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NEGATIVE IMPEDANCE CIRCUITS

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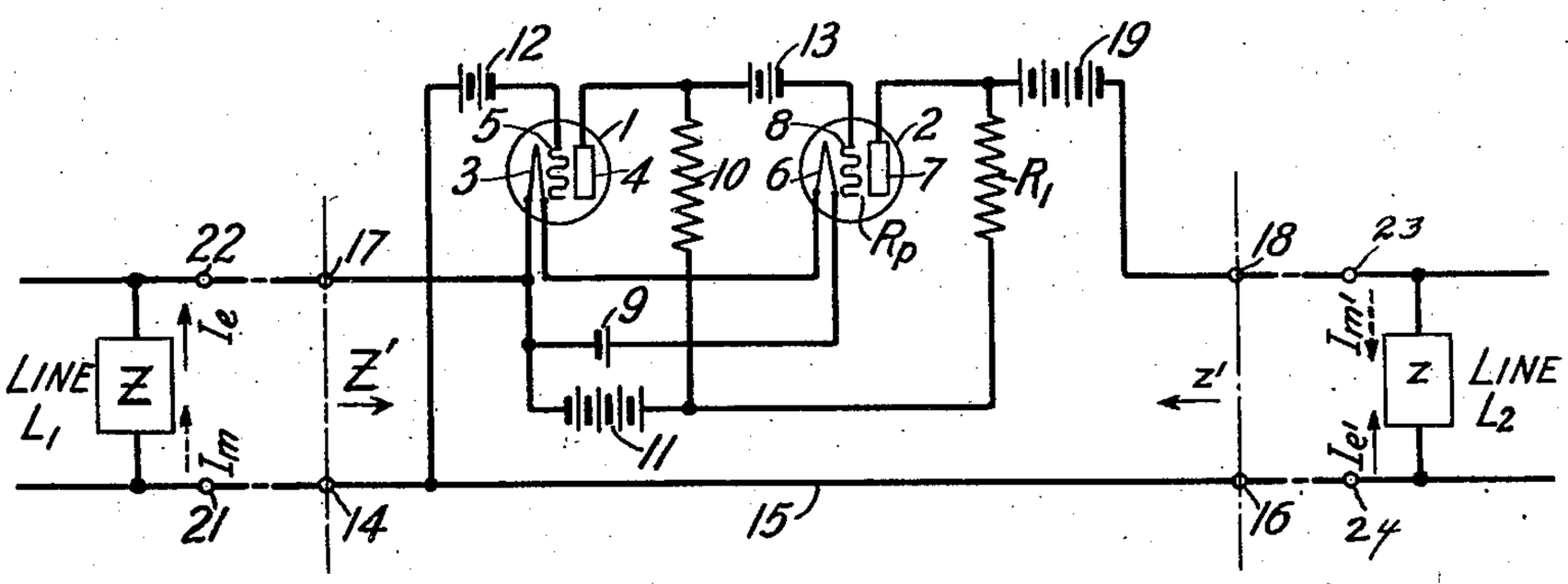

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NEGATIVE IMPEDANCE CIRCUITS

Application filed August 1, 1927. Serial No. 209,757.

This invention relates to a method and means for obtaining negative impedance and particularly to vacuum tube circuits for that purpose.

An object of the invention is to produce two different types of negative impedance in the same electrical circuit.

A further object is to combine these different types of negative impedance to produce advantageous results in a telephone repeater.

From time to time in the design of theoretical networks for special transmission purposes, solutions have been obtained which have appeared not to be physically realizable because they contained terms equivalent to negative inductances and negative capacities, as well as negative resistances. The copending application of R. C. Mathes, Serial No. 209,752, filed August 1, 1927, discloses a circuit for obtaining a negative impedance which is the negative at all frequencies of any positive impedance which can be constructed physically. This circuit in a preferred embodiment is an amplifier comprising two three-electrode vacuum tube stages, resistance-coupled, the input circuit of the first stage being short-circuited, and a feed back impedance connected between the output circuit of the last tube and the input circuit of the first tube in the amplifier. In this circuit, when an electromotive force is impressed across terminals connected respectively to the input circuit of the first tube between the grid thereof and the feed back impedance, and to the plate of the last tube, any desired negative resistance, negative inductance, negative capacity, or any complex combination thereof, may be obtained across the terminals by utilizing as the feed back impedance a similar network of positive elements having a ratio to the desired elements determined solely by the various constants of the amplifier circuit. The copending application referred to also discloses a modification of the above described circuit necessitating only one vacuum tube in which the feed back impedance is applied between the plate circuit and grid circuit of the same tube, a transformer of neglible distortion in phase and amplitude being inserted between the grid circuit and the feed back impedance, the negative impedance in this case being obtained between terminals connected respectively to the plate of the tube and one terminal of the feed back impedance.

