April 7, 1931. T. C. FRY 1,799,810

TRANSMISSION CIRCUITS

Filed Dec. 5, 1928 3 Sheets-Sheet 1

FIG. 1

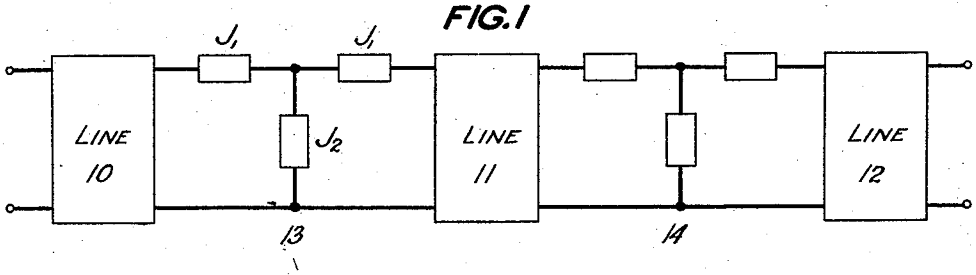

FIG. 2

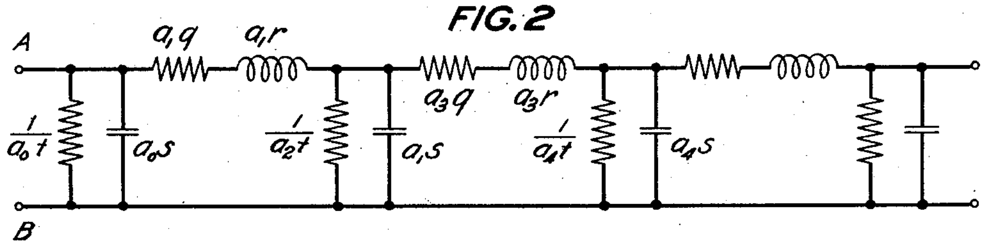

FIG. 3

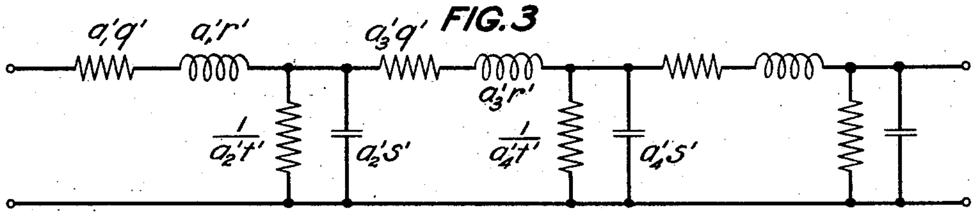

FIG. 6

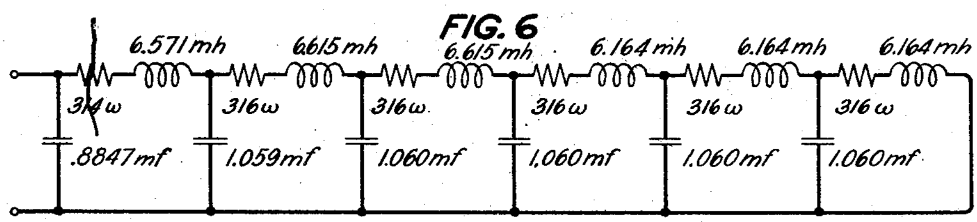

FIG. 7

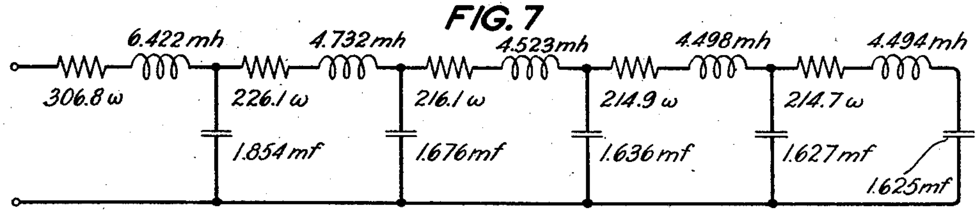

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FIG. 4

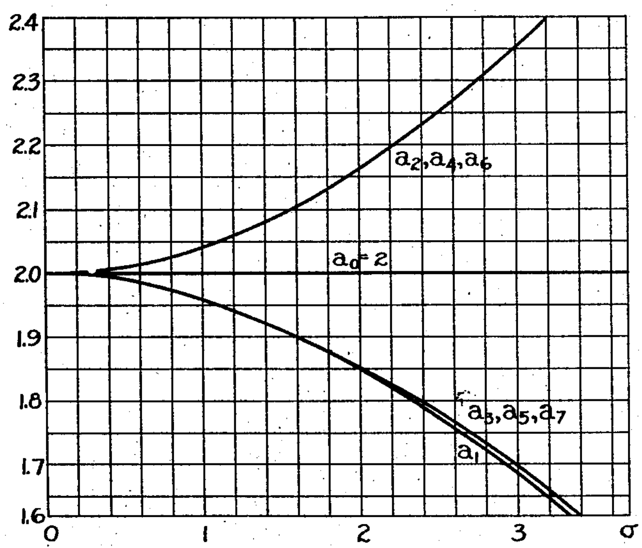
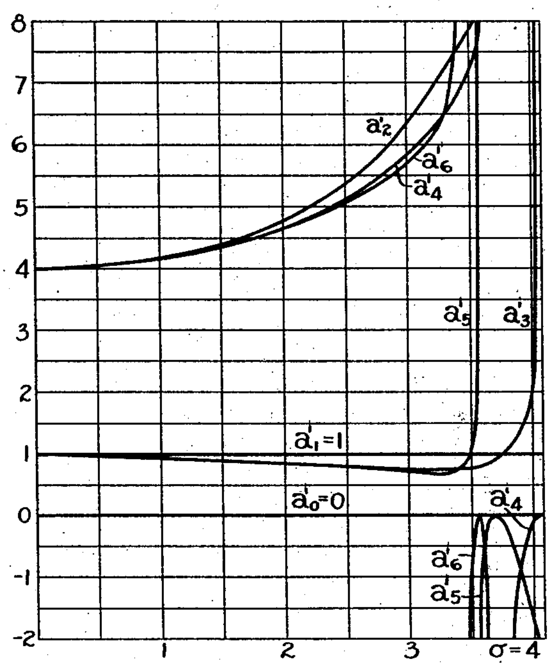


FIG. 5



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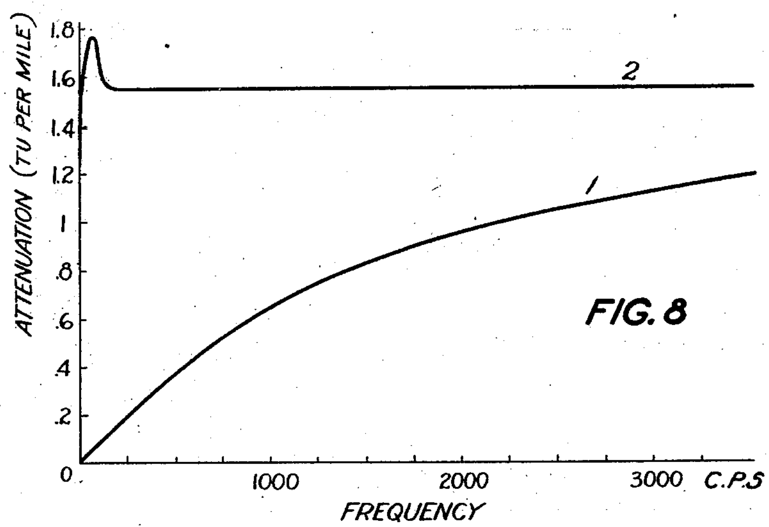


