

July 16, 1929.

O. J. ZOBEL

1,720,777

ELECTRICAL NETWORKS

Filed Sept. 9, 1926

2 Sheets-Sheet 1

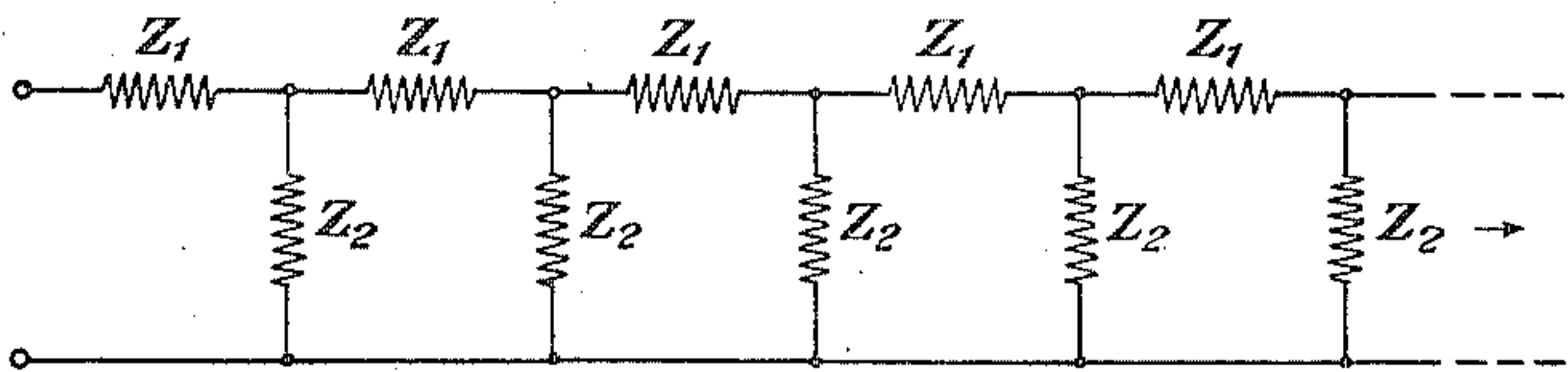


Fig. 1 Ladder-type structure

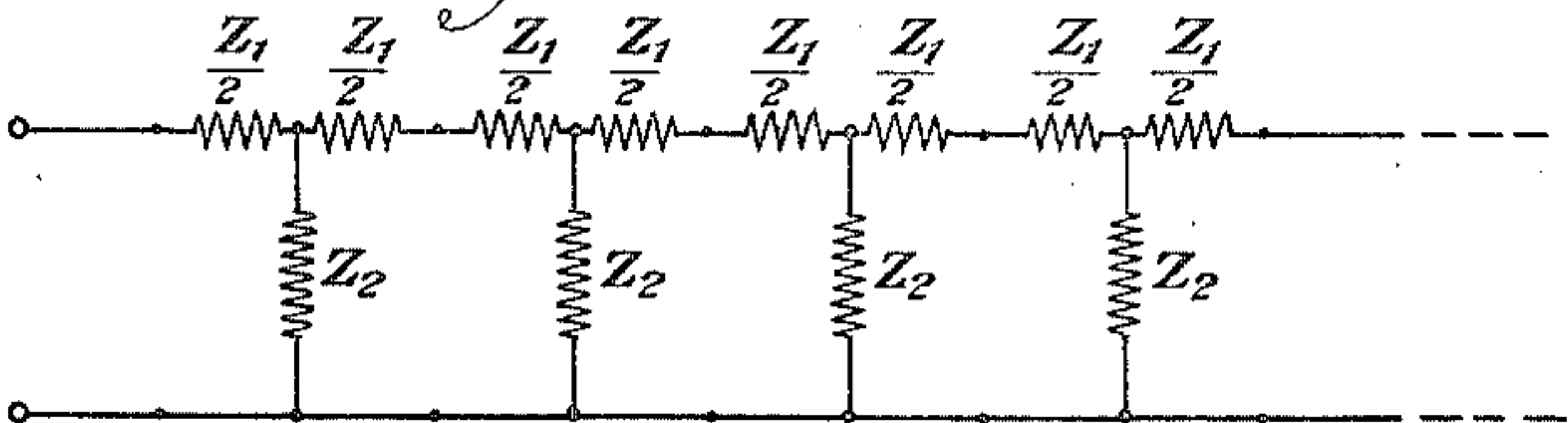


Fig. 2 Ladder-type, mid-series impedance

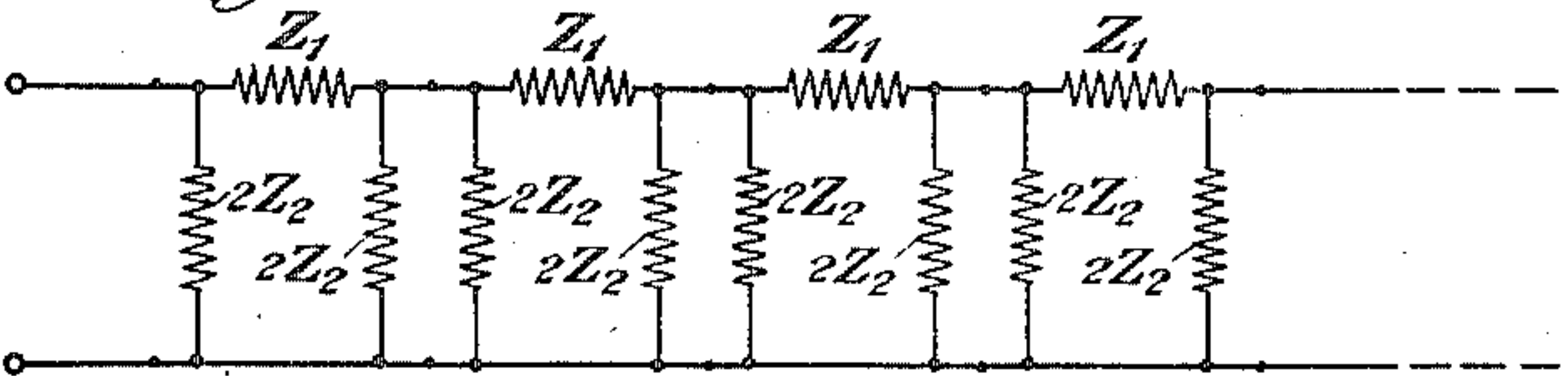


Fig. 3 Ladder-type, mid-shunt impedance

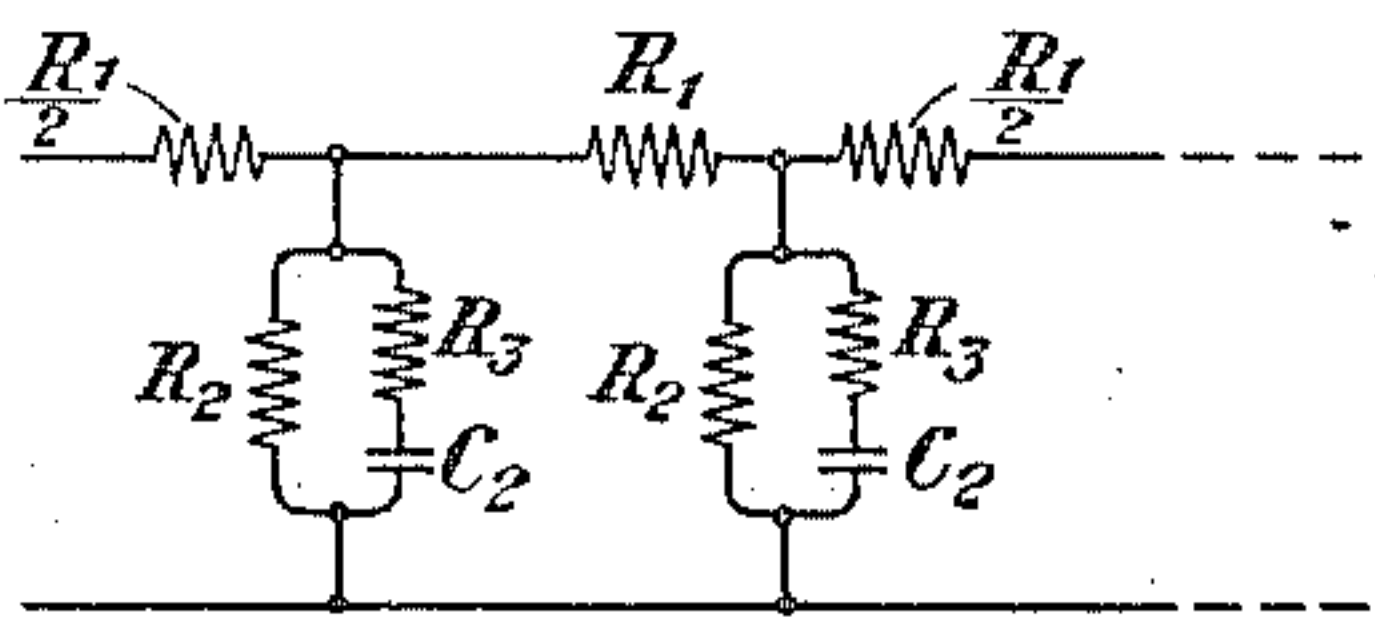


Fig. 2a

