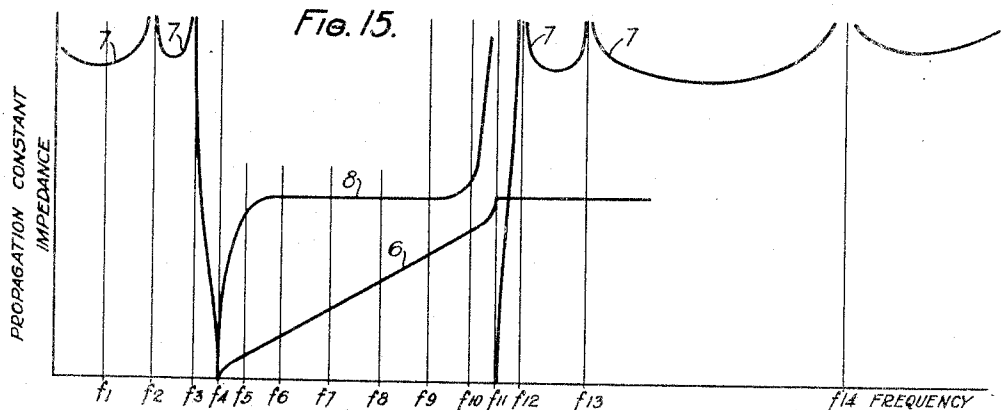


Fig. 16.

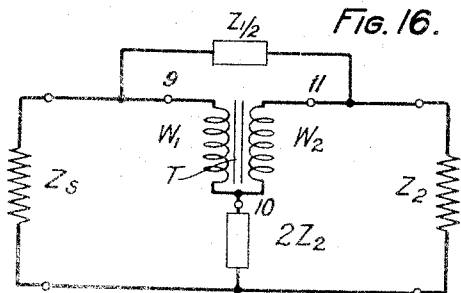


Fig. 17.

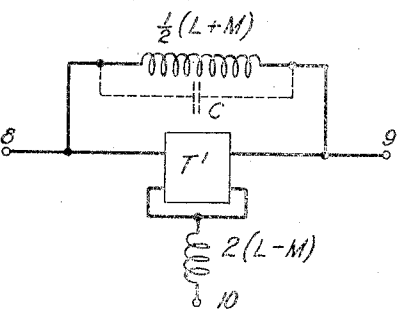
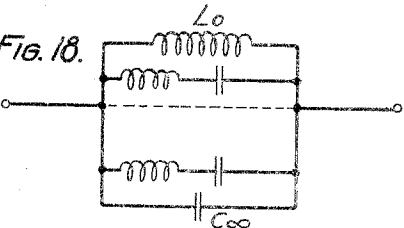


Fig. 18.



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TRANSMISSION NETWORK

Application filed July 3, 1930. Serial No. 465,522.

This invention relates to wave transmission networks and more particularly to networks having broad-band transmission characteristics such, for example, as broad-band wave filters and delay networks. It has for its principal object the improvement of the transmission characteristics of networks of this type particularly with respect to the uniformity of the delay or phase characteristics in their transmission ranges and to the maintenance of high attenuation outside the transmission range.

Other objects are the improvement of the impedance characteristics in the transmission bands and the provision of independent control of the impedance and the transmission characteristics. A further object is to reduce the number of impedance elements required to obtain a desired transmission characteristic.

These objects are achieved, in accordance with the invention, by constructing the network in the form of a single lattice section, or in a form equivalent thereto, the branches of which are reactances having multiple resonances, and by spacing the resonance frequencies in a particular manner as herein-after described.

An important general property of a symmetrical lattice network is that its propagation constant and its characteristics impedance are mutually independent. By constructing the networks of the invention in this form and using only a single section having branches of a considerable degree of structural complexity, a large number of independent design parameters are obtained which enable the impedance and the transmission characteristics to be controlled in a very precise manner. In this way it has been found possible to obtain a high degree of linearity of the phase characteristic within the transmission band coupled with uniform impedance in the same range and a very high attenuation outside the band.

Of the attached drawings,

Fig. 1 represents one schematic form of the networks of the invention;

Figs. 2 and 4 show typical forms of the network branch impedances;

Fig. 3 shows the variations of the branch impedances and illustrates the principle of band formation;

Fig. 5 is a phase characteristic used for reference purposes;

Figs. 6 and 8 illustrate typical impedance arrangements in accordance with the invention for securing linear phase characteristics;

Figs. 7 and 9 are phase characteristics for the arrangements of Figs. 6 and 8;

Fig. 10 shows the impedance arrangements of a low-pass filter having both impedance and propagation constant controls in accordance with the invention;

Fig. 11 shows the transmission characteristics corresponding to Fig. 10;

Figs. 12 and 13 illustrate the application of the invention to an all-pass network;

Figs. 14 and 15 illustrate the application of the invention to a band-pass filter;

Fig. 16 shows a schematic of a modified form of the invention; and

Figs. 17 and 18 are theoretical diagrams relating to the structure of Fig. 16.

