Nov. 16, 1926. 1,606,817

G. H. STEVENSON

ELECTRICAL NETWORK

Filed Dec. 7, 1925

Fig. 1

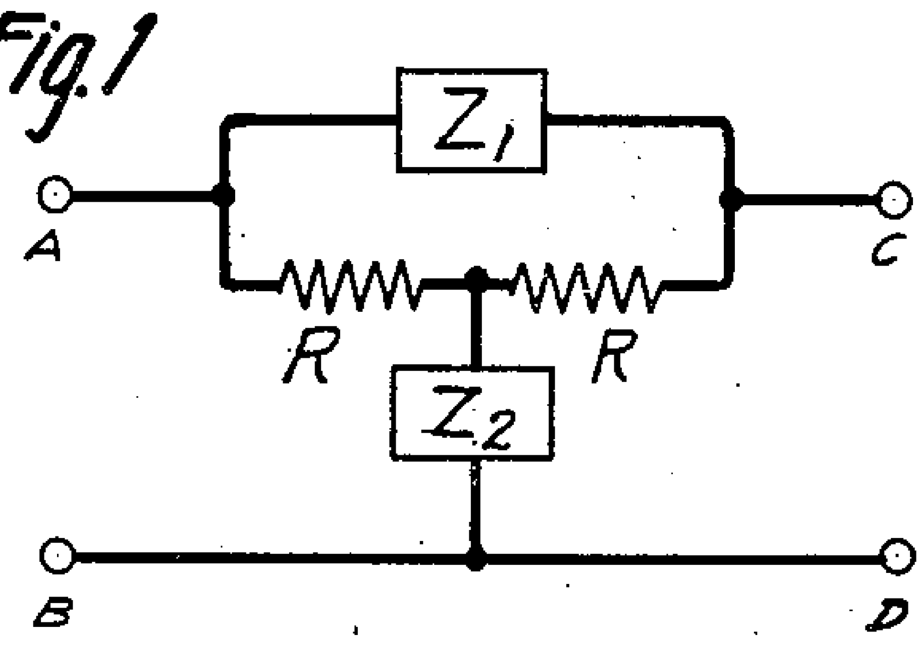


Fig. 2

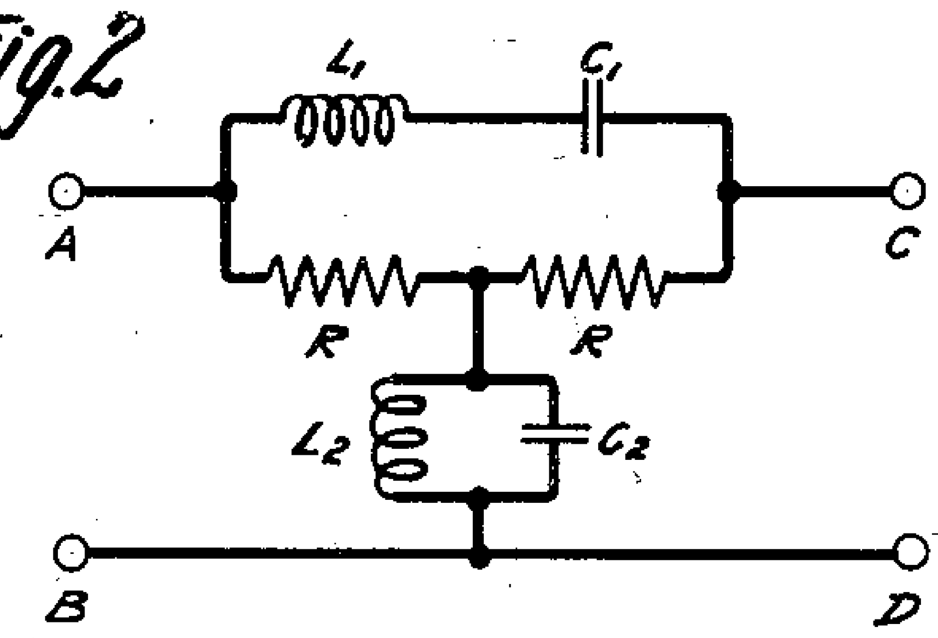


Fig. 3

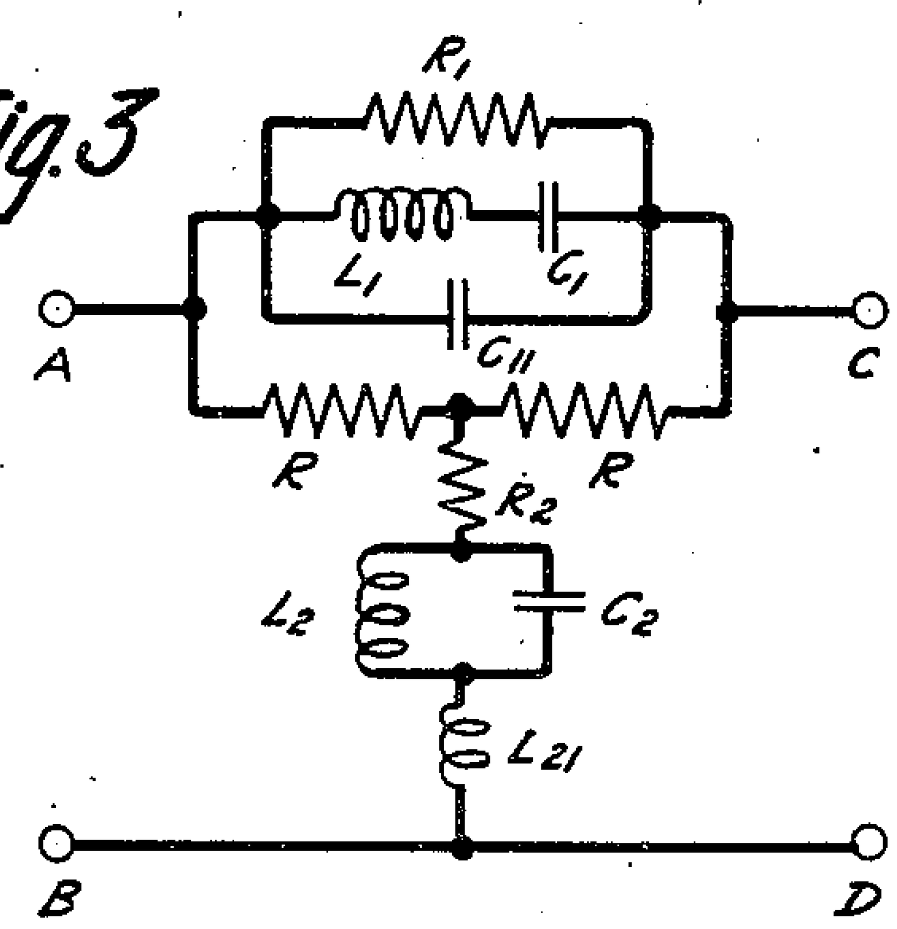