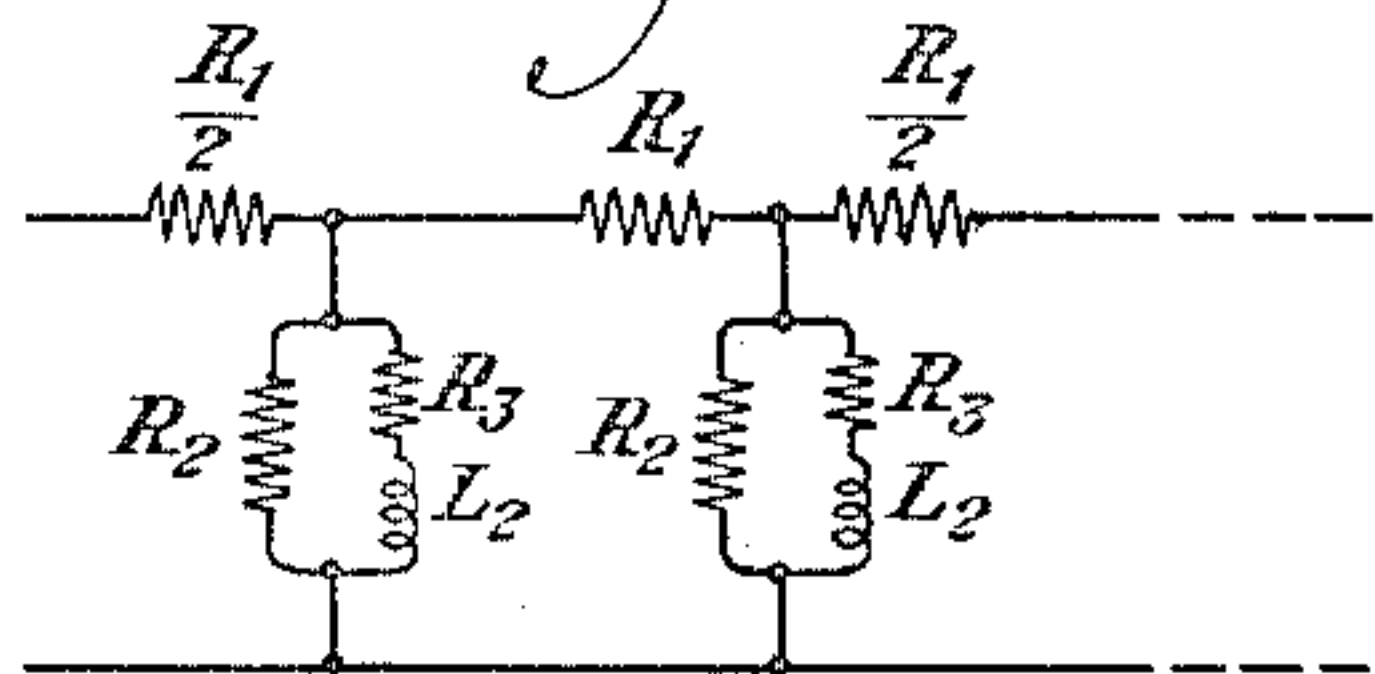


Fig. 2b

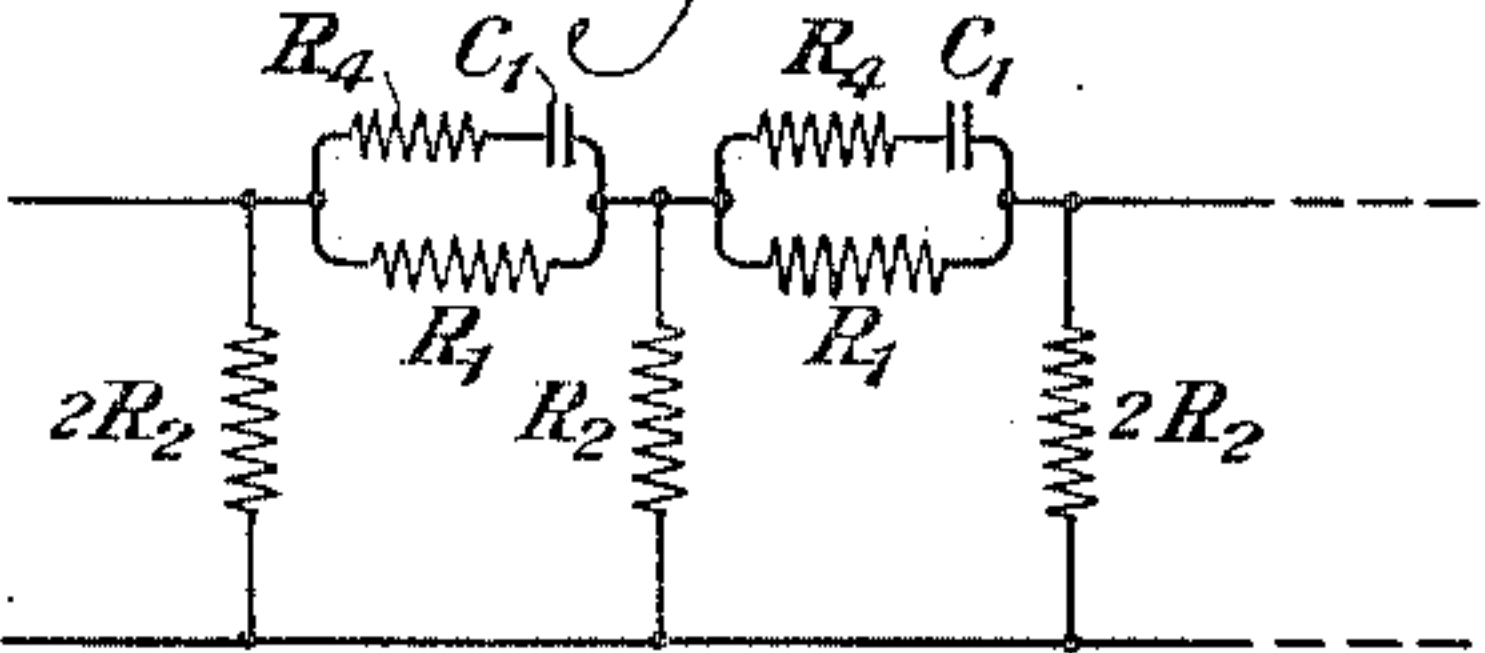


Fig. 3a

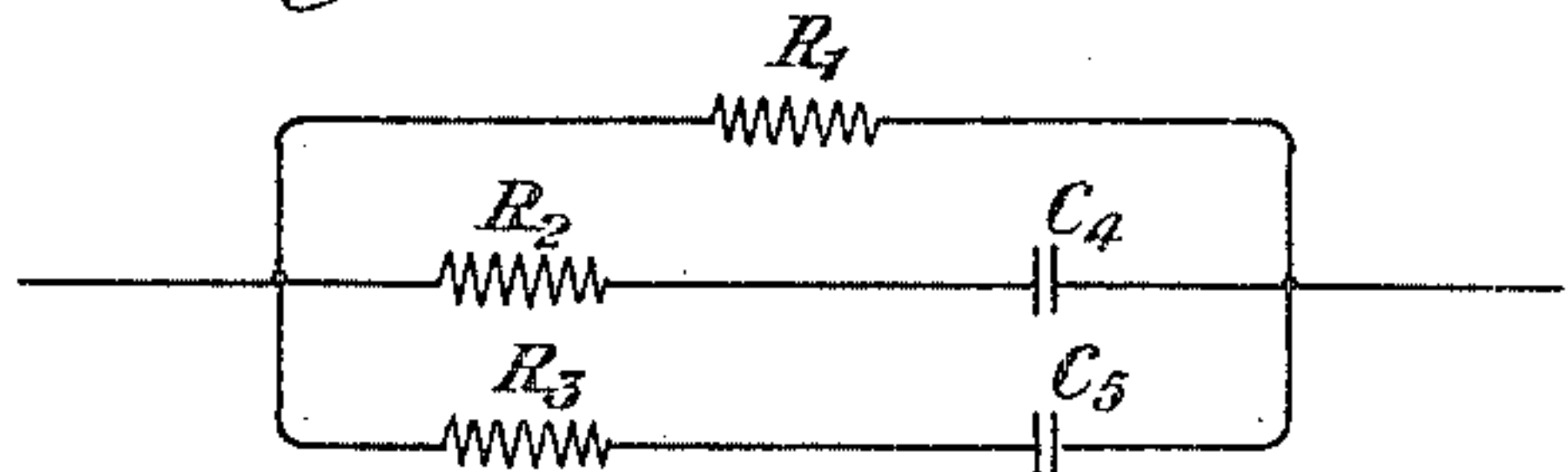


Fig. 4

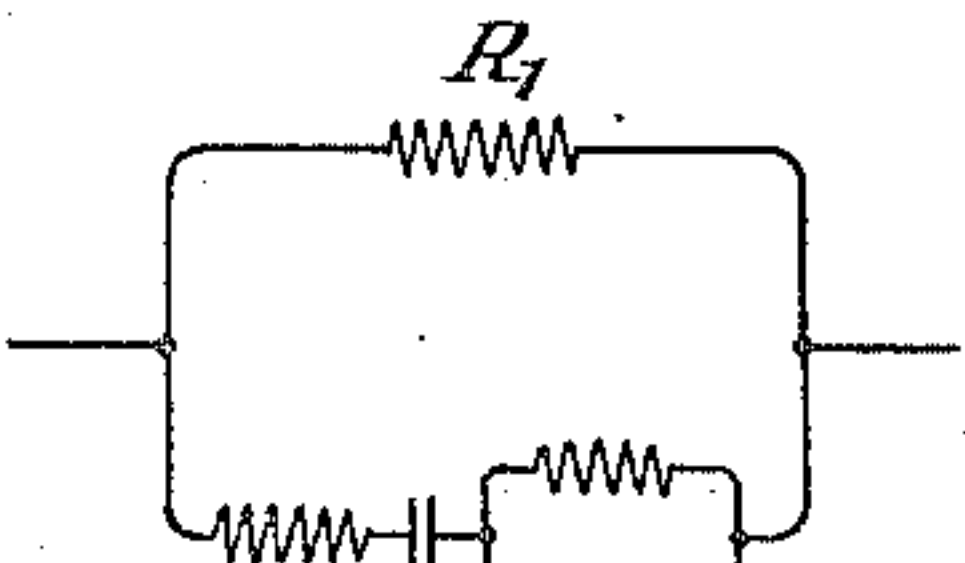


Fig. 5

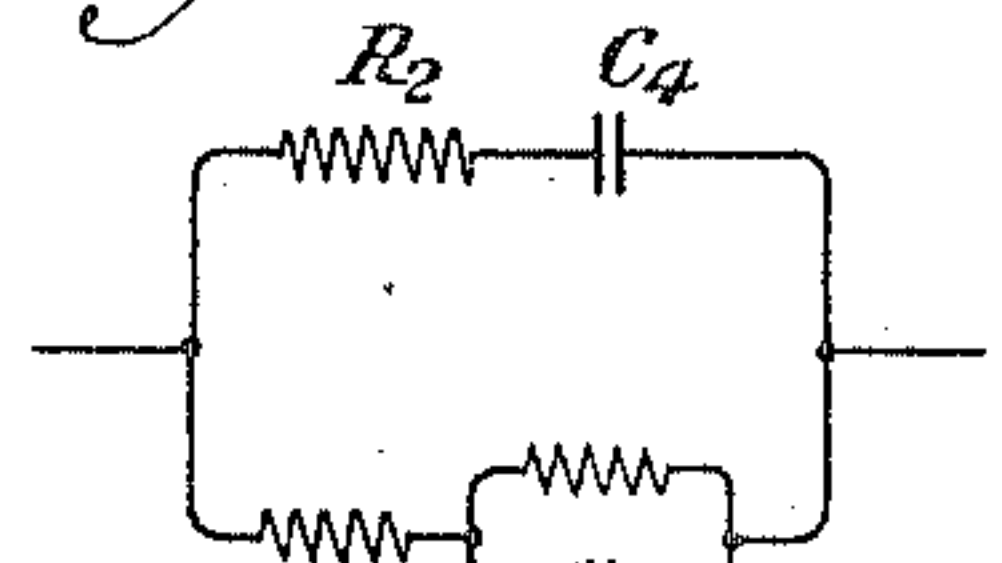


Fig. 6

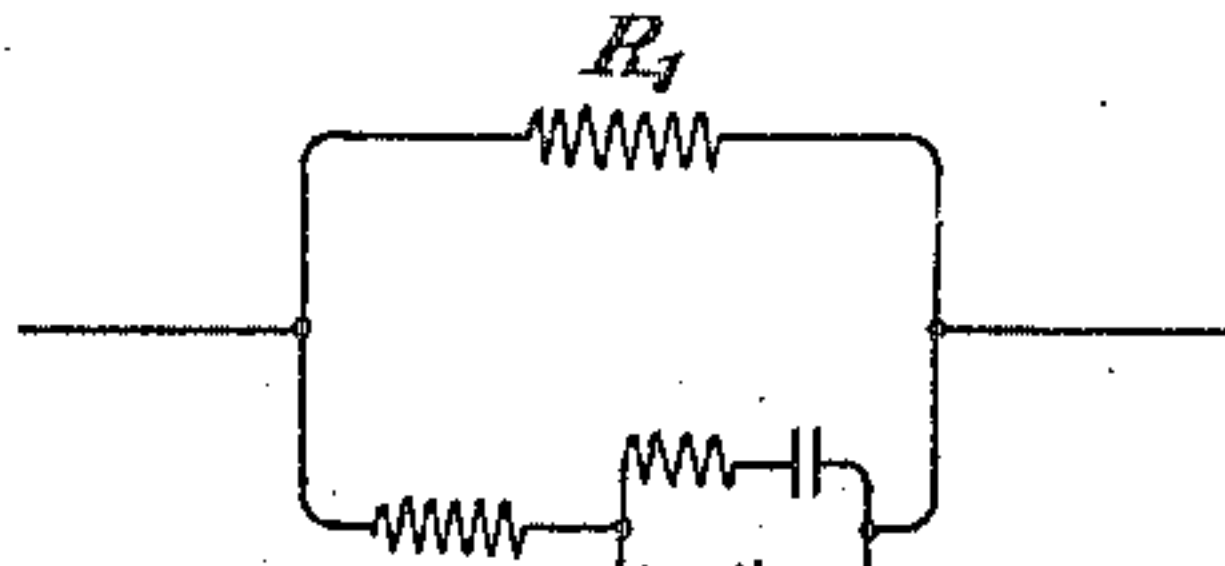


Fig. 7

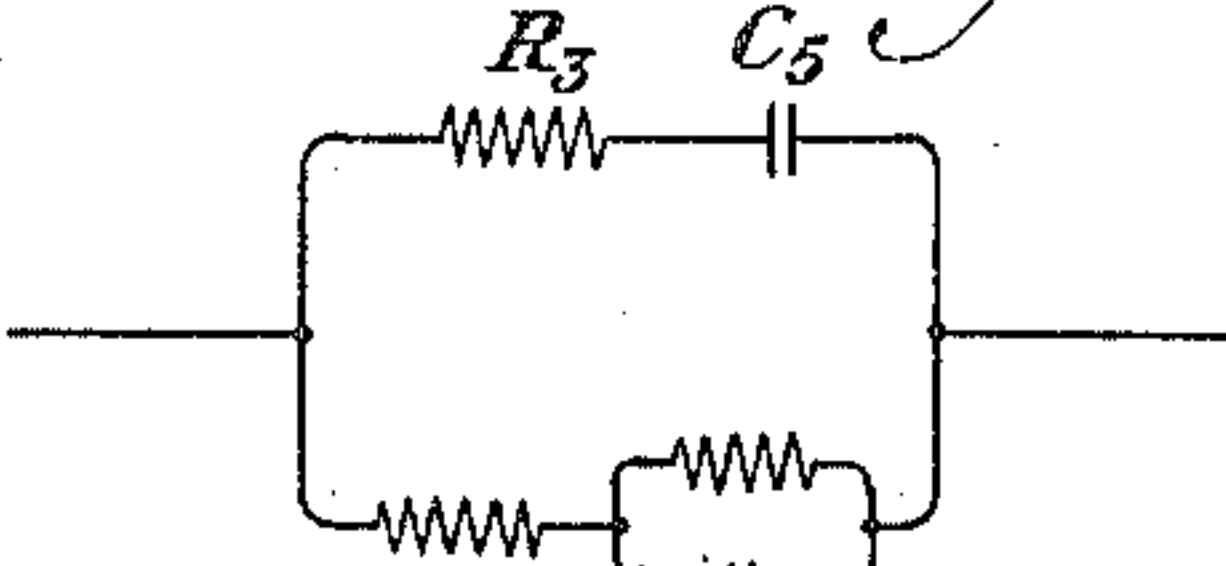


Fig. 8

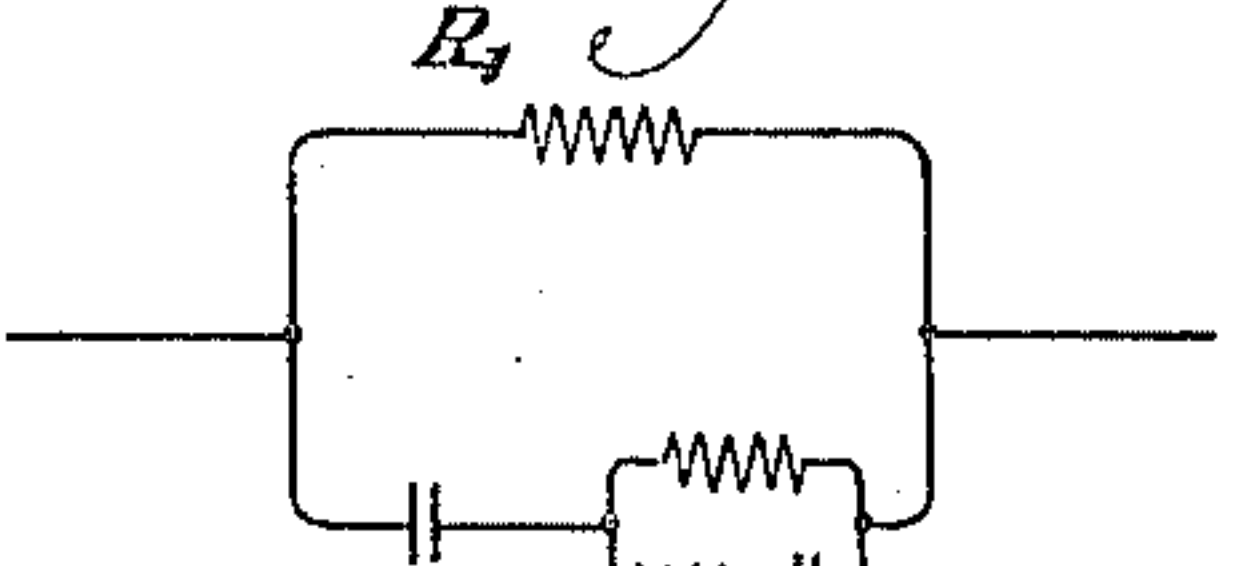


