Jan. 13, 1931. K. S. JOHNSON 1,788,526

ARTIFICIAL LINE

Filed July 11, 1928

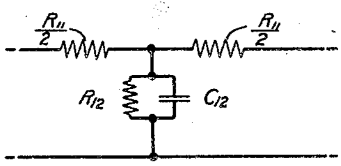

FIG. 1.

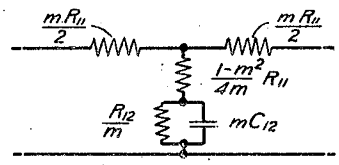

FIG. 2.

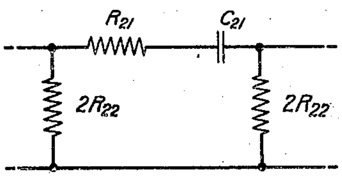

FIG. 3.

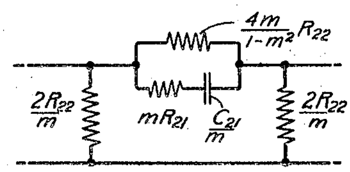

FIG. 4.

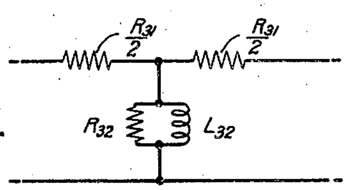

FIG. 5.

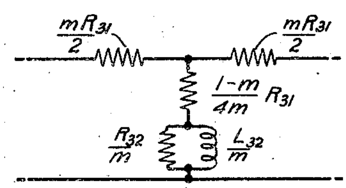

FIG. 6.

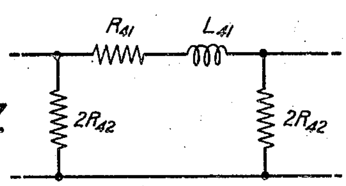

FIG. 7.

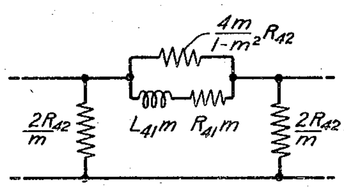

FIG. 8.

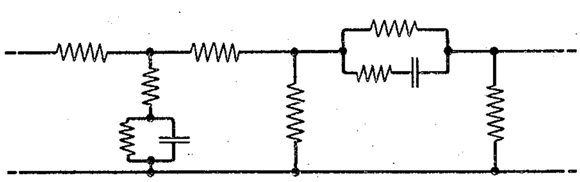

FIG. 9.

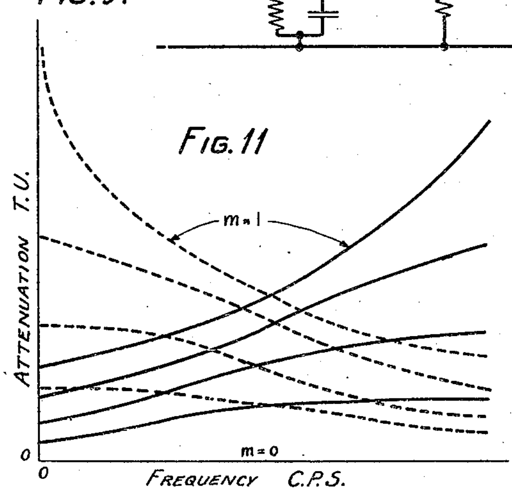

FIG. 11

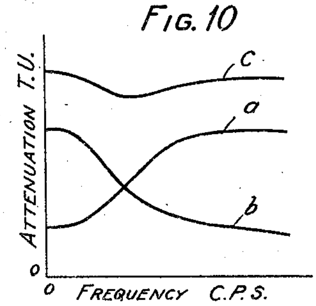

FIG. 10

INVENTOR
KENNETH S. JOHNSON
BY Joel C.R. Palmer
ATTORNEY

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UNITED STATES PATENT OFFICE

KENNETH S. JOHNSON, OF JERSEY CITY, NEW JERSEY, ASSIGNOR TO BELL TELEPHONE LABORATORIES, INCORPORATED, OF NEW YORK, N. Y., A CORPORATION OF NEW YORK

ARTIFICIAL LINE

Application filed July 11, 1928. Serial No. 291,803.