The principles of the invention will be developed and illustrated in connection with their application to the design of filters of the low-pass type and their extension to other types of networks will be discussed later. It will be assumed that the desirable characteristics are as follows:

1. A linear phase characteristic within the transmission band;

2. Substantially infinite attenuation outside the transmission band; and

3. Uniform impedance throughout the transmission band.

The lattice structure of Fig. 1, comprising two equal line impedances Z_1 and two equal impedances Z_2 connected diagonally between

the input and output terminals, represents a typical schematic form of the networks of the invention. Impedances Z_s and Z_r represent terminal loads between which the network may be connected. The propagation constant P and the characteristic impedance K of this network are given by

$$\tanh \frac{P}{2} = \sqrt{\frac{Z_1}{Z_2}} \quad (1)$$

$$\text{and} \quad K = \sqrt{Z_1 Z_2} \quad (2)$$

and are clearly independent since the ratio

$$\frac{Z_1}{Z_2}$$

may be varied without changing the value of the product $Z_1 Z_2$.

For linear phase shift it is necessary that the impedances Z_1 and Z_2 be so proportional that, within the transmission band

$$\sqrt{\frac{Z_1}{Z_2}} = \tanh \frac{P}{2} = j \tan qf \quad (3)$$

25

$$Z_1 = jX_{01} \frac{\left(1 - \frac{f^2}{f_2^2}\right) \left(1 - \frac{f^2}{f_4^2}\right) \left(1 - \frac{f^2}{f_6^2}\right) \left(1 - \frac{f^2}{f_8^2}\right) \left(1 - \frac{f^2}{f_{10}^2}\right)}{\left(1 - \frac{f^2}{f_1^2}\right) \left(1 - \frac{f^2}{f_3^2}\right) \left(1 - \frac{f^2}{f_5^2}\right) \left(1 - \frac{f^2}{f_7^2}\right) \left(1 - \frac{f^2}{f_9^2}\right)} \quad (4)$$

30

f denoting frequency and q being a constant. Infinite attenuation outside the transmission band requires that $\tanh P/2$ shall be unity in this range, or that $Z_1 = Z_2$, and constant impedance within the band further requires that the product $Z_1 Z_2$ shall be approximately constant throughout the band. The simultaneous satisfaction of all three requirements would apparently require a greater number of variables than the two impedances Z_1 and Z_2 , but in the structures of the invention the necessary number of parameters is provided by the multiple resonance frequencies of the impedances and moreover the circuit arrangements are such that the first two requirements are simultaneously met by the proportioning of one set of design parameters.

The impedances Z_1 and Z_2 are pure reactances of general type and of any degree of complexity. They may have any of a wide variety of schematic forms, for example, a chain of anti-resonant circuits, or a parallel connected system of resonant circuits, or any combination of these arrangements. Whatever the schematic form may be the value of the reactance may be expressed in terms of a numerical constant and a product of similar factors involving the resonance and anti-resonance frequencies. This general expression for the impedance of a two-terminal reactance is discussed in an article by R. M. Foster. A Reactance Theorem, Bell System Technical Journal, Vol. III, No. 2, April 1924, to which reference is made for further particulars.

In accordance with the invention the impedances Z_1 and Z_2 are so arranged that their resonance and anti-resonance frequencies provide two sets of design parameters by means of which the frequency variations of the propagation constant and the characteristic impedance are independently controlled. The constant coefficients of the reactances provide additional parameters for the control of the magnitudes. The manner in which the independent control of the characteristics is effected will be seen from the following example illustrating the general arrangement of the impedances in the case of a low-pass filter.

Let it be assumed that Z_1 consists of a chain of six anti-resonant circuits as shown in Fig. 2, the simple inductance being regarded as a circuit anti-resonant at infinite frequency. In the figure each anti-resonant circuit is designated by its frequency of anti-resonance f_1, f_2 etc. The impedance of this chain may be expressed as

in which X_{01} is the limiting value of the reactance at frequencies close to zero and the frequencies f_2, f_4, f_6 , etc. are the resonance frequencies in their successive order. The variation of the reactance is shown by the solid line curve 1 of Fig. 2 in which abscissae represent frequency and the ordinates the values of the reactance. The values are alternately infinite and zero at the successive frequencies $f_1, f_2, f_3, \dots, f_{10}$.

If it is desired that the network shall have a single pass-band extending from zero to f_6 , the necessary variation of Z_2 follows readily from the requirement that Z_1 and Z_2 shall have opposite signs everywhere throughout the band and shall have the same sign everywhere outside the band. The dotted curve 2 of Fig. 3 shows the required character of Z_2 and the schematic of Fig. 4 shows a suitable structure of Z_2 to give this variation. Z_2 has infinite values at zero frequency, f_2, f_4, f_7, f_9 and infinity and has zero values at f_1, f_3, f_5, f_8 , and f_{10} ; its value is given by

$$Z_2 = jX_{02} \frac{b_1 b_3 b_5 b_7 b_9}{b_2 b_4 b_6 b_8 b_{10}} \quad (5)$$

in which the notation b_1, b_2 etc. is used to denote factors of the type

$$\left(1 - \frac{f^2}{f_1^2}\right) \text{ etc.}$$

The factors X_{01} and X_{02} representing the limiting values of the reactances at zero frequency have values that are readily obtained by inspection. In the case of Z_1 , the reactance is inductive at zero frequency and is obviously that due to the sum of all of the inductances. For Z_2 it is the capacity reactance of the end condenser, indicated as anti-resonant at zero frequency.

From Equations 4 and 5 the expression for the propagation constant and the characteristic impedance follow:

$$\tanh \frac{P}{2} = \pm \sqrt{\frac{X_{01}}{X_{02}}} \frac{b_2 b_4 \sqrt{b_6}}{b_1 b_3 b_5}, \quad (6)$$

and

$$K = \pm j \sqrt{X_{01} X_{02}} \frac{\sqrt{b_6 b_8 b_{10}}}{b_7 b_9}, \quad (7)$$

the signs being chosen to make K positive

in the band and $\tanh \frac{P}{2}$ positive outside the band.