FIG. 8

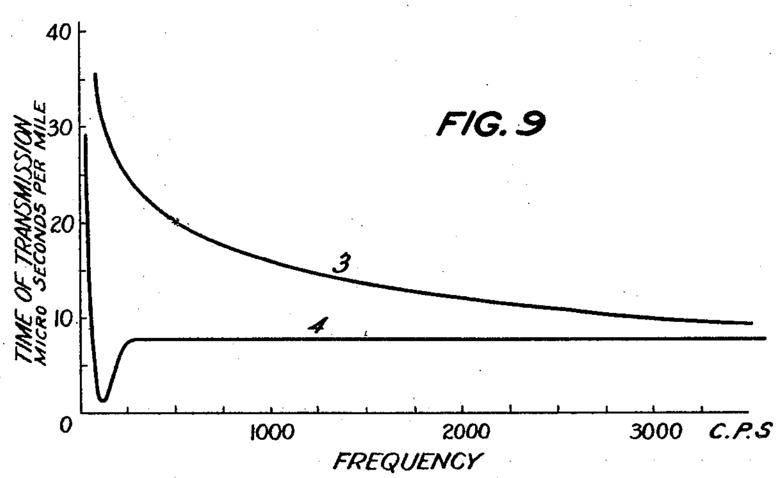


FIG. 9

FIG. 10

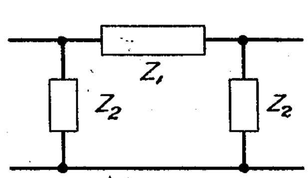
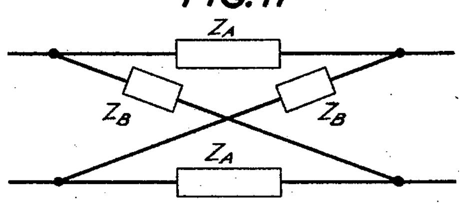


FIG. 11



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UNITED STATES PATENT OFFICE

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TRANSMISSION CIRCUITS

Application filed December 5, 1928. Serial No. 323,880.

This invention relates to electrical wave transmission, and particularly to the correction of signal distortion in systems for the transmission of information, such as telephone or telegraph systems.

