March 10, 1931. W. P. MASON 1,795,874

IMPEDANCE ELEMENT

Filed Sept. 22, 1927

FIG. 3.

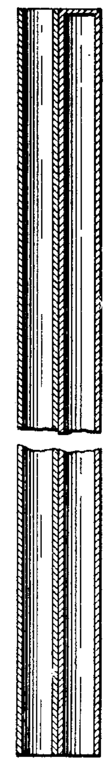
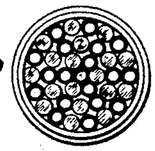


FIG. 2.



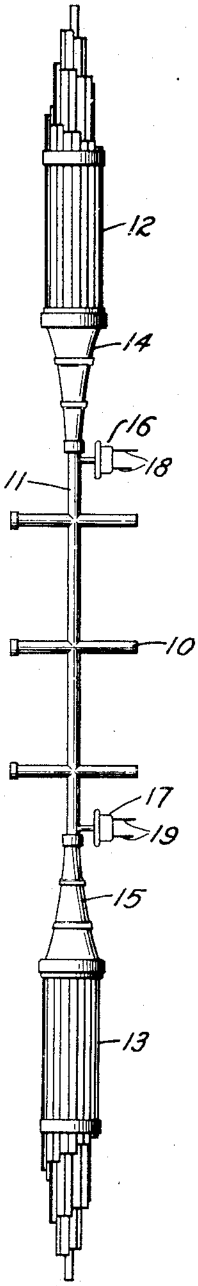
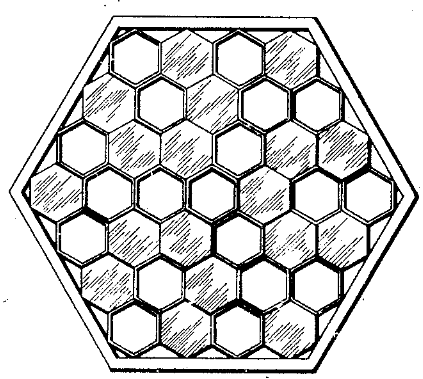


FIG. 5.

FIG. 4.



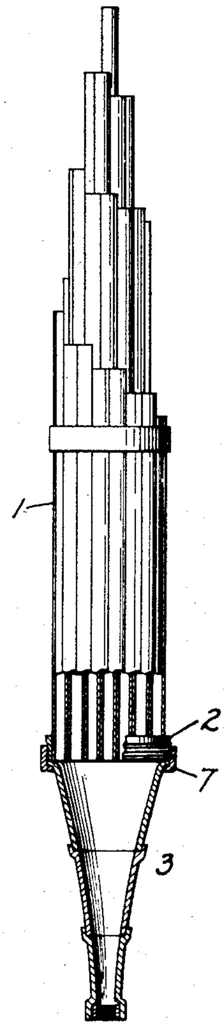


FIG. 1.

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UNITED STATES PATENT OFFICE

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IMPEDANCE ELEMENT

Application filed September 22, 1927. Serial No. 221,261.

This invention relates to the construction of impedance elements, and more particularly of acoustic impedance elements having impedance of an energy dissipative, or resistive character. An object of the invention is to secure a uniform rate of dissipation of energy, or uniform resistance, for a wide range of wave frequencies, such as is encountered in the transmission of speech or music. Another object is to secure compactness of structure and economy of material in the construction of acoustic impedance elements. A further object is to facilitate the design and construction of acoustic resistances in accordance with preassigned resistance values.