Fig. 9

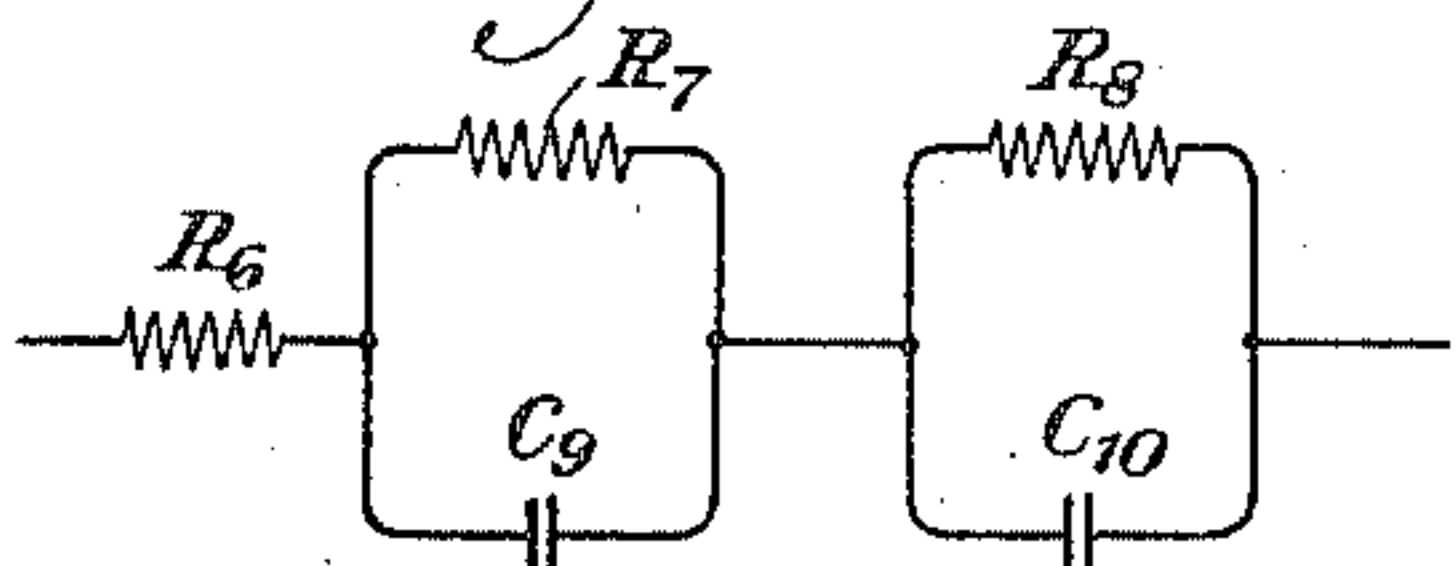


Fig. 10

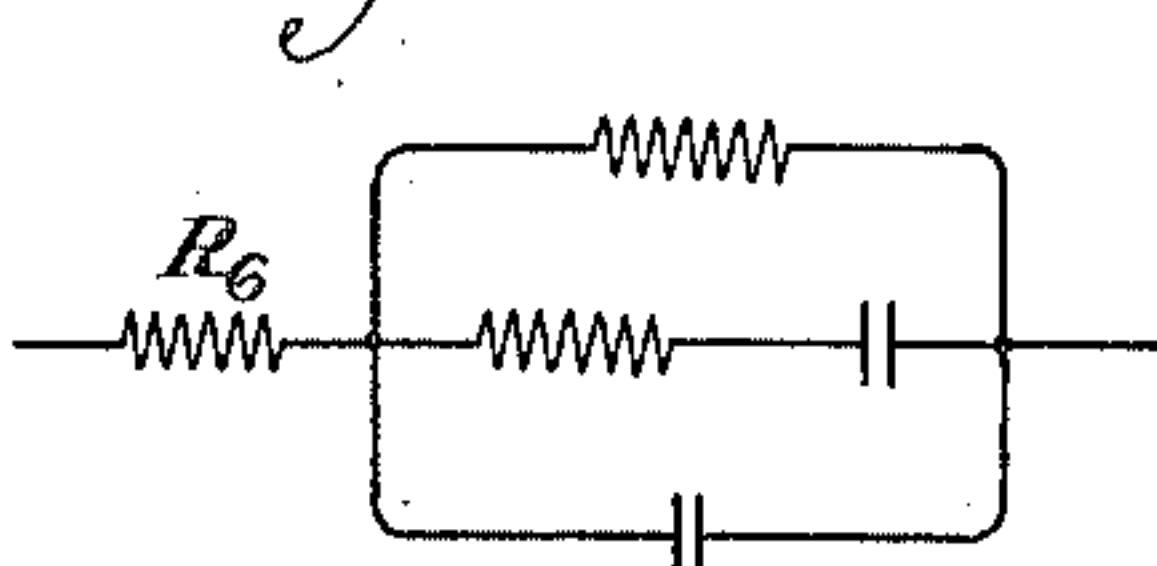


Fig. 11

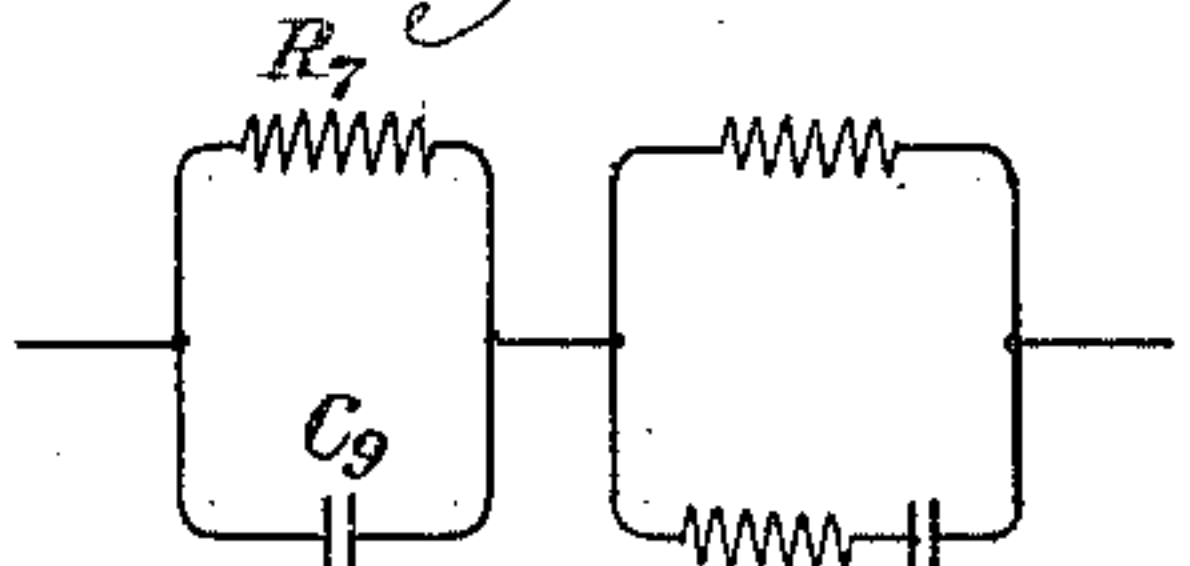


Fig. 12

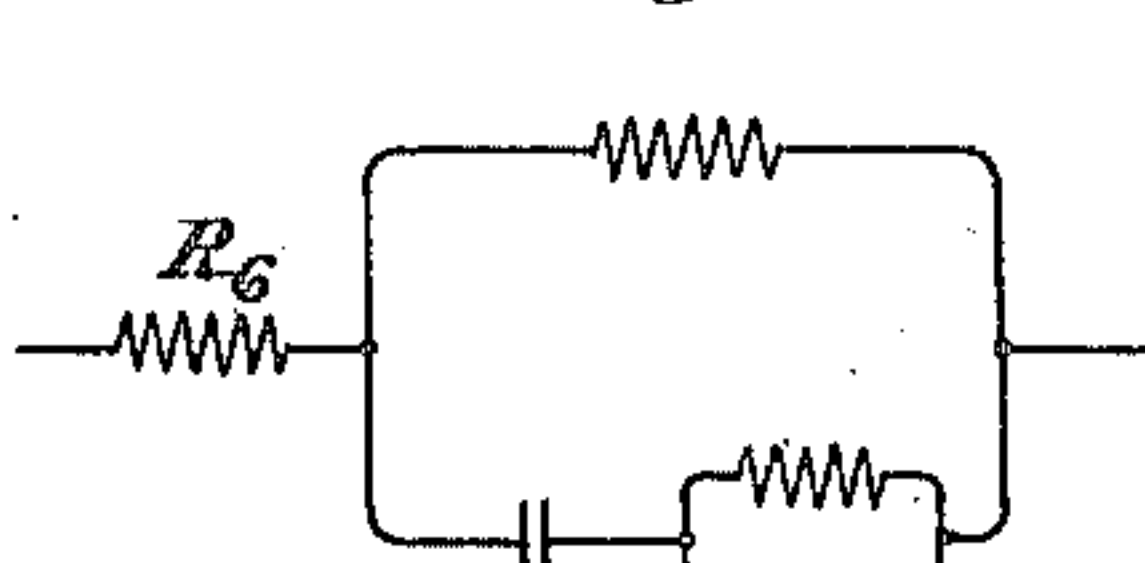


Fig. 13

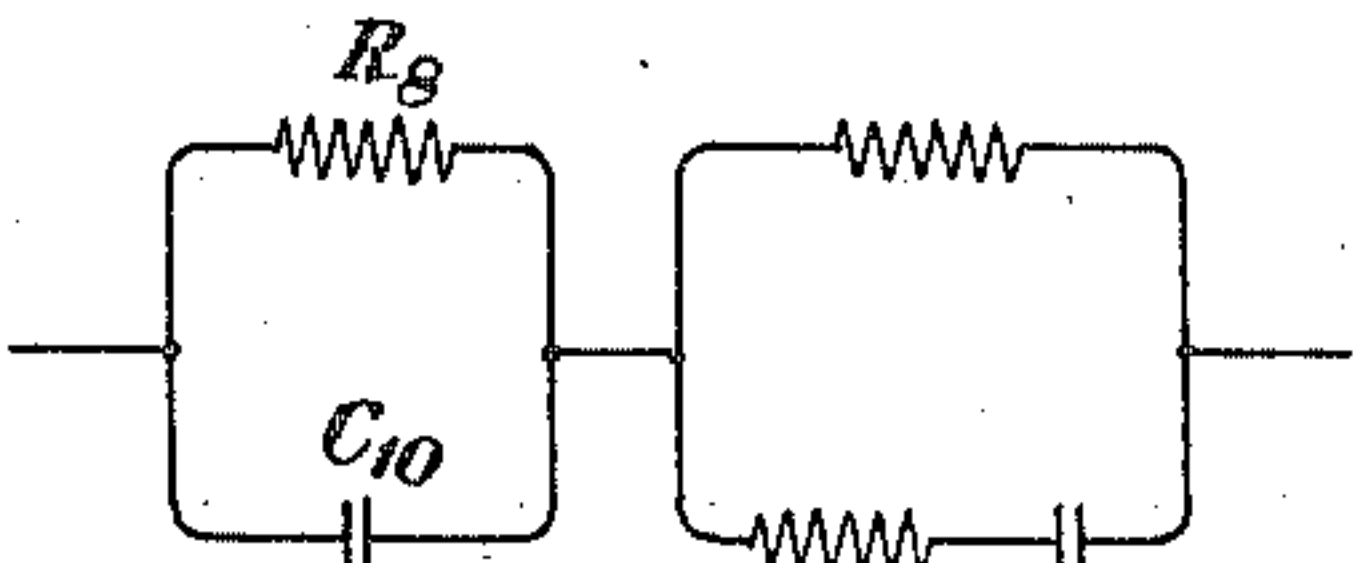


Fig. 14

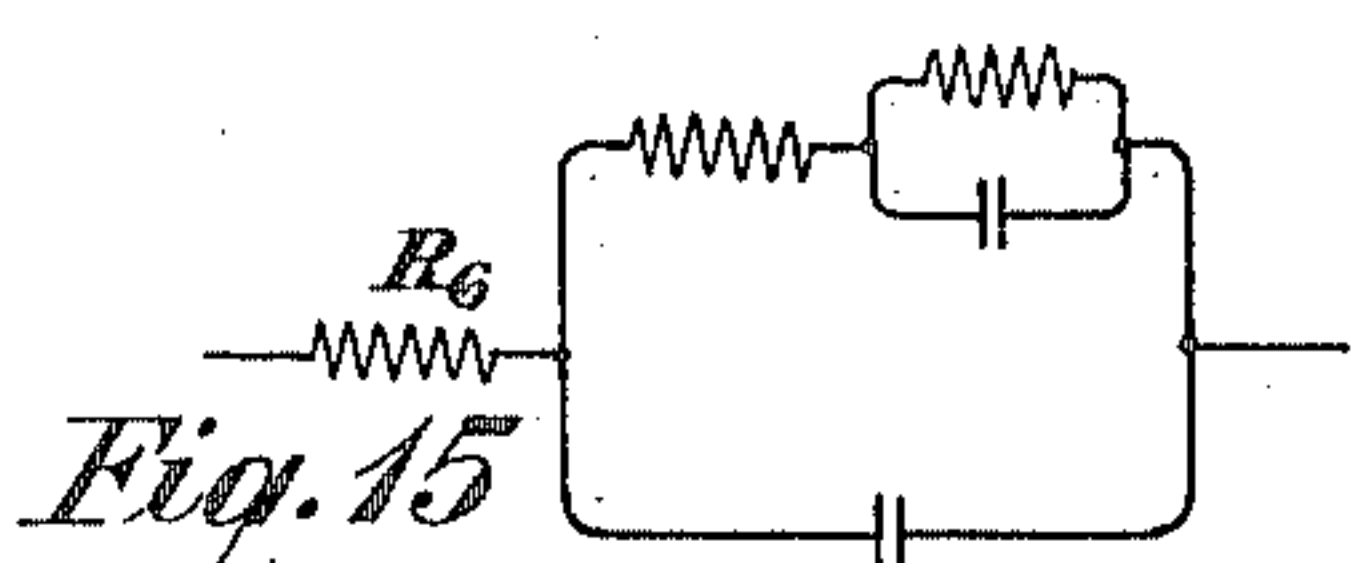


Fig. 15

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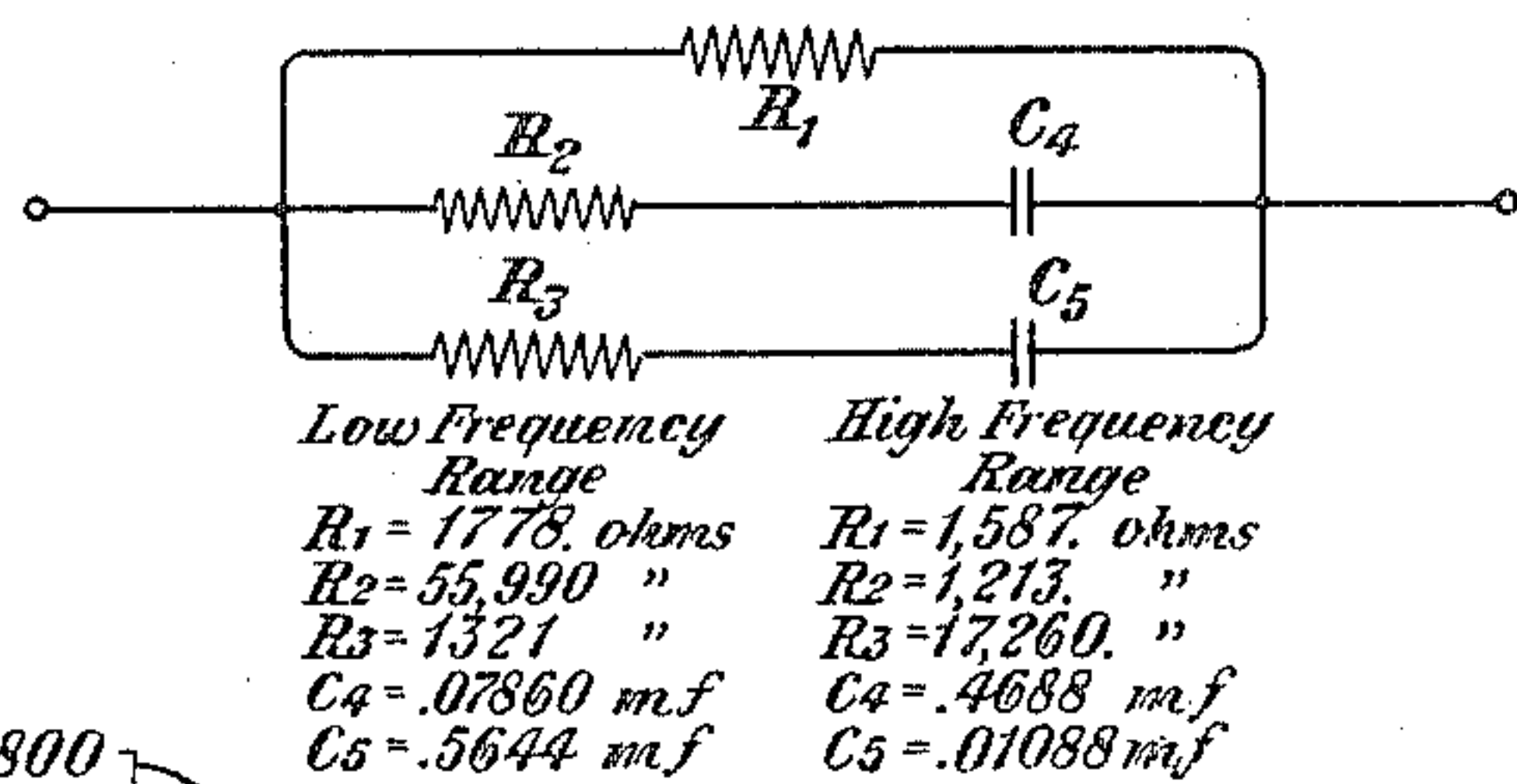