In both of the circuits of the copending application referred to, the negative impedance obtained, when utilized to balance the effect of positive impedance in an electrical circuit connected across the negative impedance terminals, has been found to be stable only when the constants of the circuits are selected so that the net impedance of the combined circuits is positive; that is the negative impedances must be less than the positive impedance of the circuit it faces in order to prevent singing.

In accordance with the present invention it has been found that either of the above described circuits besides being utilized to obtain the negative impedance having the characteristics set forth, may be utilized also at the same time to produce between two other points in the same circuit, another negative impedance having different characteristics. The new type of negative impedance is obtained across the terminals of the feed back impedance network in the amplifier circuit, and is such that when utilized to balance the positive impedances in an electrical circuit connected across these terminals, it is stable only when the net impedance of the combined circuits is negative, that is, when the constants of the circuits are so selected that the negative impedance across the feed back impedance terminals is greater than the positive impedance of the circuit it faces to prevent singing.

A practical application of the circuits of the invention is a repeater adapted for repeating a message in two opposite directions with the use of one repeater element. The repeater in this case is formed by balancing the two telephone lines against each other, one line being connected to the particular terminals of the amplifier circuit across which is produced one type of negative impedance and the other to the terminals thereof across which is produced the other type of negative impedance. The repeater thus formed acts for transmission in one direction effectively as a series booster, while for transmission in the opposite direction, it is equivalent to a shunt booster.

The various objects and features of the invention will be understood from the following detailed description thereof, when read in connection with the accompanying drawing, in which the single figure shows diagrammatically one embodiment of the invention utilized as a telephone repeater.

Referring to the drawing, the two stage amplifier shown therein comprises in the first stage a space discharge device 1 having a cathode or filament 3, an anode or plate 4, and a control electrode or grid 5, and in the second stage, a space discharge device 2 having a cathode or filament 6, a plate or anode 7, and a control electrode or grid 8. The cathodes 3 and 6 respectively of tubes 1 and 2 are connected together and may be heated by a common battery 9. The output circuit of tube 1 and the input circuit of tube 2 are coupled through coupling resistance 10 common to the circuits. The battery 11 supplies space current to the plate 4 of tube 1 through coupling resistance 10, and space current to the plate 7 of tube 2 through resistance R_1. The battery supply resistances 10 and R_1 preferably are made non-inductive and non-capacitive. The grid potentials of tubes 1 and 2 are adjusted to any suitable values by grid batteries 12 and 13 in the respective input circuits of the tubes.

The grid 5 of tube 1 is connected through branched paths respectively to a terminal 14 and through conductor 15 to a terminal 16. The cathode 3 of tube 1 is connected directly to a terminal 17. A terminal 18 is connected through a battery 19 to a point in the output circuit of tube 2 between the anode 7 thereof and the resistance R_1. The battery 19 is poled so as to oppose the effect of space current battery 11 and its voltage is made equal to the voltage of space current battery 11 less the potential drop in resistance R_1.

L_1 represents a telephone line having a characteristic impedance represented by the box $\mathcal{Z}$, the terminals 21 and 22 of which line are adapted to be connected across the terminals 14 and 17 of the amplifier circuit, and L_2 represents a second telephone line having a characteristic impedance represented by the box z, the terminals 23 and 24 of which second line are adapted to be connected across the terminals 16 and 18 of the amplifier circuit.

In explaining the operation of the circuit above described, it will be assumed that the line L_1 is coupled in suitable manner by its terminals 21 and 22 to the terminals 14 and 17 of the amplifier, and that line L_2 is connected by its terminals 23 and 24 to the terminals 16 and 18 of the amplifier circuit.