The term "impedance" is here used in a general sense to define the ratio of the effective value of a sinusoidally varying force impressed upon a medium to the effective value of the resulting velocity in the medium. In electrical systems impedance is familiarly defined as the ratio of a sinusoidal E. M. F. to the resulting current, and in mechanical systems it is the ratio of a sinusoidal mechanical force to the resulting vibrational velocity. In acoustic systems the medium by which waves are transmitted consists of a gas, generally air at normal atmospheric pressure, and the force by which the wave motion is set up is a pressure variation superimposed upon the normal pressure of the medium. The resulting velocity may be taken as the linear velocity of the gas particles that is superimposed on their normal random motion, but it is more convenient to define the velocity as the volumetric rate of flow through a given cross sectional surface of the acoustical system. In dealing with systems of the usual type in which the wave motion is guided by the walls of a conduit, the term "acoustic impedance" is used to define the ratio of the excess pressure intensity to the volumetric rate of flow, or volumetric velocity, across a given section of the conduit. The volumetric velocity may or may not be in phase with the excess pressure, and accordingly, if the familiar complex notation is used, the impedance will in general be a complex quantity. When the velocity is in phase with the pressure the impedance has a real value and is said to be resistive.

One method commonly used to obtain resistance in an acoustic system is by the use of screens or plugs of porous material such as hair, felt or cotton, or by the use of mechanical screens having a multiplicity of finely divided passages. When structures like these are used the flow of air is controlled mainly by viscosity, and the rate of energy dissipation is dependent principally upon the viscosity coefficient of the medium. Methods involving the use of porous structures are better adapted for obtaining high resistances, but on account of the irregularity of the air passages, and their small dimensions, the resistance values are not readily determined, and are generally quite variable with respect to frequency.

Acoustic systems, such as speaking tubes, wave filters, phonograph horns, and the like, are as a rule provided with conduits of substantial size to prevent the loss of energy due to viscosity. The impedances of acoustic systems are, in consequence, relatively low, and resistance elements for use in connection therewith are most frequently required to be of a correspondingly low value. To obtain acoustic resistance of low value and negligible reactance is a problem of greater difficulty, which heretofore has required the use of structures of inconveniently large dimensions, for example, large diameter tubes of very great length. As is well known, a uniform air conduit has wave transmission properties resembling those of a section of uniform electrical transmission line. If it is made increasingly long so that wave reflection from the remote end becomes unimportant, the acoustic impedance like the impedance of a uniform electrical line, approaches closely to a constant real value. The constant value of the impedance depends on the elastic and mass coefficients of the air, and is known as the characteristic acoustic impedance. The impedance of a finite tube, due to wave reflection, or resonance, oscillates with frequency about the characteristic impedance value, but, due to the absorption of energy by viscosity, the range of the variation decreases gradually to zero as the frequency rises. The maxima and minima occur at the several frequencies at which the air in the tube is resonant. When a single tube is used to provide a resistive impedance the length must be great enough to make the variations of resistance with frequency negligibly small. This results in impracticable structures involving tubes as long as 30 meters or more.

In accordance with this invention acoustic resistance elements are constructed by combining in parallel a number of small diameter tubes of progressively varying lengths. The impedance of each tubular element is characterized by periodic variations with frequency, just as in the case of a single long tube, but the fluctuations in the different tubes occur at different frequencies and mutually compensate each other. The result of this is that a high degree of uniformity is obtained at all frequencies down to a very low value. For an impedance of equal value and an equal degree of uniformity to that of a given long tube the multiple tube structure of the invention requires only a small fraction of the amount of material used in the single tube, and occupies only a very small fraction of the space. The impedance of the combination moreover is susceptible of precise computation from the dimensions of the device, since it depends almost entirely upon the readily ascertainable coefficients of mass and elasticity of air.

A particular feature of the invention by which the saving of material and space is substantially increased relates to the construction of the tubular elements of the combination. According to this feature the elements each comprise a pair of equal tubes one open at its remote end and the other closed. By this means the length of each tubular element is effectively doubled.

Another feature relates to means for varying the effective resistance of the combination so that a single resistance device may be used under different operating conditions.

