

Oct. 22, 1929.

H. NYQUIST ET AL

1,732,312

ELECTRICAL DELAY NETWORK

Filed Sept. 11, 1926

2 Sheets-Sheet 1

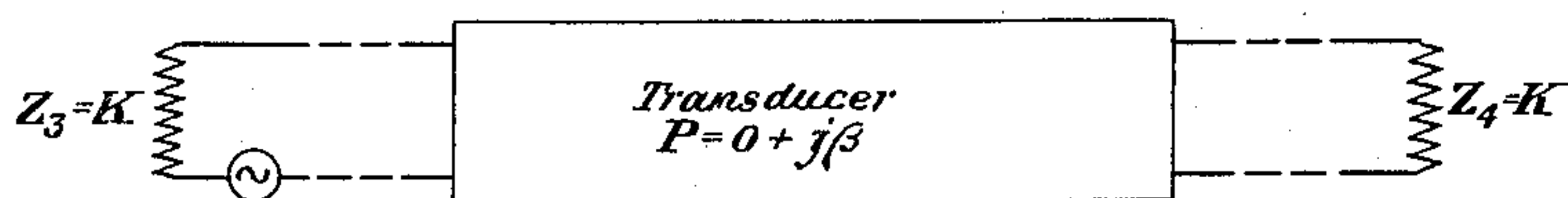


Fig. 1

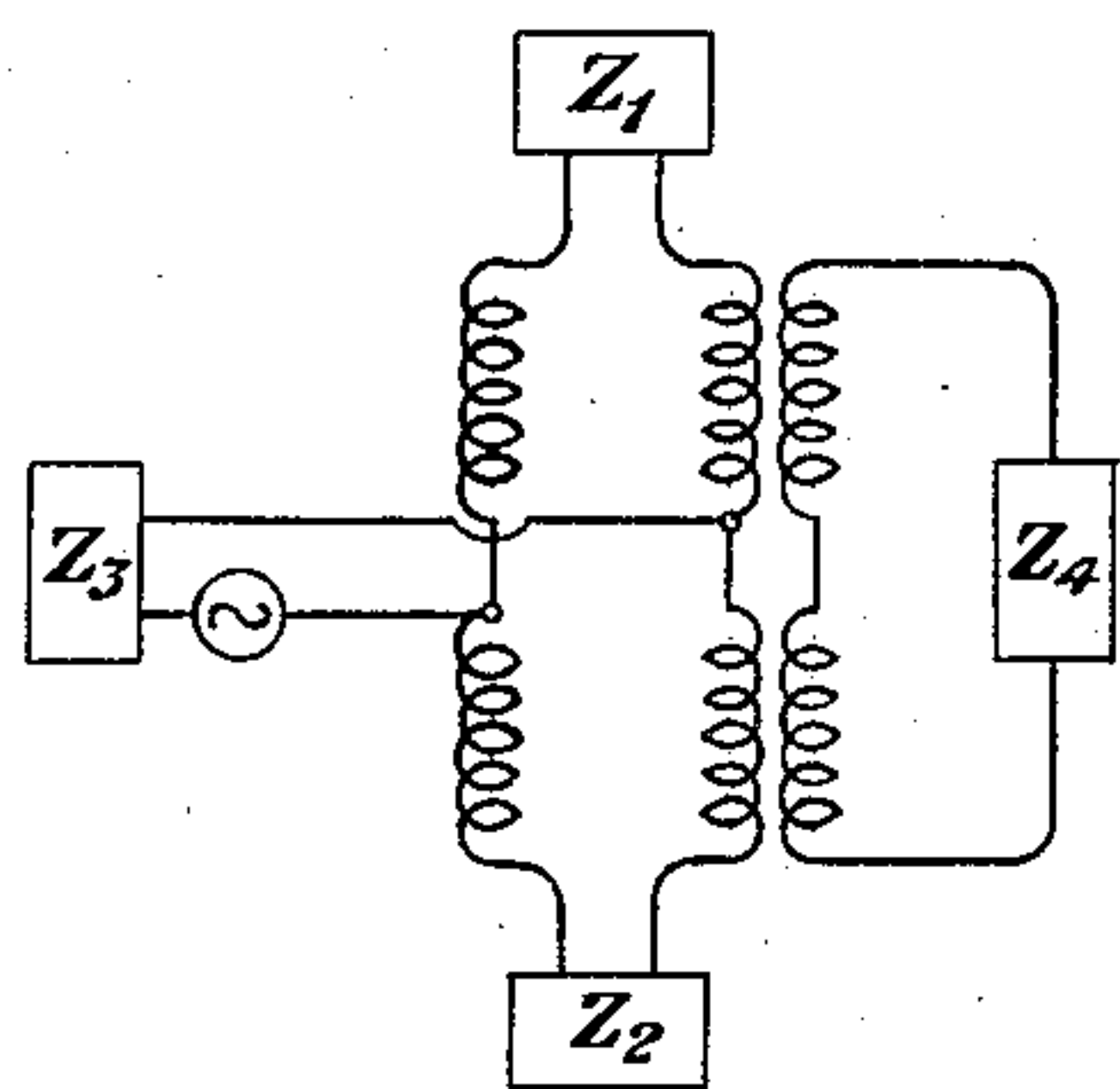


Fig. 3

