Oct. 21, 1930. R. C. MATHES 1,779,382

NEGATIVE IMPEDANCE CIRCUITS

Filed Aug. 1, 1927

FIG. 1.

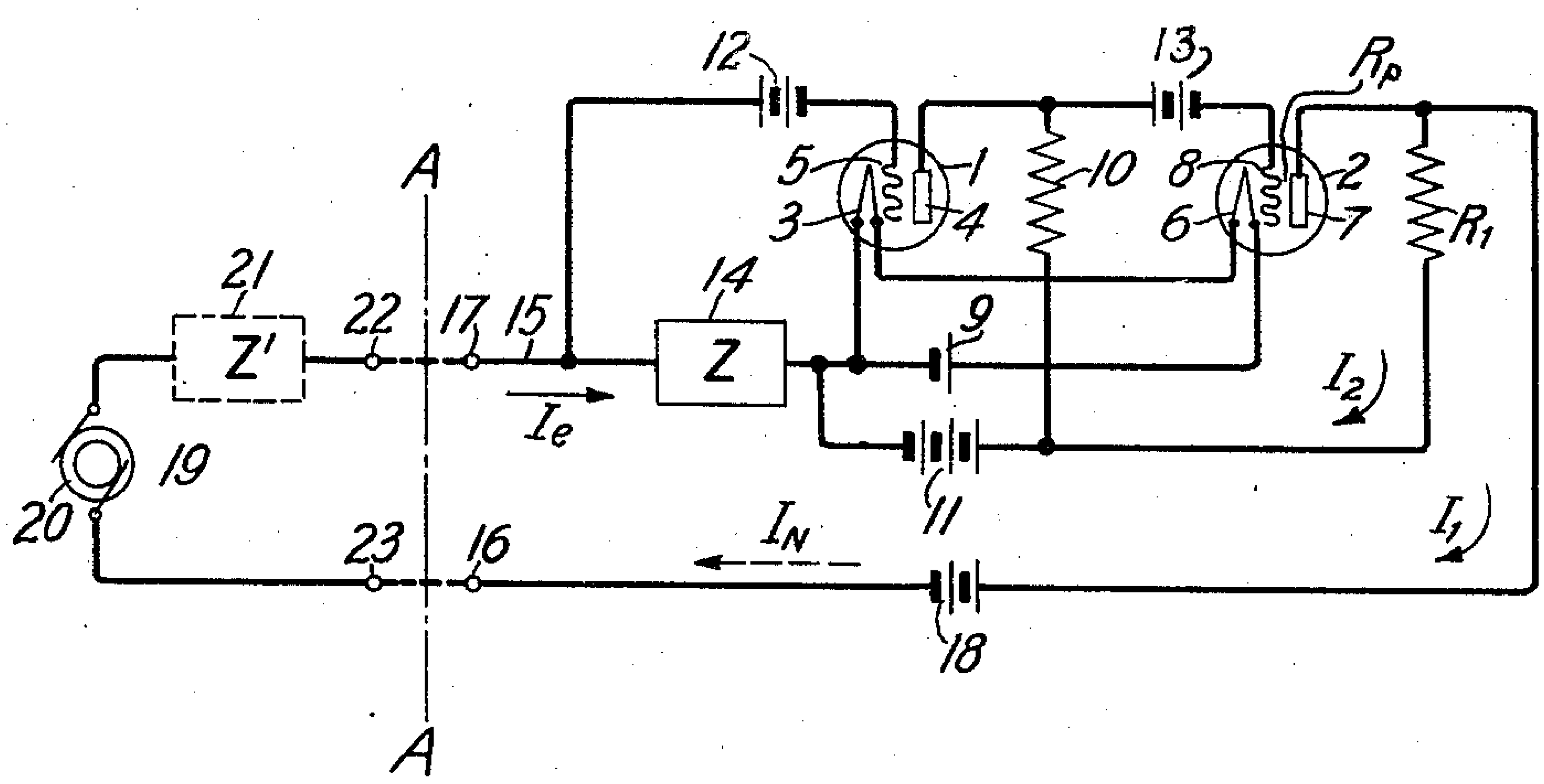
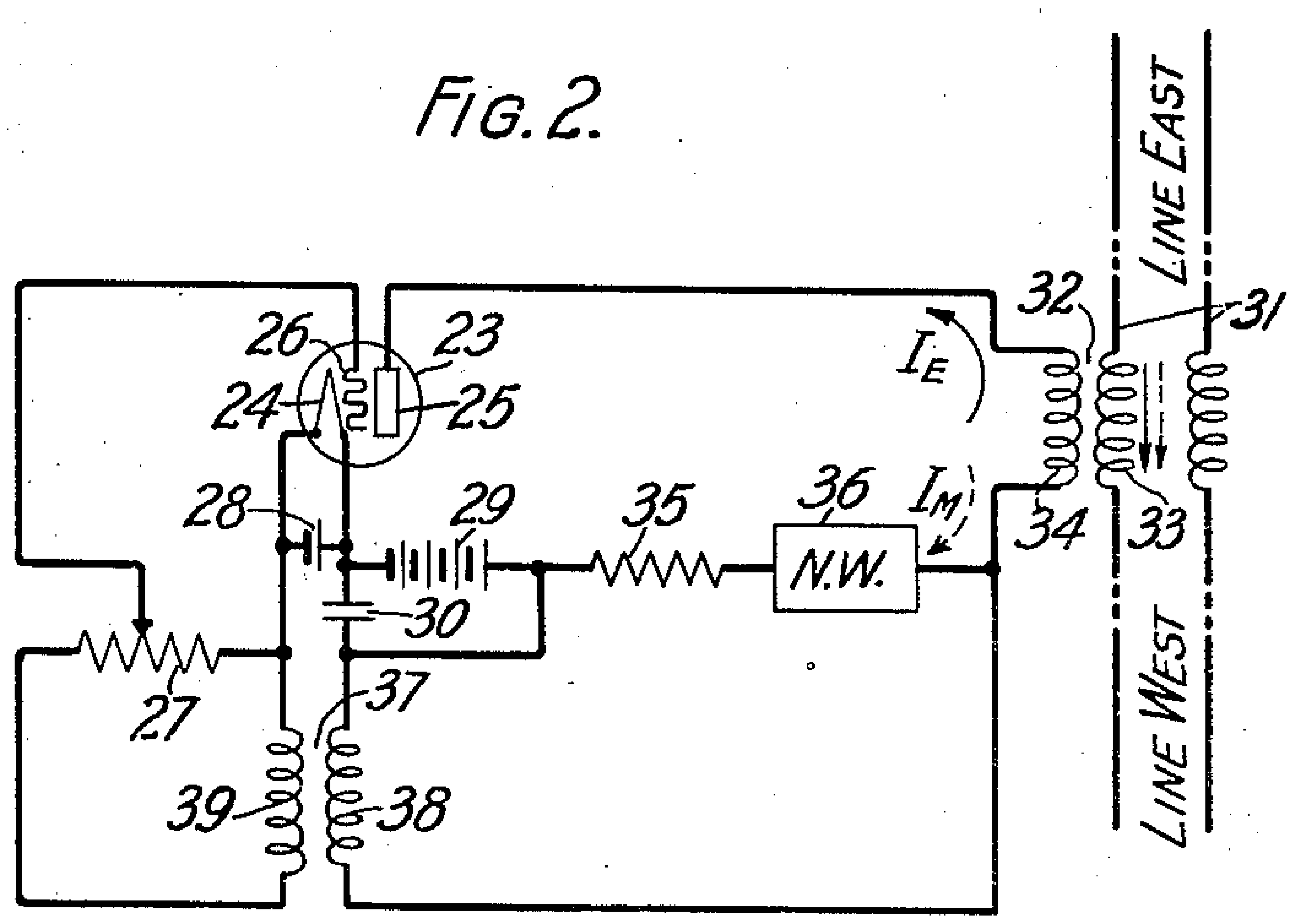



