

April 7, 1931.

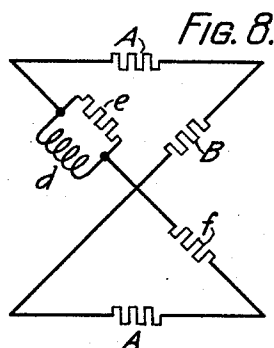
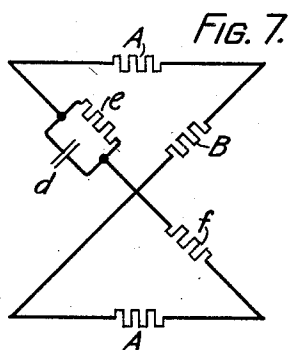
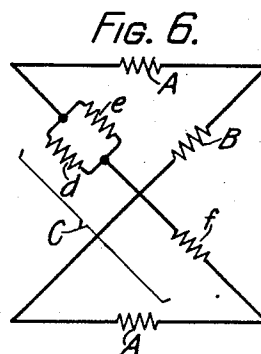
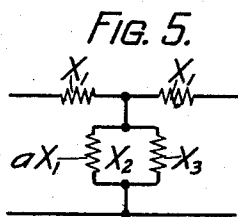
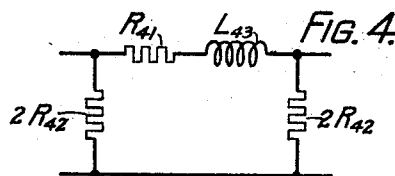
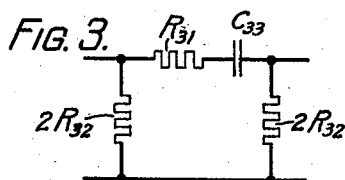
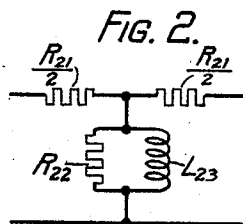
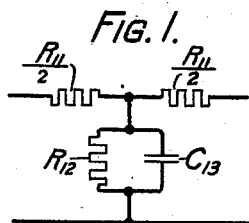
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1,799,794

ARTIFICIAL LINE

Filed Nov. 21, 1928

2 Sheets-Sheet 1



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1,799,794

ARTIFICIAL LINE

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2 Sheets-Sheet 2

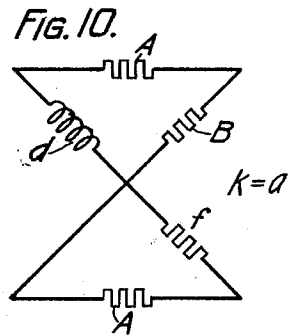
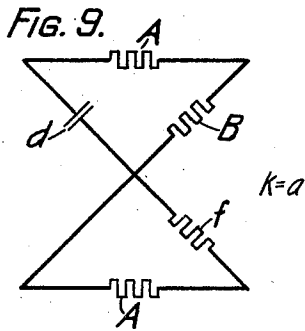


FIG. 11.

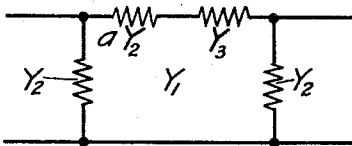


FIG. 12.

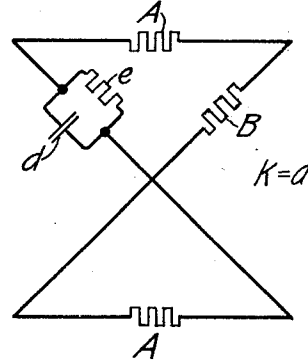
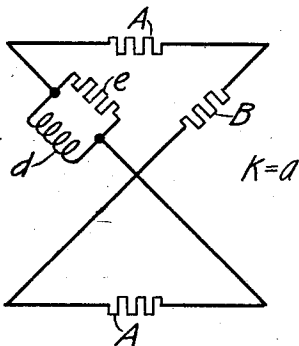


FIG. 13.



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ARTIFICIAL LINE

Application filed November 21, 1928. Serial No. 320,858.

This invention relates to artificial lines and particularly to artificial lines for simulating the transmission properties of lines having uniformly distributed constants.

5 The object of the invention is to provide artificial lines of the recurrent type having the property that their characteristic impedances are similar to the characteristic impedances of a smooth line, and, by proper adjustment of the values of the component impedances, may be made equal to the characteristic impedances of a given smooth line. Such lines may be used for balancing a
10 smooth line, as in the operation of telephone repeaters, or in duplex telegraph systems and a highly accurate balance may be obtained, provided both lines are of sufficient length or are so terminated that wave reflection from their remote ends is negligibly small.