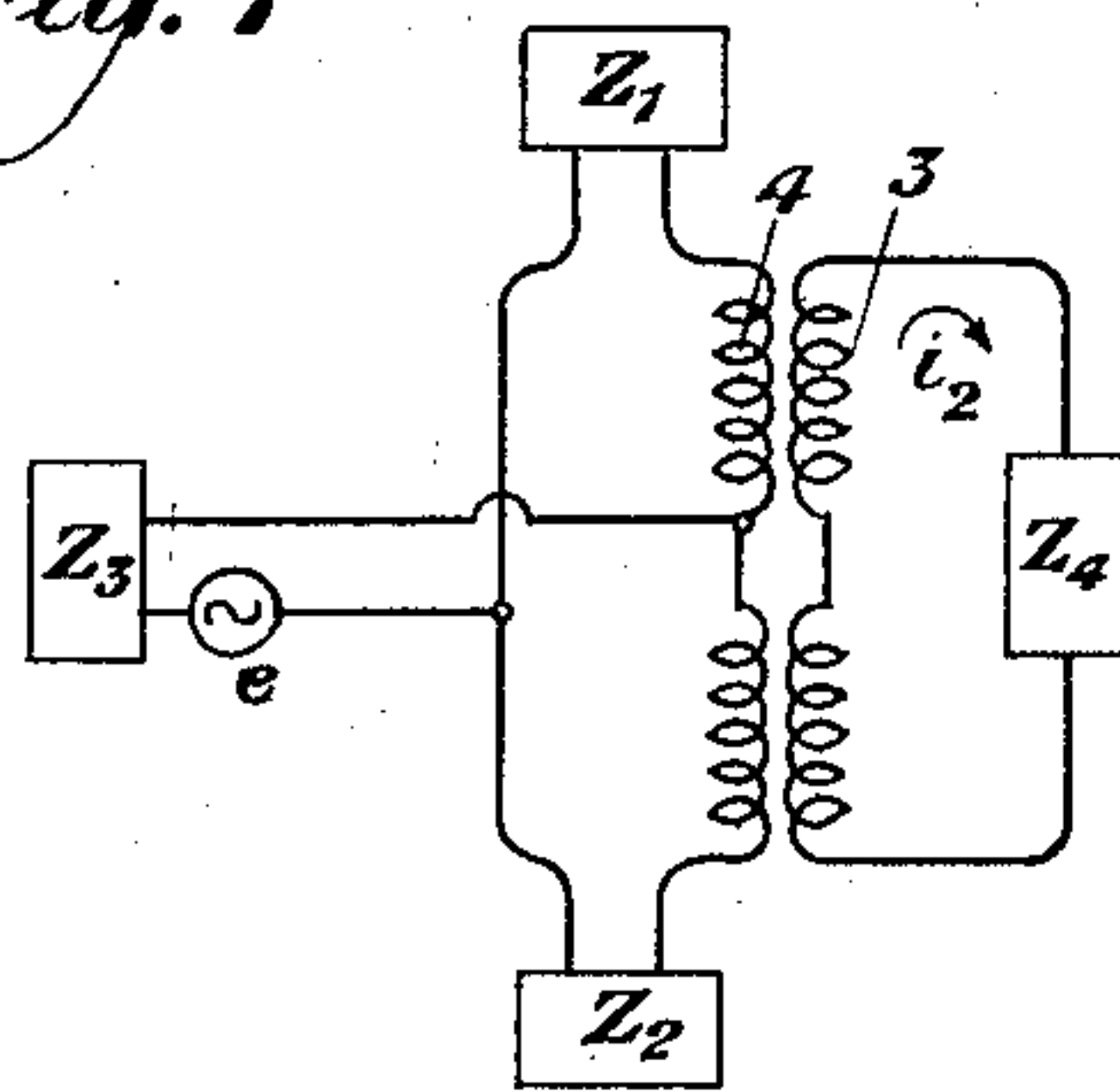


Fig. 4

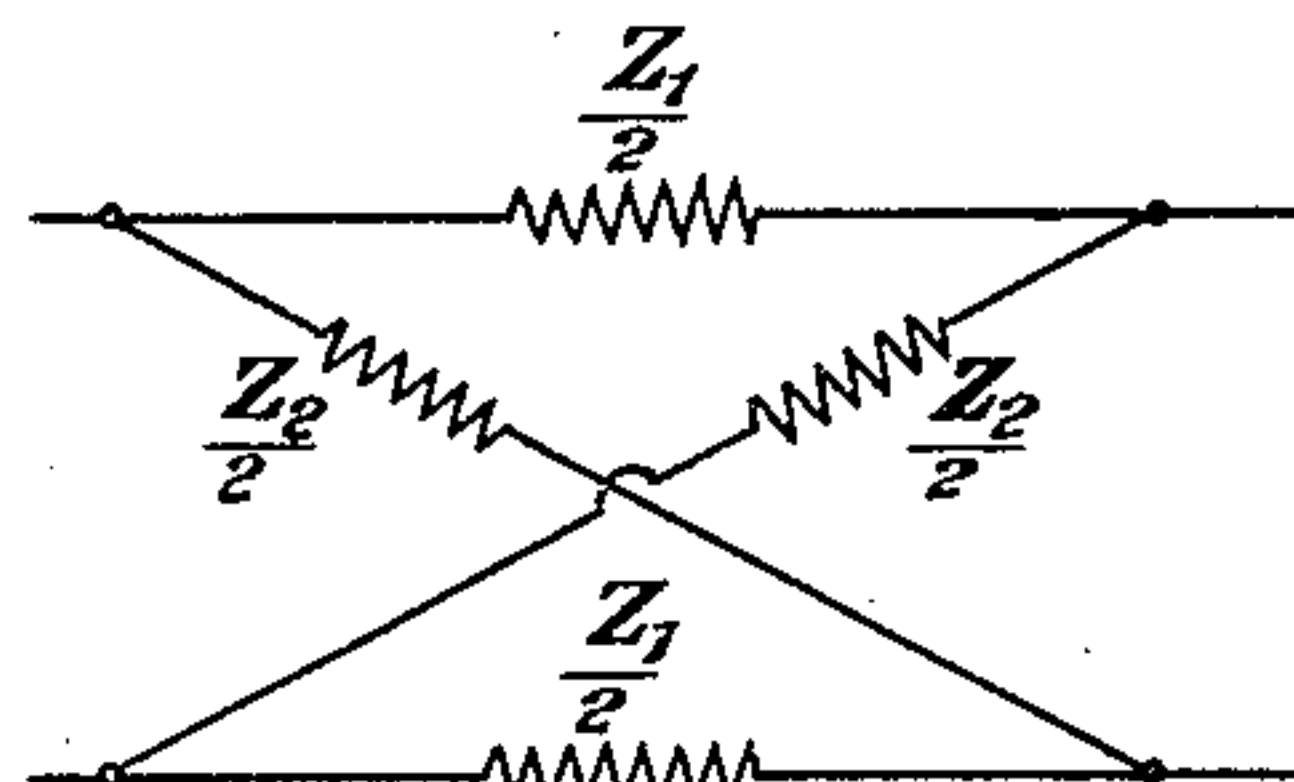


Fig. 2

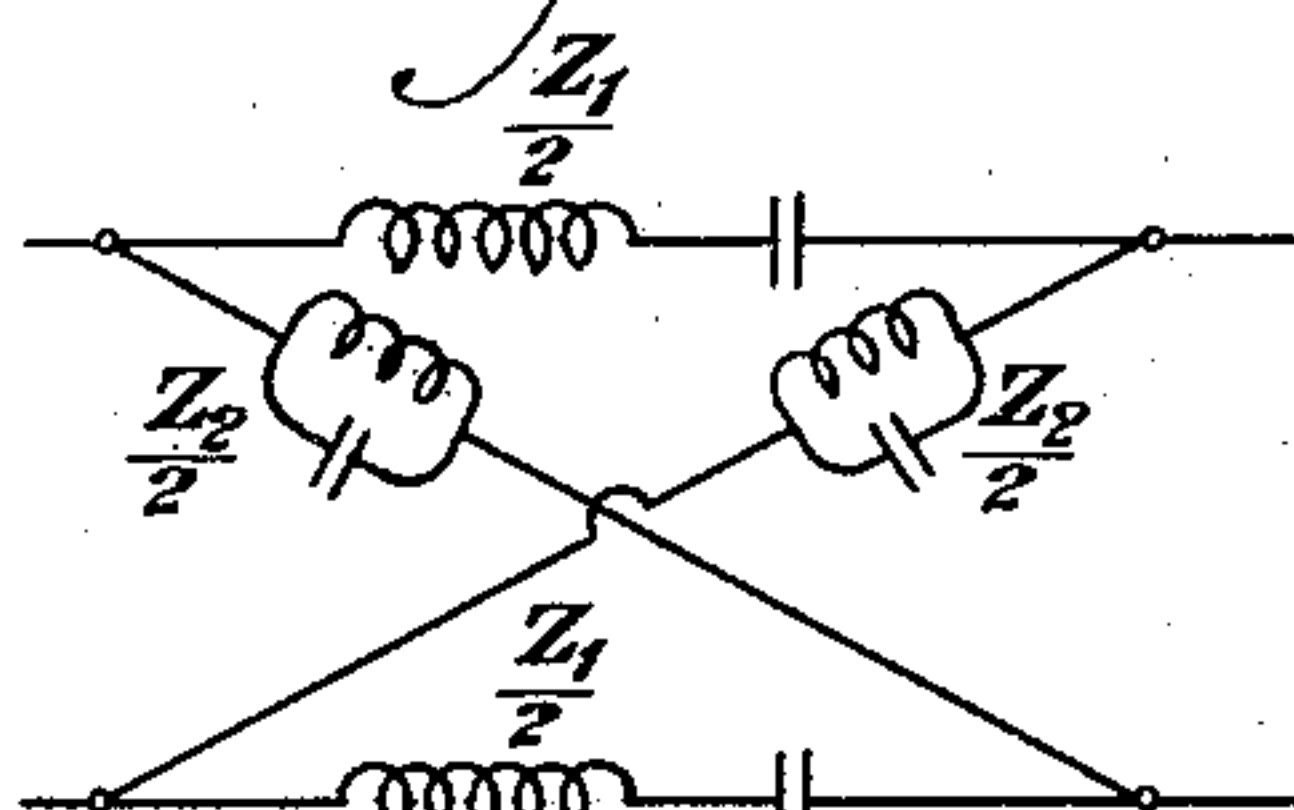


Fig. 2a

Fig. 5

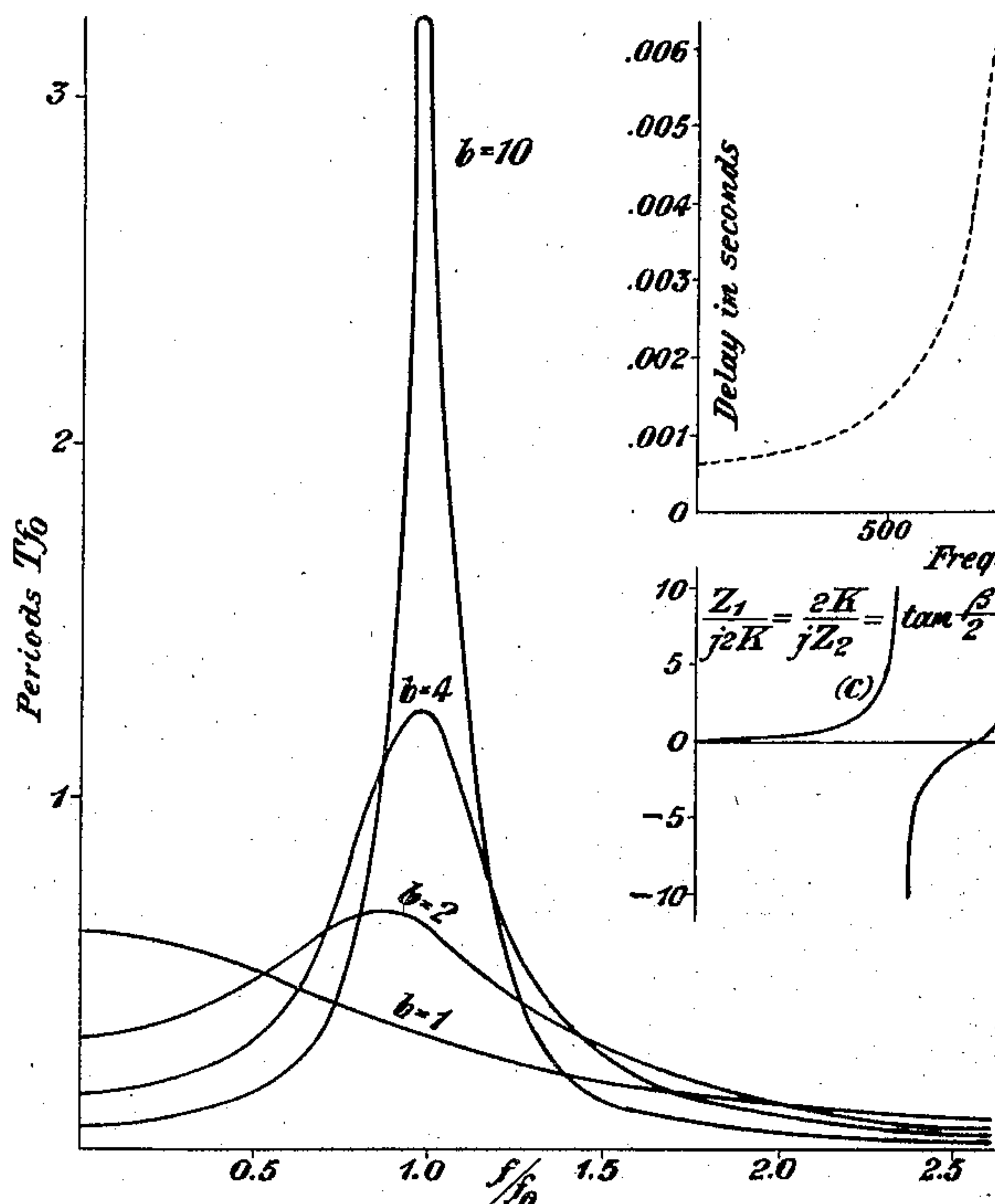
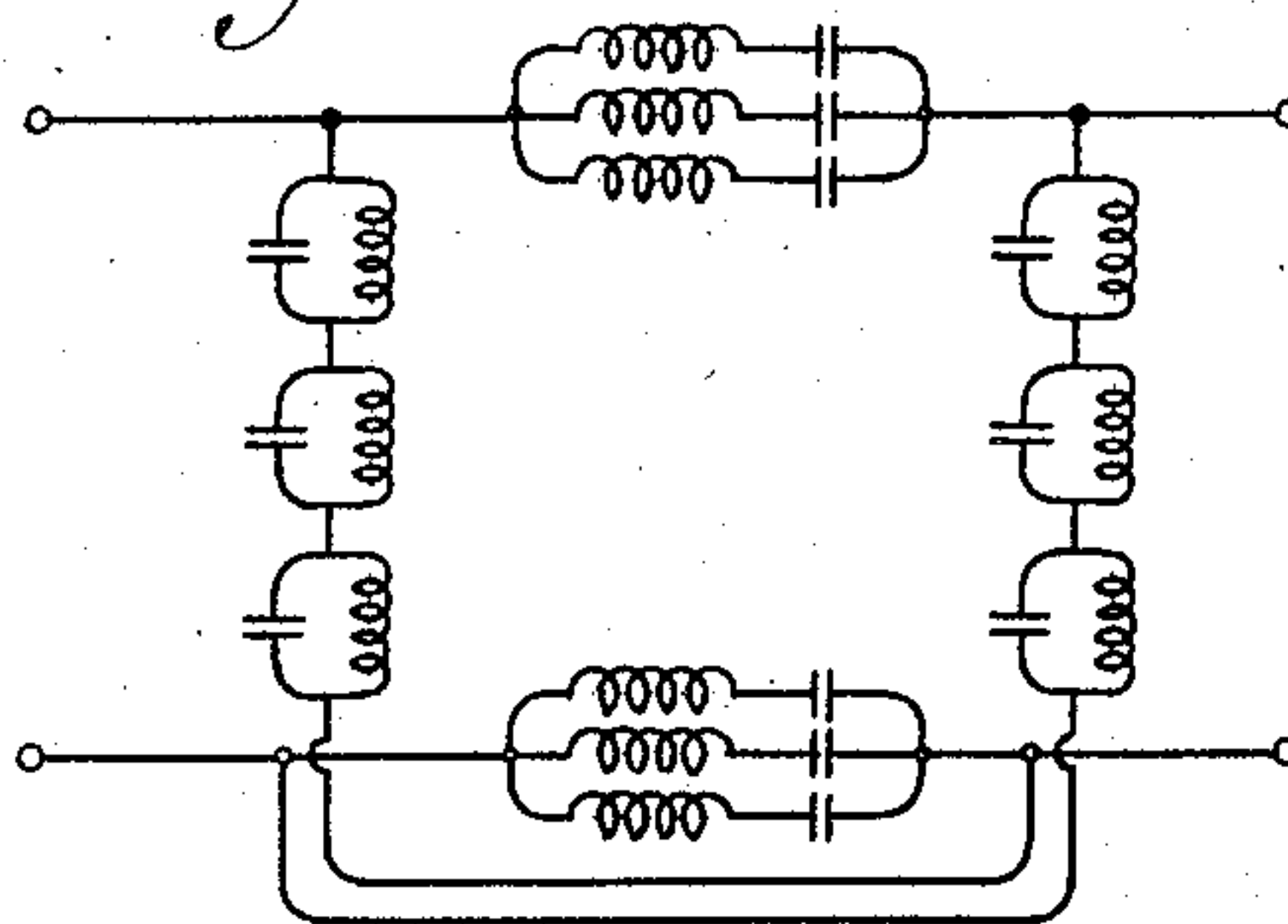