These equations show that the frequency factors b_1 to b_5 independently control the propagation constant while the factors b_7 to b_{10} inclusive modify only the characteristic impedance. Since the resonance and anti-resonance frequencies may be adjusted arbitrarily these frequency factors constitute independent design parameters which may be used for controlling the characteristics.

$$\tan m\pi x = \left[\frac{(1 - \frac{x^2}{(2a)^2})(1 - \frac{x^2}{(4a)^2}) \cdots (1 - \frac{x^2}{(1-2a)^2})}{(1 - \frac{x^2}{a^2})(1 - \frac{x^2}{(3a)^2}) \cdots (1 - \frac{x^2}{(1-a)^2})} \right] \times \left[\frac{(1 - x^2)(1 - \frac{x^2}{(1+2a)^2}) \cdots}{(1 - \frac{x^2}{(1+a)^2})(1 - \frac{x^2}{(1+3a)^2}) \cdots} \right] \quad (9)$$

Obviously as many independent frequency parameters as may be desired can be provided by simply increasing the number of anti-resonant combinations in each branch and dividing the resonance frequencies as required between the transmission and attenuation ranges.

Control of phase characteristic

The spacing of the resonance frequencies to secure the desired variations of the propagation constant and the characteristic impedance will now be considered, the requirement of linear phase shift in the band being dealt with first.

As stated by Equation 3, linearity of the phase shift characteristic requires that the ratio

$$\sqrt{\frac{Z_1}{Z_2}}$$

shall approximate the tangent of an angle proportional to frequency in the transmission range. The design of the networks of the invention to achieve this relationship is based on certain mathematical theorems relating to the approximation of the tangent function by expressions of the form given by Equation 6.

The tangent of an angle may be expressed as an infinite product as follows:

$$\tan \frac{\pi}{2} \alpha = \frac{\pi}{2} \frac{(1 - \frac{\alpha^2}{2^2})(1 - \frac{\alpha^2}{4^2}) \cdots}{(1 - \alpha^2)(1 - \frac{\alpha^2}{3^2}) \cdots} \quad (8)$$

The form of this expansion is similar to the product expression for the reactance of an infinite chain of anti-resonant circuits, the resonances and anti-resonances of which are spaced at equal frequency intervals. Exact simulation of this expansion could, of course, be achieved only by the use of an infinite reactance system, but this is not necessary since only a limited frequency range is to be considered.

The infinite expansion of Equation 8 may be written in the form

$$a = \frac{1}{2m}.$$

The first bracketed expression contains the first $2m-1$ factors and the second expression contains the remainder. If the first and second bracketed expressions be denoted by R_1 and R_2 respectively then

$$\tan m\pi x = R_1 R_2.$$

The factor R_1 will have zero and infinite values for the same values of x as the tangent function in the range from zero to $x=1-a$, but the finite values will be modified

fied in proportion to the value of R_2 . In the same range R_2 will vary continuously, but at higher values of x will oscillate between + and - infinity.

It is obvious that

$$R_2 = \prod_{n=0}^{\infty} \frac{1 - \frac{x^2}{(1+2na)^2}}{1 - \frac{x^2}{(1+(2n+1)a)^2}} \quad (10)$$

and hence that

$$R_2 = \prod_{n=0}^{\infty} \frac{(m-1/2+n)^2(m-1-mx+n)(m-1+mx+n)}{(m+1+n)^2(m-1/2-mx+n)(m-1/2mx+n)} \quad (11)$$

where the symbol Π indicates that the expression is a continued product and where n is a positive integer.

The infinite product R_2 can be replaced by a ratio of gamma functions, using a theorem given in Whittaker and Watson "Modern Analysis" 2nd edition, page 232. This gives

$$R_2 = \frac{\Gamma^4 m \Gamma(2m-2mx) \Gamma(2m+2mx)}{\Gamma^2 2m \Gamma^2(m-mx) \Gamma^2(m+mx)} \quad (12)$$

In the range of x from zero to unity the gamma functions may be replaced by their Stirling series equivalents giving.

$$\log R_2 = 1/2 \log(1-x^2) + \sum_{r=1}^{\infty} \frac{(-1)^{r-1}(2^{2r}-1)B_r}{r 2^{2r}(2r-1)m^{2r-1}} \left[2 - \frac{1}{(1+x)^{2r-1}} - \frac{1}{(1-x)^{2r-1}} \right] \quad (13)$$

which B_r denotes the r^{th} Bernoulli number.

A first approximation to the value of R_2 is obviously

$$R_2 = \sqrt{1-x^2} \quad (14)$$

which gives

$$\tan m \Pi x = R_1 \sqrt{1-x^2} = \quad (15)$$

in the range $0 < x < 1$.

Comparing this equation with the expression for the phase displacement of a low pass filter given in Equation 6 it is seen that the phase angle will approximate to a linear function of the frequency if

$$\sqrt{\frac{X_{01}}{X_{02}}} = j \frac{\Pi}{2} \frac{f}{f_a} \quad (17)$$

and if the several resonance frequencies f_1, f_2 , etc. up to the cut off frequency are spaced at equal intervals f_a .