When information is transmitted by an electric current varying in accordance with a sequence of symbols the important requirement for intelligible reception is that the time variations of the received current should correspond to that of the current originally transmitted. If it does not the received signal is distorted and its intelligibility is diminished. From the viewpoint that a signal may be considered as comprising a multiplicity of simple harmonic waves of different frequencies, signal distortion may be described in terms of the effects produced on the several single frequency component waves as they traverse the system. In linear systems, that is, in systems in which the impedances of the elements are independent of the current strength, two types of distortion are recognized. First, the component waves of different frequency may be attenuated in different ratios, and second, the component waves may travel with different velocities, with the result that they arrive at the receiving end of the system with their relative phases altered. The first type is known as amplitude distortion, and the second type as phase, or velocity, distortion. Generally both types are present in the resultant signal distortion, but each separately may be effective to diminish the intelligibility of the signal. Where a transmission line is involved the wave propagation characteristic is dependent upon the resistance, inductance, leakance, and capacity of the line. If these quantities are properly related the line by itself may be distortionless, but in most, if not all, practical cases this proper relationship does not exist and the line is productive of both amplitude distortion and phase distortion. In addition distortion may be introduced by discontinuities in the line which give rise to wave reflection. Such discontinuities may be caused by the insertion in the line or at its terminals of apparatus having an impedance different from the characteristic impedance of the line.

Complete freedom from signal distortion requires that waves of all frequencies, from zero to infinity, be propagated with the same velocity and with the same degree of attenuation, but for many purposes it has been practical to limit the frequency range in which the requirements are met to such a value that only the most important of the component waves are transmitted with uniform velocity and attenuation. In telephone lines, for example, this has been accomplished by the use of loading coils. The added inductance of the coils serves to diminish both the amplitude distortion and the phase distortion in a certain range of frequencies, but at higher frequencies the distortion is greatly increased by wave reflections at the points where the coils are inserted. The added inductance is also effective to reduce the wave propagation velocity in the frequency range in which the distortion is compensated. Continuous loading eliminates the wave reflections and, if of proper magnitude, can render the line distortionless.

Another method of correcting distortion involves the use of networks which may be connected to the ends of a line, or inserted at intermediate points, to compensate either the amplitude distortion or the phase distortion or both. With the networks heretofore available the application of this method has been restricted to the compensation of distortion in a limited frequency range and to an approximate degree of accuracy. As the frequency range is widened and as the accuracy of the compensation is increased the design of the networks tends to become very laborious and their construction complicated. By this invention compensating networks are provided, which, when combined with a given uniform transmission line, render the combination substantially completely distortionless. Amplitude distortion and phase distortion are corrected for all frequencies from zero to infinity, and with a degree of accuracy as high as may be desired.

A feature of the invention is that the impedances constituting the compensating networks comprise ladder type structures or artificial lines, each branch of which is made up of simple combinations of elements, for example resistance and inductance, or resistance and capacity. No negative impedance elements are employed. For perfect compensation these ladder type lines should, theoretically, have an infinite number of sections, but, for the reason that the currents are attenuated as they penetrate into the lines, a high degree of compensation can be obtained by the use of a small number of sections. By the simple expedient of extending the lines to include a larger number of sections any desired degree of accuracy can be achieved.

Besides having a simple structural arrangement, the networks of the invention possess the advantages that they cause no wave reflection when inserted in the line they are designed to compensate, and are applicable to the correction of lines with or without distributed shunt conductance, that is, they are adapted to compensate the distortion in a line of the general type having distributed resistance, inductance, capacity, and conductance.

The compensating networks of the invention may take various schematic forms, such as, for example, a three branch T or π, or a four branch lattice. In each case the branches consist of singly infinite ladder lines of the type mentioned above, in which the impedances are all of a simple character involving only positive coefficients.

In the detailed description which follows the invention will be described with reference to a limited number of networks representative of the preferred forms. The general method of design will also be described whereby the design and construction of other forms of network may be readily achieved.

Of the drawings:

Fig. 1 represents in schematic form a compensated transmission line in accordance with the invention;

Figs. 2 and 3 represent the impedances constituting the branches of the compensating network in Fig. 1;

Figs. 4 and 5 are charts for facilitating the design of the compensating networks;

Figs. 6 and 7 illustrate the impedances of a network of the invention for the compensation of a particular line;

Figs. 8 and 9 illustrate the degree of compensation attained in the particular example by the use of the structures of Figs. 6 and 7; and

Figs. 10 and 11 illustrate additional forms of compensating network in accordance with the invention.

The system shown in Fig. 1 comprises a uniform line divided into a plurality of sections 10, 11, 12, etc., with each of which is associated a network 13, 14, etc., designed to compensate the distortion therein. The line may be regarded as being loaded by the insertion of networks at intervals, and the combination of a line section and a compensating network may for convenience be termed a loading section. The fundamental requirements for the design of the compensating networks are readily established. First, in order that the networks may be inserted without introducing additional distortion due to wave reflection, their characteristic impedances must be the same as that of the line. Second, the attenuation and velocity characteristics of the networks must be complementary to those of the line sections respectively associated therewith so that the resultant attenuation and velocity may be constant.