FIG. 2.

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# UNITED STATES PATENT OFFICE

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## NEGATIVE IMPEDANCE CIRCUIT

Application filed August 1, 1927. Serial No. 209,752.

This invention relates to method of and means for obtaining negative impedances, and particularly to vacuum tube circuits for that purpose.

An object of the invention is to produce a general negative impedance of any desired form for which a corresponding physical structure can be built up in terms of positive impedances.

From time to time in the design of theoretical networks for special transmission purposes, formal solutions have been obtained which have appeared to be physically unrealizable because they contained terms equivalent to negative inductances and negative capacities, as well as negative resistances. In the prior art, it is old to use vacuum tube amplifier circuits for obtaining a negative resistance, such a circuit being shown, for example, in an article entitled "Resistance neutralization—an application of thermionic amplifier circuits" by Bennett and Peters, pages 234–248 of the Journal of the A. I. E. E. vol. 41 (1922). The copending applica-cation of Frank H. Graham, Serial No. 209,753, filed August 1, 1927, discloses a vacuum tube amplifier circuit having two three-electrode vacuum tube stages, resistance-coupled, and a resistance feed-back coupling between the output circuit of the last stage and the input circuit of the first stage, in which circuit a negative resistance which is a function of the constants of the circuit is obtained between terminals connected respectively to the grid of the first tube and the plate of the last tube. The prior art, however, does not so far as known disclose any circuits for obtaining negative impedances in general.

In accordance with the present invention means are provided for obtaining a negative impedance which is the negative at all frequencies of any impedance network which can be constructed physically. This means in accordance with a preferred embodiment of the invention comprises an amplifier circuit having two three-electrode vacuum tube stages, resistance-coupled, the input circuit of the first stage being short circuited, and an impedance feed-back coupling between the output circuit of the last tube and the input circuit of the first tube. When a voltage is impressed across terminals connected, respectively, to the input circuit of the first tube between the grid thereof and the feed-back coupling, and to the plate of the last tube in this circuit, a desired negative resistance, negative inductance, negative capacity, or any complex combination of them, may be obtained across these terminals by utilizing as the feed-back impedance a similar network of positive elements having a ratio to the desired elements determined solely by choice of the various constants of the amplifier circuit. The value of this negative impedance is independent over a wide range of the frequency or amplitude of the current supplied by the impressed electromotive force.

In a modification of the invention necessitating only one vacuum tube, the feed-back impedance coupling is applied between the plate circuit and grid circuit of the same tube, a transformer of negligible distortion in phase and amplitude being connected between the feed-back impedance and the grid of the tube to give the required 180° shift of voltage which is obtained by the second vacuum tube in the preferred embodiment described above.

The invention is of particular application as a shunt or series booster circuit for telephone repeater amplifiers, as the positive resistance and reactance of a telephone line may be annuled to any desired degree by coupling the circuit of the invention effectively in shunt to or in series with the line, and employing as the feed-back impedance in that circuit a network designed to imitate the impedances in the line.

The various objects and features of the invention will be understood from the following detailed description thereof, when read in connection with the accompanying drawing in which

Fig. 1 shows diagrammatically one embodiment of the invention utilizing a resistance-coupled amplifier of two stages, and

Fig. 2 shows diagrammatically a different embodiment of the invention employing only one vacuum tube as applied in a series booster circuit for a telephone repeater.

In Figs. 1 the circuit shown to the right of the dot-dash line A—A in solid lines represents the circuit of the invention for obtaining a negative impedance, and the circuit to the left of the dot-dash line A—A represents any electrical circuit the positive impedances in which may be neutralized by the circuit of the invention.

Referring to Fig. 1, a two-stage vacuum tube amplifier circuit is shown comprising in the first stage, a space discharge device 1 having a cathode or filament 3, an anode or plate 4 and a control electrode or grid 5, and in the second stage a space discharge device 2 having a cathode or filament 6, a plate or anode 7, and a control electrode or grid 8. The cathodes 3 and 6, respectively, of tubes 1 and 2 are connected together and may be heated by a common battery 9. The output circuit of tube 1 and the input circuit of tube 2 are coupled through coupling resistance 10 common to the two circuits. The battery 11 supplies space current to the plate 4 of tube 1 through resistance 10, and space current to the plate 7 of tube 2 through resistance $R_1$. The battery supply resistances 10 and $R_1$ preferably are made non-inductive and non-capacitive. The grid potentials of tubes 1 and 2 are adjusted to any suitable values by grid batteries 12 and 13 in the respective input circuits of the tubes.