The general form of the phase displacement characteristic obtainable with equal spacing of the frequencies is shown in Fig. 5 for the case $m=3$, corresponding to the network illustrated by Figs. 2, 3 and 4. The phase angle increases by $\frac{\Pi}{2}$ in each interval between the successive resonances and anti-resonances. At first the rate of increase is nearly uniform, but at frequencies close to the cut off a noticeable undulation occurs in the characteristic indicating a lower accuracy in the approximation formula in this range. A closer approximation to linearity is obtained by departing from the uniform spacing of the resonance frequencies in the region close to the cut off frequency. The desired spacings are found from a consideration of the further approximation for the expression for R_2 taking into account additional terms of the expression given in Equation 13. Manifestly the approximation for R_2 should, for design purposes, take the form

$$R_2 = \frac{\left(1 - \frac{x^2}{(1+a_1)^2}\right) \left(1 - \frac{x^2}{(1+a_3)^2}\right) \cdots \sqrt{1 - \frac{x^2}{(1+a_k)^2}}}{\left(1 - \frac{x^2}{(1+a_2)^2}\right) \left(1 - \frac{x^2}{(1+a_4)^2}\right) \cdots} \quad (18)$$

To apply this relationship to the design of

a low pass filter let $x = \frac{f}{f_c}$,

and let $af_c = \frac{f_c}{2m} = f_a$.

Then

$$\tan m \Pi \frac{f}{f_c} = \frac{\Pi}{2} \frac{f}{f_a} \frac{\left(1 - \frac{f^2}{(2f_a)^2}\right) \cdots \sqrt{1 - \frac{f^2}{f_c^2}}}{\left(1 - \frac{f^2}{f_a^2}\right) \left(1 - \frac{f^2}{(3f_a)^2}\right) \cdots \left(1 - \frac{f^2}{(f_c-f_a)^2}\right)} \quad (16)$$

where $a_1, a_2 \dots a_k$ are numerical constants determining the zero and infinite values of R_2 . This expression may be transformed to

$$R_2 = \frac{\left(1 + \frac{a_1}{1-x}\right) \left(1 + \frac{a_1}{1+x}\right) \dots \sqrt{\left(1 + \frac{a_k}{1-x}\right) \left(1 + \frac{a_k}{1+x}\right)}}{\left(1 + \frac{a_2}{1-x}\right) \left(1 + \frac{a_2}{1+x}\right) \dots} \left[Q \sqrt{1-x^2} \right] \quad (19)$$

Q being a constant involving only the numerical factors $(1+a_1)$ etc. To determine the best values of the constants a_1, a_2 etc. this expression may be identified with that of Equation 13, using n terms of the summation in the latter. This procedure gives

For the first case let $m=2$ and $k=2$ and, as before, let

$$x = \frac{2mf}{f_a}$$

$$\log \frac{\left(1 + \frac{a_1}{1-x}\right) \left(1 + \frac{a_1}{1+x}\right) \dots \sqrt{\left(1 + \frac{a_k}{1-x}\right) \left(1 + \frac{a_k}{1+x}\right)}}{\left(1 + \frac{a_2}{1-x}\right) \left(1 + \frac{a_2}{1+x}\right) \dots} + \log Q =$$

$$Q_1 + \sum_{r=1}^{r=n} \frac{(-1)^{r-1} (2^{2r}-1) B_r}{r^{2r} (2r+1) m^{2r-1}} \left(\frac{1}{(1+x)^{2r-1}} + \frac{1}{(1-x)^{2r-1}} \right) \quad (20)$$

in which Q_1 represents the sum of the constant terms in the summation. The first where f_a is the uniform frequency interval. The approximation then gives

$$\tan \frac{\Pi f}{2 f_a} = \frac{\Pi f}{2 f_a} \frac{\left(1 - \frac{f^2}{(2f_a)^2}\right) \left(1 - \frac{f^2}{(3.86f_a)^2}\right)}{\left(1 - \frac{f^2}{f_a^2}\right) \left(1 - \frac{f^2}{(3f_a)^2}\right) \sqrt{1 - \frac{f^2}{(4.22f_a)^2}}}, \quad (21)$$

term on the left hand side is equal to the sum of the logarithms of the separate factors. Each of these latter quantities may be expanded as a series of powers of

for linear phase shift the filter impedances Z_1 and Z_2 must be so designed that the ratio

$$\sqrt{\frac{Z_1}{Z_2}}$$

has the value given by the product expression in Equation 21. This requires that their zero frequency reactances X_{01} and X_{02} shall be such that

$$\sqrt{\frac{Z_{01}}{Z_{02}}} = \frac{\Pi f}{2 f_a}$$

and that the resonance and anti-resonance frequencies shall occur at frequencies $f_a, 2f_a, 3f_a, 3.86f_a$, and $4.22f_a$. The necessary character of the reactances is illustrated in Fig. 6, the solid curve corresponding to Z_1 and the dotted curve to Z_2 . The phase characteristic of a filter so constructed is shown by the solid line curve of Fig. 7, the dotted curve being that of a corresponding filter of the simple ladder type. By the arrangement described linearity of the phase characteristic up to 90% of the cut off frequency is obtained.

Figs. 8 and 9 illustrate the frequency allocation and the resulting phase characteristic for the case $m=3$ and $k=3$. For this case the resonance and anti-resonance frequencies occur at intervals proportional to the numbers 1, 2, 3, 4, 5, 5.95, 6.6, and 6.95. In Fig. 8 the solid line curve represents the reactance of Z_1 and the dotted line curve that of Z_2 as in

Number of factors	$2ma_1$	$2ma_2$	$2ma_3$	$2ma_4$	$2ma_5$
1.....	-.5				
2.....	-.14645	+.20711			
3.....	-.05032	+.67731	+.95526		
4.....	-.01897	+.86187	+1.4918	+1.7225	
5.....	-.00759	+.9372	+1.7609	+2.3079	+2.5068

The application of the foregoing princi-

the previous case. In Fig. 9 the solid line curve represents the phase displacement and the dotted line curve represents the phase displacement of a corresponding simple ladder type filter.