Consider the loading section formed by the uniform line section 10 and the network 13. The propagation constant, Γl, and the characteristic impedance Z, of the line section are given by the equations:

$$\Gamma l = l\sqrt{(R+j\omega L)(G+j\omega C)} \quad (1)$$

and

$$Z=\sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad (2)$$

where R, L, G and C denote respectively the line constants, resistance, inductance, capacity and conductance per unit length, and l is the length of the line section. The symbol ω is the usual frequency factor, 2π times the frequency. In order that the loading section shall be distortionless its propagation constant should have the form

$$\alpha+j\omega\beta$$

where α and β are both constant. The real part α of this expression represents the attenuation and its constancy corresponds to the absence of amplitude distortion. The imaginary part $\omega\beta$ represents the phase change of a wave in traversing the section, the factor β being the reciprocal of the wave velocity. Constancy of the velocity, therefore, corresponds to a linear relationship between the phase shift and the frequency. If desired values are assigned to α and β the propagation constant of the compensating network becomes

$$\alpha+j\omega\beta-\Gamma l.$$

To avoid reflections the characteristic impedance of the network should be equal to Z.

Knowing the values of the characteristic impedance and the propagation constant for the network, the values of the impedances of the various branches can be established by well-known rules. For a simple symmetrical T-network, as shown in the figure, comprising series impedances J_1, and a shunt impedance J_2, the values are

$$J_1 = Z \tanh \frac{1}{2} (\alpha + j\omega\beta - \Gamma l) \quad (3)$$

and

$$J_2 = Z \operatorname{cosech} (\alpha + j\omega\beta - \Gamma l). \quad (4)$$

These expressions for the impedances are, of course, not in a form that would enable a structure to be built having the desired impedance values, but it will be shown that they can be transformed in such manner that they can be at once identified with a simple ladder type network.

It is well known that the impedance of a ladder, or series-shunt, type of network can be expressed as a continued fraction the elements of which are alternately the impedances of the series branches and the admittances of the shunt branches. The admittance is expressed by the reciprocal of this fraction, which is also a continued fraction of the same type. If, then, the expressions for J_1 and J_2, or for their reciprocals, can be expanded as continued fractions the elements of which can be identified alternately with the impedances and admittances of known combinations of electrical elements, it follows that structures may be built having the impedances J_1 and J_2. Many such expansions are possible, but not all of them are practical. For example, the elements of the continued fraction may, in some cases, correspond to the impedances of infinitely extended networks of the ladder type and, since the continued fraction will in general have an infinite number of elements, the resulting impedance would be an infinite ladder type network each branch of which is also an infinite ladder type network. In other cases, the elements of the expansion may represent simple impedance combinations, but may involve impedance elements having negative coefficients, for example negative resistance. Structures of this type may be realized, approximately at least, by the use of certain types of amplifier circuits, but, for the purposes of the present invention, a multiplicity of these would be required and the arrangement would be extremely complicated. For simplicity, therefore, it is necessary that the elements of the continued fractions into which the impedance functions are expanded should represent simple combinations of impedance elements all having positive coefficients, or, in other words, the elements should represent simple passive impedances. It is also necessary that the expansions should either have a finite number of terms or else should be convergent. Only the latter case need be considered since the expressions for J_1 and J_2 are of a type that can be expanded only as infinite series or as infinite continued fractions.

Under certain conditions a function $F(y)$ can be expanded as a convergent continued fraction of the type

$$F(y) = a_0 + \cfrac{1}{a_1 y + \cfrac{1}{a_2 + \cfrac{1}{a_3 y + \cdots}}} \quad (5)$$

in which the coefficients a_0, a_1, etc., are all positive. These conditions refer to the mathematical properties of the function $F(y)$ and are discussed at length by Perron, "Die Lehre von den Kettenbrüchen", B. G. Teubner, Leipzig, 1913, pages 396 to 417. Fractions of this type were originally studied by Stieltjes, Annales de la Faculte des Sciences de Toulouse, Volumes VIII and IX, and are known as Stieltjes fractions. If the function $F(y)$ expresses the frequency characteristic of an impedance, y being a function of frequency the continued fraction represents a series-shunt network by which the impedance can be realized. The series branches comprise resistances of values a_0, a_2, etc., and the shunt then branches admittances of values $a_1 y$, $a_3 y$, etc. The degree of complexity of the shunt branches depends on the form of y, expressed as a function of frequency.

In many cases the expression for the impedance may be incapable of expansion as a Stieltjes fraction, that is to say, the impedance cannot be realized by a series-shunt network of the simple character described. For these cases a modified form of the Stieltjes fraction is often suitable. If the argument y is separable into two factors u and v, it may be possible to expand the function

$$\frac{F(y)}{u}$$

or the function

$$\frac{1}{uF(y)}$$

as a Stieltjes fraction. In the first, the expansion would lead to

$$F(y) = a_0 u + \cfrac{1}{a_1 v + \cfrac{1}{a_2 u + \cfrac{1}{a_3 v + \cdots}}} \quad (6)$$

and in the second to

$$F(y) = \cfrac{1}{a_0 u + \cfrac{1}{a_1 v + \cfrac{1}{a_2 u + \cdots}}} \quad (7)$$

Each of these fractions can represent series-shunt impedance structures provided u and v are functions of frequency of the same type as the impedances or admittances of known combinations of electrical elements.