The grid 5 of tube 1 is connected through grid battery 12 and the impedance network 14 to the filament 3 of tube 1, and through grid battery 12 and conductor 15 to terminal 17. Terminal 16 is connected through a battery 18 to a point in the output circuit of tube 2 between plate 7 and resistance $R_1$. The battery 18 is poled so as to oppose the action of the space current source 11 and is of such voltage as will bring terminals 16 and 17 to the same potential when no external electromotive forces are impressed.

In explaining the operation of the circuit, it will be assumed that an electromotive force E is applied across the terminals 16 and 17 of the amplifier circuit by a generator having zero internal impedance (not shown), and drives a current $I_e$ through conductor 15 in the direction indicated by the solid arrow. For tracing the operation approximately it will be assumed that the applied E. M. F. is unidirectional and the network 14 is made up of resistance elements only. A portion of this current passes through the network 14 and the resultant drop in potential therein, the network 14 being connected across the input circuit of tube 1, will be impressed on grid 5 of tube 1 and cause a decrease in the negative voltage applied thereto, and a resultant increase in the average current flowing through the coupling resistance 10 in the output circuit of tube 1. The resultant drop in potential in resistance 10 will be applied to grid 8 in tube 2 and the voltage set up therein will cause a current to flow through the output circuit of the tube in the direction from the anode to the cathode thereof. Because of the high impedance thereto of resistance $R_1$, most of this current $I_N$ will be transmitted through battery 18, terminal 16, the source of potential E (not shown), terminal 17, to conductor 15 in the direction indicated by the dotted arrow. As this current is in phase with the initial current $I_e$ produced by the source of potential E connected across the terminals 16 and 17, it is apparent that the amplifier circuit will produce across these terminals an effect which is opposite to what would be produced by positive resistance elements and which may be termed a negative resistance.

The operation and the results obtained by the circuit described above may be explained more exactly for circuits in general and for alternating E. M. F.'s by the following theory:

Let $E$ = the voltage applied across terminals 16 and 17 of the circuit of Fig. 1

$M$ = net voltage amplification from the grid of tube 1 into the plate circuit of tube 2

$R_0$ = A. C. plate circuit resistance of tube 2

$R_1$ = battery feed resistance of tube 2

$Z$ = feed-back impedance of network 14

Then $E=(R_1+Z)I_1-R_1I_2$

$MZI_1=-R_1I_1+(R_1+R_0)I_2$

or $0=-(R_1+MZ)I_1+(R_1+R_0)I_2$

$$I_1=\frac{(R_1+R_0)E}{(R_1+Z)\ (R_1+R_0)-R_1(R_1+MZ)}$$

$$\frac{E}{I_1}=\frac{R_0R_1}{R_1+R_0}+Z-\frac{R_1M}{R_1+R_0}Z$$

Let $\frac{R_1}{R_1+R_0}=A$

Then $\frac{E}{I_1}=AR_0-(AM-1)Z$

If Z is a resistance R, a negative resistance of magnitude $AMR-R-AR_0$ is obtained at the terminals where E was applied.

A negative impedance to match any impedance, say $Z_1$, over a wide frequency range, may be produced by a corresponding group of positive impedance elements. Thus, if Z is made equal to

$$\frac{AR_0+Z_1}{AM-1},$$

the impedance

$$\frac{E}{I_1}=-Z_1$$

is obtained. It is thus possible to get not only negative resistance but negative inductance, negative capacity, and any complex combination of them desired. This is done by building up a similar network of positive impedance elements having the ratio to the desired elements of

$$\frac{1}{AM-1},$$

putting in series with it resistance

$$\frac{AR_0}{AM-1},$$

and using the whole as the feed-back impedance in the circuit described. If AM is made equal to 2 then the network should have an impedance equal to that of the external circuit plus the impedance of the plate circuit of the tube for complete neutralization of positive impedance.