Attenuation characteristic

It may be shown that the frequency allocation described above not only provides for a linear phase shift in the band but also satisfies the requirement for a substantially infinite attenuation outside the band. Referring to Equations 9 to 13, a procedure similar to that used for the evaluation of R_2 in the range $0 < x < 1$ gives for R_1 in the range $x > 1$ the following expression

$$\log \frac{1}{R_1} = -j\frac{\Pi}{2} + 1/2 \log (1-x^2) + \sum_{r=1}^{\infty} \frac{(-1)^{r-1}(2^r-1)B_r}{r2^{2r}(2^r-1)m^{2r-1}} \left[2 - \frac{1}{(1+x)^{2r-1}} - \frac{1}{(1-x)^{2r-1}} \right] \quad (22)$$

Except for the factor

$$-j\frac{\Pi}{2}$$

which serves merely to make the value real in the range $x > 1$, this expression is exactly the same as the expression for R_2 . Examination of Equation 20 shows that the various logarithmic terms of the expression used to approximate R_2 are such that their series expansions are convergent for x greater than unity as well as for x less than unity. It follows then, from Equation 22, that this same expression will approximate the value of $1/R_1$ in the range x greater than unity. If the product approximation for R_2 be denoted by G_2 , then the product of R_1 and G_1 will be approximate unity for values of x greater than unity. From this it is clear that if the filter impedances are designed in the manner described so that

$$\sqrt{\frac{Z_1}{Z_2}} = R_1 G_2 = \tanh \frac{P}{2}$$

then $\tanh \frac{P}{2}$ will be unity and P will be sub-

stantially infinite when $x > 1$, that is at frequencies above the cut off. The impedance structure of the invention thus not only provides for linear phase shift in the transmission band but also for substantially infinite attenuation outside the band.

Characteristic impedance

In the examples discussed above the filter branch impedances have no resonance or anti-resonance frequencies outside the transmission band and, therefore, have no independent

means for controlling their characteristic impedances. For the case illustrated in Fig. 6 the characteristic impedance has the value

$$K = \frac{\pm j\sqrt{X_{01}X_{02}}}{\sqrt{1-\frac{f^2}{f_c^2}}}, \quad 70$$

f_c being the cut off frequency, and for the case illustrated by Fig. 8

$$K = \pm j\sqrt{X_{01}X_{02}}\sqrt{1-\frac{f^2}{f_c^2}}. \quad 80$$

The form in the first case corresponds to the

mid-shunt impedance of a simple series shunt low-pass filter and the latter to the mid series impedance. The variation of the impedance is due to the cut off factor

$$\sqrt{1-\frac{f^2}{f_c^2}} \quad 95$$

in each case. To compensate this variation it is necessary to introduce into the expression for the impedance additional factors which will substantially cancel the effect of this factor. These factors involve resonance and anti-resonance frequencies above the cut off frequency.

It has been shown that the distribution of the resonance and anti-resonance frequencies within the band gives a substantially constant value, equal to unity, of the propagation constant in the attenuation range. If these frequencies were absent the variation of the attenuation would be that due to the cut off factor

$$\sqrt{1-\frac{f^2}{f_c^2}}. \quad 115$$

It is evident therefore that the distribution described has the effect of cancelling the cut off factor. The effect of this factor on the impedance may be cancelled by a somewhat similar distribution of the resonance and anti-resonance frequencies above the cut off, but in this case the spacing is based on an inverse frequency scale.

If the reciprocal of frequency be denoted by t and if the reciprocals of the successive resonance and anti-resonance frequencies above the cut off be designated $t_1, t_2, \dots, t_n$ then the general expression for the charac-

teristic impedance, see Equation 6, may be written as the cut off frequency the impedance, of course, becomes infinite. Curve 4 shows the general

$$K = \sqrt{X_{01} X_{02}} \frac{t_1^2 t_2^2 \dots t_{n-1}^2}{t_1^2 t_2^2 \dots t_n^2} \frac{1}{t} \frac{(1 - \frac{f^2}{f_{n-1}^2}) \dots (1 - \frac{f^2}{f_2^2})}{(1 - \frac{f^2}{f_n^2}) \dots (1 - \frac{f^2}{f_1^2})} \sqrt{1 - \frac{f^2}{f_c^2}} \quad (23)$$

This expression is in the same form as the expression for the propagation constant in terms of frequency and will be approximately constant in the transmission band if the t 's are given the same values as the frequencies controlling the propagation constant. The more remote of the impedance controlling frequencies are spaced uniformly on an inverse frequency scale while those closer to the cut off are spaced on this scale in accordance with the values given in the table above. The values of the impedance controlling frequencies, when their number has been specified in any given case, are most readily calculated by determining first the corresponding frequencies in the transmission band on the assumption that they are to be used for control of the propagation constant and then inverting the values with respect to the cut off frequency.