Fig. 4

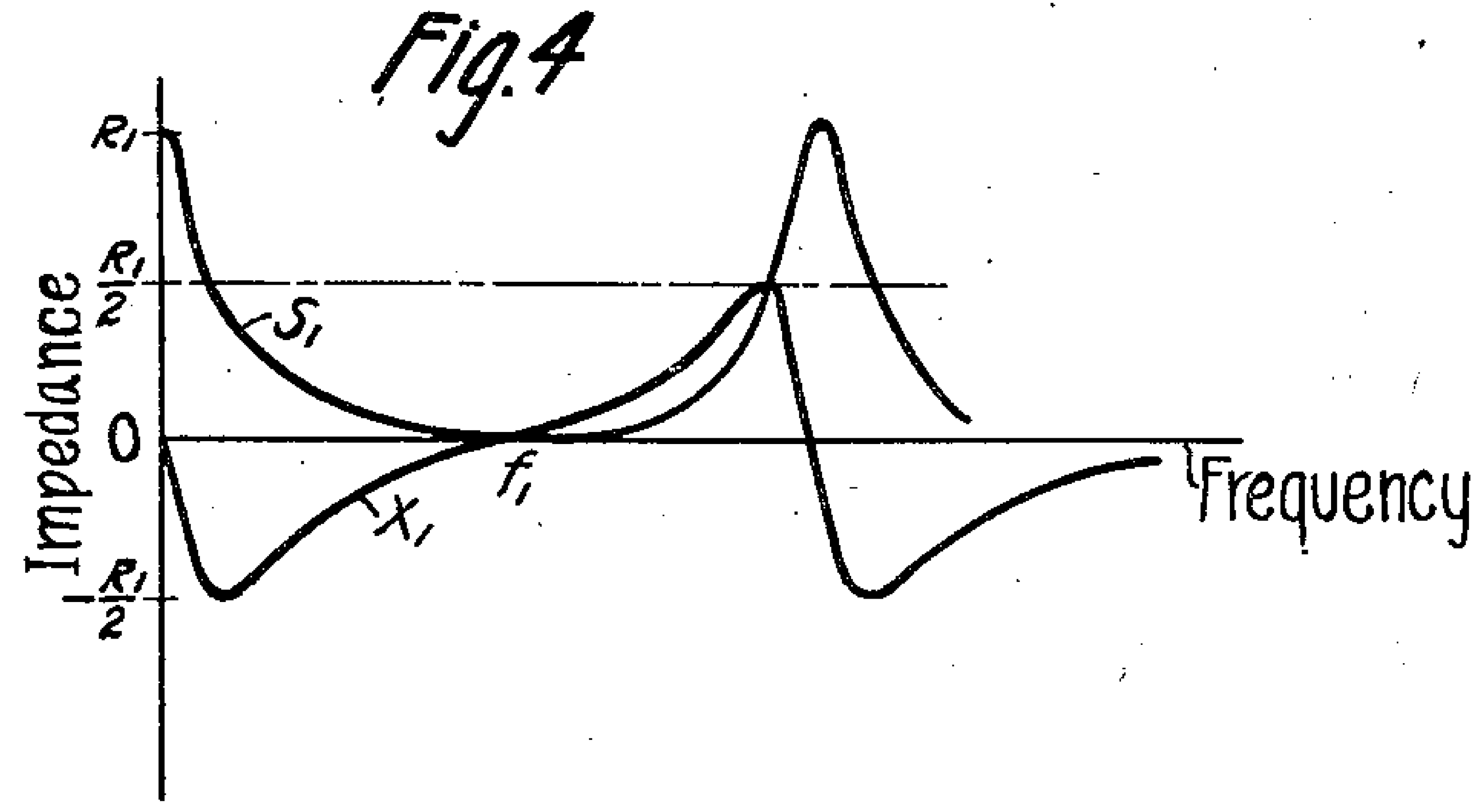
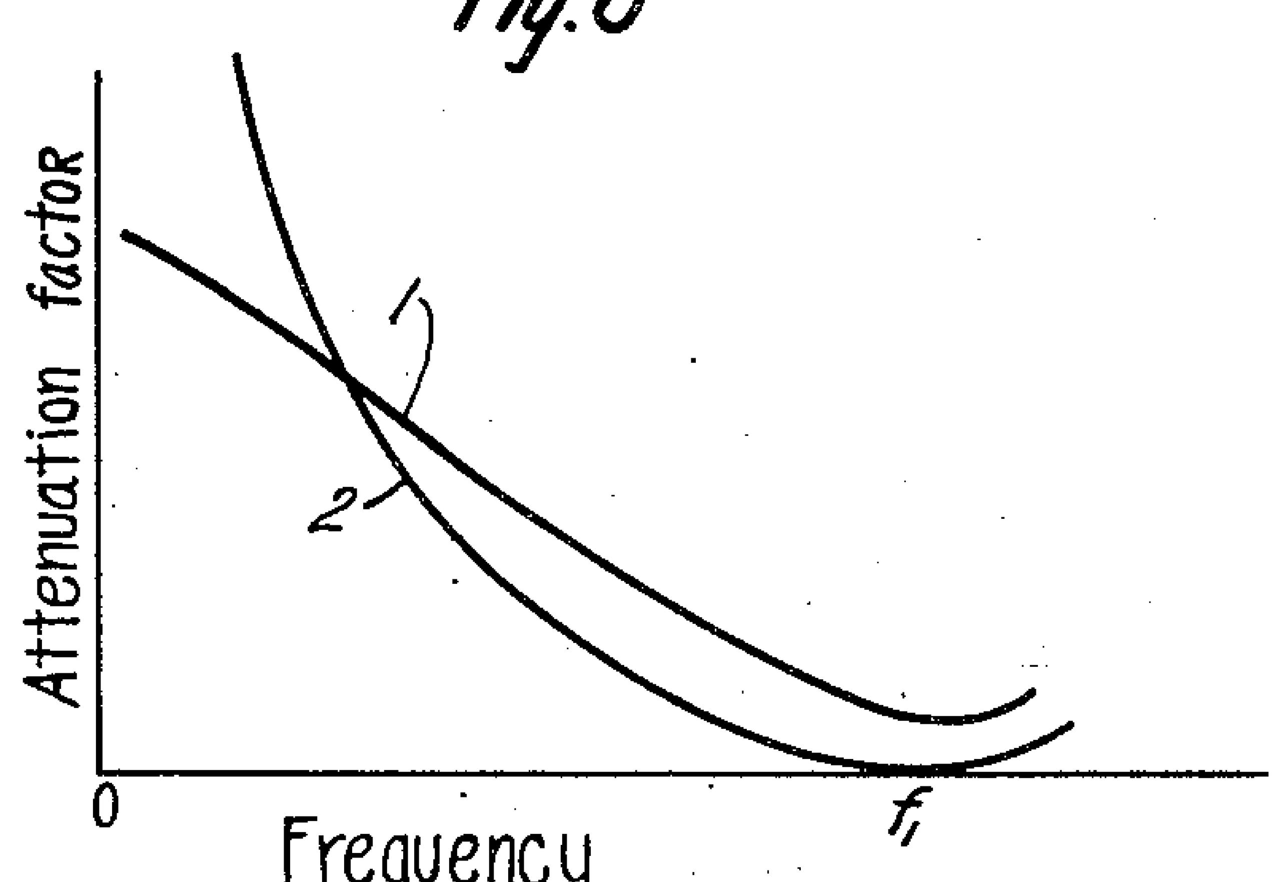