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ELECTRICAL NETWORKS

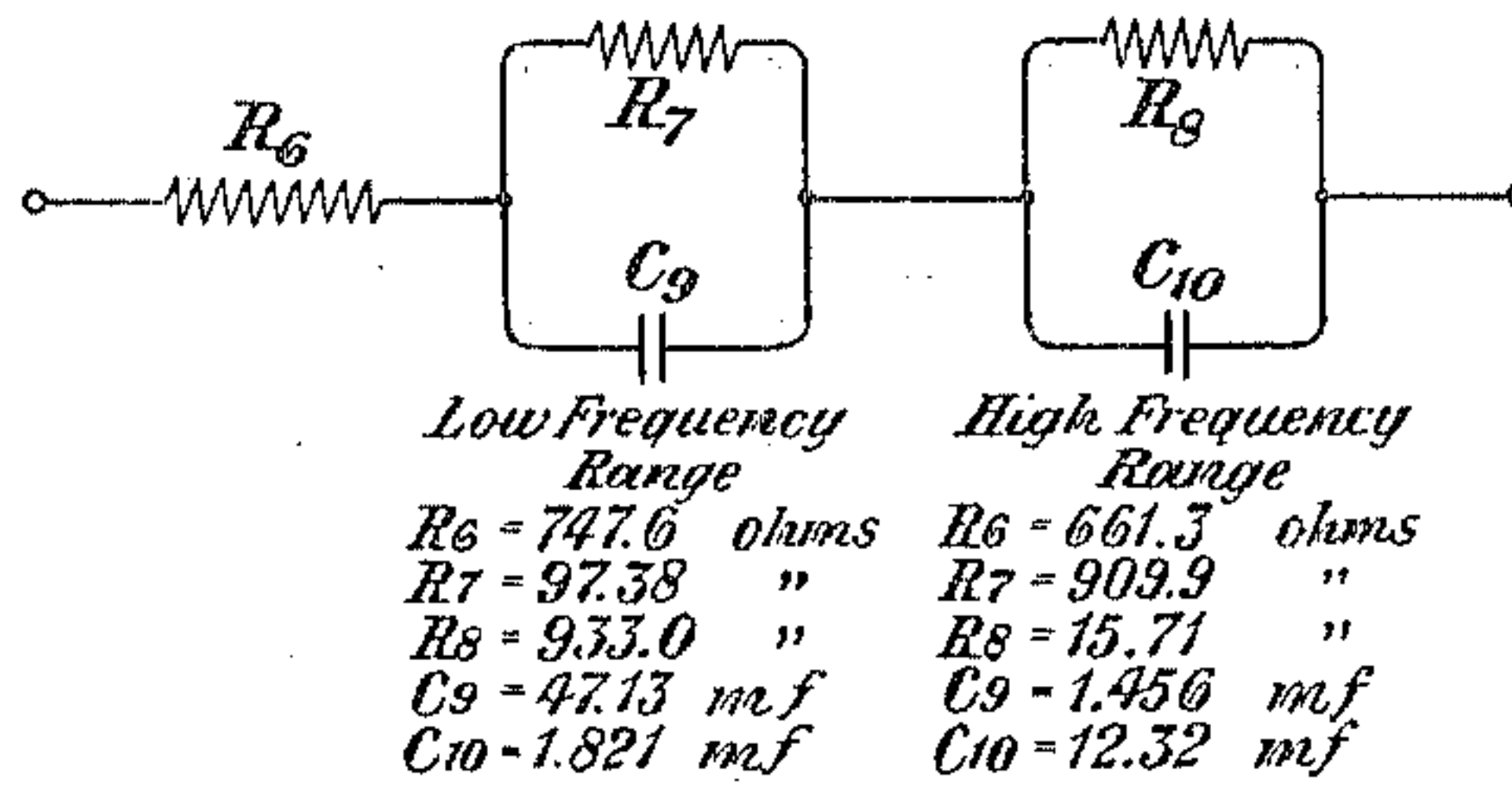
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2 Sheets-Sheet 2



Low Frequency Range	High Frequency Range
$R_1 = 1778 \text{ ohms}$	$R_1 = 1587 \text{ ohms}$
$R_2 = 55,990 \text{ "}$	$R_2 = 1,213 \text{ "}$
$R_3 = 1321 \text{ "}$	$R_3 = 17,260 \text{ "}$
$C_4 = .07860 \text{ mf}$	$C_4 = .4688 \text{ mf}$
$C_5 = .5644 \text{ mf}$	$C_5 = .01088 \text{ mf}$

Fig. 16



Low Frequency Range	High Frequency Range
$R_6 = 747.6 \text{ ohms}$	$R_6 = 661.3 \text{ ohms}$
$R_7 = 97.38 \text{ "}$	$R_7 = 909.9 \text{ "}$
$R_8 = 933.0 \text{ "}$	$R_8 = 15.71 \text{ "}$
$C_9 = 47.13 \text{ mf}$	$C_9 = 1.456 \text{ mf}$
$C_{10} = 1.821 \text{ mf}$	$C_{10} = 12.32 \text{ mf}$