To simplify the explanation of the operation of the circuit, it will be assumed that the impedance z of line L_2 is zero and that an electromotive force e is applied by the line L_2 to terminals 16 and 13 of the amplifier circuit in such a manner as to drive a current I_e in the direction indicated by the solid arrow to the left of the figure through conductor 15, impedance $\mathcal{Z}$ of telephone line L_1, and through R_1 and R_p (the internal resistance of the space path of tube 2) in parallel. It is also assumed that the impedance $\mathcal{Z}$ of line L_1 connected across the terminals 21 and 22 of the amplifier has a finite value, no electromotive force being applied across these terminals. The resultant drop of potential in impedance $\mathcal{Z}$ due to the current flowing therein will be impressed on the input circuit of the amplifier across grid 5 and cathode 3 of tube 1. This voltage will be amplified in the amplifier circuit, and the voltage set up thereby in tube 2 of the amplifier will cause a current to flow in the output circuit of that tube in the direction away from the anode 7. Because of the high impedance of resistance R_1 thereto, most of this current, which will be designated at I_m, is transmitted through battery 19, the impedance z of line L_2, conductor 15, and impedance $\mathcal{Z}$ of telephone line L_1 in the same direction, as indicated by the dotted arrow, as the original current I_e due to the voltage e impressed across the terminals 16 and 18.

The effect, therefore, is to produce a current flowing through the positive impedance in the circuit (which is

$$\mathcal{Z}+\frac{R_pR_1}{R_p+R_1},$$

where R_p is the A. C. plate circuit resistance of tube 2), which current is in phase with that due to the voltage e alone applied across the terminals 16 and 18. When the total current is greater than the latter current the effect is one of inserting across these terminals an impedance z', which, as will be explained later, may be made any desired negative impedance. The value of the impedance z', as is shown in the copending application of R. C. Mathes referred to, depends solely on the values of the constants of the amplifier circuit, and may be expressed in terms of the resistance R_1, the impedance $\mathcal{Z}$, the net volt-

age amplification M from the grid of tube 1 into the plate circuit of tube 2, and the A. C. plate circuit resistance R_p of tube 2, as

$$z' = AR_p - (AM-1)\mathcal{Z}$$

where

$$A = \frac{R_1}{R_1 + R_p}$$

The elements in $\mathcal{Z}$ may comprise resistances, capacities, inductances or any complex combination of these elements, and the negative impedance z' will be the negative of their sum for all frequencies. Any desired value of negative impedance for z' may be obtained at terminals 16 and 18 by suitably selecting the relative values of the elements given in the equations.

It has been determined, however, that to prevent "singing" effects in the circuit described, it is necessary that the circuit elements be so selected that the negative of the impedance z' is in general less than the positive impedance z in the circuit which it faces, in other words, so that the net impedance of the combined circuits is positive.

The portion of the circuit of the drawing which has been described above is identical with that disclosed and claimed in the copending application of R. C. Mathes referred to, supra.

Now in accordance with this invention means are provided for obtaining between two other points in the same circuit, a negative impedance which has a different characteristic than that obtained between the terminals 16 and 18 as described. These points in the circuit are the terminals of the feed back impedance network, which are designated 14 and 17 in the drawing, and across which the impedance $\mathcal{Z}$ of the line L_1 is connected.

As in the preceding case, to simplify explanation of the operation of this part of this portion of the circuit in obtaining this different type of negative impedance, it will be assumed that an electromotive force e' is applied by the telephone line L_1 across the terminals 14 and 17 of the amplifier circuit, and that the impedance $\mathcal{Z}$ of the line L_1 equals zero. It is assumed also that the impedance z of line L_2 connected across the terminals 16 and 18 of the amplifier has a finite value, no electromotive force in this case being applied across these terminals.

It will be assumed that the electromotive force e' drives a current $I_{e'}$ through the circuit including R_p and R_1 in parallel, and impedance z of telephone line L_2 in the direction indicated by the solid arrow to the right of the drawing. The resultant drop in potential in impedance z will cause an increase in the negative potential of grid 5 of tube 1, resulting in a voltage being set up in the last tube of the amplifier, tube 2, which will cause a current to flow in the output circuit of that tube in the direction away from its anode. Because of the high impedance of resistance R_1 thereto, most of this current, which may be designated as $I_{m'}$, will flow through the battery 19 and the impedance z of telephone line L_2, as indicated by the dotted arrow, in the direction opposite to that of the original current $I_{e'}$ due to the voltage e' impressed on the amplifier by telephone line L_1.