Fig. 5



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UNITED STATES PATENT OFFICE.

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ELECTRICAL NETWORK.

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This invention relates to electrical networks, and more particularly to transmission equalizing networks of the type known as constant resistance networks.

Various forms of constant resistance equalizing networks are described in the copending applications of O. J. Zobel, Serial No. 580,769, filed August 9, 1922, and Serial No. 722,506, filed June 26, 1924, in which the general properties of the type, and its application to the equalization of the attenuation in transmission systems are also discussed.

The present invention is an improved form of network, the object of the improvement being the reduction of the number of elements required for the construction of a network of symmetrical structure and characteristics, to provide a specified attenuation correction.

A characteristic feature of constant resistance equalizing networks is the use of a pair of two-terminal impedance networks, including reactance elements, in which the coefficients of the impedance elements and the interconnections between the elements, are related in an inverse manner, so that the product of the two impedances is a constant quantity, having the electrical dimensions of a resistance squared. Two impedances so related are termed inverse networks of constant resistance product, or more simply, inverse networks.

In the constant resistance equalizing networks of the types heretofore known it has been necessary to use two pairs of inverse networks to secure symmetry with respect to the structural form and the electrical properties. In accordance with the present invention symmetry, both in form and in transmision characteristics, is obtained with a single pair of inverse networks.

The nature of the invention will be more fully understood by referring to the following detailed description and to the drawing, forming a part thereof, in which—

Fig. 1 shows in schematic, the most general form of the improved network of the invention;

Figs. 2 and 3 show two specific forms of network constructed in accordance with the invention; and

Figs. 4 and 5 are illustrative of the properties of the specific form shown in Fig. 3.

As shown in Fig. 1, the network in its most general form comprises a pair of equal resistances connected in series in one side of a transmission line, and a pair of inverse networks, one of which Z_1, is bridged across the outer terminals of the two resistances, and the other of which, Z_2, is connected in shunt between the junction point of the two resistances and the other side of the line. The line terminals are designated A, B, C and D.

The relationships between the impedance Z_1, Z_2, and R necessary to impart the constant resistance characteristic to the network may be found by examining the formulæ for the image impedances of the network, which in accordance with known principles, may be derived from the open circuit and short circuit impedances measured at each pair of the network terminals. For a comprehensive account of the image parameters of a network and of their relation to the transmission characteristics, reference is made to the following article: Transmission characteristics of electric wave filters, by O. J. Zobel, Bell System Technical Journal, Vol. III, No. 4, October, 1924.

Briefly, the image impedances are such that when the network is terminated in impendances respectively equal to the image impedances for the corresponding terminals there are no wave reflections at the terminal junction points. Under this condition the output current is determined solely by the magnitude of the image impedances and the attenuation factor of the network.