Figs. 10 and 11 illustrate the character of the branch impedances and the transmission characteristics of a low-pass filter embodying the features of impedance control and control of the propagation constant. In the transmission band the number and arrangement of the controlling frequencies is the same as in the case illustrated by Figs. 6 and 7 for which $m=2$ and $k=2$. In the attenuation range the same number of impedance controlling frequencies is used and the spacings are inversely related. In all there are 9 critical frequencies the values being proportional to the numbers 1, 2, 3, 3.86, 4.22,

$$4.22 \left(\frac{4.22}{3.86} \right), 4.22 \left(\frac{4.22}{3} \right), 4.22 \left(\frac{4.22}{2} \right),$$

and

$$4.22 \left(\frac{4.22}{1} \right).$$

The solid line curve of Fig. 10 shows the variation of Z_1 with frequency and the dotted line curve shows the variation of Z_2 . The several critical frequencies are designated $f_1 \dots f_9$ inclusive, the cut off frequencies being f_c . Above the cut off frequency the two impedances have substantially the same value, as indicated by the substantial coincidence of the curves. In Fig. 10 curve 3 shows the variation of the phase displacement with frequency, this curve being the same as that of Fig. 7. Curve 5 shows the variation of the characteristic impedance in the transmission range and illustrates the high degree of uniformity obtained by the invention. At

character of the total transmission loss characteristic in the attenuation range when the filter is connected between terminal impedances equal to the constant value of the characteristic impedance. The peaks indicated at frequencies f_6, f_7, f_8 and f_9 do not appear in the attenuation constant of the filter itself, the value of which does not involve these frequencies. These peaks appear, however, in the over-all transmission loss since, at the frequencies indicated, both branch impedances are simultaneously resonant or anti-resonant and therefore suppress the transmission of currents when the filter is connected between finite impedances. The attenuation characteristic per se has a form inverse to that of curve 5 and is more or less constant at a very high value. The scale of the ordinates of curve 4 is quite arbitrary, the purpose of the curve being to indicate the character of the transmission loss and the effect of the impedance controlling frequencies in sharpening the cut off.

Having given the frequency variation of the branch impedances Z_1 and Z_2 in terms of the resonance and anti-resonance frequencies, appropriate structural forms may be determined by the rules set forth in the before mentioned article by Foster. The numerical values of the elements computed from the chosen values of the frequencies and from the values of the zero frequency reactances. As already shown these latter must be so related that

$$\sqrt{\frac{X_{01}}{X_{02}}} = j \frac{\pi f}{2 f_a},$$

defining the zero frequency slope of the phase characteristic, where f_a denotes the uniform interval between the critical frequencies in the transmission range. An additional relationship between X_{01} and X_{02} is necessary and this is provided by

$$j \sqrt{X_{01} X_{02}} = K_0$$

where K_0 is the zero frequency value of the characteristic impedance and its substantially constant value throughout the band.

In the case discussed above the number of impedance controlling frequencies is equal to the number of frequencies controlling the propagation constant. It will be understood that this is not an essential condition, but that the numbers in each category may be varied at will.

All-pass delay networks

The linearity of the phase characteristics of low-pass filters designed in accordance with the invention makes them suitable for use as delay networks, but if perfect impedance uniformity is desired it is preferable to use networks of the all-pass type. In all-pass networks of the lattice type of Fig. 1 the branch impedances are so constructed that Z_1 and Z_2 constitute inverse reactances having a constant product. The characteristic impedance is thus constant at all frequencies and is a pure resistance. Linearity of the phase shift in a given frequency range may be obtained with a fair degree of accuracy by so constructing the branch impedances that they have a large number of evenly spaced resonance and anti-resonance frequencies in this range. However, unless a very large number of critical frequencies is used the even spacing of the frequencies results in an undulatory characteristic such as is shown in Fig. 5.

A very high degree of linearity is obtained in accordance with the invention by using a frequency distribution similar to that used in the filter illustrated by Figs. 10 and 11. The arrangement differs from that of the filter in that the cut off frequency is omitted, the upper critical frequencies of Z_1 being stepped up one interval so that Z_1 and Z_2 are completely inverse with respect to each other.

It has been shown in connection with the low-pass filter design that the impedance controlling factors involving the frequencies above the cut off have the effect of cancelling the cut off factor. It follows then that the effect of this factor may be simulated by the inverse of the product of the impedance controlling factors. The arrangement of the frequencies described above for the all-pass network, while eliminating the cut off factor from the propagation constant, introduces in place thereof the inverse of the product of the impedance controlling factors and therefore gives substantially the same phase displacement as in the filter.

The character of the branch impedances and the resulting phase characteristic of an all-pass network obtained in this way are illustrated respectively by Figs. 12 and 13. For comparison purposes the system of critical frequencies is the same as that of the low-pass filter of Figs. 10 and 11 except that the cut off frequency f_5 is omitted. In Fig. 12 the solid line curve represents the variation of Z_1 and the dotted line curve that of Z_2 , the two being inversely related at all frequencies. The phase characteristic represented by curve 5 of Fig. 13 is linear over a somewhat greater range than that of the corresponding filter and does not exhibit the discontinuity at the cut off frequency.

To obtain a large delay the number of the uniformly spaced frequencies in the lower

range may be increased as desired. Increasing the number of non-uniformly spaced frequencies results in a greater degree of linearity, that is, in greater freedom from undulations of the characteristic in the range of the uniformly spaced frequencies. The numbers of critical frequencies below and above the hypothetical cut off frequency may be chosen at will, but it is preferable that not more than two in the range below the hypothetical cut off should be spaced on the non-uniform basis.