The first step in the design of networks to represent J_1 and J_2 is to change the expressions for the impedances in terms of frequency into one or other of the forms

$$\frac{QJ}{u}=F(u.v)$$

or

$$uQJ=\frac{1}{F(u.v)}$$

where Q is a constant factor and the function $F(u.v)$ is such that it can be expanded as a Stieltjes fraction. To simplify the appearance of the equations it is convenient to introduce a new notation in which

$$z=\frac{L}{CR-LG}(G+j\omega C)$$

and

$$\sigma=\frac{CR-LG}{\sqrt{CL}}l.$$

In terms of this notation the impedances of the T-network branches become

$$\sqrt{\frac{C}{L}}J_1=\sqrt{(z+1)/z}\tanh\frac{\sigma}{2}\left[\frac{\alpha}{\sigma}+j\omega\frac{\beta}{\sigma}-\sqrt{z\ (z+1)}\right] \quad (8)$$

and

$$\sqrt{\frac{C}{L}}J_2=\sqrt{(z+1)/z}\ \text{cosech}\ \sigma\left[\frac{\alpha}{\sigma}+j\omega\frac{\beta}{\sigma}-\sqrt{z(z+1)}\right]. \quad (9)$$

Further steps in the transformation require that definite values be assigned to α and β, the attenuation and phase constants of the loading section. The propagation constant of the smooth line, given by Equation (1), is a complex quantity, the real and the imaginary components of which are both variable with frequency. The real component representing attenuation increases with frequency from the value

$$l\sqrt{RG}$$

at zero frequency to the value

$$l\frac{RC+LG}{2\sqrt{LC}}$$

at infinite frequency. The imaginary component likewise ranges between the values

$$\omega l\frac{RC+LG}{2\sqrt{RG}}$$

at zero frequency and

$$\omega l\sqrt{LC}$$

at infinite frequency. The velocity of propagation increases in the same manner as the attenuation and has the value

$$\sqrt{LC}$$

at infinite frequency.

Purely physical considerations suggest that the constant attenuation of the loading section should be made equal to the maximum attenuation in the uniform line, and that the uniform effective velocity should also equal the maximum velocity in the uniform line. Other values of the attenuation may be chosen provided they lead to forms for the impedance functions that can be expanded as Stieltjes fractions, but it is manifest that the constant attenuation cannot be less than the maximum line attenuation. The choice of the uniform effective velocity is also restricted for the reason that a certain definite time interval is required for any wave to traverse the uniform line and no network placed at the end of the line can do anything to reduce this time interval.

The choice of the values of the attenuation and phase constants to correspond to the values of the uniform line constants at infinite frequency gives the following values for α and β:

$$\alpha=\frac{RC+LG}{2\sqrt{LC}}l,$$

$$\beta=l\sqrt{LC},$$

which, if substituted in Equations (8) and (9), give

$$\sqrt{\frac{C}{L}}J_1=\sqrt{(z+1)/z}\tanh\frac{\sigma}{2}\left[1+\frac{z}{2}-\sqrt{z(z+1)}\right], \quad (10)$$

and

$$\sqrt{\frac{C}{L}}J_1=\sqrt{(z+1)/z}\ \text{cosech}\ \sigma\left[1+\frac{z}{2}-\sqrt{z(z+1)}\right]. \quad (11)$$

Introducing the further transformation $y=z(4z+4)$, the following equations are obtained:

$$\frac{\sigma}{8\sqrt{\frac{C}{L}}J_1z}=\frac{\sigma}{4\sqrt{y}}\coth\frac{\sigma}{4}(\sqrt{y+1}-\sqrt{y}), \quad (12)$$

$$\frac{2}{\sigma\sqrt{\frac{C}{L}}J_2z}=\frac{4}{\sigma\sqrt{y}}\sinh\frac{\sigma}{2}(\sqrt{y+1}-\sqrt{y}), \quad (13)$$

which express the impedances as functions of y capable of expansion as Stieltjes fractions. The values of J_1 and J_2 determined from the expansion are

$$J_1=\frac{\sigma}{8}\sqrt{\frac{L}{C}}\cdot\cfrac{1}{a_0z+\cfrac{1}{a_1(4z+4)+\cfrac{1}{a_2z+\cdots}}}\qquad(14)$$

$$J_2=\frac{2}{\sigma}\sqrt{\frac{L}{C}}\cdot\cfrac{1}{a_0'z+\cfrac{1}{a_1'(4z+4)+\cfrac{1}{a_2'z+\cdots}}}\qquad(15)$$

in which the coefficients a_0, a_0', etc. are numerical quantities.