15 Artificial lines having the above noted property are disclosed in U. S. patent to G. A. Campbell No. 1,643,332, dated September 27, 1927, these lines being of the lattice or bridge type contemplated by the present invention and having as their component
20 branch impedances resistive inductances and conductive condensers in accordance with the usual method of separating the component impedances of a uniform line.

25 In the improved lines of the present invention lattice sections are used comprising impedance branches having elements of only two types, namely, resistances and condensers or resistances and inductance coils. A feature of the invention is that the lattice
30 structures are not symmetrical, the two diagonal branches having unequal impedances.

Referring to the drawings, Figs. 1 to 4 inclusive illustrate previously known forms of ladder type networks;

Fig. 5 is a generalized T section of the forms of Figs. 1 and 2;

Fig. 6 illustrates a generalized lattice or bridge section of a form of the present invention, which is electrically equivalent to the T in Fig. 5;

Figs. 7 to 10 inclusive show specific lattice sections embodying the invention;

Fig. 11 illustrates a generalized form of

the π type sections of Figs. 3 and 4; and Figs. 12 and 13 illustrate lattice sections embodying the invention, which can be made equivalent to the π section of Fig. 11.

A smooth line is one whose impedance elements are uniformly distributed in character, and is exemplified by non-loaded open wire lines and cables. The characteristic impedance of a smooth line is the impedance measured at the input terminal of a line of infinite length, or of finite length if the remote end is terminated by an impedance which is equal to the characteristic impedance at all frequencies. The characteristic impedance varies with frequency and is expressed by the formula:

$$Z_L = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, \quad (1)$$

where

Z_L = the characteristic impedance,

R = the resistance in ohms per loop mile;

L = the inductance in henries per loop mile;

G = the leakage conductance in ohms per loop mile;

C = the capacity in farads per loop mile;

$\omega = 2\pi$ times the frequency.

An inspection of Equation (1) shows that at zero frequency the characteristic impedance is equal to

$$\sqrt{\frac{R}{G}}$$

while at infinite frequency it is equal to

$$\sqrt{\frac{L}{C}};$$

at intermediate frequencies it lies between these two limiting values.

The artificial lines of the present invention are composed of lumped impedance elements arranged in lattice structures, the characteristic impedances of which can be made to satisfy Equation (1) at all frequencies.

Figs. 1, 2, 3 and 4 illustrate sections of lines whose characteristic impedances are equal to that of a smooth line, and which satisfy Equation (1). The lines are assumed to be indefinitely long, that is, to consist of an in-

finite number of sections so that no waves are reflected from the end. In practice, however, it is found that the reflection effects are negligible in a line composed of several sections. A line terminated in the characteristic impedance at the remote end is free from wave reflections regardless of the number of sections used.

In terms of the smooth line constants the elements in Fig. 1 are:

$$\begin{aligned} \frac{R_{11}}{2} &= \sqrt{\frac{L}{C}} \\ R_{12} &= \frac{RC - LG}{2G\sqrt{LC}} \\ C_{13} &= \frac{2C\sqrt{LC}}{RC - LG} \end{aligned} \quad (2)$$

The elements in Fig. 2 are:

$$\begin{aligned} \frac{R_{21}}{2} &= \sqrt{\frac{R}{G}} \\ R_{22} &= \frac{LG - RC}{2C\sqrt{RG}} \\ L_{23} &= \frac{LG - RC}{2G\sqrt{RG}} \end{aligned} \quad (3)$$

The elements in Fig. 3 are:

$$\begin{aligned} R_{31} &= \frac{2L\sqrt{RG}}{RC - LG} \\ 2R_{32} &= \sqrt{\frac{R}{G}} \\ C_{33} &= \frac{RC - LG}{2R\sqrt{RG}} \end{aligned} \quad (4)$$

In Fig. 4 the elements are:

$$\begin{aligned} R_{41} &= \frac{2R\sqrt{LC}}{LG - RC} \\ 2R_{42} &= \sqrt{\frac{L}{C}} \\ L_{43} &= \frac{2L\sqrt{LC}}{LG - RC} \end{aligned} \quad (5)$$

The sections in Figs. 1 and 2 are said to have "mid-series" terminations because the terminating arms are series impedances equal to half the series impedance of a full section, while the sections in Figs. 3 and 4 have "mid-shunt" terminations because the terminating arms correspond to half the shunt admittance of a full section. The structures in Figs. 1 and 3 are suitable for simulating the impedances of smooth lines in which

$$\frac{R}{L} > \frac{G}{C}$$

while the networks in Figs. 2 and 4 are of use when the relationship

$$\frac{R}{L} < \frac{G}{C}$$

obtains. These networks are described in British Patent 223,336 of October 23, 1924, to A. C. Bartlett.