This invention relates to artificial lines and more particularly to artificial lines for simulating the impedance of a line having uniformly distributed constants.

The invention provides artificial lines of the recurrent type having the property that their characteristic impedances are similar to the characteristic impedance of a smooth line, and by proper adjustment of the values of the component impedances, may be made equal to the characteristic impedance of a given smooth line.

Such lines may be used for balancing a smooth line, as in the operation of telephone repeaters, of the duplex telegraph systems, and a highly accurate balance may be attained, provided that both lines are of such length or are so terminated, that wave reflection from their remote ends is negligibly small.

The invention also provides artificial lines of the type described above, the attenuation characteristics of which may be varied within wide limits, thereby enabling the lines to be used as attenuation equalizers, or for the simulation of both the impedance and the propagation characteristics of a smooth line.

These objects are attained by recurrent line structures in which resistive impedances and complex, resistive and reactive, impedances alternate with each other in series and shunt relation with respect to the line. The complex impedance branches are of the three-element type, comprising two resistances and one reactance, which may be either inductance or capacity. In certain embodiments of the invention the complex impedances constitute the series impedances of the line, and in other embodiments they are used as shunt impedances. Composite structures may also be used in which portions of both types of lines are used in combination. Variation of the attenuation characteristic, without affecting the characteristic impedance, is made possible by the use of three-element impedance branches in the line construction.

The invention will be better understood from the following detailed description and by reference to the drawing, of which Figs. 1, 3, 5 and 7 represent sections of previously known artificial lines for simulating the characteristic impedance on smooth lines. Since they form the basis from which the various forms of the present invention are derived, it is convenient to describe them as prototypes. Figs. 2, 4, 6 and 8 represent artificial lines embodying the invention. Fig. 9 shows a composite network embodying the invention. Fig. 10 illustrates attenuation characteristics of the composite network of Fig. 9 and its component parts. Fig. 11 illustrates variations in attenuation characteristics obtainable in the networks of this invention.

A smooth line is one whose impedance elements are uniformly distributed in character, and is exemplified by non-loaded open wire lines and cables. The characteristic impedance of a smooth line is the impedance measured at the input terminals of a line of infinite length, or of finite length if the remote end is terminated by an impedance which is equal to the characteristic impedance at all frequencies. The characteristic impedance varies with frequency and is expressed by the formula:

$$Z_L=\sqrt{\frac{R+j\omega L}{G+j\omega C}}, \quad (1)$$

where Z_L = the characteristic impedance,
R = resistance in ohms per loop mile,
L = inductance in henries per loop mile,
G = leakage conductance in ohms per loop mile,
C = capacity in microfarad per loop mile,
$\omega = 2\pi$ times the frequency.

The artificial lines of the present invention consist of lumped impedance structures of the series-shunt type, the characteristic impedances of which may be made to have the value expressed by equation (1). Consider the prototype structures in Figs. 1, 3, 5 and 7. Figs. 1 and 5 represent symmetrical T sections of lines composed of impedance arms disposed in alternate series-shunt relation, said to have mid-series termination inasmuch as the terminating arms consist of series impedances equal to half the impedance of full series arm. Similarly, the sections of Figs. 3 and 7 are said to have mid-shunt terminations since the terminating arms are shunt admittances equal to half the admittance of a full shunt arm. The impedance measured at the input terminals of these structures are designated respectively, mid-series impedance and mid-shunt impedance. The elements of the structure in Fig. 1 are determined from the smooth line constants by the formulæ:

$$R_{11}=2\sqrt{\frac{L}{C}}, \quad (2)$$

$$R_{12}=\frac{RC-LG}{2G\sqrt{LC}}, \quad (3)$$

$$C_{12}=\frac{2C\sqrt{LC}}{RC-LG}. \quad (4)$$

The elements of the section of Fig. 3 are expressed by the formulæ:

$$R_{21}=\frac{2L\sqrt{GR}}{RC-LG}, \quad (5)$$

$$R_{22}=\frac{1}{2}\sqrt{\frac{R}{G}}, \quad (6)$$

$$C_{21}=\frac{RC-LG}{2R\sqrt{GR}}. \quad (7)$$

These networks are suitable for simulating the impedances of lines for which RC−LG is positive, that is, for which

$$\frac{R}{L}>\frac{G}{C}.$$

The structures of Figs. 5 and 7 are of use for simulating the impedence of those smooth lines which satisfy the condition