The procedure for the evaluation of the coefficients a_0, a_1, a_0', a_1', etc. is explained in my earlier Patent 1,570,215, issued January 19, 1926, in which the design of impedance network for other purposes is discussed. The method of evaluation is also described in Perron's textbook referred to above. In the above expansions the coefficients are functions of the quantity σ and therefore of the length l of the section of uniform line that is to be compensated. The values of the first seven coefficients in the expansion for J_1 for different values of σ are given by the curves of Fig. 4, and the first six coefficients in the expansion for J_2 are similarly plotted in Fig. 5. In the expansion for J_1 the first coefficient a_0 has the constant value 2; the even coefficients a_2, a_4, a_6, etc. have values which increase with σ and which are sufficiently alike to be represented by a single curve; and the odd coefficients likewise converge towards a single constant. In the expansion for J_2 the first coefficient a_0' is zero. The remaining coefficients are positive for a range of values of σ and at higher values are negative. It may be shown that so long as σ does not exceed the value $\sigma_{max}=\pi$ all of the coefficients of the expansion are positive. Negative values of the coefficients would correspond to an impedance structure using negative elements and hence to one that could be realized only by the use of negative impedance devices such as amplifiers. In order that the network may be of the passive type it is necessary, therefore, that the uniform line sections should not exceed in length the value for which $\sigma=\pi$, which corresponds to a length of line equal to

$$l_{max}=\pi\frac{\sqrt{CL}}{CR-LG}.\qquad(16)$$

The factors z and $(4z+4)$ are functions of the frequency. In this case they are linear functions of the same type as the impedance of a simple series combination of resistance and inductance, or the admittance of a resistance shunted by a condenser. When the multiplication of the fractions by the initial factors is carried out the elements may be identified with the impedances and admittances of the simple combinations.

The network for the impedance J_1 is shown in Fig. 2, the desired impedance being that between terminals A and B. The first branch is a shunt admittance of value

$$\frac{8}{\sigma}\sqrt{\frac{C}{L}}a_0z$$

which upon the substitution of the value of z becomes

$$\frac{1}{a_0t}+j\omega a_0s$$

where

$$t=\frac{8}{\sigma}\sqrt{\frac{C}{L}}\frac{LG}{CR-LG}\qquad(17)$$

and

$$s=\frac{8}{\sigma}\sqrt{\frac{C}{L}}\frac{LC}{CR-LG}.$$

This represents the admittance of a capacity of value a_0s shunted by a resistance

$$\frac{1}{a_0t}.$$

The remaining shunt branches are of the same form, the values of the elements being given by the products of the factors s and t and the appropriate coefficient a_2, a_4, a_6, etc. The series branches of the network likewise comprises resistances of respective values a_1q, a_3q, etc. in series with inductances a_1r, a_3r, etc. where

$$q=\frac{\sigma}{2}\sqrt{\frac{L}{C}}\frac{CR}{CR-LG}$$

and

$$r=\frac{\sigma}{2}\sqrt{\frac{L}{C}}\frac{LC}{CR-LG}.\qquad(18)$$

The network for the impedance J_2 is shown in Fig. 3 between the terminals C and D. It is of the same type as the network for J_1 but starts with a series branch instead of a shunt branch. The resistances of the successive series branches are given by $a_1'q'$, $a_3'q'$, etc., and the inductances by $a_1'r'$, a_3r', etc., where

$$q'=\frac{8}{\sigma}\sqrt{\frac{L}{C}}\frac{CR}{CR-LG}$$

and $\qquad(19)$

$$r'=\frac{8}{\sigma}\sqrt{\frac{L}{C}}\frac{LC}{CR-LG}.$$

The successive shunt resistances are

$$\frac{1}{a_2't'},\ \frac{1}{a_4't'},$$

etc., and the shunt capacities $a_2's'$, $a_4's'$, etc., where

$$s'=\frac{\sigma}{2}\sqrt{\frac{C}{L}}\frac{LC}{CR-LG}$$

and (20)

$$t'=\frac{\sigma}{2}\sqrt{\frac{C}{L}}\frac{LG}{CR-LG}$$

The numerical design of a network of the type described above for the correction of distortion in any given line requires only the knowledge of the line constants R, L, C and G. The line is preferably divided into a number of sections each of the maximum length given by Equation (16), and a network provided for each section. The schematic system of Fig. 1 shows the networks inserted at intervals along the line, but this is not necessary and the networks may conveniently be grouped together at one end of the line. This follows from the fact that the networks have the same characteristic impedance as the line and consequently may be inserted anywhere without causing wave reflections. The values of the factors q, r, s and t, in the expression for J_1 and the corresponding factors in J_2 are computed from Equations (17) to (20) and the coefficients a_0, a_1, etc., may be obtained from the curves of Figs. 4 and 5.