Generally the image impedance has a frequency characteristic such that it cannot be simulated by any simple combination of physical elements, but in the networks of the present invention the image impedances are constant resistances and may be very readily simulated. It follows, then, that if the network is terminated at each end in resistances corresponding to the respective image impedances, the impedance looking into the network at either pair of terminals is a constant resistance, and further if terminated in this manner at one end the impedance measured at the other end is a constant resistance.

The image impedance, of the network of Fig. 1 at terminals A B is given by the equation

$$W=\sqrt{X_a Y_a} \quad (1)$$

in which X_a and Y_a are respectively the open circuit and the short circuit impedances of the network measured at terminals A B, and in which W is the image impedance. On account of the symmetry of the structure the image impedances at both pairs of terminals are equal.

In terms of Z_1, Z_2, and R the impedances X_a and Y_a are given by the equations

$$X_a=\frac{(R+Z_1)(R+Z_2)+RZ_2}{2R+Z_1}, \quad (2)$$

and

$$Y_a=\frac{R[Z_1Z_2+Z_1(R+Z_2)]}{(R+Z_1)(R+Z_2)+RZ_2}, \quad (3)$$

from which it follows that

$$W^2=X_aY_a=\frac{R[Z_1Z_2+Z_1(R+Z_2)]}{2R+Z_1}. \quad (4)$$

Now, impedances Z_1 and Z_2 are inversely related, and if their magnitudes be so chosen that their constant value product is equal to R^2, it will be found that equation 4 reduces to

$$W=R. \quad (5)$$

The rules for designing a pair of impedance networks, so that they may be inversely related to each other by the constant factor R^2, will be elucidated by a consideration of the networks shown in Figs. 2 and 3. In the network of Fig. 2 the impedance Z_1 comprises an inductance L_1 and a capacity C_{1}, in terms of which the impedance is expressed by

$$Z_1=j\omega L_1+\frac{1}{j\omega C_1}. \quad (6)$$

Now, since it is stipulated that

$$Z_1Z_2=R^2, \quad (7)$$

or

$$Y_2=\frac{Z_1}{R^2},$$

in which Y_2 is the admittance corresponding to Z_2, the value of Y_2 may at once be written as

$$Y_2=j\omega\frac{L_1}{R^2}+\frac{1}{j\omega C_1R^2}. \quad (8)$$

The form of equation 5 indicates that the admittance comprises two parallel paths, the component admittances being simply additive. Further, the first term on the right hand side has the electrical dimensions and the characteristic frequency variation of the admittance of a capacity, while the second term, in like manner, corresponds to an inductance. It is therefore possible to express the value of Y_2 in the form

$$Y_2=j\omega C_2+\frac{1}{j\omega L_2}, \quad (9)$$

in which

$$C_2=\frac{L_1}{R^2},$$

and

$$L_2=C_1R^2.$$

In this case the inverse impedance, Z_2, of Z_1 consists of an inductance connected in parallel with a capacity, the value of the inductance being inversely related to the value of the capacity in impedance Z_1 by the factor R^2, and the capacity being correspondingly related to the inductance L_1. For the more complicated network shown in Fig. 3, the admittance Y_1 of impedance Z_1 is expressed by the formula

$$Y_1=\frac{1}{R_1}+j\omega C_{11}+\frac{1}{j\omega L_1+\frac{1}{j\omega C_1}}. \quad (10)$$

By equation 7 the value of impedance Z_2 for the inverse relationship is

$$Z=Y_1R^2=\frac{R^2}{R_1}+j\omega C_{11}R^2+\frac{1}{\frac{j\omega L_1}{R^2}+\frac{1}{j\omega C_1R^2}}, \quad (11)$$

which is identified with the value

$$Z_2=R_2+j\omega L_{21}+\frac{1}{j\omega C_2+\frac{1}{j\omega L_2}} \quad (12)$$

when

$$R_2R_1=\frac{L_{21}}{C_{11}}=\frac{L_2}{C_2}=\frac{L_1}{C_1}=R^2. \quad (13)$$

evidently equation 12 corresponds to the impedance of a network comprising a resistance, an inductance, and an anti-resonant inductance-capacity pair, all connected in series, as shown in the figure.

Out of these two examples, the following rules, which are general in their application, may be stated: Additive impedances in the one network are replaced by additive admittances in the inverse network, and vice versa; inductive elements in the one network are replaced by capacitive elements in the inverse network, and vice versa; and resistances are replaced by resistances, the products of the impedances of each pair of corresponding elements being equal to R_2.