These and other features of the invention will be more clearly understood from the following detailed description and the accompanying drawings, of which

Fig. 1 shows an acoustic resistance device in accordance with the invention;

Fig. 2 is an end view of the device of Fig. 1 illustrating features of the tube arrangement;

Fig. 3 shows a tubular element embodying a feature of the invention;

Fig. 4 is an enlarged detail illustrating a preferred embodiment of the invention; and

Fig. 5 illustrates the application of the invention in connection with an acoustic wave transmission system.

Referring to Fig. 1, the device there illustrated comprises a group of tubular elements 1 of progressively increasing lengths, which, as shown by Fig. 2, are stacked together in an assembly of approximately circular cross section. The tubes may be of brass or other suitable constructional material. The assembly comprises in all 37 tubes, the longest of which is about twice as long as the shortest. At one end of the assembly the tubes are brought together to a common mouth and are held together by a clamping ring 2. Additional clamping rings may be used for binding the tubes together at other points, or other suitable methods may be used. The tubes, all of which are open at the mouth end, may be either open or closed at their remote ends, or they may be grouped in pairs of equal length, each pair comprising an open and a closed tube. In the figure the tubes are arranged in the last mentioned manner, as indicated by Fig. 2 which shows an end view of the device of Fig. 1 as seen from above. A longitudinal section of a paired tubular element is shown in Fig. 3.

For the purpose of adapting the device to any particular acoustic system, a tapered connecting tube 3 is provided, which is clamped to the mouth of the device by the screwed coupling ring 7. The connecting tube 3 should preferably have an exponential taper, and further, should taper very slowly in order that it may be able to transmit freely waves of the lowest speech frequencies. It is advantageous to construct the tube in several sections as shown, one or more of which may be removed as desired. This permits the device to be used in connection with acoustic conduits of different sizes, and at the same time provides means for varying the effective value of the resistance of the device.

The acoustic properties of the structure will be understood from the following discussion of the theoretical principles involved. The characteristic impedance of a uniform tube, which is the value about which the actual impedance oscillates, is defined as the impedance of an infinitely great length of the tube. In other words the characteristic impedance of a tube is the value of the impedance when the effects of wave reflection at the ends are nullified. Its value, denoted by Z_k, for a tube of cross sectional area S is given accurately by the following formula

$$Z_k=\frac{\sqrt{P_0\gamma\rho}}{S}\left[1+x-jx\right] \quad (1)$$

in which

$$x=\frac{d}{2S}\sqrt{\frac{\mu}{2\omega\rho}}\left[1+\sqrt{\frac{5}{2}}\left(\sqrt{\gamma}-\frac{1}{\sqrt{\gamma}}\right)\right],$$

P_0 = the normal pressure intensity in the medium,
μ = the coefficient of viscosity,
ρ = the density,
d = the perimeter of the tube,
$\omega = 2\pi$ times the wave frequency,
γ = ratio of specific heats.

The derivation of Equation 1 is discussed in the Bell System Technical Journal, Volume VI, No. 2, April, 1927, pages 258 to 262. The principal part of the impedance is given by the factor

$$\frac{1}{S}\sqrt{P_0\gamma\rho}\ ,$$

which is the value of the characteristic impedance when there is no energy dissipation. The term x represents the fractional change due to the presence of dissipation.

The wave propagation constant for unit length of the infinite line is given by the formula

$$P=j\frac{\omega}{c}\left[1+x-jx\right]\equiv\alpha+j\beta \quad (2)$$

in which the propagation constant is denoted by P, and in which c denotes the velocity of sound in the medium. The real part α of the propagation constant measures the fractional diminution of the wave pressure, or of the volumetric velocity, per unit length of the tube. This part is termed the attenuation constant. The imaginary part β measures the change in phase as the wave traverses the tube, and is called the phase constant.