Fig. 17

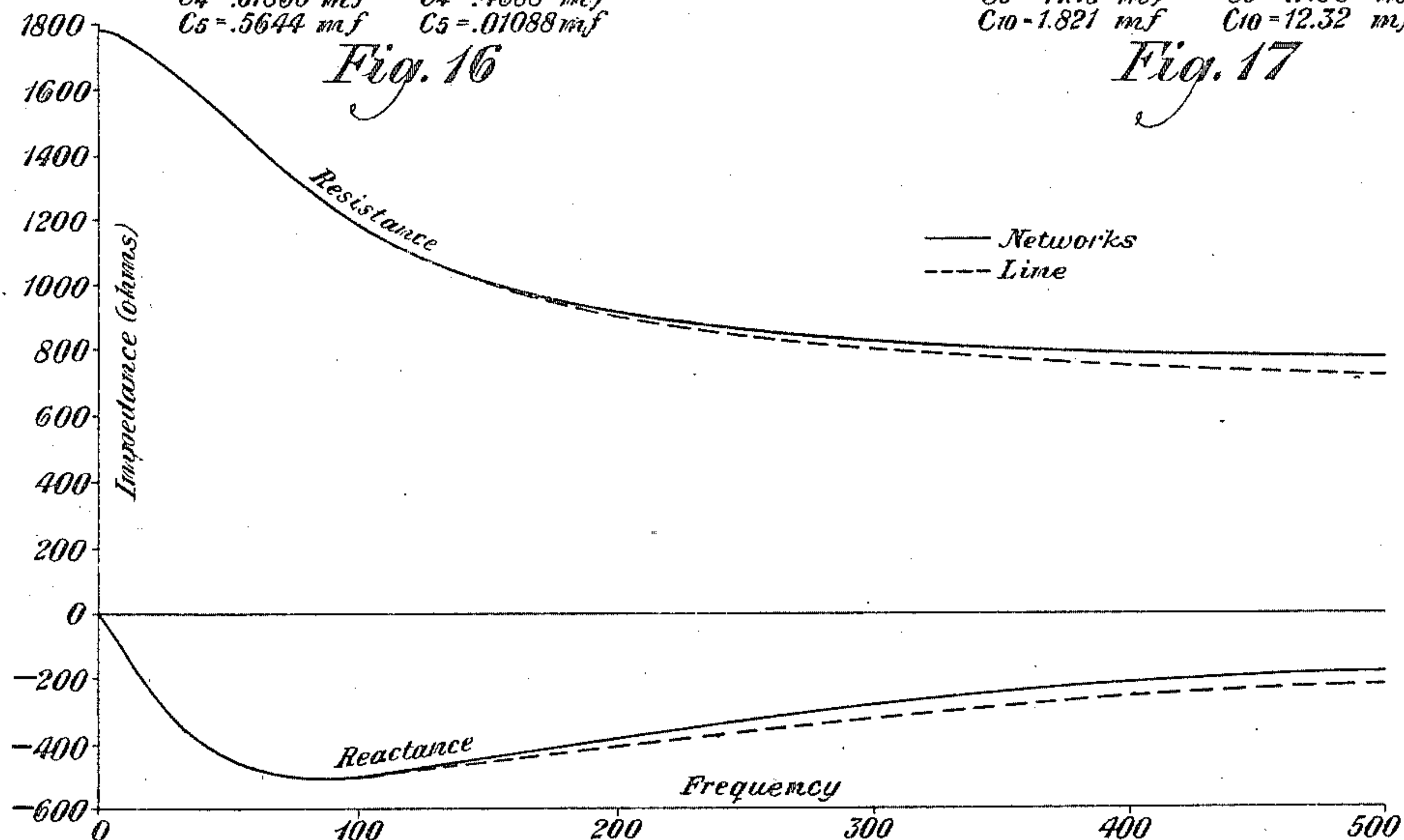


Fig. 18

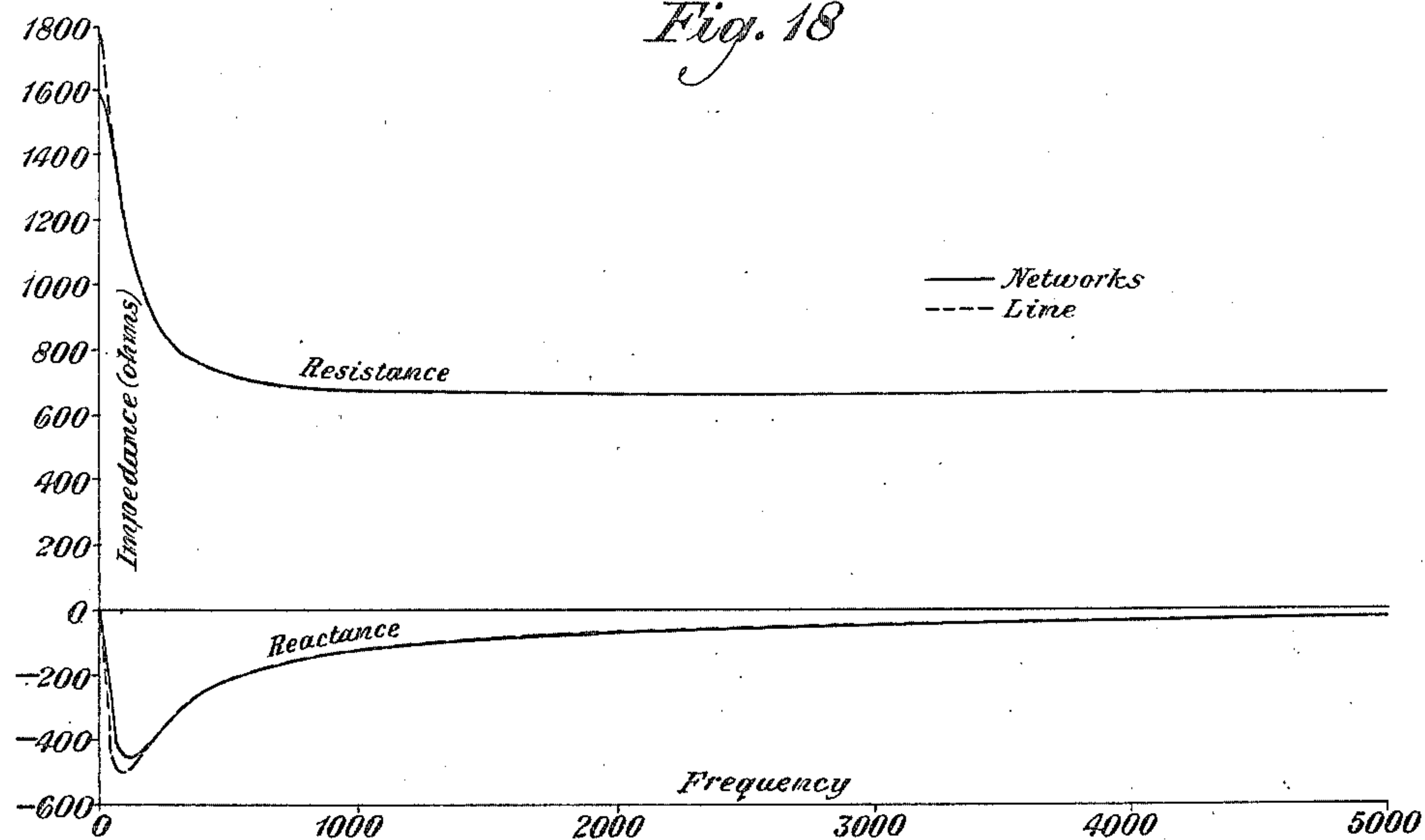


Fig. 19

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ELECTRICAL NETWORK.

Application filed September 9, 1926. Serial No. 134,515.

The principal object of the present invention is to provide an electrical network of definite and practicable design that shall simulate the impedance of a given smooth transmission line or any electrical conductor system having an impedance-frequency characteristic similar to such a line.

The invention will be better understood by reference to the accompanying drawing in which Figures 1 to 3 and Figs. 2^a, 2^b and 3^a are circuit drawings to assist in explaining the invention; Figs. 4 and 10 represent two forms of networks embodying the ideas of my invention, Figs. 5 to 9 are equivalents of Fig. 4; Figs. 11 to 15 are equivalents of Fig. 10; Figs. 16 and 17 are networks on which actual calculations have been made for illustrative purposes and Figs. 18 and 19 are curves showing the degree to which the calculated networks of Figs. 16 and 17 agree with the smooth line they were intended to simulate.

In the consideration of wave filters and artificial lines it has become useful to speak of electrical structures known as ladder-type structures and shown in a generalized form in Fig. 1 of the drawing as consisting of an iterative impedance, each section comprising series impedance and shunt impedance. The properties of these structures are described broadly in various places such as "Transmission Circuits for Telephonic Communication", by K. S. Johnson, p. 121 et seq., where they are classified as mid-series and mid-shunt iterative impedances. These two forms are shown generically in Figs. 2 and 3 from which it is seen that Fig. 2 is made up of a succession of T structures and Fig. 3 of a succession of π structures. The extent to which such structures may be made to simulate a smooth line depends on the number of sections used but it is known that certain of these two distinct ladder-type structures have at some termination in each a characteristic impedance equal to that of a smooth line, such as an open wire or a cable signaling line, namely

$$K = \sqrt{\frac{R + i2\pi Lf}{G + i2\pi Cf}}, \quad (1)$$

and propagation constants, which can be varied over a wide range, and differing, in gen-

eral, from the propagation constant of the smooth line. In one this termination is at mid-series, in the other, it is at mid-shunt, so that mid-series and mid-shunt sections, respectively, have the characteristic impedance K. Where $R/L \geq G/C$, as in most telephone lines, the mid-series section has a series impedance of resistance and a shunt impedance of two distinct resistances and one capacity, as shown in Fig. 2^a. In the mid-shunt section the positions of these types of impedances are interchanged, as shown in Fig. 3^a. Where $R/L \leq G/C$, the sections have the same forms but with capacities replaced by inductances, as shown in Fig. 2^b which corresponds to Fig. 2^a.

As pointed out above, the degree of simulation obtainable in any case, depends on the number of sections, n , in such ladder structures and becomes exact for all frequencies only when n is infinity. However, a certain degree of simulation is obtained even with one or with two sections and in both of these cases a substitute and equivalent network can be found using a smaller number of resistance elements. The transformations for finding such substitute networks for a few simple cases are given in my article on "Theory and design of uniform and composite wave-filters", Bell System Technical Journal, January 1923, p. 45. As an illustration, let us consider first the important case where $R/L \geq G/C$. Take one section of either kind above and terminate one end by a resistance of any magnitude thus giving a two-terminal impedance network. By certain impedance transformations, this network, as far as its driving-point impedance is concerned, can be reduced to a minimum of three elements, two resistances and one capacity, having the form either of a resistance in parallel with one branch of series resistance and capacity, or a resistance in series with one branch of parallel resistance and capacity.

Next take two sections of either kind, or both, join them together, and then terminate by a resistance. This two-terminal network can be reduced, without altering its impedance, to one of five elements, three resistances and two capacities. One form it may have is a resistance in parallel with two branches of

series resistance and capacity, as shown in Fig. 4. Another form is that of a resistance in series with two branches of parallel resistance and capacity, as shown in Fig. 10. Figs. 5 to 9 are five other impedance forms equivalent to Fig. 4, and Figs. 11 to 15 are five equivalent to Fig. 10.

With three sections, the reduced network would have seven elements. The forms corresponding to the two above have three branches in each case. We may continue this to any number of sections and arrive at the following conclusion:

The driving-point impedance of n sections having the characteristic impedance K (mid-series, mid-shunt, or both,) and terminated by any resistance is identically that of a reduced network having $2n+1$ elements, $(n+1)$ resistances and n capacities. One form which these elements may have is a resistance in parallel with n branches of series resistance and capacity. Another form is that of a resistance in series with n branches of parallel resistance and capacity. For n greater than 1 there are other equivalent forms, the number increasing very rapidly with n .

For the case where $R/L \leq G/C$ the same general conclusions apply wherein all capacities are replaced by inductances.

In all these reduced networks the number of resistances has been diminished to a minimum, a fact of practical importance.

Since each section has attenuation, the larger the number of sections taken the less the effect due to the imperfect resistance termination and the more accurately does the driving-point impedance approach that of the desired characteristic impedance K . Hence, obviously, the more parameters in the reduced networks the better the possible impedance approximation.

The above was based upon the assumption that the line parameters R , L , G , and C are independent of frequency, as that is the basis of the ladder-type structures. However, the forms of the reduced networks would still be valid even when the line parameters varied somewhat with frequency, as they do. Such variations are no hindrance to the design method given below.

Method of design.