The propagation constant, denoted by P, of the network of Fig. 1 may be found in terms of the impedance coefficients of the elements from the following equation

$$\operatorname{Tanh} P=\sqrt{\frac{Y_a}{X_a}} \quad (14)$$

which holds for any symmetrical network. Substituting in this equation the values of X_a and Y_a given by equations 2 and 3, and utilizing the inverse relationship Z_1 and Z_2 to simplify the expression, the value of Tanh P is found to be

$$\operatorname{Tanh} P=\frac{Z_1+ZR}{Z_1+Z(R+Z_2)} \quad (15)$$

The propagation constant P is a complex quantity having real and imaginary components defined by

$$P=\alpha+j\beta,$$

the real component α being the attenuation factor, and the imaginary component β being the phase constant of the network. To determine the values of α and β, an explicit formula for P may first be obtained from equation 15, using the relationship

$$e^{2P}=\frac{1+\tanh P}{1-\tanh P}, \quad (16)$$

the value of P so found being

$$P=\log_e\left(1+\frac{Z_1}{R}\right). \quad (17)$$

If the total resistance and the total reactance of Z_1 be denoted by S_1 and X_1 respectively, the value of P becomes

$$P=\log_e\left(1+\frac{S_1}{R}+j\frac{X_1}{R}\right) \quad (18)$$

from which it follows that

$$\alpha=\log_e\sqrt{\left(1+\frac{S_1}{R}\right)^2+\left(\frac{X_1}{R}\right)^2} \quad (19)$$

and

$$\beta=\tan^{-1}\frac{X_1}{R+S_1}.$$

When the equalizing network is inserted between two sections of a transmission line or system, the impedance of each of which is resistive and equal to R, there will be no reflection effects at the junction points, and the change produced in the received current will be controlled entirely by the propagation constant P of the network. In this case the ratio of the absolute value of the received current, $|I_{r0}|$, before the network is inserted; to the modified value, $|I_r|$, due to the insertion of the network is given by the equation

$$\frac{I_{r0}}{I_r}=e^{\alpha}=\sqrt{\left(1+\frac{S_1}{R}\right)^2+\left(\frac{X_1}{R}\right)^2} \quad (20)$$

If the two terminating impedances are not resistive and equal to R the relationship between the received currents in the two cases will be less simple than that given by equations 20 on account of reflection effects. It is generally possible, however, to find a point in any transmission system at which the impedance on both sides, or at least on one side, of the section is substantially a constant resistance. In the latter case the reflection effects are unchanged by the insertion of the network, and the ratio of the two currents is again in accordance with equation 20.

The design problem is that of finding a network structure which will have a specified attenuation characteristic. As a rule an exact solution cannot be found, but by successive trials it is generally possible to select a network which will provide the requisite attenuation through a substantial frequency range with as great accuracy as may be desired.

Equations 20 may be transformed to

$$\sqrt{S_1^2+X_1^2}=R\sqrt{e^{2\alpha}-1}, \quad (21)$$

which gives the absolute value of the impedance Z_1, in terms of the constant resistance R and the specified attenuation factor α. The quantity on the right hand side of equation 21 may be computed for several frequencies from the specified attenuation characteristic, and a curve, showing the computed values plotted against frequency, is also the curve showing the required frequency variation of impedance Z_1.

The selection of the proper network is facilitated by a familiarity with the characteristics of two-terminal impedance networks. The general characteristics of such networks, when composed entirely of reactive elements, are described by R. M. Foster in the Bell System Technical Journal, Vol. III, No. 2, April, 1924, in an article entitled "A reactance theorem". The dominating characteristics of reactive networks is the alternate recurrence of resonance and anti-resonance frequencies, and the uniformly positive rate of increase of reactance with frequency.

The network corresponding to Z_1 in Fig. 3 is of the above mentioned type, but having a resistance R_1 added in parallel to the reactive elements. The effect of the added resistance is illustrated in Figure 4 which shows the frequency variation of the two components S_1 and X_1 of the impedance.

At the frequencies of anti-resonance, or the frequencies for which the impedance of the reactive group is infinite, the effective resistance is equal to R_1 and the reactance is zero; at the resonance frequencies both S_1 and X_1 are zero.