Fig. 7

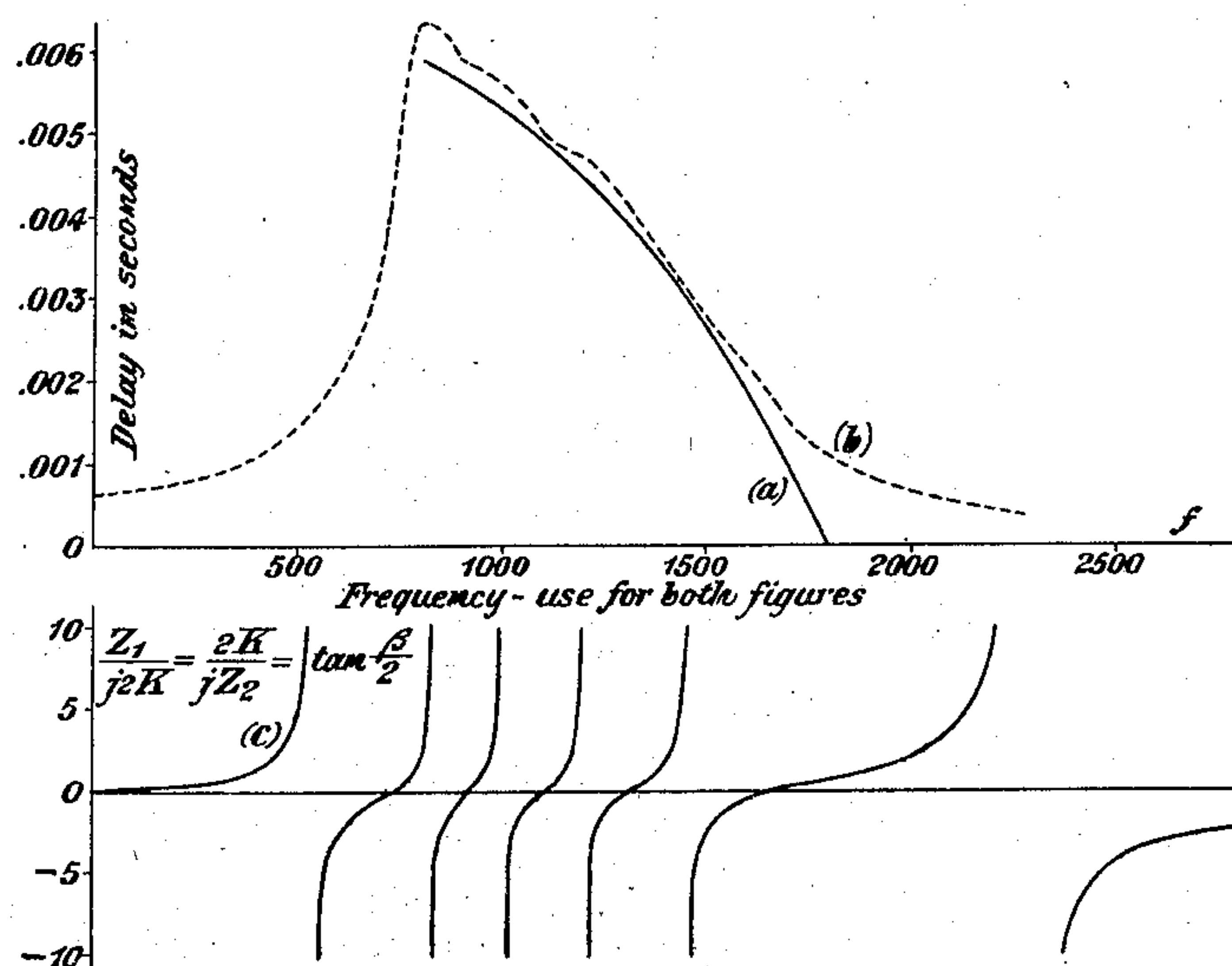


Fig. 6

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2 Sheets-Sheet 2

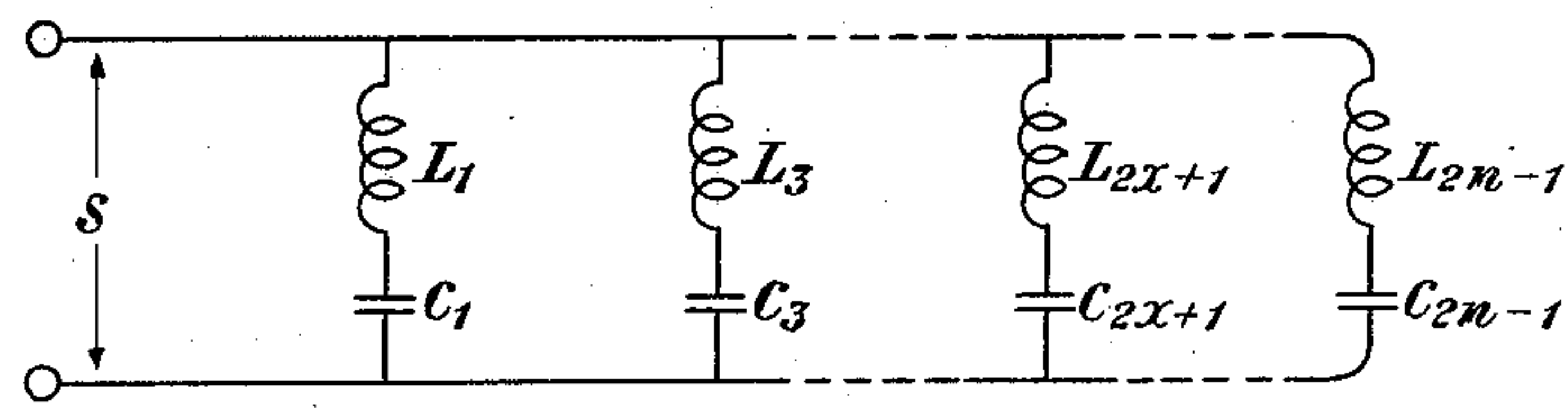


Fig. 8

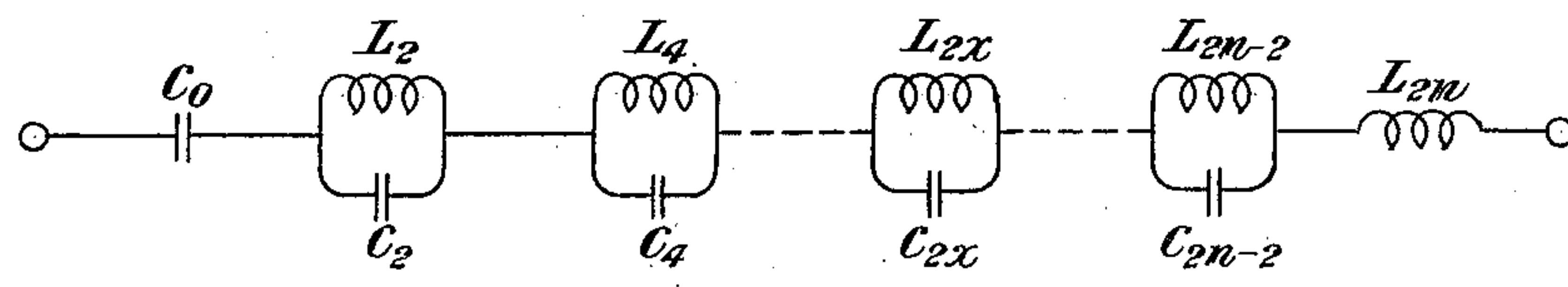


Fig. 9

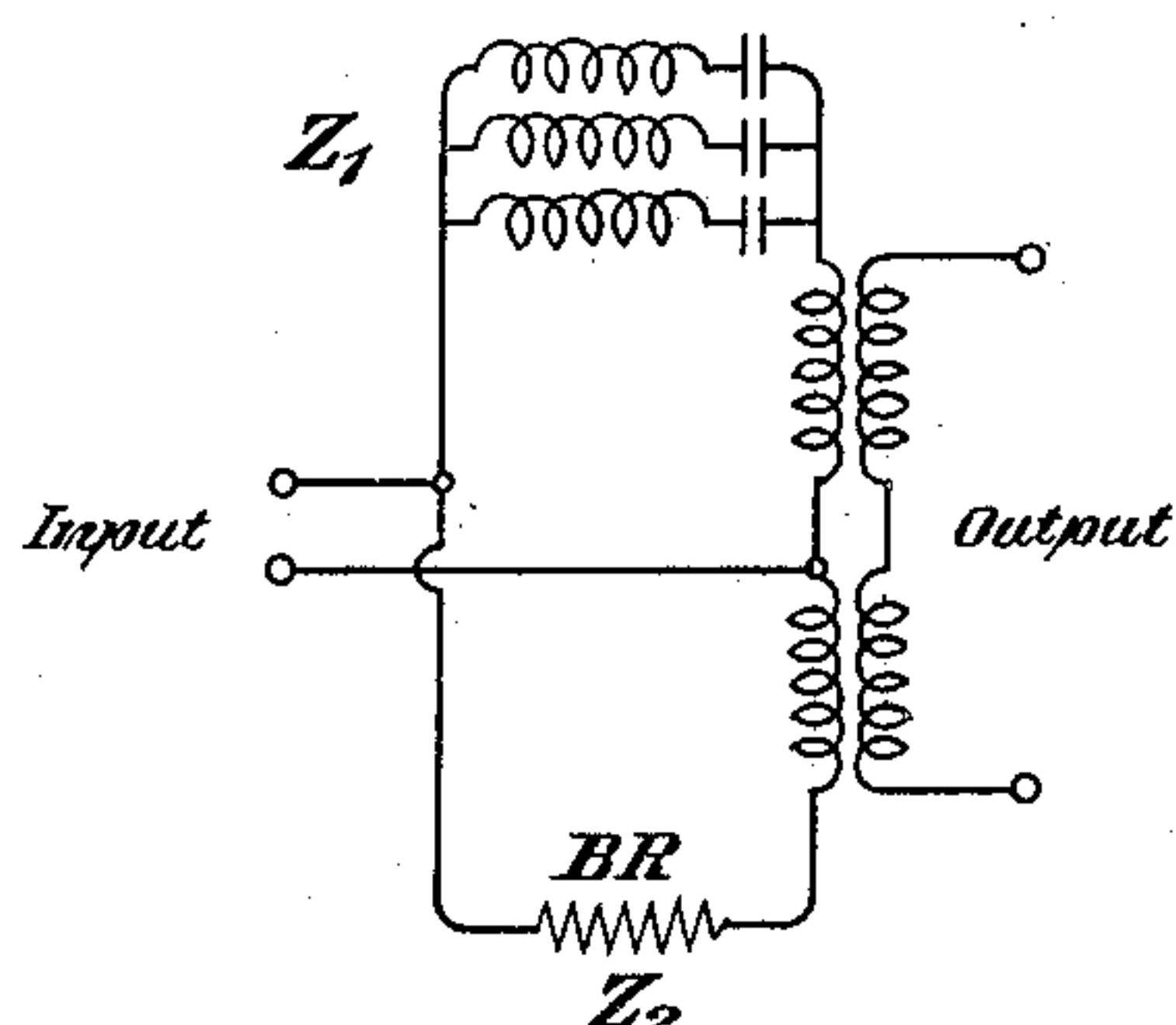


Fig. 10

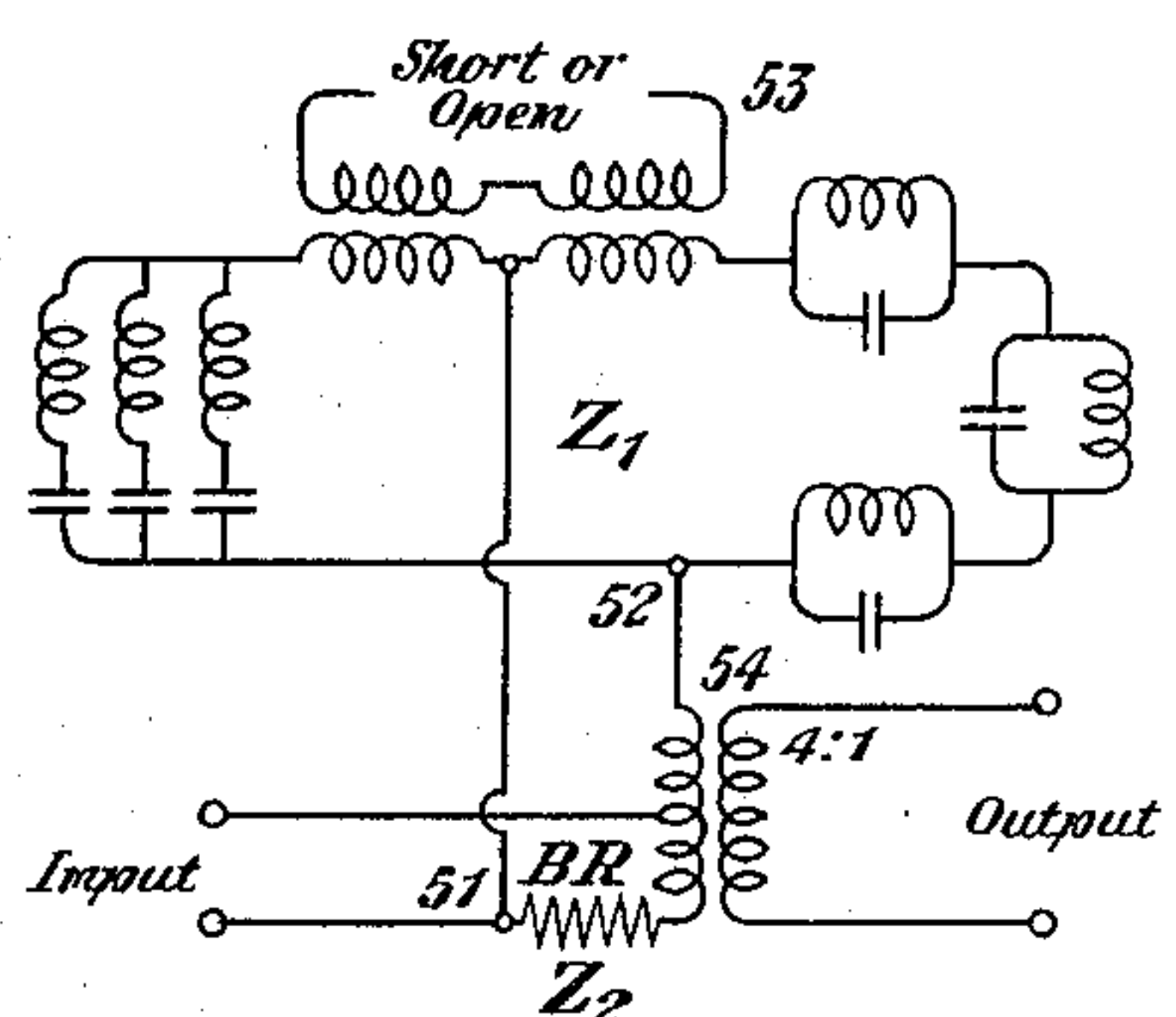


Fig. 11

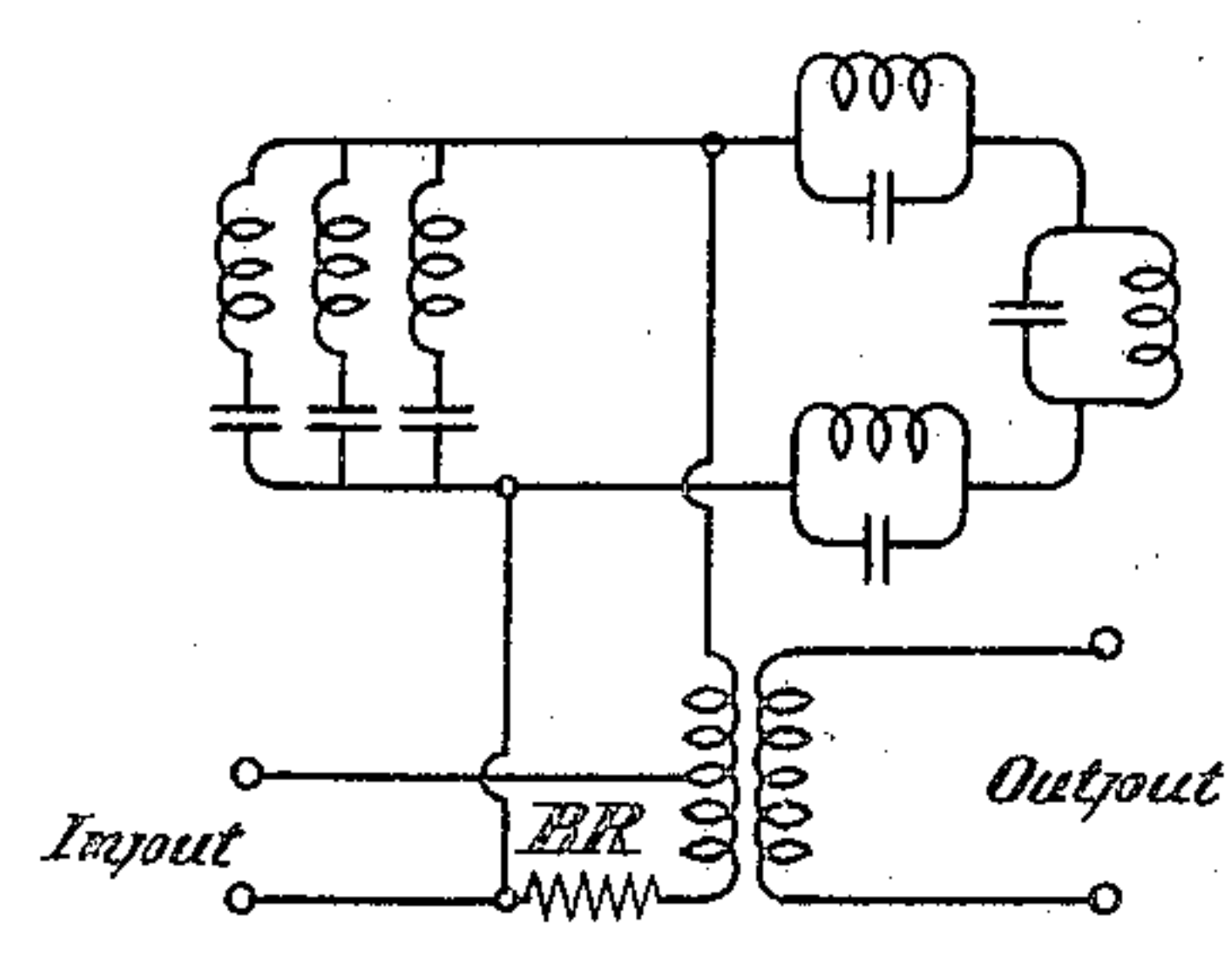


Fig. 12

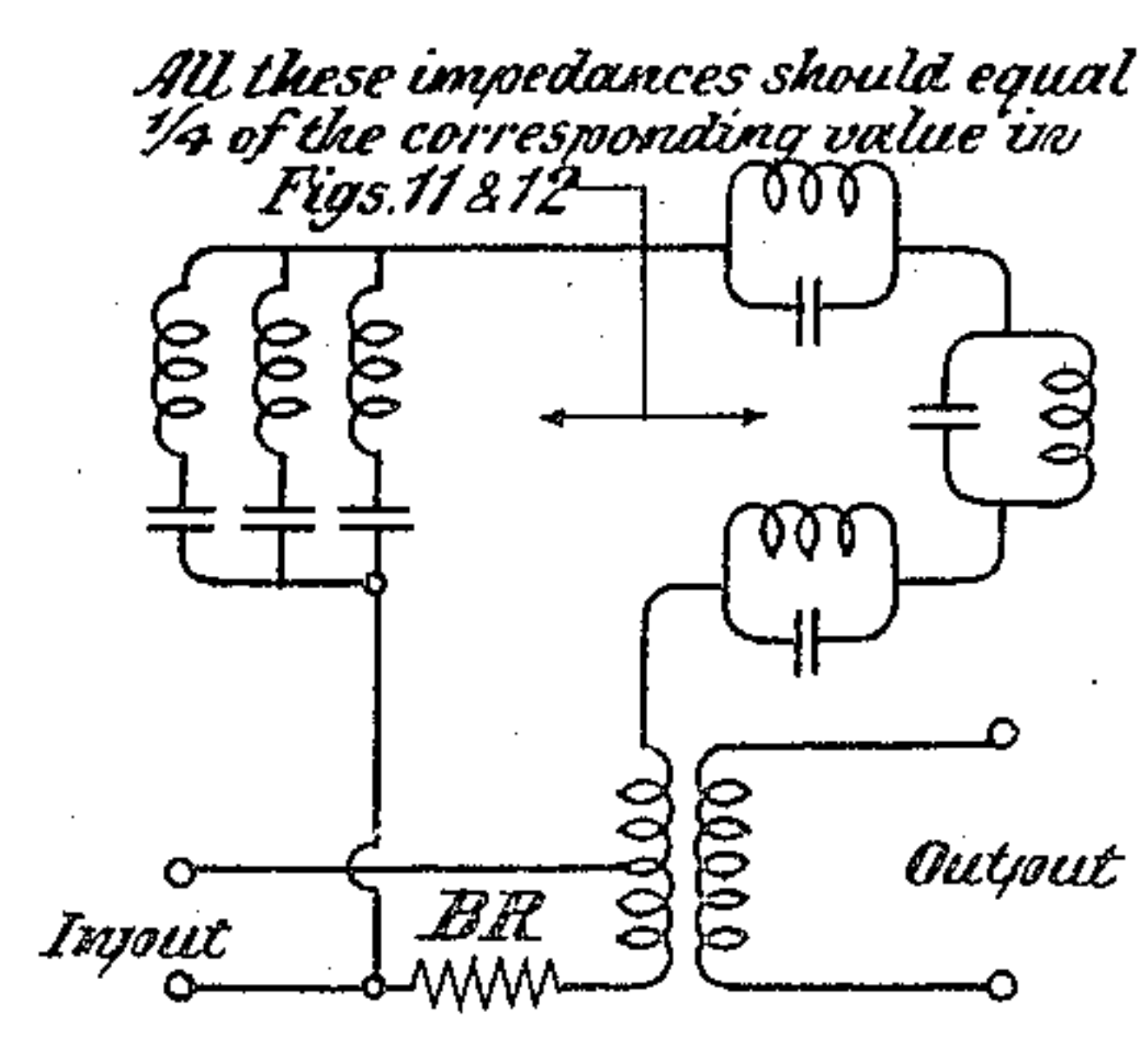


Fig. 13

All these impedances should equal 1/4 of the corresponding value in Figs. 11 & 12

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ELECTRICAL DELAY NETWORK

Application filed September 11, 1926. Serial No. 134,928.

This invention relates to electrical delay networks and has for its object, the design of devices for producing delay in the transmission of wave motion. More particularly it relates to the correction of that form of distortion in a wave transmission system resulting from the difference in velocity of waves of different frequencies and accomplishes this by the introduction of an additional delay, varying with frequency in such manner as to give an approximately equal delay over the frequency range of interest.

Another object is to design such a delay network which shall function with a given number of degrees of freedom but with a fewer number of elements than heretofore. Still a further object is to give a method of design and design formulæ of ready applicability to this network of reduced number of elements. The subject-matter of this invention is a development and extension of certain parts of the disclosure in application of Nyquist, Serial No. 90,656, filed February 25, 1926.