The actual impedance of a finite tube of length L, depends not only upon the characteristic impedance but on the reflection effects due to the manner in which the tube is terminated. Only two types of termination need be considered, first when the end of the tube is closed, and, second when the end is open. The impedance, Z_c, of a closed tube is given by the equation

$$Z_c=Z_k\left[\frac{1+e^{-2PL}}{1-e^{-2PL}}\right]=Z_k \coth PL, \quad (3)$$

and the impedance, Z_0, of an open tube is

$$Z_0=Z_k\left[\frac{1-e^{-2PL}}{1+e^{-2PL}}\right]=Z_k \tanh PL. \quad (4)$$

The factors within the brackets take account of the reflections at the end of the tubes. As the frequency varies the impedance, Z_0 or Z_c, oscillates between maximum values Z_k coth αL, and minima Z_k tanh αL. For an open tube the maxima occur at frequencies proportional to the numbers 1, 3, 5, etc., the first maximum occurring at the frequency for which the length of the tube is a quarter wave length. The minima occur at frequencies proportional to the numbers 2, 4, 6, etc. For a closed tube the frequencies of maximum and minimum impedance are interchanged. At high frequencies the values of tanh αL and coth αL both converge to unity and hence the tube impedances converge to the value Z_k. The tube behaves as though it were a vibratory system having an infinite number of resonance and anti-resonance frequencies following each other in alternation at regular intervals. The high impedances occur at the anti-resonance points and the low impedances at the resonance points.

The impedance of a combination of a number of tubes in parallel may be computed by combining the impedances of the individual tubes, as expressed by Equations 3 or 4, in the manner appropriate for a parallel connected system. For the case in which the tubes increase in length by equal increments a comparatively simple formula for the resultant impedance results. If n denotes the number of tubes, l the length of the shortest tube, and Δl the increment of length, and if it be assumed that the tubes are all open ended, the impedance of the combination is given by the equation.

$$\frac{1}{Z}=\frac{1}{Z_k}\left[\frac{1+e^{-2Pl}}{1-e^{-2Pl}}+\frac{1+e^{-2P(l+\Delta l)}}{1-e^{-2P(l+\Delta l)}}+\cdots\frac{1+e^{-2P(l+\overline{n-1}\Delta l)}}{1-e^{-2P(l+\overline{n-1}\Delta l)}}\right]. \quad (5)$$

Each of the terms within the brackets may be expanded into an infinite series, of which only the first two terms in each case are of importance. Neglecting the unimportant terms and summing up those retained, the following approximate formula is found:

$$\frac{1}{Z}=\frac{n}{Z_k}\left[1+\frac{2}{n}e^{-2Pl}\frac{1-e^{-2Pn\Delta l}}{1-e^{-2P\Delta l}}\right]. \quad (6)$$

The first term on the right hand side of Equation 6 indicates that the impedance of the combination has an average value equal to $\frac{1}{n}$ of the characteristic impedance of each tube. The second term represents the oscillations in the value of the impedance as the frequency varies. The validity of Equation 6 as an expression for the impedance depends upon the structure being such that the rejection of the higher terms in the expansion of Equation 5 is justified. Mathematically the justification requires that the quantity $2e^{-4Pl}$ should be small compared with n for the lowest frequency at which the device is to be used. In the structure, since it is desired to keep the length of the device small, this condition is achieved by the use of small diameter tubes. For example, it is found that for an air column in a tube 50 cm. long and 0.4 cm. diameter, the real part of the quantity $2e^{-Pl}$ has the value 1.6 at a frequency of 200 C. P. S. Where the number of tubes employed is fairly large, the value 1.6 is small enough to justify ignoring the higher terms.