Now if the constants of the amplifier circuit are so selected that the current $I_{m'}$ driven through the impedance z by the voltage set up in tube 2 of the amplifier is made larger than the current $I_{e'}$ due to the voltage e' impressed across the terminals 14 and 17, the resultant current in impedance z of telephone line L_2 will flow in the direction opposite to that due to the impressed electromotive force e'. This means that in effect a negative impedance $\mathcal{Z}'$ is produced across the terminals 14 and 17 facing the impressed E. M. F. This negative impedance $\mathcal{Z}'$, it has been found, has such characteristics that it must in general be greater in magnitude than the positive impedance $\mathcal{Z}$ it faces to prevent singing in the circuit. When the constants of the circuit are so selected that the current $I_{m'}$ is less than the current $I_{e'}$ the resultant current flowing through the impedance z is decreased so that the effect is one of multiplying an impedance by a value greater than unity.

The circuit illustrated and described above forms a novel form of 21 type repeater in which the impedances $\mathcal{Z}$ and z, respectively, of two telephone lines are balanced against each other. For proper operation, the constants of the amplifier circuit and that of the telephone line are selected so that its net impedance is negative for $Z+Z'$ and positive for $z+z'$. Accordingly, this produces the effect that for transmission from line L_2 line L_1 the signaling currents have the same direction with the amplifier in the circuit as with it out, whereas for transmission from line L_1 to line L_2, the signaling current in the receiving line L_2 is opposite in direction to that which would be obtained if the repeater were omitted. Therefore, for transmission in one direction the amplifier arrangement operates as a series booster, while for transmission in the opposite direction, the amplifier is equivalent to a shunt booster.

The theory on which the operation of the above described circuit is based will be clear from the following analysis, in which the characters used in the equations are the same as those referred to in the previous description.

The fundamental equations for the circuits shown in the drawing are:

$$z' = -K\mathcal{Z} + R \quad (1)$$

$$z = -K\mathcal{Z}' - R \quad (2)$$

where

$$K=\frac{(M-1)R_1-R_p}{R_1+R_p} \quad (3)$$

and

$$R=\frac{R_1R_p}{R_1+R_p} \quad (4)$$

From Equation (4) it is seen that R is always a positive resistance. K is a positive real number when

$$MR_1>R_1+R_p \quad (5)$$

From Equation (1) it is seen that z' can be made the negative of any positive impedance $\mathcal{Z}_0$ by making

$$\mathcal{Z}=\frac{\mathcal{Z}_o}{K}+\frac{R}{K} \quad (6)$$

similarly from Equation (2)

$$\mathcal{Z}'=-\mathcal{Z}_o-\frac{R}{K}$$

if

$$z=K\mathcal{Z}_o \quad (7)$$

so that by adding the resistance $\frac{R}{K}$ in series with $\mathcal{Z}'$, the impedance is made $-\mathcal{Z}_o$ as desired.

From the foregoing, it is seen that any desired negative impedance may be obtained as a driving point impedance from either of two points in the circuit of the drawing. This negative impedance may be used as another impedance would be so long as the system retains approximately linear current-voltage relations, i. e., the tubes are not overloaded and the circuit does not sing.

We will now consider the repeater application in which $\mathcal{Z}$ and z are the positive impedances of telephone lines. Here the conditions for all repeater gains, i. e.,

$$z=-z';\ \mathcal{Z}=-\mathcal{Z}' \quad (8)$$

will be examined. Substituting from Equation (8) in Equations (1) and (2), both equations reduce to

$$K\mathcal{Z}=z+R \quad (9)$$

The resistance $\frac{R}{K}$ can of course be added on the $\mathcal{Z}'$ side of the network becoming a part of $\mathcal{Z}$, which multiplied by K balances off R in (9). Therefore, Equation (8) can be satisfied provided the two line impedances have a real number as their ratio. K is then made equal to this ratio by the proper choice of M, R_0 and R_1 in Equation (3).