The objects of this invention will be better understood from the following specification and the accompanying drawing in which Figure 1 represents a transmission line with a correcting transducer of a form elsewhere described; Figs. 2 and 2^a show details of the transducer of Fig. 1; Figs. 3 and 4 give forms which the network of our invention may take; Fig. 5 shows a composite section of lattice network; Figs. 6 and 7 are plotted curves showing certain characteristics, and which curves are used in the design of our new network; Figs. 8 and 9 give forms which certain parts of that network may take; Figs. 10, 11, 12 and 13 show modified forms of our circuit.

Referring more specifically to Fig. 1, there

is shown a transducer of the constant "K" type, i. e., the characteristic impedance is independent of frequency. Let this be a transducer of no attenuation and total phase shift β and whose propagation constant is therefore given by $P = 0 + j\beta$, where $j = \sqrt{-1}$. Such a transducer may be realized if we have n sections of lattice type networks of the kind shown generically in Fig. 2 and more particularly in Fig. 2^a, in which all elements are free from resistance. For such a transducer it can be shown that the characteristic impedance is $K = \sqrt{Z_1 Z_2 / 4}$ and if K is to be maintained constant this can be assured by making

$$Z_1 = j 2 K \tan \frac{\beta}{2} \quad (1)$$

$$Z_2 = 2 K / \left(j \tan \frac{\beta}{2} \right) = -j 2 K \cot \frac{\beta}{2}$$

We will show now that the circuit of Fig. 1 can be replaced by the hybrid coil arrangement of Fig. 3 or the simpler one of Fig. 4, both of these possessing the transformer connections characteristic of the so-called 22-type repeater circuit. In these figures, Z_3 and Z_4 represent the impedance of the incoming and outgoing lines.

It can be shown by use of ordinary circuit-al equations that the current i_2 in Fig. 4 is given by

$$i_2 = \frac{E}{2n} \frac{Z_1 - Z_3}{Z_1 \left(Z_2 + Z_3 + \frac{Z_4}{4n^2} \right) + Z_2 \left(Z_3 + \frac{Z_4}{4n^2} \right) + \frac{Z_3 Z_4}{n^2}}$$

where n is the ratio of turns in the windings 3 and 4. In this case let $n = 1/2$. Now substituting K for Z_3 and Z_4 and the above values