Theoretically an infinite number of sections of each of the ladder structures is required, but in practice only a few are necessary. The effect of using finite structures will be illustrated in a numerical example. Consider the case of a uniform line having the following constants;

R=43 ohms per loop mile
L=9x10^{-4} henry per loop mile
C=6.2x10^{-8} farads per mile
G=0

This corresponds to a number 16 gauge paper insulated telephone cable. The maximum length of line that can be compensated by a single T-network is found to be 8.8 miles, in which case $\sigma=\pi$. The resulting structure for J_1 is shown in Fig. 6, the values of the impedance elements being indicated in the drawing. The structure for J_2 is shown in Fig. 7. The first three sections of each network were calculated by the method described above, the remaining sections being added to provide an approximately correct termination. The values of the elements in the end sections were arrived at by noting that as the line is extended the sections tend towards uniformity. The transmission characteristics of the loading section comprising an 8.8 mile length of uniform line and a compensating network are illustrated by the curves of Figs. 8 and 9. Fig. 8 shows attenuation as a function of frequency; curve 1 represents the attenuation in the line and curve 2 the attenuation in the loading section. Fig. 9 shows the time of transmission as a function of frequency, curve 3 corresponding to the line and curve 4 to the loading section. The compensation is substantially complete except at frequencies below 200 c.p.s. where there is a slight increase in the attenuation and a diminution in the time interval. The reason for the residual distortion is that the attenuation in the ladder structures of the compensating network is small at low frequencies and the waves are able to penetrate to the ends of the structures, where they are reflected. The residual distortion can be reduced to any desired degree by increasing the number of sections, or by adding other impedances that will approximate to an infinite extension.

Other forms of the distortion correcting networks of the invention are shown schematically in Figs. 10 and 11. The network of Fig. 10 is of the π-type, comprising a series branch of impedance Z_1 and equal shunt branch impedances Z_2. The requirements that the network shall have a characteristic impedance equal to that of a given line and a propagation constant complementary to that of the line lead to values of Z_1 and Z_2 as follows:

$$Z_1=Z\sinh\,(\alpha+j\omega\beta-\Gamma l)$$

and

$$Z_2=Z\coth\frac{1}{2}(\alpha+j\omega\beta-\Gamma l).$$

If the same constant values are assigned to α and β as in the case of the T-network these equations may be rewritten in the forms

$$Z_1=\frac{\sigma}{8}\sqrt{\frac{L}{C}}(4z+4)\left[\frac{4}{\sigma\sqrt{y}}\sinh\frac{\sigma}{2}(\sqrt{y+1}-\sqrt{y})\right] \quad (21)$$

and

$$Z_2=\frac{2}{\sigma}\sqrt{\frac{L}{C}}(4z+4)\left[\frac{\sigma}{4\sqrt{y}}\coth\frac{\sigma}{4}(\sqrt{y+1}-\sqrt{y})\right] \quad (22)$$

where σ, z and y, have the same meanings as before. The functions of y in these equations are identical with those appearing in Equations (12) and (13) and accordingly may be replaced by Stieltjes fractions having the same coefficients. The values of Z_1 and Z_2 expressed as continued fractions are

$$Z_1=\frac{\sigma}{8}\sqrt{\frac{L}{C}}\cfrac{1}{a_1'z+\cfrac{1}{a_2'(4z+4)+\cfrac{1}{a_3'z+'}}} \quad (23)$$

and

$$Z_2=\frac{2}{\sigma}\sqrt{\frac{L}{C}}\left[a_0(4z+4)+\cfrac{1}{a_1z+\cfrac{1}{a_2(4z+4)+'}}\right. \quad (24)$$

which, upon carrying through the multiplication by the initial factors, may be identified with the impedances of ladder structures of the same types as in the T-network. The π-network is subject to the same limitation as to the length of line that may be compensated as applies to the T-network. This follows from the fact that the expansion of Z_1 involves the coefficients a_0', a_1', etc., which become negative for large values of σ. Since the expansion for Z_2 involves the coefficients a_0, a_1, etc., which are always positive, the design of the network for Z_2 is not thus restricted.

A type of correcting network that is not subject to this limitation is shown in Fig. 11. This is the "lattice" or "bridge" type having one pair of arms of impedance Z_a and another pair of impedance Z_b. In accordance with the requirements for complete compensation the branch impedances have the values

$$Z_a=Z \tanh \frac{1}{2}(\alpha+j\omega\beta-\Gamma l) \quad (25)$$

and

$$Z_b=Z \coth \frac{1}{2}(\alpha+j\omega\beta-\Gamma l) \quad (26)$$

Assuming the same values of α and β as before, the impedances Z_a and Z_b are respectively identical with the impedances J_1 of the T-network and Z_2 of the π-network, and may therefore be realized in the ladder type structures already found for these impedances. The design of these networks has already been shown to be free from any restriction as to the value of σ, or the length of the line.