As a particular case, if the two lines have the same impedance, and it is

$$K=1 \text{ and } \mathcal{Z}=z+R \quad (10)$$

in this case

$$M=2\left(1+\frac{R_o}{R_1}\right) \quad (11)$$

The type of repeater disclosed can very conveniently be used to give different gains in the two directions. The currents obtained are easily calculated since the same current must flow through both $\mathcal{Z}$ and z. With an electromotive force e in $\mathcal{Z}$, the current is

$$i=\frac{e(M-1)}{M\mathcal{Z}-(\mathcal{Z}+z+R_O)} \quad (12)$$

while with an electromotive force e in impedance z, the current is

$$i'=-(M-1)i \quad (13)$$

so that changing M changes the gains in the two directions giving an unsymmetrical repeater.

If M=2 then $i=-i'$, so that this is the condition for equal amplification in the two directions. The Equation 12 shows, however, that a resistance in series with z or $\mathcal{Z}$ can be used to control the gain. As this does not change M the gain in the two directions is changed by the same amount.

It is to be understood, that in accordance with the invention, any suitable means other than the means shown may be utilized for separating the A. C. and D. C. paths in the output circuit of the tubes for obtaining the two types of negative impedance, for example, condensers, repeating coils or inductances may be used for this purpose.

The telephone repeater formed by balancing two telephone lines against each other, as described above, utilizes a resistance coupled amplifier of two stages for obtaining the two impedances of different characteristics facing the telephone lines. It is apparent, however, that the type of circuit disclosed in Fig. 2 of the Mathes application referred to, in which only one vacuum tube is used, the impedance feed back being applied between the plate and grid circuits of the same tube through a transformer of negligible distortion in phase and amplitude, may be utilized also for forming the novel form of repeater described above. If such an amplifier is used, one of the telephone lines should be connected across the feed back terminals and the other line connected between the plate of the tube and the primary winding of the transformer.

The system above illustrated and described should be considered merely as typical and not as limiting the invention, the scope of which is defined in the appended claims.

What is claimed is:

1. The method of obtaining the negative of a given reactive impedance in a circuit comprising the given impedance and a source of electromotive force comprising deriving a voltage from the circuit, proportioned with respect to the value of said reactive impedance, amplifying the voltage and applying the amplified voltage to the original circuit

in out of phase relation to the source of voltage therein, the amplified voltage being of sufficient magnitude to produce in the circuit current of greater amplitude than would be produced, by the said source of voltage.

2. The method of obtaining the negative of a given reactive impedance in a circuit including the given impedance and a source of voltage comprising deriving a voltage from the drop of potential in said impedance, amplifying the derived voltage and applying the amplified voltage to the circuit in out of phase relation to the voltage of said source, the amplified voltage being of sufficient magnitude to produce in the circuit current of greater amplitude than would be produced by the said source of voltage.

3. The method of securing the negative of a desired reactive impedance between a pair of terminals in a circuit comprising constructing a positive impedance similar in characteristics to the impedance the negative of which is desired, deriving a voltage from the drop of potential in said positive impedance in circuit with a source of voltage, amplifying the derived voltage and applying the amplified voltage to said circuit in series with said pair of terminals and said positive impedance in out of phase relation to the voltage of said source of voltage, the amplified voltage being of sufficient magnitude to produce in the circuit current of greater amplitude than would be produced by the said source of voltage acting through said positive impedance.

4. The method of effectively increasing the effective value of a given reactive impedance, comprising deriving a voltage from a circuit including the given reactive impedance and a source of potential, proportioned with respect to the value of said given impedance, amplifying the derived voltage, and applying the amplified voltage to said circuit in out-of-phase relation to the voltage of said source, the amplification being of a degree to produce in the circuit a smaller current flow than would be produced by said source of voltage acting through the given impedance.

5. A combination for obtaining a negative impedance of certain definite characteristics between the terminals of an electrical circuit including a source of electromotive force, said combination comprising an amplifier circuit consisting of a space discharge device having an input circuit and an output circuit, an impedance network in said input circuit, the terminals of said network being connected across the terminals of said electrical circuit and across said source of electromotive force independently of said amplifier circuit, and means for feeding back the amplified voltages from said output circuit of said device to the input circuit thereof across said impedance network in phase with the electromotive force impressed thereon, the characteristics of said negative impedance thus obtained between said terminals of said electrical circuit being such that said negative impedance is stable only when the net impedance of the combined circuits is negative.