The variation of the impedance corresponding to the variation of the second term of Equation 6 is more readily estimated from a simpler form of the equation obtained by standard trigonometrical transformations. The modified equation for the impedance is as follows:

$$\frac{1}{Z}=\frac{n}{Z_k}\left[1+\frac{2}{n}e^{-P(l+l_n)}\frac{\sinh Pn\Delta l}{\sinh P\Delta l}\right], \quad (7)$$

in which l_n denotes the length of the longest tube of the group. The second term within the brackets is a complex quantity, since P is complex, but in estimating its importance it is sufficient to determine its modulus value. Substituting the value $\alpha+j\beta$ for P, and simplifying the expression, the value of the modulus is found to be

$$\frac{2}{n}e^{-\alpha(l+l_n)}\sqrt{\frac{\sinh^2\alpha n\Delta l+\sin^2\beta n\Delta l}{\sinh^2\alpha\Delta l+\sin^2\beta\Delta l}}. \quad (8)$$

The above formula is convenient as a guide in the design of acoustic resistances in accordance with the invention. The numerical value diminishes very rapidly with frequency, consequently it is necessary to consider its effect only at the lowest frequency at which the device will be used. At this frequency the value should not exceed the limit of error prescribed by the operating conditions. For the design of acoustic resistances in which the medium is air at normal atmospheric temperature and pressure, the following numerical values of the basic coefficients, expressed in C. G. S. units, may be used:

$$P_0=10^6,$$
$$\mu=1.86\times10^{-4},$$
$$\rho=1.29\times10^{-3},$$
$$\gamma=1.4,$$
$$c=34500.$$

The value of x in Equations 1 and 2 corresponding to these coefficients is given by

$$x=.206\frac{d}{S\sqrt{\omega}}.$$

A convenient design rule is that the length of the shortest tube should be at least equal to half the wave length corresponding to the lowest frequency of the range desired, and that the longest tube should be from 2 to 3 times as long as the shortest. In a device for use in a speech frequency system tubes of inside diameter as low as 0.2 cm. may be used but satisfactory results may also be obtained with tubes of considerably greater diameter. The number of tubes determines the rapidity with which the resistance reaches a constant value and it is therefore desirable to use as large a number of tubes as possible. Considerations of the desired resistance value and of the permissible minimum diameter of the tubes, however, place a limit on the number of tubes that may be used.

In the device illustrated in Fig. 1 the tubular elements are combined in pairs each pair comprising one open and one closed tube. The impedance of the combination is readily found from Equations 3 and 4 to be as follows:

$$\frac{1}{Z_e}=\frac{1}{Z_k}\left[\coth PL+\tanh PL\right],$$

or

$$Z_e=\frac{Z_k}{2}\tanh 2PL, \quad (9)$$

in which Z_e denotes the impedance of the combination, and L the length of the tubes. A comparison of Equations 9 and 4 shows that the combination is equivalent to a single open tube of twice the length and having one half the characteristic impedance of each of the component tubes. A combination of n double elements of this type, graded in length in the manner described, has a normal impedance equal to $\frac{Z_k}{2n}$, and in uniformity is equivalent to a combination of n tubes of the double length. By doubling the effective length of each element in this manner, a greater degree of uniformity is obtained, or, alternatively, the overall length of the assembly may be reduced to approximately one half.

If circular tubes are used the interstices between the tubes constitute an additional series of paths which contribute a resistance in shunt to the resistance of the main paths. The amount of the correction, due to the shunt resistance may be estimated roughly by the foregoing formulæ, or if desired the interstices may be filled up. Since the interstices are very small the parallel impedance due to them is large and their effect on the total resistance is small.

One method of eliminating the interstices is shown in Fig. 4 which shows an end view of an assembly in which the tubes are of hexagonal cross section. Tubes of this form stack up without leaving any air spaces between them, and consequently no correction is required. The assembly of Fig. 4 comprises, like that of Fig. 1, 37 tubes which are arranged in 18 paired elements of the type of Fig. 3, together with a single open tube in the center. This center tube may either form part of the resistance proper or it may be used as a gauge tube in connection with an indicating device such as a telephone receiver for measuring or indicating the pressure variations at the input end of the resistance.