In the foregoing analysis the quantities z and σ involve the factor $(RC-LG)$, which may be positive or negative depending on the relative values of the line constants. In almost every practical case the values of the line constants are such that RC is greater than LG, and the only cases in which the magnitudes would be reversed are represented by lines which are impractical either from the economic or from the efficiency standpoint. Even in these cases however, the compensating networks of the invention are applicable. The coefficients a_0, a_0', etc., of the various expansions are positive for negative values of σ, with the same limitations as described above, and the factors p, q, p', q', etc., since they involve the product or the quotient of σ and z are also positive regardless of the relative magnitudes of RC and LG. For the particular case RC=LG the correcting networks disappear, the line being already distortionless.

In the claims which follow the phrase singly infinite is used to describe the ladder networks that constitute the impedances of the correcting networks of the invention. This distinguishes from more complicated types in which there is not only an infinite number of sections but each branch is also constituted by an infinitely extended structure.

What is claimed is:

1. A distortion correcting network for use with a uniform transmission line of characteristic impedance Z and propagation constant P, comprising a plurality of branches, each of said branches being constituted by a ladder network including only passive impedance elements, and said ladder networks being initial portions of singly infinite artificial lines having impedances defined by the product of Z and a hyperbolic function of (P_0-P), where (P_0-P) is a propagation constant complementary to P, whereby the correcting network is adapted to compensate the distortion in the line at all frequencies.

2. A distortion correcting network for use with a uniform transmission line of characteristic impedance Z and propagation constant P, comprising a plurality of branches each constituted by a ladder network including only passive impedance elements, two of said branches being initial portions of singly infinite artificial lines having impedances defined by

$$Z \tanh \frac{1}{2}(P_0-P)$$

where (P_0-P) is a propagation constant complementary to P, and the remaining branches having impedances related thereto whereby the correcting network has the propagation constant (P_0-P), and is adapted to compensate the distortion in the line at all frequencies.

3. A distortion correcting network for use with a uniform transmission line of characteristic impedance Z and propagation constant P, comprising a plurality of branches each constituted by a ladder network including only passive impedance elements, two of said branches being initial portions of singly infinite artificial lines having impedances defined by

$$Z \coth \frac{1}{2}(P_0-P),$$

where (P_0-P) is a propagation constant complementary to P, and the remaining branches having impedances related thereto whereby the correcting network has the propagation constant (P_0-P), and is adapted to compensate the distortion in the line at all frequencies.

4. A distortion correcting network for use with a uniform transmission line of characteristic impedance Z and propagation constant P comprising a lattice, or bridge, network, the branches of which each comprise a ladder network including only passive impedance elements, one pair of oppositely disposed branches being initial portions of

singly infinite artificial lines having impedances defined by

$$Z \tanh \frac{1}{2}(P_0-P),$$

and the other pair having impedances substantially equal to

$$Z \coth \frac{1}{2}(P_0-P),$$

where (P_0-P) is a propagation constant complementary to P, whereby the correcting network is adapted to compensate the distortion in the line at all frequencies.

5. A distortion correcting network of the type set forth in claim 4 in which the propagation constant (P_0-P) has such value that the quantity P_0, which represents the propagation constant of the distortionless combination of the line and the correcting network corresponds to the attenuation and propagation velocity in the line at infinite frequency.

6. A distortion correcting network for use with a uniform transmission line having distributed shunt conductance in addition to series resistance and inductance and shunt capacity, said network comprising a lattice network the branches of which are constituted by ladder type networks including only passive elements, one pair of opposing branches having impedances substantially equal to

$$Z \tanh \frac{1}{2}(P_0-P),$$

and the other branches having impedances substantially equal to

$$Z \coth \frac{1}{2}(P_0-P),$$

where Z and P denote respectively the characteristic impedance and the propagation constant of the uniform line, and where (P_0-P) is a propagation constant complementary to P, whereby the correcting network is adapted to compensate the distortion in the line at all frequencies.

7. A distortion correcting network in accordance with claim 1 in which the series branches of the ladder networks are each constituted by a series combination of resistance and inductance, and in which the shunt branches are each formed by a capacity with or without a resistance in shunt thereto.

8. A distortion correcting network for use with a uniform transmission line of characteristic impedance Z and propagation constant P, comprising a T-network, the branches of which each comprise a ladder network including only passive impedance elements, the series branches being initial portions of singly infinite artificial lines having impedances defined by

$$Z \tanh \frac{1}{2}(P_0-P),$$

and the shunt branch having an impedance substantially equal to

$$Z \operatorname{cosech} (P_0-P)$$

where (P_0-P) is a propagation constant complementary to P, whereby the correcting network is adapted to compensate the distortion in the line at all frequencies.

9. A distortion correcting network for use with a uniform transmission line of characteristic impedance Z and propagation constant P, comprising a π-network the branches of which each comprise a ladder network including only passive impedance elements, the series branch of said π-network being an initial portion of a singly infinite artificial line having an impedance defined by

$$Z \sinh (P_0-P)$$

and the shunt branches having impedances substantially equal to

$$Z \coth \frac{1}{2}(P_0-P),$$

where (P_0-P) is a propagation constant complementary to P, whereby the correcting network is adapted to compensate the distortion in the line at all frequencies.

In witness whereof, I hereunto subscribe my name this 4th day of December, 1928.

THORNTON C. FRY.