The impedance of networks will, in general, depend on the frequency and for any given network containing reactances may be expressed as a function of the frequency. Here the problem is to find proper values for the various elements making up the chosen

form of network in order that it shall approach as closely as possible the impedance of the smooth line or other electrical system to be simulated. To do this, the network elements will now be derived in terms of its impedances at arbitrary frequencies.

Consider, for brevity, only the case where $R/L \geq G/C$, as the other is similar, and for definiteness let the network structure be that of a resistance in parallel with n branches of series resistance and capacity. Its admittance, $1/z$, is readily seen to have the form

$$\frac{1}{z} = \frac{1}{a_0} + \frac{if b_1}{1 + if a_1} + \dots + \frac{if b_n}{1 + if a_n} \quad (2)$$

This gives an impedance expression of the form

$$z = \frac{A_0 + if A_1 + \dots + (if)^n A_n}{1 + if B_1 + \dots + (if)^n B_n} = r + ix \quad (3)$$

which must correspond also to any of the other equivalent structures. Here r and x are the components of the impedance to be simulated. In both Equations (2) and (3) the a 's and b 's and the A 's and B 's are constants for a given network (of which Fig. 4 would be an example with $n=2$); also, in these equations f is the frequency, an independent variable, and $i = \sqrt{-1}$. The impedance z , according to Equation (3), is the ratio of two polynomials of n th degree in (if) , with $2n+1$ coefficients which occur linearly.

The important observation to make here is that these coefficients, $A_0, \dots, B_n$, which are functions of the network elements, are exactly equal in number to the number, $2n+1$, of those elements. For example, in Fig. 4, the A 's and the B 's are functions of the R 's and the C 's and there are five A 's and B 's in Equation (3) when $n=2$ and there are five R 's and C 's in Fig. 4. Hence, in general, if these coefficients, the A 's and the B 's, can be determined, the network elements (for example, the R 's and C 's of Fig. 4) are fixed in terms of them.

The method of deriving these coefficients, and thus the network elements, directly from the network's impedances at arbitrary frequencies is as follows: First, clear Equation (3) of fractions and group separately only real and imaginary parts containing the coefficients in the left-hand member which must equal the right, $r + ix$. Assuming merely for convenience that n is even, the form of this equation will be

$$[A_0 - f^2 A_2 + \dots (if)^n A_n + f x B_1 + f^2 x B_2 + \dots - (if)^n x B_n] + i[f A_1 - \dots i^{n-2} f^{n-1} A_{n-1} - f x B_1 + f^2 x B_2 + \dots - (if)^n x B_n] = r + ix \quad (4)$$

Since the real and imaginary parts must separately satisfy the equation we have the equations (simultaneous at any frequency)

$$A_0 - f^2 A_2 + \dots - (if)^n r B_n = r, \quad f A_1 + \dots - (if)^n x B_n = x. \quad (5)$$

At any given frequency these are linear equations in the coefficients $A_0, \dots, B_n$.

Remembering that r and x are the resistance and reactance components of the impedance of the smooth line or other conductor system to be simulated, they will have definite values which may be ascertained (or computed) for any given value or values of the frequency f . Accordingly, fix r and x , as obtained by measurements or calculations, at a number of frequencies so as to obtain $2n+1$ consistent equations. We then have a system of simultaneous linear equations from which to obtain the coefficients by ordinary methods. From these coefficients the network elements can be found; they will be physical provided the network is capable of having the impedances assumed.

Other ways of setting up the system of equations would be to consider (5) as non-simultaneous equations or use their sum or difference. While such assumptions would lead to solutions for the coefficients there would be no guarantee that the resulting network would have any of the impedance components assumed. They might give good results, however.

The above mode of procedure for fitting a network to an impedance characteristic which it can have is new, but it is straightforward and one which is obviously valid whenever the number of independent coefficients of powers of (if) in the impedance expression equals the number of independent network elements. This condition is known to hold for a large variety of networks, but not for some others.

Derivation of formulae for 5-element networks.

The method just described can now be used to obtain design formulae and, as an illustra-

tion, they will be used for the 5-element structures of Figs. 4 to 15. These networks will be considered as sufficiently typical and are also practical. They correspond initially to two ladder-type sections terminated by a resistance.

The networks have an impedance expression given by

$$z = \frac{g + ifh - mf^2}{1 + ifn - sf^2} = r + ix. \quad (6)$$

Clearing fractions gives the simultaneous equations

$$\begin{aligned} g - f^2 m + fx \cdot n + f^2 r \cdot s &= r, \\ f \cdot h - fr \cdot n + f^2 x \cdot s &= x. \end{aligned} \quad (7)$$

Two sets of formulae will be derived for these coefficients; one containing the impedance at zero frequency, the other that at infinite frequency, in addition to the impedances at any two intermediate frequencies f_1 and f_2 .

Lower frequency range.

The data are

$$\begin{aligned} f_0 &= 0, \quad r_0 + i0, \\ f_1, \quad r_1 + i x_1, \\ f_2, \quad r_2 + i x_2. \end{aligned} \quad (8)$$

Substitution in (7) gives

$$\begin{aligned} g &= r_0, \\ g - f_1^2 m + f_1 x_1 \cdot n + f_1^2 r_1 \cdot s &= r_1, \\ g - f_2^2 m + f_2 x_2 \cdot n + f_2^2 r_2 \cdot s &= r_2, \\ f_1 \cdot h - f_1 r_1 \cdot n + f_1^2 x_1 \cdot s &= x_1, \\ f_2 \cdot h - f_2 r_2 \cdot n + f_2^2 x_2 \cdot s &= x_2. \end{aligned} \quad (9)$$

The solution of (9) may be put as

$$\begin{aligned} g &= r_0, \\ h &= r_1 n - f_1 x_1 s + x_1 / f_1, \\ m &= x_1 n / f_1 + r_1 s + (r_0 - r_1) / f_1^2, \\ n &= \frac{(f_2^2 - f_1^2) [f_1 x_1 (r_2 - r_0) - f_2 x_2 (r_1 - r_0)]}{D}, \\ s &= \frac{[f_1^2 (r_0 - r_2) - f_2^2 (r_0 - r_1)] (r_1 - r_2) + (f_1 x_2 - f_2 x_1)^2}{D}, \end{aligned} \quad (10)$$

where

$$D = f_1 f_2 (f_1 f_2 [(r_1 - r_2)^2 + x_1^2 + x_2^2] - (f_1^2 + f_2^2) x_1 x_2).$$

The coefficients g, h, m, n , and s are all positive in a physical structure,

Upper frequency range.

The data are

$$\begin{aligned} f_1, r_1 + ix_1, \\ f_2, r_2 + ix_2, \\ f_\infty = \infty, r_\infty + i \cdot 0. \end{aligned} \quad (11)$$

$$\begin{aligned} g &= r_1 - f_1 x_1 n - f_1^2 (r_1 - r_\infty) s, \\ h &= r_1 n - f_1 x_1 s + x_1 / f_1, \\ m &= r_\infty s, \\ n &= \frac{f_1 f_2 (f_1 x_1 - f_2 x_2) (r_1 - r_2) + (f_1 x_2 - f_2 x_1) [f_1^2 (r_1 - r_\infty) - f_2^2 (r_2 - r_\infty)]}{D}, \\ s &= \frac{f_1 f_2 (r_1 - r_2)^2 + (f_1 x_1 - f_2 x_2) (f_2 x_1 - f_1 x_2)}{D}, \end{aligned} \quad (13)$$

where

$$D = f_1 f_2 [(f_1^2 (r_1 - r_\infty) - f_2^2 (r_2 - r_\infty)) [r_1 - r_2] + (f_1 x_1 - f_2 x_2)^2).$$

The coefficients are all positive.

It should be pointed out that still a third set of formulæ like (10) and (13) could be obtained by taking as the third arbitrary frequency neither zero nor infinity and using but one of the two Equations (7), their sum, or their difference. Other combinations of linear equations may be used as the basis for an approximation to the given impedance data. For example, the two equations of (7) may be used a different number of times and even at different frequencies. Again, we might use at the lowest arbitrary frequency the difference of Equations (7), at the next higher frequency their sum, and so on alternately until 5 equations have been set up to include data at five arbitrary frequencies. The solutions for all such cases are obtained most readily direct from the linear equations chosen and will not be given here in explicit form.

Network elements.

Having determined the coefficients g , h , m , n , and s , the network elements are found from them. Here it will be sufficient to obtain such explicit formulæ only for Figs. 4, 10, 16 and 17 since when the elements of these two structures are known those of the other equivalent forms can be obtained if desired by an application of the equivalent network transformation formulæ of my article in the Bell System Journal mentioned above.

If one writes down the impedance of the network of Fig. 4 or Fig. 16 in terms of its R 's and C 's and simplifies the resulting expression and compares coefficients with Equation (6), the following relations will become apparent:

$$\begin{aligned} g &= R_1, \\ h &= 2\pi R_1 (R_2 C_4 + R_3 C_5), \\ m &= 4\pi^2 R_1 R_2 R_3 C_4 C_5, \\ n &= 2\pi [R_1 (C_4 + C_5) + R_2 C_4 + R_3 C_5], \\ s &= 4\pi^2 [R_1 (R_2 + R_3) + R_2 R_3] C_4 C_5. \end{aligned} \quad (14)$$

The simultaneous equations here are the same as those of (9) except for the replacement of the relation $g = r_0$ by

$$-m + r_\infty s = 0 \quad (12)$$

The solution for this case is

Whence,

$$\begin{aligned} R_1 &= g, \\ R_2 &= \frac{gm(u-v)}{[m-gs+2\pi(gn-h)u]v}, \\ R_3 &= \frac{gm(u-v)}{[gs-m-2\pi(gn-h)v]u}, \\ C_4 &= u/R_2, \\ C_5 &= v/R_3, \end{aligned} \quad (15)$$

where

$$\frac{u}{v} = (h \pm \sqrt{h^2 - 4gm}) / 4\pi g.$$

The network of Fig. 17 (or Fig. 10) gives

$$\begin{aligned} g &= R_6 + R_7 + R_8, \\ h &= 2\pi [R_6 (R_7 C_9 + R_8 C_{10}) + R_7 R_8 (C_9 + C_{10})], \\ m &= 4\pi^2 R_6 R_7 R_8 C_9 C_{10}, \\ n &= 2\pi (R_7 C_9 + R_8 C_{10}), \\ s &= 4\pi^2 R_7 R_8 C_9 C_{10}. \end{aligned} \quad (16)$$

Whence,

$$\begin{aligned} R_6 &= m/s, \\ R_7 &= [gt - h + m(n-t)/s] / (t-w), \\ R_8 &= [h - gw - m(n-w)/s] / (t-w), \\ C_9 &= t/2\pi R_7, \\ C_{10} &= w/2\pi R_8, \end{aligned} \quad (17)$$

where

$$\frac{t}{w} = (n \pm \sqrt{n^2 - 4s}) / 2.$$

Application of formulæ.