The normal value of the resistance is substantially equal to the characteristic impedance of a single tube whose cross sectional area is equal to the sum of the areas of the component tubes. One of the important uses of the resistance device is to provide the proper termination or damping for an acoustic system, but, if the area of the conduit of the acoustic system is smaller than the effective area of the device, the proper termination will not be provided unless some transforming means is introduced to effect the matching of the impedances. The necessary transforming means is provided by the exponentially tapered tube 3. As is well known, the characteristic acoustic impedance of an exponentially tapered sound conduit at any cross section is inversely proportional to the area at that cross section, and is constant and resistive at all frequencies above a certain critical frequency determined by the rate of taper. The tapered tube connection brings about the desired impedance matching by transforming the resistance of the device in the inverse ratio of its end areas. To adapt the device to a system of larger cross sectional area, an expanding tapered tube may obviously be used. By making the tapered tube of a number of detachable sections in the manner shown in Fig. 1, the device may be adapted to a variety of systems, or may be used under other conditions as a variable resistance.

The effect of a departure from the uniform law of variation of the length of the tubes is to produce a superimposed variation on the normal value of the resistance. For example, if all of the tubes were of the same length the superimposed variation would be periodic with respect to frequency, the frequency interval between successive maxima corresponding to the intervals between the successive tube resonances, which in this case occur at the same frequencies in all tubes. If the tube lengths vary progressively by some regular law other than the arithmetic progression, a similar periodic variation will result, but the amount of the variation will be less than in the extreme case mentioned above. For certain purposes a non-uniform resistance characteristic may be desirable, and, within limits, may be secured by grading the lengths of the tubes in different manners.

In Fig. 5 the use of the invention in connection with an acoustic link in a telephone system is illustrated. The acoustic link comprises an acoustic wave filter 10, of the multiband-pass type described in the co-pending application of W. P. Mason, Serial No. 201,525 filed June 25, 1927. The main conduit 11 of the filter is connected to resistance devices 12 and 13 by means of the tapered tubes 14 and 15. At the ends of the conduit 11 short side branch tubes of small diameter connect to telephone receivers 16 and 17 in which the telephone lines 18 and 19 terminate. The telephone receivers serve to convert the electrical waves into acoustic pressure variations and also to reconvert the acoustic waves, after transmission through the filter, into electrical oscillations. Telephone receivers of the electro-magnetic type are capable of translating the wave energy in both directions and permit two-way transmission through the systems. If transmission in one direction only is desired one of the receivers may be replaced by a microphone. Since the telephone instruments are essentially of high mechanical impedance, they are not adapted to provide, by themselves, the proper termination for the filter. This however, is provided by the resistance devices and the selective properties of the filter are thereby properly developed. An acoustic link of the type described above may be used for eliminating superimposed noise currents from a telephone circuit. The multi-band-pass filter 10 can be so designed that the bands are very closely spaced in the frequency scale, and are separated by narrow bands of very great attenuation spaced at harmonic intervals. The attenuating ranges may be so located that they suppress the noise making harmonics of stray currents, induced, for example, by power transmission lines. At the same time the narrowness of the attenuation bands prevents the loss of important speech frequencies.

The resistance elements may also be used for terminating other types of acoustic links in telephone systems. Instead of the wave filter 10 in the system of Fig. 5, the acoustic link may consist of a simple uniform tube, its purpose being to effect a delay in the transmission of the speech waves. The uniformity of delay, that is characteristic of an infinitely extended uniform tube, is realized in a finite tube only when wave reflection at the ends is eliminated.

The devices of the invention are not restricted to the use described above, but may be used under many conditions where acoustic resistance is desired, or where the damping of the energy of acoustic oscillations is required. It is to be observed, moreover, that the principles of the invention are applicable not only to acoustic devices but to wave transmission lines and combinations thereof in general, whether the medium be gaseous, solid, or the conduction of an electrical transmission line. Accordingly the scope of the invention is not to be limited to the particular devices described but only as defined by the appended claims.