Fig. 5 shows a generalized T network of the form of Figs. 1 and 2 where the arms X_1 represent the series impedance and X_2 the shunt impedance. It may be shown that in general the lattice network of Fig. 6 is completely equivalent to the ladder network of Fig. 5 when

$$X_1 = A \quad (6)$$

and

$$X_2 = \frac{BC - A^2}{2A + B + C} \quad (7)$$

where A is the impedance of the two line arms of the lattice and B and C are the impedances of the diagonal branches respectively. When the network of Fig. 5 is of the type shown on Figs. 1 and 2, the impedance X_1 is a resistance and the shunt impedance X_2 may be considered to be composed of two impedances in parallel, one a resistance designated aX_1 , where a is a constant, and the other a reactance designated X_3 .

Now assuming that the impedance B in the network of Fig. 6 is a resistance of value kX_1 , and substituting in Formula (7) the components of X_2 , the following equation is obtained:

$$\frac{1}{aX_1} + \frac{1}{X_3} = \frac{2X_1 + kX_1 + C}{kX_1C - X_1^2} \quad (8)$$

from which

$$C = \frac{X_1[X_3(2a + ka + 1) + aX_1]}{akX_1 + X_3(k - a)} \quad (9)$$

Simplification and rearrangement of the Equation (9) gives

$$C = \frac{X_1}{k} + \frac{1}{\frac{k^2}{(k+1)^2 X_3} + \frac{(k-a)k}{a(k+1)^2 X_1}} \quad (10)$$

This equation represents the impedance of a combination comprising a resistance in series with a resistance and a reactance connected in parallel, and shows that the branch C can be physically realized. The conditions under which the lattice network of Fig. 6 is equivalent to the T of Fig. 5 are therefore as follows:

$$\begin{aligned} A &= X_1 \\ B &= kX_1 \\ d &= \frac{(k+1)^2}{k^2} X_3 \\ e &= \frac{a(k+1)^2}{k(k-a)} X_1 \\ f &= \frac{X_1}{k} \end{aligned} \quad (11)$$

where d , e and f are the component elements of the impedance arm C. This network is physically realizable so long as $k > a$. The value of a follows from the relationship between R_{11} and R_{12} in Fig. 1, or between the corresponding elements in Fig. 2. By substituting in formulæ (11) the values that X_1 and X_3 have in the network of Fig. 1, the elements of a lattice section equivalent to the network of Fig. 1 are determined. The values are:

$$\begin{aligned} A &= \sqrt{\frac{L}{C}} \\ B &= k \sqrt{\frac{L}{C}} \\ d &= \frac{k^2}{(k+1)^2} \frac{2C\sqrt{LC}}{RC-LG} \\ e &= \frac{a(k+1)^2}{k(k-a)} \sqrt{\frac{L}{C}} \\ f &= \frac{1}{k} \sqrt{\frac{L}{C}} \end{aligned} \quad (12)$$

The value of the coefficient a in this case is

$$a = \frac{RC-LG}{2LG}.$$

The structure of the equivalent lattice section is shown in Fig. 7.

By substituting in formulæ (11) the values that X_1 and X_3 have in Fig. 2 and the value of a determined from Fig. 2 in a manner similar to that for evaluating it from Fig. 1, the following values are obtained for the elements of the equivalent lattice section:

$$\begin{aligned} A &= \sqrt{\frac{R}{G}} \\ B &= k \sqrt{\frac{R}{G}} \\ d &= \frac{(k+1)^2}{k^2} \frac{LG-RC}{2G\sqrt{RG}} \\ e &= \frac{a(k+1)^2}{k(k-a)} \sqrt{\frac{R}{G}} \\ f &= \frac{1}{k} \sqrt{\frac{R}{G}} \end{aligned} \quad (13)$$

where

$$a = \frac{LG-RC}{2RC}.$$

This section is illustrated in Fig. 8.