It may be observed that the all-pass networks of the invention simulate in a limited frequency range the impedance and the propagation constant of a length of smooth line having uniformly distributed series inductance and shunt capacity. They may also be made to simulate a dissipative smooth line if each of the inductances is connected in series with a resistance such that the inductance to resistance ratio is the same as that of the line and if each capacity is provided with a shunt conductance similarly proportional in accordance with the line conductance.

Band-pass filters

The rules for spacing the critical frequencies in a band-pass filter in accordance with the invention are similar to the rules for the spacing in the case of a low-pass filter and will be stated without proof. The branch impedances of a typical band-pass filter are shown in Fig. 14, there being in this case fourteen critical frequencies including the two cut off frequencies. As before the impedance Z_1 is indicated by the solid line curve and Z_2 by the dotted line curve. The cut off frequencies are f_4 and f_{11} , the location of the band being indicated by the shaded portion of the frequency axis. The impedance controlling frequencies are f_1, f_2, f_3 , below the lower cut off and f_{12}, f_{13} , and f_{14} above the upper cut off. Frequencies f_5, f_6, f_7, f_8, f_9 , and f_{10} , within the band control the propagation constant.

The impedance controlling frequencies below the lower cut off are spaced on the same basis as the frequencies in the transmission band of a low-pass filter having this off frequency.

The impedance controlling frequencies above the upper cut off are spaced on the same basis as the impedance controlling frequencies of a low-pass filter having a cut off at the upper cut off frequency.

The frequencies within the band are spaced uniformly except at band limits where they are crowded closer together.

In the region near the upper cut off the values of the non-uniformly spaced frequencies are determined in accordance with the product expression of equation 18 using the coefficients a_1, a_2 etc. of the table already

given. The factor $2m$ by which the constants given in the table are to be divided is such that $2m-1$ is the number of uniform frequency intervals occurring in the band.

the bridged-T type the bridging branch having the impedance

$$\frac{Z_1}{2}$$

The spacing of the frequencies near the lower cut off is found in the same way except that the signs of the constants given in the table are reversed.

and the central branch having the impedance $2Z_2$. The two branches are connected by means of a unit ratio transformer T the windings W_1 and W_2 of which are connected series aiding between their outer terminals, that is to produce a reversal of the E. M. F. If the transformer T is so constructed that the windings, besides being equal, have very large inductances and substantially perfect coupling, the network of Fig. 16 will be substantially equivalent to that of Fig. 1 with respect to its impedance and transmission characteristics. For complete equivalence it is necessary that T should be a perfect transformer, that is, one having infinitely great winding inductances and unity coupling. The equivalence of the networks of Figs. 1 and 16 can be achieved with a very high degree of exactness by making use of a circuit equivalence illustrated by Fig. 17. It may be shown that the unity ratio transformer T of Fig. 16, the terminals of which are designated 8, 9, and 10 is equivalent to the bridged-T circuit of Fig. 17 the terminals of which are correspondingly identified. In Fig. 17 T' is a perfect transformer of unity ratio. The bridge circuit connected between terminals 8 and 9 comprises an inductance equal to $1/2(L+M)$, L being the inductance of each of the windings W_1 and W_2 of the transformer T and M being their mutual inductance. In parallel with the bridging inductance is a capacity C representing the effective capacity between windings W_1 and W_2 . The central branch between the transformer T' and terminal 10 comprises a small inductance $2(L-M)$.

As in the other types of networks, the numbers of the critical frequencies in the various categories may be chosen at will.

The forms of the transmission and impedance characteristics of the band-pass filter having the branch impedances of Fig. 14 are illustrated in Fig. 15. Curve 6 represents the variation of the phase component of the propagation constant in the band. Except close to the cut off frequencies this characteristic is linear to a high degree of accuracy. Curve 7 represents the total transmission loss characteristic of the filter when connected between finite constant impedances. The peaks at the impedance controlling frequencies are of the same character as those in curve 4 of Fig. 11 and do not appear directly in the propagation constant. Curve 8 represents the characteristic impedance in the band. For the particular case illustrated it is zero at the lower cut off and infinite at the upper cut off, but manifestly, by the use of other frequency arrangements in the branch impedances, the value at either cut off may be made zero or infinity as desired. Throughout the greater part of the band the impedance is substantially constant.

High-pass filters

Since the transmission band of a high-pass filter is infinitely broad, any attempt to approximate a linear phase shift (other than zero) throughout this range would demand a network containing an infinite number of elements. However, linear phase shift can be secured over a finite frequency band near the cut off, while satisfying the usual attenuation and impedance requirements, by a combination of the methods employed for band-pass and all-pass structures. The arrangement of propagation constant factors near the cut off should follow the pattern adopted for the lower group of propagation constant controlling factors in a band-pass filter. The disposition of the other factors may be similar to that employed for the all-pass delay network. The pattern adopted for the impedance controlling factors in the lower attenuating range of a band-pass filter is obviously appropriate for the high-pass configuration also.