What is claimed is:

1. An acoustic impedance device comprising a plurality of tubular elements of graded lengths constituting acoustic wave paths, and means for impressing acoustic wave pressure upon said paths simultaneously, the cross-sections and the number of said tubular elements being so proportioned with respect to a predetermined resistance that said device has an impedance substantially equal to said predetermined resistance over a wide frequency range which extends down to a very low frequency.

2. An acoustic impedance device comprising a plurality of tubular elements of lengths

graded in uniform steps constituting acoustic wave paths, and means for impressing acoustic wave pressure upon said paths simultaneously, the ratio of inner surface area to cross-sectional area of each element being large, whereby sound waves are substantially damped.

3. An acoustic impedance device according to claim 1 in which the tubular elements comprise pairs of equal tubes, one of each pair being closed at one end and the other open at both ends.

4. An acoustic impedance device comprising a plurality of tubular elements of progressively increasing lengths constituting acoustic wave paths, said paths being joined together at one end in a common mouth opening, and an air chamber in front of said mouth opening whereby acoustic wave pressure may be applied to said paths, the lengths of said elements being sufficient to substantially damp sound waves therein.

5. An acoustic impedance device comprising a plurality of tubular elements of progressively increasing lengths constituting acoustic wave paths, said tubular elements being stacked parallel to each other to form a close packed assembly of minimum cross-sectional periphery, and being connected at one end to a common air chamber whereby acoustic wave pressure may be applied to all of said paths simultaneously, the viscosity of said paths being substantial whereby sound waves are damped therein.

6. An acoustic impedance device in accordance with claim 5 in which the tubular elements comprise tubes of hexagonal cross-section and are stacked together in honeycomb fashion whereby interstices between the tubes are eliminated.

7. An acoustic impedance device comprising a plurality of tubular elements of progressively increasing lengths constituting acoustic wave paths, said paths being joined together in a common mouth opening, and an exponentially tapered tube connected to said mouth as a means for impressing acoustic wave pressure upon said paths simultaneously, the cross-sections of said elements being small enough in comparison with their lengths to produce a substantial degree of damping.

8. An acoustic impedance device in accordance with claim 7 in which the tapered connecting tube comprises a plurality of detachable sections of progressively increasing diameters.

9. An acoustic impedance device comprising a plurality of tubular elements of progressively increasing lengths constituting acoustic wave paths, means for impressing acoustic wave pressure upon said paths simultaneously, and means for varying the acoustic impedance presented by said paths to the impressed wave pressure.

10. An acoustic impedance device comprising a parallel connected system of multiple resonant acoustic elements which are so proportioned with respect to each other that the resonances occur at progressively increasing frequencies, and that the frequency intervals between the successive resonances of the system are small compared with the intervals between the resonances of each element, each of said elements including an air conduit of such small cross-section that the air vibration at resonance frequencies is substantially damped by viscosity.

11. An acoustic device in accordance with claim 10 in which each element includes an air conduit the diameter of which is of the order of 0.4 cm.

12. An acoustic wave impedance device comprising a connected system of linear elements of progressively increasing lengths, constituting wave transmission channels in which energy storage and energy dissipation properties are distributed uniformly along the length, said elements being connected at one end to a common wave input means whereby wave energy is impressed upon all of the transmission channels simultaneously, and being terminated at the other ends to produce full wave reflection, the cross-section of each element being small enough in comparison with its length to produce a substantial degree of damping due to viscosity.

13. A wave impedance device comprising a connected system of linear elements of progressively increasing lengths, constituting wave transmission channels in which energy storage and energy dissipation properties are distributed uniformly along the length, said elements being connected at one end to a common input means whereby wave energy is impressed upon all of the transmission channels, each of said elements providing a pair of equal transmission channels, one channel of the pair being closed at the end remote from the common junction whereby wave motion is stopped at said closed end, and the other channel being open at the end remote from the common junction whereby there obtains freedom of wave motion at said open end.

In witness whereof, I hereunto subscribe my name this 21st day of September, A. D. 1927.

WARREN P. MASON.