for Z_1 and Z_2 , the preceding equation becomes:

ter there is reflection with a change of phase of π radians.

$$i_2 = E \frac{j2K \left(\tan \frac{\beta}{2} + \cot \frac{\beta}{2} \right)}{j2K \tan \frac{\beta}{2} \left(-j2K \cot \frac{\beta}{2} + K + K \right) - j2K \left(\cot \frac{\beta}{2} \right) (K + K) + 4K^2}$$

$$i_2 = \frac{E \left(\tan \frac{\beta}{2} + \cot \frac{\beta}{2} \right)}{\tan \frac{\beta}{2} \left(2K - j2K \cot \frac{\beta}{2} \right) - 2K \cot \frac{\beta}{2} - j2K}$$

$$= \frac{E \left(\tan \frac{\beta}{2} + \cot \frac{\beta}{2} \right)}{2K \left(\tan \frac{\beta}{2} - \cot \frac{\beta}{2} - j2 \right)}$$

but

$$\tan \frac{\beta}{2} = \frac{\sin \beta}{(1 + \cos \beta)}$$

and

$$\cot \frac{\beta}{2} = \frac{\sin \beta}{(1 - \cos \beta)}$$

$$i_2 = \frac{E}{-2K} \frac{1}{\cos \beta + j \sin \beta}$$

but the value for the output current from the far terminals of the sections of lattice network of Fig. 1 is,

$$i_1 = \frac{E}{2K} e^{-j\beta} = \frac{E}{2K} \frac{1}{\cos \beta + j \sin \beta}$$

and by reversing a terminal $i_1 = i_2$.

Thus it is seen that a plurality of tandem sections of non-dissipative lattice network, each with its 4 members, serving as a transducer in Fig. 1, can be replaced by a hybrid coil and two members having non-dissipative reactance Z_1 and Z_2 .

Referring again to Fig. 4, it will be observed that a wave coming in from e and entering the impedance Z_1 , will be reflected at the end of Z_1 and traverse it again. If the phase displacement corresponding to one traversal of Z_1 is $\beta/2$ then the total phase displacement will be β . If now Z_2 is such that its total phase displacement differs from this by the angle π then the two reflected components will annul each other so far as the current back to the source is concerned but will be additive in the secondary of the transformer. Thus, the wave is propagated forward but with a delay or phase displacement determined by Z_1 and Z_2 .