For the special condition, $k=a$, the impedance e becomes infinite and the network has one less element. In this case the element

values of the lattice equivalent of Fig. 1 are:

$$\begin{aligned} A &= \sqrt{\frac{L}{C}} \\ B &= a \sqrt{\frac{L}{C}} \\ d &= \frac{a^2}{(a+1)^2} \frac{2C\sqrt{LC}}{RC-LG} \\ e &= \infty \\ f &= \frac{1}{a} \sqrt{\frac{L}{C}} \end{aligned} \quad (14)$$

where

$$a = \frac{RC-LG}{2LG}.$$

The network is illustrated in Fig. 9.

The elements of the lattice equivalent to the T in Fig. 2 for the condition $k=a$, are:

$$\begin{aligned} A &= \sqrt{\frac{R}{G}} \\ B &= a \sqrt{\frac{R}{G}} \\ d &= \frac{(a+1)^2}{a^2} \frac{LG-RC}{2G\sqrt{RG}} \\ e &= \infty \\ f &= \frac{1}{a} \sqrt{\frac{R}{G}} \end{aligned} \quad (15)$$

where

$$a = \frac{LG-RC}{2RC}.$$

The network is illustrated in Fig. 10.

In a similar manner a lattice network equivalent to the π type networks of Figs. 3 and 4 may be determined. A general form of the π networks is shown in Fig. 11, where Y_1 represents the series impedance and Y_2 the shunt impedances of the specific structures. The series impedance Y_1 is separated into its components, a resistance, (aY_2), and a reactance Y_3 . The lattice will have the same general structure as that obtained from a T network, which is represented by Fig. 6. The values of the elements are then found to be:

$$\begin{aligned} A &= Y_2 \\ B &= kY_2 \\ d &= \frac{(k+1)^2}{(ka+2k+1)^2} Y_3 \\ e &= \frac{(k+1)^2}{k(ka+2k+1)} Y_2 \\ f &= \frac{a-k}{(ka+2k+1)} Y_2 \end{aligned} \quad (16)$$

These structures are realizable when $k < a$.

When the values of Y_2 and Y_3 correspond-

ing to Fig. 3 are substituted in Equations (16), the lattice equivalent to Fig. 3 takes the form of Fig. 7 and the element values are:

$$\begin{aligned}
 A &= \sqrt{\frac{R}{G}} \\
 B &= k \sqrt{\frac{R}{G}} \\
 d &= \frac{(ka+2k+1)^2}{(k+1)^2} \frac{RC-LG}{2R\sqrt{RG}} \\
 e &= \frac{(k+1)^2}{k(ka+2k+1)} \sqrt{\frac{R}{G}} \\
 f &= \frac{(a-k)\sqrt{\frac{R}{G}}}{(ka+2k+1)} \quad , \quad (17)
 \end{aligned}$$

in which

$$a = \frac{2LG}{RC-LG}$$

The lattice equivalent to the π in Fig. 4 has the form of Fig. 8, and the elements are evaluated as follows:

$$\begin{aligned}
 A &= \sqrt{\frac{L}{C}} \\
 B &= k \sqrt{\frac{L}{C}} \\
 d &= \frac{(k+1)^2}{(ka+2k+1)^2} \frac{2L\sqrt{LC}}{LG-RC} \\
 e &= \frac{(k+1)^2}{k(ka+2k+1)} \sqrt{\frac{L}{C}} \\
 f &= \frac{(a-k)\sqrt{\frac{L}{C}}}{(ka+2k+1)} \quad , \quad (18)
 \end{aligned}$$

where

$$a = \frac{2RC}{LG-RC}$$

If $k=a$ the element f becomes zero and there is one less element present. Figs. 12 and 13 illustrate lattice networks obtained from the π structures of Figs. 3 and 4 respectively, for the special condition $a=k$. In Fig. 12 the elements are:

$$\begin{aligned}
 A &= \sqrt{\frac{R}{G}} \\
 B &= a \sqrt{\frac{R}{G}} \\
 d &= (a+1)^2 \frac{RC-LG}{2R\sqrt{RG}} \\
 e &= \frac{1}{a} \sqrt{\frac{R}{G}} \quad . \quad (19)
 \end{aligned}$$

In Fig. 13 the elements are expressed by:

$$\begin{aligned}
 A &= \sqrt{\frac{L}{C}} \\
 B &= a \sqrt{\frac{L}{C}} \\
 d &= \frac{1}{(a+1)^2} \frac{2L\sqrt{LC}}{LG-RC} \\
 e &= \frac{1}{a} \sqrt{\frac{L}{C}} \quad . \quad (20)
 \end{aligned}$$

It is obvious that in general there are an infinite number of lattice networks which may be made equivalent to each T or π network, since there are an infinite number of values of k which may be used.