6. Means for obtaining a negative impedance between two terminals of an electrical circuit comprising a three-electrode space discharge device having an input circuit and an output circuit, means comprising an impedance network having terminals for feeding back the amplified voltages produced in the output circuit of said space discharge device to the input circuit thereof, means for producing a 180° phase shift of the fed back voltages, and a conductive connection independent of said device from each of said electrical circuit terminals to a different terminal of said impedance network.

7. Means for obtaining a negative impedance between two terminals of an electrical circuit comprising an impedance network connected between said terminals by connections of negligible impedance, an amplifier consisting of an even number of vacuum tube stages and having an input circuit and an output circuit, said impedance network being connected directly in said output circuit and directly in said input circuit, and serving as a feed-back coupling therebetween.

8. Means for obtaining a negative impedance of certain definite characteristics between two terminals of an electrical circuit, comprising an amplifier consisting of an even number of space discharge devices resistance coupled, the output of the last space discharge device in said amplifier being coupled to the input circuit of the first device in said amplifier through an impedance network, said terminals of said electrical circuit being connected across said impedance network, independently of said amplifier, the negative impedance obtained between said terminals having such characteristics that it is stable only when its value, which is determined solely by the constants of said amplifier, exceeds that of the positive impedance in said electrical circuit which it faces.

9. Means for obtaining a negative impedance of certain definite characteristics between two terminals of an electrical circuit comprising an amplifier consisting of an even number of space discharge devices, resistance coupled, each of said space discharge devices having an input and an output circuit and having a cathode, an anode and a control electrode, an impedance network coupling the output circuit of the last space discharge device in said amplifier to the input circuit of the first space discharge device in said amplifier, a connection from one of said terminals to said input circuit of said first discharge device between the cathode thereof and one end of said impedance net-

work, and a second connection from the other of said terminals to said input circuit of said first device between the cathode thereof and the other end of said impedance network.

10. A circuit for repeating signaling currents in either direction between two lines comprising an amplifier providing between two terminals thereof a type of negative impedance having certain characteristics and between two other terminals thereof a type of negative impedance having different characteristics, one of said lines being connected between the first mentioned two terminals and the other of said lines being connected between the other of said terminals, said certain characteristics of the first mentioned negative impedance being such that it is stable only when it is greater than the positive impedance of the line which it faces, and said different characteristics of the last mentioned negative impedance being such that it is stable only when it is less than the positive impedance of the line it faces.

11. A telephone repeater for repeating signaling currents in either direction between two lines comprising a single amplifier circuit, providing between two terminals thereof connected across one of said lines a negative impedance which for stability necessitates that it should be greater than the positive impedance of the line it faces, and providing between two other terminals thereof a negative impedance which for stability necessitates that it should be less than the positive impedance of the line it faces.

12. A repeater for repeating signaling currents in either direction between two telephone lines comprising an amplifier and having such characteristics that for transmission between the lines in one direction the current has the same direction in the receiving line with the amplifier in as with it out and for transmission in the opposite direction, the current in the receiving line is opposite in direction to that which would be obtained if the amplifier were omitted and the two lines were directly connected to each other.

13. A repeater circuit comprising a single amplifier for repeating signaling currents in either direction between two lines, said amplifier being arranged to give any desired ratio between the amplifications of the signaling currents in the two directions of transmission.

14. A repeater circuit comprising a single amplifier for repeating signaling currents in either direction between two lines, and including separate means for producing unequal amplification in the two directions of transmission.

15. The method of obtaining the negative of a given impedance including a reactive element in a circuit including said given impedance and a source of electromotive force, comprising deriving a voltage from said circuit proportioned with respect to the value of said impedance, amplifying the derived voltage, and applying the amplified voltage to said circuit in out-of-phase relation to the voltage of said source, the amplification of said voltage being of a degree to produce in said circuit the effect of said impedance multiplied by a real negative factor.

In witness whereof, I hereunto subscribe my name this 28 day of July, A. D. 1927.

HOMER W. DUDLEY.