$$\frac{R}{L}<\frac{G}{C}.$$

In Fig. 5, the elements of the structure can be expressed by the formulæ:

$$R_{31}=2\sqrt{\frac{R}{G}}, \quad (8)$$

$$R_{32}=\frac{LG-RC}{2C\sqrt{RG}}, \quad (9)$$

$$L_{32}=\frac{LG-RC}{2G\sqrt{RG}}. \quad (10)$$

The elements of the structures illustrated in Fig. 7 are given by:

$$R_{41}=\frac{2R\sqrt{LC}}{LG-RC}, \quad (11)$$

$$R_{42}=\frac{1}{2}\sqrt{\frac{L}{C}}, \quad (12)$$

$$L_{41}=\frac{2L\sqrt{LC}}{LG-RC}. \quad (13)$$

For further information relative to these structures reference is made to British Patent No. 223,336, of October 23, 1924.

The sections of lines shown in Figs. 2, 4, 6 and 8 which represent forms of the present invention are derived from the structures of Figs. 1, 3, 5 and 7, respectively as prototypes. As indicated in the drawings, the values of the elements of the derived sections are obtained from the elements of the respective prototypes by the application of a numerical factor "m". Consequently they are designated m-derived structures of the prototypes. A feature of any of these m-derived networks is that its characteristic impedance is the same as that of its prototype for any value of m, when the impedance is measured either at mid-series or mid-shunt according to the terminations of the prototype. Thus the mid-series characteristic impedance of the network in Fig. 2 is the same as the mid-series characteristic impedance of the network in Fig. 1. Likewise it follows that the mid-shunt characteristic impedance of the prototype of Fig. 3 and of the m-derived type of Fig 4 are equal and simulate the smooth line characteristic impedance at all frequencies. The same reasoning applies to the structures illustrated in Figs. 5, 6, 7 and 8. An inspection of the values of the elements of the m-derived networks shows that they are physically realizable when the value of m satisfies the condition $0<m<1$.

While the characteristic impedances of the m-derived types are the same as those of their respective prototypes, there is a difference between the attenuation characteristics. The prototypes all have infinite attenuation either at zero or at infinite frequency whereas the m-derived types do not. Fig. 10 illustrates the type of variation of the attenuation with frequency in the networks of the invention. The ordinates represent attenuation measured in "transmission units". The transmission unit, TU, is a logarithmic ratio of the power attenuation in the network. A treatment of this unit may be found in Chapter II of "Transmission Circuits for Telephonic Communication" by K. S. Johnson published by D. Van Nostrand Company, New York. Curve a represents a characteristic of the structure of either Fig. 2 or Fig. 8. Curve b is representative of the type of attenuation characteristic obtainable in the structures shown in either Figs. 4 or 6.

When the networks of the invention are used for simulating or for balancing the impedance of a given smooth line, the simulation or the balance can be effected to any desired degree of accuracy by the use of a relatively small number of sections, provided the overall attenuation in the simulating line is sufficient to render the effect of reflection at its end remote from the input terminals, negligibly small. On account of their greater attenuation at low frequencies the networks of Figs. 4 and 6 are better adapted to simulate the line impedance at low frequencies than at high, whereas the structures of Figs. 2 and 8 are better at high frequencies. By combining a section of the type shown in Fig. 2, however, with one of the type shown in Fig. 4, a composite section is obtained in which the attenuation is nearly uniform at all frequencies. Such a network is shown in Fig. 9 and the attenuation characteristic of the composite section is shown by curve c of Fig. 10. An artificial line made up of composite sections of this type is therefore effective to simulate or balance the impedance of a smooth line with a nearly uniform accuracy at all frequencies, and for a given degree of precision in the simulation it will require a total number of single sections less than half the number required in a line using sections of only one type.

The networks of Figs. 4 and 6 provide attenuations which diminish as the frequency increases, this being a variation of an inverse type to the variation of the attenuation in a smooth line. These networks may therefore be used to affect an equalization of the attenuation in a smooth line, and moreover, may be inserted at any point therein without causing reflection loss. By the choice of a suitable value of the factor m, or by the use of a plurality of sections for which the factor has different values, the compensation of the attenuation variation may be effected with a satisfactory degree of precision over any given range of frequencies. By plotting the attenuation characteristics for various values of m, a family of curves similar to curve b of Fig. 10 is readily obtained by means of which the proper selection and combination of sections to give a desired equalization is facilitated.