One way in which this phase difference of an angle π can be realized is by use of the relationship of Equations (1) and this may be obtained in a particular case by using one half of the transducer of Fig. 1 as Z_1 and as Z_2 , the one of these being short-circuited at the further end and the other being open-circuited. In the former case, as is well known, there is a reflection of the current wave without change of phase and in the lat-

Although we have spoken of thus replacing n sections of lattice network, it will be observed that the treatment is not limited to a plurality of sections but would hold equally well for a single section, and in this case, the impedances Z_1 and Z_2 would each correspond to one of the four members of the lattice thus requiring only half as many elements of inductance and capacity as in the lattice. This may be looked upon as due to the fact that in Figs. 3 or 4, the waves, because of reflection, must traverse the impedances Z_1 and Z_2 twice, thus introducing the required retardation or phase displacement. An essential part of our invention consists in the economy of required elements for a given delay introduced in this way and the importance of that feature may be more evident when it is recognized that n sections of such lattice networks can be replaced by a single lattice section, the members being made complex, as shown in Fig. 5, and containing the same number of elements as the n sections. By the hybrid coil arrangement, then, the given delay or phase displacement may be obtained by the use of two of the four complex members or their equivalents and thus the number of elements reduced to one half.

The method which we have devised for designing the electrical networks Z_1 and Z_2 for any particular case is illustrated by the following: Suppose a medium heavy loaded side circuit, 222 miles long, is to be corrected for transients over the range 800-1800 cycles for picture transmission. The delay characteristics of this line are such that to obtain substantially equal delay for all frequencies in the range of interest it would be desirable to introduce a network whose delay characteristic is given by curve a of Fig. 6. By means of six sections of lattice type networks of the kind shown in Fig. 2^a, a delay curve, as shown in curve b of Fig. 6, was obtained and this is a sufficiently close approximation to that of curve a over the specified frequency range, and our problem is that of finding the constants of the impedances Z_1 and Z_2 of Fig.

4 which will give a result equivalent to the six section lattice network. These six sections, taken together, have the following constants determined by the method disclosed in application of Nyquist, Serial No. 751,146, filed November 20, 1924, which became Patent No. 1,675,460, July 3, 1928:

$$\begin{array}{ll} b_1=5 & f_1=800 \\ b_2=7 & f_2=900 \\ b_3=5 & f_3=1050 \\ b_4=5 & f_4=1200 \\ b_5=5 & f_5=1300 \\ b_6=5 & f_6=1500 \end{array}$$

where b is a constant depending on the capacities and inductances of any one section and determines the amount of delay for that section at different frequencies. This is shown in Fig. 7 where Tf_0 is plotted against f/f_0 for different values of b . Here T represents the time of delay of a section of the form of Fig. 2^a and f_0 represents the resonant frequency of the members of the section. Also $f_1, f_2, f_3 \dots$ in the table denote the resonant frequency of the members of the network in question and means the same as f_0 in Fig. 7.

The delay, introduced by any one section, can be found from the following considerations: For the lattice type network

$$\cosh P = 1 + \frac{2Z_1}{Z_2 - Z_1} \text{ and } K = \sqrt{\frac{Z_1 Z_2}{4}}$$

If we keep K constant by making $Z_1 = KZ$ and $\frac{Z_2}{4} = \frac{K}{Z}$ then

$$\cosh P = 1 + \frac{2Z}{\frac{4}{Z} - Z}$$

and

$$\tanh \frac{P}{2} = \sqrt{\frac{\cosh P - 1}{\cosh P + 1}} = \frac{Z}{2}$$

$$S = \frac{-jH(p_1^2 - p^2)(p_3^2 - p^2) \dots (p_{2x+1}^2 - p^2) \dots (p_{2n-1}^2 - p^2)}{p(p_{2x}^2 - p^2) \dots (p_{2x-2}^2 - p^2) \dots (p_{2n-2}^2 - p^2)} \quad (4)$$

Let Z be the series resonant circuit, then

$$\frac{Z}{2} = jf \frac{b}{2f_0} + \frac{bf_0}{j2f}; \left(b = \frac{1}{2\pi f_0 C} \right)$$

$$\tanh \frac{P}{2} = \frac{jb}{2} \left(\frac{f}{f_0} - \frac{f_0}{f} \right)$$

Since there is no attenuation, $P = j\beta_n$

$$\therefore \tan \frac{\beta_n}{2} = \frac{b}{2} \left(\frac{f}{f_0} - \frac{f_0}{f} \right)$$

$$\beta_n = 2 \tan^{-1} \frac{b}{2} \left(\frac{f}{f_0} - \frac{f_0}{f} \right)$$

The time of delay is given by

$$T = \frac{d\beta}{dp} = \frac{d\beta}{2\pi df} = \frac{\frac{b}{2\pi f_0} \left(1 + \frac{f_0^2}{f^2} \right)}{1 + \frac{b^2}{4} \left(\frac{f}{f_0} - \frac{f_0}{f} \right)^2}$$

from which the curves of Fig. 7 can be plotted.

Let it be desired to replace the six section network by the equivalent hybrid coil arrangement which uses only two members Z_1 and Z_2 , as in Fig. 4.

$$\frac{Z_1}{j2K} = \frac{2K}{jZ_2} = \tan \frac{\beta}{2} \quad (1)$$

where

$$\frac{\beta}{2} = \sum_{n=1}^6 \frac{\beta_n}{2} \text{ and } \beta_n/2 = \tan^{-1} \left[\frac{b_n \left(\frac{f}{f_n} - \frac{f_n}{f} \right)}{2} \right]$$

for each section of network.

The Equation (1) plotted vs. frequency, f , gives a curve proportional directly or inversely to the impedances which are desired (see curve C of Fig. 6). Whenever β is zero or a multiple of π , the impedance is zero or infinite. These frequencies can be determined by plotting β vs. f and observing at what values of f the curve of β crosses $0, \pi, 2\pi, 3\pi$, etc.

There are several ways to simulate Z_1 and Z_2 . One method is that using Foster's "Reactance theorem", published in Bell System Technical Journal, Vol. III, No. 2, p. 259, which states: The driving point impedance S of a finite resistanceless network whose resonant frequencies (i. e., $S=0$) are

$$f_1 = \frac{p_1}{2\pi}, f_3 = \frac{p_3}{2\pi}, f_5 = \frac{p_5}{2\pi}, \text{ etc.},$$

and whose anti-resonant frequencies (i. e., $S=\infty$) are:

$$f_0 = \frac{p_0}{2\pi}, f_2 = \frac{p_2}{2\pi}, f_4 = \frac{p_4}{2\pi}, \text{ etc.},$$

is given by the equation:

$$S = \frac{-jH(p_1^2 - p^2)(p_3^2 - p^2) \dots (p_{2x+1}^2 - p^2) \dots (p_{2n-1}^2 - p^2)}{p(p_{2x}^2 - p^2) \dots (p_{2x-2}^2 - p^2) \dots (p_{2n-2}^2 - p^2)} \quad (4)$$

Where H is a constant factor ≥ 0 and $0 = p_0 \leq p_1 \leq p_2 \leq \dots \leq p_{2n-1} \leq p_{2n} = \infty$. H may be calculated from the value of S at some particular frequency from the curve C of Fig. 6 and preferably at other than a resonant or anti-resonant frequency.

In order to cover all possible cases, p_1 may sometimes equal p_0 and p_{2n-1} may become infinite while Hp_{2n-1}^2 is maintained finite.

For example, if the impedance to be constructed is resonant at zero and infinity, then we will let $p_0 = p_1 = 0$ and $p_{2n-1} = p_{2n} = \infty$. This is illustrated by the calculation of Z_1 following. We do not actually use either p_0 or p_{2n} in this case but p_{2n-1} must represent the highest resonant frequency and there must be a frequency $p_{2n} > p_{2n-1}$. We can think of them as $p_{2n-1} = \infty$ and $p_{2n} = \infty + 1 = \infty$ for the sake of convenience. Similarly if the desired impedance were resonant at zero and

anti-resonant at infinity we would have $p_0 = p_1 = 0$, $p_{2n} = \infty$ but p_{2n-1} would be the highest resonant frequency and not infinity. Keeping the above in mind together with the fact that resonant frequencies have odd subscripts and anti-resonant frequencies have even subscripts, it is possible to assign the proper values for p_0 , p_1 , etc. . . . from any given impedance curve. S may be represented, as in Fig. 8, by parallel resonant circuits each having a coil and condenser in series resonating at the frequencies f_1, f_3, f_5 , etc., or, as in Fig. 9, by a series of anti-resonant circuits each having a coil and condenser in parallel resonating at f_0, f_2, f_4 , etc., as shown in Figs. 8 and 9.

It should be pointed out that the impedances Z_1 and Z_2 of Fig. 4 may be of the form of Fig. 8 or Fig. 9, as described, and both of them may be of the same form or they may be of different forms. Also, other equivalent forms may be used, but the discussion here will, for simplicity, be confined to those of Figs. 8 and 9. In any case, however, the values of the inductances and capacities must be so chosen that for one of them there shall be a series of resonant frequencies, $p_1, p_3, \dots, p_{2n+1}$, with an anti-resonant frequency, p_{2n} , between each of the resonant frequencies and that for the other, a similar condition holds with the further condition that the anti-resonant frequencies of the one shall be the same as the resonant frequency of the other.

While the constants of these two circuits may be obtained from Foster's theorem, we shall now show that certain constants for either Z_1 or Z_2 made in the form shown above can be calculated from the formulæ (5) and (6), which are much simpler:

$$L_{2x+1} = \frac{KT_{2x+1}}{2} \text{ and } C_{2x+1} = \frac{2}{p_{2x+1}^2 KT_{2x+1}} \quad (5)$$

for resonant circuits in parallel or

$$L_{2x} = \frac{8K}{p_{2x}^2 T_{2x}} \text{ and } C_{2x} = \frac{T_{2x}}{8K} \quad (6)$$

for anti-resonant circuits in series.

In the above T_{2x+1} is the delay in seconds $\frac{d\beta}{dp}$, at the frequency f_{2x+1} ; and T_{2x} is the delay at the frequency f_{2x} .