The schematic form of the networks of the invention shown in Fig. 1 since it comprises two impedances Z_1 and two impedances Z_2 involves a duplication of elements. Fig. 16 illustrates a modified structure in which this duplication is avoided. This network is of

It follows then that the networks of Figs. 1 and 16 will be exactly equivalent if the impedance $Z_1/2$ of the latter is shunted by a negative inductance equal to $-1/2(L+M)$ and a negative capacity equal to $-C$ and if the impedance $2Z_2$ has connected in series therewith a negative inductance equal to $-2(L-M)$. If the transformer is wound with bifilar windings this latter inductance is substantially zero and only the former need be considered. The effect of adding a negative inductance and a negative capacity in parallel with the bridging impedance $Z_1/2$ can be obtained exactly provided that this impedance is such that its value is zero both at zero frequency and at infinite frequency. In that case the impedance of the branch may take the form shown in Fig. 18, comprising a group of parallel connected circuits one of which is a simple inductance L_0 , another is a simple capacity C_∞ , and the others are simple resonant combinations. The negative shunt inductance may then be effectively

introduced by increasing the value of L_0 in the proper proportion and the negative capacity may be introduced by diminishing the value of C_∞ .

5 The use of the schematic form of Fig. 16 for the networks of the invention involves a substantial saving of elements, but a further important economy arises from the fact that, in either form, the impedance and the
10 transmission characteristics of the networks of the invention are independently controlled. By the introduction of the independent frequency parameters a given degree of precision in the achievement of a desired set of characteristics is obtained with
15 a smaller number of elements than heretofore possible.

It is to be noted that the two general types of the network shown in Figs. 1 and 16 with
20 their connected terminal loads both constitute six-branch networks of the well known Wheatstone bridge or "Star Delta" type. In the lattice form the load impedances are non-contiguous branches and in the bridged-
25 T form they are branches terminating at a common point.

What is claimed is:

1. In a wave transmission network comprising a plurality of impedance branches,
30 two impedances adapted to determine the transmission characteristics of the network, said impedances each having a plurality of critical frequencies defining resonances and anti-resonances and being proportioned to
35 provide a broad band in which transmission is substantially undistorted, the said critical frequencies being evenly spaced throughout the major portion of the band and being progressively more closely spaced
40 towards the band limits in the remainder of the band to provide a substantially linear phase characteristic throughout the band.

2. In a wave transmission network comprising a plurality of impedance branches,
45 two impedances adapted to determine the transmission characteristics of the network, said impedances each having a plurality of critical frequencies defining resonances and anti-resonances and being proportioned to
50 provide a free transmission band and an attenuation band, the said critical frequencies being evenly spaced throughout the major portion of the band and being progressively more closely spaced towards the band limits
55 in the remainder of the band to provide a substantially linear phase characteristic throughout the band.

3. In a wave transmission network comprising a plurality of impedances, two impedances adapted to determine the transmission characteristics of the network, said impedances each having a plurality of critical frequencies defining resonances and anti-resonances and being proportioned to provide
60 a free transmission band and attenuation at

frequencies above the transmission band, said critical frequencies being spaced on an inverse frequency scale in the attenuation range remote from the band limit and on a progressively closer scale near to the band
70 limit to provide a substantially uniform characteristic impedance throughout the band.

4. The method of obtaining a linear phase characteristic in a broad frequency range in a wave transmission network having multiple
75 resonant branch impedances which comprises locating the resonance and anti-resonance frequencies of said impedances with respect to one another to provide free transmission in the desired range and spacing the said
80 resonance and anti-resonance frequencies evenly throughout the major portion of said range and progressively more closely towards the limits of said range in the remainder of the range.

5. A wave transmission network comprising two pairs of equal impedances arranged to form a symmetrical lattice structure, said impedances comprising multiple resonant reactances, the resonances of the one pair coinciding with the anti-resonances of the other pair in a preassigned broad frequency range to provide free transmission, and the said
90 resonances and anti-resonances being spaced at uniform frequencies throughout the major portion of said range and being spaced at progressively closer frequencies towards the limits of the remainder of the range to provide a linear phase characteristic throughout the range.

6. A wave transmission network comprising two pairs of equal impedances arranged to form a symmetrical lattice structure, said impedances comprising multiple resonant reactances resonant and anti-resonant at a plurality of common frequencies and being proportioned with respect to each other to provide free transmission in a broad range extending from zero frequency, said resonance and anti-resonance frequencies being
110 spaced at equal intervals in the lower frequency range and at progressively smaller intervals in an intermediate frequency range up to a preassigned limit to provide a linear phase characteristic in the lower frequency range, and at progressively increasing intervals above the preassigned limit.

7. A low-pass wave filter network comprising two pairs of equal impedances arranged to form a symmetrical lattice structure, said impedances comprising multiple resonant reactances, the resonances and anti-resonances of the respective pairs of impedances being coordinated to provide a single low-pass band and being spaced throughout the lower part of the band at uniform intervals and at progressively closer intervals towards the band limit to provide a linear phase characteristic in the band.

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8. A filter in accordance with claim 7 in which the branch impedances have additional resonance and anti-resonance frequencies spaced at progressively increasing frequency intervals above the band limit whereby the characteristic impedance is made substantially uniform throughout the band.

9. The arrangement described in claim 1 wherein the said two impedances are coupled by a high efficiency transformer to constitute a bridged-T network.

10. The arrangement described in claim 1 wherein the said two impedances are coupled by a high efficiency transformer to constitute a bridged-T network and in which at least one of said impedances is modified to neutralize the effect of the transformer impedances.

11. The arrangement described in claim 3 wherein the said two impedances are coupled by a high efficiency transformer to constitute a bridged-T network.

12. The arrangement described in claim 3 wherein the said two impedances are coupled by a high efficiency transformer to constitute a bridged-T network and in which at least one of said impedances is modified to neutralize the effect of the transformer impedances.

In witness whereof, I hereunto subscribe my name this 1st day of July, 1930.

HENDRIK W. BODE.

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