From the above, it is apparent that the circuit of the invention may be utilized to obtain a negative impedance which is the negative of any combination of positive impedances that may be constructed. For example, let it be assumed that the electrical circuit 19, shown to the left of the dot-dash line A—A in Fig. 1, is connected by its terminals 22 and 23 in any suitable manner across the terminals 16 and 17 of the circuit to the right of the dot-dash line A—A. In circuit 19, a source of electromotive force driving a current therethrough is represented by the generator 20, which is assumed to have zero internal impedance, and box 21 represents a network of the positive impedances in the circuit. Then if the feed-back impedance network 14 is made up of positive impedances imitating the positive impedances in network 21 in electrical circuit 19 and having the ratio to the elements of network 21 as given by the equations above, it is apparent that the effect of the positive impedances in network 21 in the circuit 19 may be substantially annulled. By also annulling the plate circuit impedance as described very large currents may be caused to flow as compared with the currents which flow when the negative impedance circuit is removed.

In accordance with the invention, the arrangement of Fig. 1 may be modified by inserting in place of the battery 18 in the conductor connecting the plate 7 of tube 2 with the terminal 16, a condenser, which will prevent the direct currents in the output circuit of tube 2 from being transferred to the feedback circuit. In that case, of course, it will be necessary to neutralize the effect of this condenser by inserting another condenser of suitable value in series with the impedance network 14, and with a suitable high resistance leak around it. This leak is needed to maintain the proper D. C. potential on the first grid. It should be of such a high resistance that it will have negligible effect on the generated negative impedance within the desired frequency range.

In Fig. 2 is shown a modification of the negative impedance arrangement of Fig. 1 utilized as a series booster circuit for a telephone repeater. Referring to Fig. 2, this arrangement comprises the space discharge device 23 having a cathode or filament 24, an anode or plate 25 and a control electrode or grid 26. The input circuit of tube 23 is connected through a portion of an adjustable high resistance 27, through a path extending from grid 23 through a portion of resistance 27 to filament 24. The cathode 24 is heated to incandescence by the battery 28. The plate 25 of tube 23 is supplied with space current from the battery 29. A condenser 30 is connected across the terminals of battery 29 to provide a path for alternating current around it. The output circuit of tube 23 is coupled to the telephone line 31 through a transformer 32 which has one winding 33 in series with one line wire and another winding 34 which is in series in the output circuit of tube 23. The output circuit of tube 23 includes in series, filament 24 of tube 23, space current battery 29, resistance 35, impedance network 36, winding 34 of transformer 32, plate 25 of tube 23 and the space path of tube 23. The output circuit of tube 23 is coupled to the input circuit of that tube through a transformer 37 having one winding 37 connected across the part of the output circuit containing resistance 35 and impedance network 36 in series, and a second winding connected across the terminals of resistance 27. The transformer 37 is designed to have negligible distortion both in phase and amplitude. The variable high resistance 27 is provided to give a convenient adjustment of the magnitude of the negative resistance. It should be made so high in value as to make negligible the load across the output winding 39 of transformer 37.

The arrangement of Fig. 2 operates in exactly the same manner as Fig. 1 if the windings 38 and 39 of transformer 37 are poled so as to produce the same reversal of phase that tube 2 did in Fig. 1. The factor M in the formula for negative resistance given above would be for this case the net combined voltage ratio due to the transformer 37 and the single tube 23 between the voltage generated in the plate circuit of the tube and the voltage developed across the feed-back impedances comprising resistance 35 and network 36.

The resistance 35 in the circuit of Fig. 2 should be given such a value as to cause a resistance feed-back which will neutralize the positive resistance of the plate circuit of tube 23. Then if network 36 is constructed of elements imitating the corresponding positive impedance elements in line 31, whether these elements be resistances, capacities or inductances, or combinations thereof, on a suitable scale to balance the sum of the line impedances as measured through transformer 37, these line impedances will be substantially annulled as to their effect on the signaling currents. The network 36 should preferably be connected in through a transformer identical with (or in proper ratio to) transformer 32 to neutralize its effect on the currents to be amplified. To obtain maximum efficiency for this circuit, of course, the transformer 37 should be one which will cause minimum distortion both in phase and amplitude on the fed-back currents.