These Equations (5) and (6) are fundamental in our method of design and constitute our design formulæ. However, the boundary conditions introduce certain other relations which must be used and we will now derive these. Looking at Fig. 8 it will be apparent that if this network is to be used for Z_1 , it must have, according to curve C of Fig. 6, a zero impedance at zero frequency (i. e., $C_1 = \infty$) and it must have zero impedance at infinite frequency (i. e., $L_{2n-1} = 0$). Also, if Fig. 9 is to be used for Z_1 , the same impedance

conditions must hold (i. e., $C_0 = \infty$ and $L_{2n} = 0$). Furthermore, in Fig. 8, the impedance at infinite frequency will be that of capacity C_{2n-1} since L_{2n-1} has been made zero and since all other capacities have series inductance. This impedance will be

$$\frac{1}{jpC_{2n-1}}$$

and on giving this value to S in Equation (4) and allowing p to approach infinity, it is seen that

$$C_{2n-1} = \frac{1}{Hp_{2n-1}^2}$$

At first glance, it looks as though C_{2n-1} would be zero because $p_{2n-1} = \infty$, but H includes the factor

$$\frac{1}{p_{2n-1}^2 - p^2},$$

which when multiplied by p_{2n-1}^2 , gives a finite value for C_{2n-1} . It will be shown below that $L_1 = KT_0$.

If Fig. 9 is to be used as Z_2 , then at infinite frequency the impedance must be infinite, which will be the impedance of L_{2n} , since all other inductances are shunted by capacities. This impedance will be jpL_{2n} and, on giving this value to S in Equation (4) and allowing p to approach infinity, it is seen that $L_{2n} = H$. It will be shown below that $C_0 = \frac{T_0}{4K}$.

Bringing these relations together for ready reference we have:

For

$$\left. \begin{aligned} Z_1, \quad C_1 = C_0 = \infty \quad L_{2n} = L_{2n-1} = 0 \\ L_1 = KT_0 \quad C_{2n-1} = \frac{1}{Hp_{2n-1}^2} \end{aligned} \right\} \quad (7)$$

and for

$$\left. \begin{aligned} Z_2, \quad C_0 = \frac{T_0}{4K} \quad L_{2n} = H \end{aligned} \right\}$$

To prove Equations (5) and (6), let Z_2 be represented by $S = j2K \cot \beta/2$, then from Equation (1), it follows that:

$$\beta = 2 \cot^{-1} \frac{jS}{2K} \text{ or } 2 \tan^{-1} \frac{2K}{jS}$$

$$T = \frac{d\beta}{dp} = \frac{-2d\left(\frac{jS}{2K}\right)/dp}{1 + \left(\frac{jS}{2K}\right)^2} \text{ or } \frac{2d\left(\frac{2K}{jS}\right)/dp}{1 + \left(\frac{2K}{jS}\right)^2} \quad (8)$$

Writing the expression for S from the above figure we get

$$S = \frac{1}{jpC_0} + \sum_{x=1}^{x=n-1} \left(\frac{jp p_{2x}^2 L_{2x}}{p_{2x}^2 - p^2} \right) + jp L_{2n}$$

from Fig. 9, and:

$$\frac{1}{S} = \sum_{x=0}^{x=n-1} \frac{jp}{L_{2x+1}(p_{2x+1}^2 - p^2)}$$

from Fig. 8.

Substituting the value of S (from Fig. 9) into the first form of Equation 8 for delay and simplifying gives

Values of T_0 , T_{2x} , and T_{2x+1} from the delay curve (b), Fig. 6 may be used to compute C_0 , C_{2x} , L_{2x} , C_{2x+1} , L_{2x+1} by the above equations.

$$T = \frac{\frac{1}{K} \left(\frac{1}{p^2 C_0} + \left(\sum_{x=1}^{n-1} \frac{p_{2x}^2 L_{2x} (p^2 + p_{2x}^2)}{(p_{2x}^2 - p^2)^2} \right) + L_{2n} \right)}{1 + \frac{1}{4K^2} \left(\frac{1}{-p C_0} + \left(\sum_{x=1}^{n-1} \frac{p_{2x}^2 L_{2x} p}{p_{2x}^2 - p^2} \right) + p L_{2n} \right)^2} \quad (9)$$

As the frequency $p/2\pi$ approaches zero, the term containing C_0 becomes large compared to the other terms in both numerator and denominator so that they may be neglected.

Therefore

$$T_0 = 4KC_0 \quad (10) \text{ see (7)}$$

Similarly as the frequency $\frac{p}{2\pi}$ approaches

$H(=L_{2n})$ may be determined from the expression for S at any frequency. The first two columns relate to Fig. 9 and the last two to Fig. 8. It depends on the value of K or the sizes of coils and condensers available whether the resonant or the anti-resonant arrangement is the cheaper.

Values for $Z_2 = S$, taking $K = 600$ ohms

$C_0 = .2792 \times 10^{-6} \text{fd.}$	$L_0 = \infty$ henry	$C_1 = .1736 \times 10^{-6} \text{fd.}$	$L_1 = .5040$
$C_2 = .8358 \times 10^{-6} \text{fd.}$	$L_2 = .05721$	$C_3 = .01986 \times 10^{-6} \text{fd.}$	$L_3 = 1.851$
$C_4 = 1.212 \times 10^{-6} \text{fd.}$	$L_4 = .02511$	$C_5 = .0153 \times 10^{-6} \text{fd.}$	$L_5 = 1.653$
$C_6 = 1.031 \times 10^{-6} \text{fd.}$	$L_6 = .02052$	$C_7 = .01260 \times 10^{-6} \text{fd.}$	$L_7 = 1.401$
$C_8 = .8396 \times 10^{-6} \text{fd.}$	$L_8 = .01753$	$C_9 = .01309 \times 10^{-6} \text{fd.}$	$L_9 = .9150$
$C_{10} = .3542 \times 10^{-6} \text{fd.}$	$L_{10} = .02570$	$C_{11} = .04195 \times 10^{-6} \text{fd.}$	$L_{11} = .1170$
$C_{12} = 0$	$L_{12} = .07372$		

any particular value $\frac{p_{2x}}{2\pi}$ the term containing $2x$ subscripts becomes large compared to the others so that they may be neglected.

Hence

$$T_{2x} = \frac{8K}{p_{2x}^2 L_{2x}} = 8KC_{2x} \quad (6)$$

Substituting the value of $\frac{1}{S}$ (from Fig. 8) into the second expression for delay and simplifying gives:

$$T = \frac{4K \sum_{x=0}^{n-1} \frac{p^2 + p_{2x+1}^2}{L_{2x+1} (p_{2x+1}^2 - p^2)^2}}{1 + 4K^2 \left(\sum_{x=0}^{n-1} \frac{p}{L_{2x+1} (p_{2x+1}^2 - p^2)} \right)^2} \quad (11)$$

By reasoning similar to that above we see that at the frequency

$$p = p_{2x+1} \quad T_{2x+1} = \frac{2L_{2x+1}}{K} = \frac{2}{p_{2x+1}^2 K C_{2x+1}} \quad (5)$$

Let us now use these equations to find the constants of Z_2 . Since Z_2 is anti-resonant at zero and infinity we know that $f_0 = 0$ and $f_{2n} = \infty$. The various frequencies, as found from curve C of Fig. 6, are as follows:

$f_0 = 0$	
$f_2 = 728$	$f_1 = 538$
$f_4 = 912$	$f_3 = 830$
$f_6 = 1094$	$f_5 = 998$
$f_8 = 1312$	$f_7 = 1198$
$f_{10} = 1668$	$f_9 = 1454$
$f_{12} = \infty$	$f_{11} = 2272$

We can solve for the constants of Z_1 , in the same way letting Z_1 be represented by $S = j2K \tan \beta/2$

$$\beta = 2 \cot^{-1} \frac{j2K}{S} \text{ or } 2 \tan^{-1} \frac{S}{j2K}$$

$$T = \frac{-2d\left(\frac{j2K}{S}\right)/dp}{1 + \left(\frac{j2K}{S}\right)^2} \text{ or } \frac{2d\left(\frac{S}{j2K}\right)/dp}{1 + \left(\frac{S}{j2K}\right)^2} \quad (12)$$

Since Z_1 is proportional to the curve for $\tan \frac{\beta}{2}$ the frequencies are as follows:

$f_0 = 0$	
$f_1 = 0$	$f_2 = 538$
$f_3 = 728$	$f_4 = 830$
$f_5 = 912$	$f_6 = 998$
$f_7 = 1094$	$f_8 = 1198$
$f_9 = 1312$	$f_{10} = 1454$
$f_{11} = 1668$	$f_{12} = 2272$
$f_{13} = \infty$	$f_{14} = \infty$

As already pointed out, for Z_1

$$C_1 = \infty \quad L_{2n-1} = 0 \text{ in Fig. 8}$$

$$C_0 = \infty \quad L_{2n} = 0 \text{ in Fig. 9}$$

in order that the impedance be zero for the frequencies 0 and ∞ .