It is furthermore possible to obtain another group of lattice structures identical in configuration to those already described, by applying a numerical factor to each of the series arms, this operation being compensated for in the bridge arms, whereby the characteristic impedance of the structure at all frequencies remains unchanged. This operation, however, results in the attenuation characteristic differing from that of the original lattice according to the particular value of the numerical factor chosen. In consequence, since there are an infinite number of possible numerical factors there are an infinite number of attenuation characteristics possible in the same type of structure, the characteristic impedance remaining unchanged.

In balancing a smooth line by an artificial line composed of network sections such as are embodied in this invention, the number of sections, in theory, should be infinitely large, to prevent wave reflections at the junction of the two lines. In practice, however, two or three sections are generally found sufficient to afford an accurate balance without serious wave reflections.

What is claimed is:

1. An artificial line for simulating the impedance of a smooth line, composed of recurrent sections of lattice network each section having in series an equal resistive impedance in each side of said line and two bridging impedances, one connecting between an input terminal and an output terminal on opposite sides of said line, the other connecting between the remaining input and output terminals, one of said bridging impedances being purely resistive, the other comprising a single reactive element.

2. An artificial line of recurrent lattice type sections for simulating the impedance of a smooth line, each section comprising a pair of equal resistive branches and a pair of impedance branches interconnecting the terminals of said resistive branches, one of said impedance branches comprising three

impedance elements, the other, one impedance element.

7. In combination, a smooth line having a characteristic impedance

$$Z_L = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

and an artificial line of recurrent lattice sections, each section having a series resistance in each side of the line equal to

$$\sqrt{\frac{L}{C}},$$

and two bridging arms interconnecting the terminals of said series resistances, one of said bridging arms consisting of a resistance, the other being a three-element arm comprising a reactance and two resistances, the elements of said bridging arms being so related to said series resistances that the characteristic impedance of said artificial line is equal to Z_L , in which R is the series resistance of said smooth line per unit length, G the shunt conductance per unit length, L , the series inductance per unit length and C , the shunt capacity per unit length.

8. The combination of claim 6 in which said three-element arm is a resistance in series with the parallel combination of a resistance and a reactance.

9. The combination of claim 7 in which said three-element arm is a resistance in series with the parallel combination of a resistance and a reactance.

In witness whereof, I hereunto subscribe my name this 20th day of November, 1928.

ARTHUR W. HORTON, JR.

3. A lattice network comprising a pair of equal resistive branches disposed in series with the line, and a pair of impedance branches interconnecting the terminals of said resistive branches, one of said branches being a three-element branch comprising two resistance elements and one reactive element, the other consisting of a resistance element, the elements of said section being so proportioned that the characteristic impedance is

$$Z_L = \sqrt{\frac{R + j\omega L}{G + j\omega C}},$$

where R , L , G and C are constants.

4. A lattice network comprising a pair of equal resistance branches and a pair of impedance branches interconnecting the terminals of said resistance branches, one of said impedance branches being resistive, the other being a three-element branch comprising the parallel combination of a resistance and reactance, in series with a resistance, the branches of said network being so proportioned with respect to the constants of a given smooth line that its characteristic impedance is equal to that of said smooth line.

5. A lattice network comprising a pair of equal resistance branches and a pair of impedance branches interconnecting the terminals of said resistance branches, one of said impedance branches being resistive, the other comprising a resistance and reactance in series relation, the branches of said network being so proportioned with respect to the constants of a smooth line that its characteristic impedance is equal to that of said smooth line.

6. In combination, a smooth line having a characteristic impedance

$$Z_L = \sqrt{\frac{R + j\omega L}{G + j\omega C}},$$

and an artificial line of recurrent lattice sections, each section having a series resistance in each side of the line equal to

$$\sqrt{\frac{R}{G}},$$

and two bridging arms interconnecting the terminals of said series resistance, one of said bridging arms consisting of a resistance, the other being a three-element arm comprising a reactance and two resistances, the elements of said bridging arms being so related to said series resistances that the characteristic impedance of said artificial line is equal to Z_L , in which R is the series resistance of said smooth line per unit length, G the shunt conductance per unit length, L , the series inductance per unit length and C , the shunt capacity per unit length.