The reactance X_1, instead of ranging between positive and negative infinite values, is confined to the limits

$$+\frac{1}{2}R_1$$

and

$$-\frac{1}{2}R_1$$

and the variation between these values becomes more and more nearly linear as the value of R_1 diminishes. The presence of the shunt resistance R_1 has therefore the effect of limiting the attenuation range of the network and of reducing the amount of curvature in the attenuation characteristics in the range between the points of maximum attenuation and of minimum attenuation.

A portion of the attenuation characteristic of the network of Fig. 3 is shown in Fig. 5 in which curve 1 shows the variation of the attenuation factor α with respect to frequency. For the purpose of a qualitative comparison the variation of α for the case in which R_1 is infinite is shown by curve 2.

Networks of the type shown in Fig. 3 have been found useful for compensating the attenuation of uniform transmission lines at high frequencies, the attenuation in such lines being a substantially linear function of the frequency.

The invention is not useful in this field alone, however, but finds application widely wherever it is desired to modify the transmission characteristics of wave systems or apparatus in general.

In the appended claims, by which the scope of the invention is pointed out, the term inverse network is used to define a network which is so related to a given network, in its structure and in the composition of the elements, that, for the impedance of each element, and each group of elements, in the given network, it presents a corresponding admittance, the ratio of the impedance in the one case to the corresponding admittance in the other being a constant quantity.

A pair of inverse networks defines two networks one of which is inversely related to the other, whereby the impedance of the one bears a constant ratio to the admittance of the other, or the product of the two impedances is a constant quantity.

What is claimed is:

1. A wave transmission network having a pair of input terminals and a pair of output terminals said network comprising an electrical path between each input terminal and a corresponding output terminal, a pair of equal resistances connected in series in one of said paths, a general impedance Z_1 connected in parallel with said resistances between the outer terminals thereof and a second general impedance Z_2 connected between the junction point of said resistances and a point in the other of said paths, the impedances Z_1 and Z_2 having a product that is a constant quantity at all frequencies.

2. A wave transmission network having a pair of input terminals and a pair of output terminals said network comprising an electrical path between each input terminal and a corresponding output terminal, one of said paths including a pair of equal resistances of resistance R connected in series and a general impedance Z_1 in parallel therewith, and a second general impedance Z_2 included in a shunt path between the junction point of said resistances and a point in the other of said paths, the impedances Z_1 and Z_2 having a constant product equal to R^2.

3. A wave transmission network having a pair of input terminals and a pair of output terminals, said network comprising electrical paths between each input terminal and a respectively associated output terminal, a pair of equal resistances of resistance R included in series in one of said paths, an impedance network including reactive elements connected in parallel with said resistances between the outer terminals thereof, and a network, inversely related to said impedance network by the constant impedance product equal to R^2, connected in a shunt path between the junction point of said resistances and a point in the other of said paths.

4. A wave transmission network having a pair of input terminals and a pair of output terminals, said network comprising an electrical path between each input terminal and a respectively associated output terminal, a pair of equal resistances of resistance R included in series in one of said paths, and a pair of inverse networks of constant impedance product equal to R^2, one of said networks being connected in parallel with said resistances between the outer terminals thereof, the other of said networks being connected between the junction point of said resistances and a point in the other of said electrical paths, and the first mentioned network having an impedance Z_1 designed in accordance with the formula

$$P=\log_e\left(1+\frac{Z_1}{R}\right)$$

in which P is a preassigned propagation constant.

5. A wave transmission network having a pair of input terminals and a pair of output terminals, said network comprising

an electrical path between each input terminal and a respectively associated output terminal, a pair of equal resistances of resistance R included in series in one of said paths, a general impedance Z_1 connected in parallel with said resistances, a second impedance Z_2 inversely related to Z_1 by the constant factor R^2, included in a path between the junction front of said resistances and the other of said paths, and said network being designed to have a preassigned attenuation, denoted by α, by proportioning the impedance Z_1 in accordance with the formula

$$\alpha=\log_e\sqrt{\left(1+\frac{S_1}{R}\right)^2+\left(\frac{X_1}{R}\right)^2}$$

in which S_1 and X_1 are respectively the effective resistance and the effective reactance of the impedance Z_1.

In witness whereof, I hereunto subscribe my name this 5th day of December A. D., 1925.

GEORGE H. STEVENSON.