Now proceeding as before:

essary and are determined by the lattice net-

$$\frac{1}{S} = \frac{1}{jpL_1} + \left(\sum_{x=1}^{x=n-2} \frac{jp}{L_{2x+1}(p^2_{2x+1} - p^2)} \right) + jpC_{2n-1}$$

for Fig. 8, and

$$S = \sum_{x=1}^{x=n-1} \frac{jp p^2_{2x} L_{2x}}{p^2_{2x} - p^2} \text{ for Fig. 9.}$$

Substituting the value of $\frac{1}{S}$ into the first form of (12) and simplifying gives:

$$T = \frac{4K \left(\frac{1}{p^2 L_1} + \left(\sum_{x=1}^{x=n-2} \frac{p^2 + p^2_{2x+1}}{L_{2x+1}(p^2_{2x+1} - p^2)^2} \right) + C_{2n-1} \right)}{1 + 4K^2 \left(\frac{1}{-pL_1} + \left(\sum_{x=1}^{x=n-2} \frac{p}{L_{2x+1}(p^2_{2x+1} - p^2)} \right) + pC_{2n-1} \right)^2} \quad (13)$$

At the frequency $p=0$ terms not containing L_1 are negligible:

$$T_0 = \frac{L_1}{K} \quad (14) \text{ see (7)}$$

when $p=p_{2x+1}$ terms not containing L_{2x+1} are negligible:

$$T_{2x+1} = \frac{2L_{2x+1}}{K} = \frac{2}{K p^2_{2x+1} C_{2x+1}} \quad (5)$$

Substituting the value of S into the second expression for delay and simplifying gives:

$$T = \frac{\frac{1}{K} \sum_{x=1}^{x=n-1} \frac{p_{2x} L_{2x} (p^2_{2x} + p^2)}{(p^2_{2x} - p^2)^2}}{1 + \frac{1}{4K^2} \left(\sum_{x=1}^{x=n-1} \frac{p^2_{2x} p L_{2x}}{p^2_{2x} - p^2} \right)^2}$$

whence

$$T_{2x} = \frac{8K}{p^2_{2x} L_{2x}} = 8K C_{2x} \quad (6)$$

Values for Z_1

$C_0 = \infty$	$L_0 = \infty$	$C_1 = \infty \text{ fd.}$	$L_1 = .402$
$C_2 = 1.4 \times 10^{-6} \text{ fd.}$	$L_2 = .06667 \text{ henry}$	$C_3 = .03970 \times 10^{-6}$	$L_3 = 4.012$
$C_4 = 5.142 \times 10^{-6} \text{ fd.}$	$L_4 = .007148$	$C_5 = .01744 \times 10^{-6}$	$L_5 = 1.746$
$C_6 = 4.592 \times 10^{-6} \text{ fd.}$	$L_6 = .005537$	$C_7 = .01425 \times 10^{-6}$	$L_7 = 1.485$
$C_8 = 3.892 \times 10^{-6} \text{ fd.}$	$L_8 = .004535$	$C_9 = .01217 \times 10^{-6}$	$L_9 = 1.209$
$C_{10} = 2.542 \times 10^{-6} \text{ fd.}$	$L_{10} = .004713$	$C_{11} = .01785 \times 10^{-6}$	$L_{11} = .510$
$C_{12} = .325 \times 10^{-6} \text{ fd.}$	$L_{12} = .0151$	$C_{13} = .05119 \times 10^{-6}$	$L_{13} = 0$
$C_{14} = 0$	$L_{14} = 0$		

The same considerations govern the choice between series anti-resonant circuits and parallel resonant circuits for Z_1 as for Z_2 .

The question may be raised why use is made of the curve b of Fig. 6 instead of curve a , since the latter relates to the line to be corrected and since the former calls for the design of the lattice network which shall have a delay curve similar to curve a . The reason for this is, that the values for b_n and f_n are nec-

work which gives curve b . It is not necessary to completely design the lattice network but the values of b_n and f_n for the various sections must be determined. A method of attack which eliminates the use of the tandem sections and curve b has been developed, but is much more difficult and cumbersome and we therefore consider the designing of the lattice

network and the use of the resulting curve b as an important step in our method of design.

In order to make more evident the procedure which we follow in order to design the delay or phase compensator which is described in this specification, we give below a brief summary of the steps which may be taken. First, a plot of the desired delay characteristic, such as that of curve a , Fig. 6, is made. Then a lattice network of a desired number of sections, which will give a delay characteristic approximating that of curve a , is designed, this being done in accordance with the method disclosed in the Patent 1,675,460 referred to above. In this way, the curve b of Fig. 6 is obtained. We then find β , the phase shift, at various frequencies corresponding to this curve b and plot the values of $\tan \frac{\beta}{2}$ from these values thus obtaining the curve C of Fig. 6. The ordinates of this curve C will not only be equal to $\tan \frac{\beta}{2}$, but, as

shown in this specification, will also be equal to $\frac{Z_1}{j2K}$ and $\frac{2K}{jZ_2}$. From this curve C there will be located immediately certain points at which $\tan \frac{\beta}{2}$ is equal to zero or to infinity, these corresponding to frequencies where β is a multiple of 180° and corresponds also to the frequencies where the new network will be resonant or anti-resonant. The delay, T ,

at each of these resonant and anti-resonant frequencies is now obtained by reference to curve *b*. It will now be desirable to select the form of impedance, such as that of Fig. 8 or Fig. 9, and inspect this for the boundary conditions, and also to determine the value of the constant *K*, and, finally, the value of *T* and *K* is substituted in relations containing *T*, *K*, *L*, *C* and the frequency as given in the Equations 5 and 6, (first form), such substitution yielding the particular values of *L* and *C* which will give a network of the desired phase characteristic.

What is claimed is:

1. A phase delay compensator adapted to be associated with a transmission line and comprising a transformer and a complex impedance, said impedance having resonant and anti-resonant frequencies, the constants of the impedance being so chosen as to give the same delay as an equivalent *n* section lattice network at each of the aforesaid resonant and anti-resonant frequencies.

2. A phase delay compensator adapted to be associated with a transmission line and comprising a transformer and a complex impedance, said impedance having resonant and anti-resonant frequencies, these frequencies being those at which the phase shift of an equivalent *n* section lattice network is a multiple of 180°, and the values of the inductances and capacities in said impedance being determined by the impedance of the transmission line and the delay to be obtained at the resonant and anti-resonant frequencies.

3. A phase compensator adapted to be associated with a transmission line and being the equivalent of an *n* section lattice network and comprising a hybrid coil and a complex impedance, said impedance having resonant and anti-resonant frequencies, the constants of the impedance being determined by the delay in said equivalent lattice network at each resonant and anti-resonant frequency of the complex impedance, and by the impedance of the transmission line.

4. A phase compensator adapted to be associated with a transmission line and being the equivalent of an *n* section lattice network and comprising a hybrid coil and a complex impedance, said impedance consisting of *n* parallel branches of series inductance and capacity and having resonant and anti-resonant frequencies, the values of the inductances and capacities being determined by the delay of the equivalent lattice network at each resonant and anti-resonant frequency and the impedance of the transmission line to yield the same delay as in the lattice network.

5. A phase compensator adapted to be associated with a transmission line and being the equivalent of an *n* section lattice network and comprising a hybrid coil and a complex impedance, said impedance comprising *n* sec-

tions in series of shunt inductance and capacity and having resonant and anti-resonant frequencies, the values of the inductances and capacities being determined by the delay of the equivalent lattice network at each resonant and anti-resonant frequency and the impedance of the transmission line to yield the same delay as in the lattice network.

6. A phase compensator adapted to be associated with a transmission line and comprising a hybrid coil and two complex impedances and being the equivalent of an *n* section lattice delay network, one impedance having resonant and anti-resonant frequencies, the other having equal anti-resonant and resonant frequencies, the constants of the inductances and capacities being determined by the delay in the equivalent lattice network at each resonant and anti-resonant frequency.

7. A phase delay compensator adapted to be associated with a transmission line and comprising a transformer and a complex impedance, the delay characteristic being made equal to that of a previously chosen *n* section lattice network, said impedance having resonant and anti-resonant frequencies, the elements of the impedance having such values as to yield the same delay at the aforesaid resonant and anti-resonant frequencies as in the *n* section lattice network.

8. In an electrical delay network to simulate an *n* section lattice delay network, the combination of a hybrid coil with output terminals unaffected directly by current introduced at the input terminals, and a network to receive current from the input terminals and reflect it back, the system being adapted to make the reflected current waves effective on the output terminals, the elements of the network being so chosen that the over-all delay characteristic shall be that of the *n* section lattice network.

9. In an electrical wave transmission system, the combination of a transmission line with delay characteristics and a delay compensator, said compensator comprising a hybrid coil with output terminals unaffected directly by current introduced at the input terminals, and a network to receive current from the input terminals and reflect it back, the system being adapted to make the reflected current waves effective on the output terminals, the elements of the network being so chosen that the over-all characteristic shall be the inverse of that of the transmission line.

10. In an electrical delay network to simulate an *n* section lattice delay network, the combination of a hybrid coil with output terminals unaffected directly by current introduced at the input terminals, and a network to receive current from the input terminals and to reflect it back, the system being adapted to make the reflected current waves effective on the output terminals, the network having half as many elements as in the lat-

tice network and having resonant and anti-resonant frequencies, the elements of the network having such values as to yield the same delay at the aforesaid resonant and anti-resonant frequencies as in the lattice network.

11. A delay network comprising a hybrid coil with output terminals unaffected directly by current introduced at the input terminals, and two complex impedances to receive and divide current from the input terminals and reflect it back, the one impedance reflecting with change of phase and the other without change of phase to make the reflected waves effective on the output terminals.

12. A delay network comprising a hybrid coil with output terminals unaffected directly by current introduced at the input terminals, and two complex impedances to receive and divide current from the input terminals and reflect it back, the one impedance being resonant and anti-resonant at certain frequencies, and the other impedance being anti-resonant and resonant at the same frequencies.

13. A delay network to simulate an n section lattice delay network and comprising a hybrid coil with output terminals unaffected by current introduced at the input terminals and two complex impedances to receive and divide current from the input terminals and reflect it back, the one impedance reflecting with a change of phase and the other without change of phase to make the reflected waves effective in the output terminals, the elements of the impedances being half as great in number as in the lattice network and having values to yield the same delay as in the lattice network.

14. A delay network comprising a hybrid coil with output terminals unaffected directly by current introduced at the input terminals, and two complex impedances to receive and divide current from the input terminals and reflect it back, the two impedances reflecting with a difference in phase of 180° to make the reflected waves effective on the output terminals.

15. In combination, a transmission line and a phase delay compensator of a delay characteristic equivalent to that of an n section lattice network, said compensator comprising a transformer and a complex impedance, the number of elements of the complex impedance being half that of the lattice network, and its constants having values determined by plotting the desired delay characteristic, finding the lattice network of n sections whose delay characteristic approximates this, locating the resonant and anti-resonant frequencies for said complex impedance, finding the delay at each of said resonant and anti-resonant frequencies, and obtaining the constants of the complex impedance from this delay and from the characteristic impedance of the line.

16. A phase compensator of a delay char-

acteristic equivalent to a delay network of n lattice sections and comprising a transformer and a complex impedance, the constants of the complex impedance having values determined by plotting the desired delay characteristic, finding the equivalent lattice network of n sections, finding the angular shift as a function of frequency for this lattice network, locating the frequencies where this shift is a multiple of 180° , finding the delay at each of these frequencies, and obtaining the inductances and capacities of the impedance from this delay and the characteristic impedance of the lattice network.

In testimony whereof, we have signed our names to this specification this 10th day of September, 1926.

HARRY NYQUIST.
KENNETH W. PFLEGER.