The effect of the factor m on the attenuation characteristic can be determined from an examination of the propagation constant for any series-shunt type of line which is expressible as:

$$P=\cosh^{-1}\left(1+\frac{Z_1}{Z_2}\right)=2\tanh^{-1}\frac{1}{\sqrt{1+4\frac{Z_2}{Z_1}}}, \quad (14)$$

where P = the propagation constant per section of line,

Z_1 = the impedance of a full series arm of the section,

Z_2 = the impedance of a full shunt arm of the section. The propagation constant, P, is a complex quantity which may be written:

$$P=A+jB \quad (15)$$

where B, the imaginary component measures the phase shift per section, and A, the real component, measures the attenuation per section. Consider the network of Fig. 2 in which at zero frequency:

$$Z_1=mR_{11}, \quad (16)$$

$$Z_2=\left(\frac{1-m^2}{4m}\right)R_{11}+\frac{R_{12}}{m}, \quad (17)$$

and

$$\frac{Z_1}{Z_2}=\frac{4m^2R_{11}}{(1-m^2)\ R_{11}+4R_{12}}, \quad (18)$$

$$\text{When } m=1, \frac{Z_1}{Z_2}=\frac{R_{11}}{R_{12}}.$$

$$\text{When } m=0, \frac{Z_1}{Z_2}=0.$$

At infinite frequency,

$$Z_1=mR_{11}, \quad (19)$$

$$Z_2=\frac{(1-m^2)}{4m}R_{11}, \quad (20)$$

and

$$\frac{Z_1}{Z_2}=\frac{4m^2}{1-m^2}. \quad (21)$$

$$\text{When } m=1, \frac{Z_1}{Z_2}=\infty.$$

$$\text{When } m=0, \frac{Z_1}{Z_2}=0.$$

From Equation 14 it is seen that the propagation constant and hence the attenuation, increases with

$$\frac{Z_1}{Z_2}.$$

Thus for $m=1$, the attenuation characteristic of the structure of Fig. 2 is represented by a curve having a finite value at zero frequency and rising to an infinite value at infinite frequency. This is represented in Fig. 11 by the full line curve ($m=1$). The attenuation for $m=0$ is represented by the base line. For values of m ranging between 0 and 1, there will be a family of curves represented by the full lines. These curves are also representative of the attenuation characteristics of the section in Fig. 8. It can be shown in a similar manner that the attenuations of the structures in Figs. 4 and 6 are as represented by the family of dotted line curves. It is noted that for the value of $m=1$, the m-derived networks reduce to the prototypes.

A further use of the composite section of Fig. 9 is to insert into a smooth line varying amounts of nearly uniform attenuation. The attenuation can be increased by increasing the number of composite sections.

What is claimed is:

1. An artificial line of the recurrent type having a characteristic impedance equal to that of a uniform smooth line at all frequencies, comprising resistance branches alternat-

ing in series-shunt relation with branches having three impedance elements, two of which are resistances and the third a reactance.

2. An artificial line of the recurrent type having a characteristic impedance equal to that of a uniform smooth line at all frequencies, comprising a symmetrical three-branch section, two of the branches of said section consisting of equal resistances and the third being a three-element impedance consisting of two resistances and a reactance.

3. An impedance network for simulating the impedance of a given smooth line, comprising a pair of sections each of the type described in claim 1, one having attenuation which increases, the other, which decreases with increasing frequency.

4. In combination with a smooth line, an impedance network comprising two symmetrical three-branch sections of the type described in claim 2, one of which has two equal resistance branches disposed in shunt to the line, and the other of which has two equal resistance branches disposed in series, each section having a characteristic impedance equal to that of said smooth line.

5. An artificial line having a characteristic impedance equal to that of a uniform smooth line at all frequencies, comprising a pair of symmetrical sections, one of which has a series arm composed of two resistances and a reactance and two shunt arms composed of equal resistances, the other of which has a shunt arm composed of two resistances and a reactance and two series arms composed of equal resistances.

In witness whereof, I hereunto subscribe my name this 10th day of July, 1928.

KENNETH S. JOHNSON.