It is, of course, apparent that the negative impedance arrangement of Fig. 1 could be substituted for the negative impedance arrangement of Fig. 2, in which case the terminals 16 and 17 of the former would be connected to the terminals of the winding 34 of transformer 32. Either the arrangement of Fig. 1 or the arrangement of Fig. 2 could be used as a shunt booster repeater if the negative impedance terminals are connected by a transformer in shunt of the line 30 instead of in series with that line as in the arrangement described. In this case the network 36 would be made to imitate the impedance of the two parts of the line in parallel as seen through the transformer 37.

Although the circuit of the invention has been illustrated in Fig. 1 and described as consisting of two vacuum tube stages, it is to be understood that it is not so limited as in accordance with the invention any even number of vacuum tubes may be utilized therein to produce a negative impedance or an odd number of tubes in conjunction with a phase reversing transformer as shown in Fig. 2.

In the practical use of the circuit of the invention to neutralize the positive impedance of circuits connected across the negative impedance terminals, to prevent "singing" effects it is desirable to so select the values of the elements in these circuits that the negative impedance obtained therefrom is less than the positive impedance of the circuit it faces, that is, so that the net impedance of the combination as a whole is positive.

The system above illustrated and described should be considered merely as typical, and not as limiting the invention, the scope of which is defined in the appended claims.

What is claimed is:

1. A circuit for neutralizing the effect of the impedances in a load circuit, comprising a space discharge device having a cathode, an anode and a control electrode and having an input and an output circuit, means comprising a transformer having windings coupling said output circuit to said load circuit, said output circuit comprising in series between said cathode and said anode, a source of space current for said device, a resistance element, an impedance network and one of said windings of said transformer, said input circuit comprising a variable high resistance connected between said control electrode and said cathode, a second transformer of negligible distortion both in phase and amplitude coupling said output circuit to said input circuit, said second transformer having one winding connected across said resistance element and said impedance network in said output circuit, and another winding connected across the terminals of said high resistance in said input circuit, said windings of said second transformer being poled so as to produce a 180° phase-shift in the voltages fed-back therethrough, said resistance element in said output circuit having such a value as to produce a feed-back to said input circuit sufficient to balance the positive resistance of said output circuit, and said network being constructed of impedance elements imitating in impedance characteristics the impedances in said load circuit, and of such values as to balance the sum of said load circuit impedances as measured through said second transformer.

2. A circuit for effectively producing the negative of an impedance between the terminals of an electrical circuit including a source of alternating electromotive force, said circuit comprising a space discharge amplifying device having a cathode, an anode and a control electrode, and having an input and an output circuit, an impedance network connected in series with said terminals between said anode and said cathode in said output circuit, and a single transformer coupling said output circuit to said input circuit and comprising a primary winding connected across said impedance network and a secondary winding connected in said input circuit between said cathode and said control electrode.

3. A circuit in accordance with claim 2 and in which the windings of said single transformer are poled so as to produce substantially a 180° phase shift in the amplified waves transmitted therethrough, and said impedance network comprises elements imitating the impedance characteristics of and proportioned with respect to said reactive impedance.

4. A circuit for effectively producing the negative of a reactive impedance between the terminals of an electrical circuit including a source of alternating current waves, said circuit comprising a space discharge amplifying device having a cathode and anode and a control electrode, and having an input and an output circuit, an impedance network connected in series with a resistance between

said cathode and anode in said output circuit, a single transformer coupling said output circuit and said input circuit, said transformer having a primary winding connected across said resistance and said impedance network and a secondary winding connected in said input circuit between said cathode and said control electrode, the windings of said transformer being poled so as to produce substantially a 180° phase shift in the alternating current waves fed back therethrough from said output circuit to said input circuit, and means for preventing transmission of direct current from said output circuit to said electrical circuit.

5. A circuit in accordance with claim 4 and in which said resistance has a value such as to produce a feed-back to said input circuit sufficient to balance the positive resistance of said output circuit, and in which said impedance network comprises elements imitating in impedance characteristics said reactive impedance and of such values as to balance said reactive impedance as measured through said transformer.

In witness whereof, I hereunto subscribe my name this 28 day of July A. D., 1927.

ROBERT C. MATHES.