While these formulæ might be applied to any electrical system to be simulated I will, for illustrative purposes, apply them to a non-loaded 104 mil (No. 12-gauge) open wire telephone side circuit. The impedance data used will be the characteristic impedances of such a line at various frequencies as computed from

$$K = \sqrt{\frac{R + i2\pi Lf}{G + i2\pi Cf}}$$

They could as well have been obtained from measurements.

For the frequency range 0-5000 cycles/sec. at 20° C. the line parameters per loop mile are very accurately given by

$$\begin{aligned} R &= 10.12 + .004 (f/1000) + .032 (f/1000)^2 \text{ ohms,} \\ L &= .00366 \text{ henrys,} \\ G &= (3.20 + f/1000) 10^{-6} \text{ ohms,} \\ C &= .00837 \text{ mf.} \end{aligned} \quad (18)$$

The leakance, G , is for wet weather conditions.

whence

$$\begin{aligned} g &= 1587, & n &= 9,537.1 \cdot 10^{-6}, \\ h &= 7.5439, & s &= 10.117 \cdot 10^{-6}, \\ m &= .0066904. \end{aligned} \quad (22)$$

These give

$$\begin{aligned} R_1 &= 1587 \text{ ohms, } C_4 = .4688 \text{ mf.,} \\ R_2 &= 1213 \text{ ohms, } C_5 = .01088 \text{ mf.,} \\ R_3 &= 17260 \text{ ohms,} \end{aligned} \quad (23)$$

and

$$\begin{aligned} R_6 &= 661.3 \text{ ohms, } C_9 = 1.456 \text{ mf.,} \\ R_7 &= 909.9 \text{ ohms, } C_{10} = 12.32 \text{ mf.,} \\ R_8 &= 15.71 \text{ ohms.} \end{aligned} \quad (24)$$

Comparison of the impedances, z , of these two different range networks with that, K , of the smooth line are given in Tables I and II, and on Figs. 18 and 19. A column is added in the tables giving the percentage impedance-departure, P , as computed from

$$P = \left| \frac{z - K}{K} \right| 100 \quad (25)$$

Table I.—Lower frequency range.

Frequency cycles/sec.	Line impedance (ohms)	Network impedance (ohms)	Percentage impedance-departure
0	1778 -i 0	1778 -i 0	0
10	1760 -i123.1	1760 -i124.4	.074
20	1713 -i232.7	1713 -i232.7	0
40	1575 -i389.7	1578 -i385.1	.333
70	1359 -i485.6	1365 -i486.2	.418
100	1194 -i495.5	1194 -i495.6	0
200	917.5 -i405.3	918.2 -i374.7	3.05
500	727.6 -i218.1	779.7 -i175.5	8.86
1000	680.6 -i115.5	755.8 -i 90.0	11.50
2000	666.5 -i 56.8	749.7 -i 45.3	12.56
3000	663.7 -i 36.7	748.5 -i 30.2	12.80
5000	662.3 -i 21.2	747.9 -i 18.1	12.93
∞	661.3 -i 0	747.6 -i 0	13.05

Table II.—Upper frequency range.

Frequency cycles/sec.	Line impedance (ohms)	Network impedance (ohms)	Percentage impedance-departure
0	1778 -i 0	1587 -i 0	10.74
10	1760 -i123.1	1581 -i 75.4	10.50
20	1713 -i232.7	1562 -i147.7	10.02
40	1575 -i389.7	1490 -i273.4	8.67
70	1359 -i485.6	1350 -i397.0	6.14
100	1194 -i495.5	1214 -i449.2	3.90
200	917.5 -i405.3	917.5 -i405.3	0
500	727.6 -i218.1	722.5 -i213.7	.89
1000	680.6 -i115.5	680.6 -i115.5	0
2000	666.5 -i 56.8	666.9 -i 60.0	.49
3000	663.7 -i 36.7	663.9 -i 40.4	.55
5000	662.3 -i 21.2	662.2 -i 24.4	.48
∞	661.3 -i 0	661.3 -i 0	0

It will be seen that over the frequency ranges for which the networks are designed their impedance departures are quite small.

Note that the design is not limited to a case where R , L , G and C for the smooth line are constants but is applicable to any line or two-terminal electrical conductor system for which the impedance is a function of frequency and is approximately the same which it would be if those values were constants. In illustration of this statement it may be noticed that in the above Equations (18), R is not a constant but is a function of frequency; however, for frequencies not too great, R has nearly the constant value 10.12; similarly for G . In many cases of a conductor system the values of R , L , G and C themselves may not be known, as when the system is a composite line or contains intermediate apparatus; only the measured impedance may be known, and to proceed with the design of the network according to the present disclosure it would be necessary to know only the values of r and x for each of $n+1$ values of f according to the notation of this specification.

Assuming that it is desired to have a network which shall give particularly good simulation in the lower frequency range, the data in (8) will be taken as

$$\begin{aligned} f_0 &= 0, & r_0 &= 1778, \\ f_1 &= 20, & r_1 + ix_1 &= 1713 - i232.7, \\ f_2 &= 100, & r_2 + ix_2 &= 1194 - i495.5. \end{aligned}$$

giving in (10)

$$\begin{aligned} g &= 1778, & n &= .039514, \\ h &= 57.485, & s &= .30791 \cdot 10^{-3}, \\ m &= .2302. \end{aligned} \quad (19)$$

The network elements for Figs. 16 and 17 corresponding to these parameters are from (15) and (17), respectively,

$$\begin{aligned} R_1 &= 1778 \text{ ohms, } C_4 = .07860 \text{ mf.,} \\ R_2 &= 55,990 \text{ ohms, } C_5 = .5644 \text{ mf.,} \\ R_3 &= 1321 \text{ ohms,} \end{aligned} \quad (20)$$

and

$$\begin{aligned} R_6 &= 747.6 \text{ ohms, } C_9 = 47.13 \text{ mf.,} \\ R_7 &= 97.38 \text{ ohms, } C_{10} = 1,821 \text{ mf.,} \\ R_8 &= 933.0 \text{ ohms.} \end{aligned} \quad (21)$$

The capacities in Fig. 17 are considerably larger than those in Fig. 16.

For a network especially good in the upper frequency range we may take

$$\begin{aligned} f_1 &= 200, & r_1 + ix_1 &= 917.5 - i405.3, \\ f_2 &= 1000, & r_2 + ix_2 &= 680.6 - i115.5, \\ f_\infty &= \infty, & r_\infty &= 661.3, \end{aligned}$$

The departures of the networks designed for the upper frequency range might be further decreased at the intermediate frequencies by making slight changes in the data or frequencies used.

It is to be understood that the example given is for illustrative purposes only, and that if the requirements of simulation are more exacting, then networks of a larger number of elements would be used. However, I have shown how to find the values to be assigned to the $(2n+1)$ elements which is the smallest number of elements which can be associated together to be an impedance which is the equivalent at all frequencies of a ladder-type network of n sections with a terminating resistance, each of said sections comprising four elements as indicated in Figs. 2^a, 2^b or 3^a, and which simulates, with a high order of accuracy, the impedance characteristics of a smooth line. Furthermore, I have disclosed a method of design which enables one to design a network of a limited number of elements which shall have the least departures from a smooth line at those frequencies or range of frequencies where closest matching is desired.

An essential feature of my method resides in using a plurality of coefficients which are related in a complicated manner to the constants of the network to be designed, but which coefficients do not appear as powers or as products of each other. The method of treatment is such as to yield simultaneous equations which are linear in these coefficients, thus enabling these latter to be readily evaluated. Later, the constants themselves may be obtained from the coefficients.

As has been shown in this specification and in the drawings, there are numerous equivalent networks that can be designed all in accordance with the principles of my invention to simulate a given smooth line or other two-terminal conductor system. For example, the networks of Figs. 5 to 9 are all precisely equivalent to Fig. 4, and the networks of Figs. 11 to 15 are all precisely equivalent to Fig. 10. For the sake of clearness in this specification I mention and discuss Fig. 4 principally, with the understanding that it is representative of the whole class of networks that may be designed to simulate a given conductor system such as mentioned above.

What is claimed is:

1. A network of $n+1$ parallel branches between a pair of terminals, where n is a positive integer not less than 2, one such branch consisting of a resistance, and each remaining branch consisting of a resistance and a capacity in series, said resistances and capacities having values that make the impedance across said terminals precisely the same as for a given two-terminal conductor system at $n+1$ different pre-assigned frequencies, said conductor system having an impedance-

frequency characteristic similar to that of a smooth line, and said network having a close approximation to the same impedance-frequency characteristic over the frequency range within the extreme values of said $n+1$ different frequencies.

2. A network of three parallel branches between a pair of terminals, one such branch consisting of a resistance and each remaining branch consisting of a resistance and a capacity in series, said resistances and capacities having values that make the impedance across said terminals precisely the same as for a given two-terminal conductor system at three different pre-assigned frequencies, said conductor system having an impedance-frequency characteristic similar to that of a smooth line, and said network having a close approximation to the same impedance-frequency characteristic over the frequency range within the extreme values of said three frequencies.

3. A network of $n+1$ parallel branches between a pair of terminals, where n is a positive integer not less than 2, one such branch consisting of a resistance, and each remaining branch consisting of a resistance and a capacity in series, said resistances and capacities having values that make the impedance across said terminals precisely the same as for a given two-terminal conductor system at $n+1$ different pre-assigned frequencies, said conductor system having an impedance-frequency characteristic similar to that of a smooth line, and said network having a close approximation to the same impedance-frequency characteristic over the frequency range within the extreme values of said $n+1$ different frequencies, one of said $n+1$ different frequencies being at an extreme value on the positive number scale.

4. A network of $n+1$ parallel branches between a pair of terminals, where n is a positive integer not less than 2, one such branch consisting of a resistance, and each remaining branch consisting of a resistance and a capacity in series, said resistances and capacities having values that make the impedance across said terminals precisely the same at all frequencies as for a ladder-type recurrent network of n sections terminated by a resistance and with three resistances and one capacity per section, such ladder-type network when its sections are repeated indefinitely being precisely equivalent to a given two-terminal conductor system at $n+1$ different pre-assigned frequencies, said conductor system having an impedance-frequency characteristic similar to that of a smooth line.

5. A network of $n+1$ parallel branches between a pair of terminals, where n is a positive integer not less than 2, one such branch consisting of a resistance, and each remaining branch consisting of a resistance and a capacity in series, said resistances and capacities

having values typified by Equations (15) for the case of $n=2$.

series, the values of said R's and C's being in accordance with Equations (15). 25

6. A network of $n+1$ resistances and n like reactance elements between a pair of terminals, where n is a positive integer not less than 2, said resistances and reactance elements having values that make the impedance across said terminals precisely the same as for a given two-terminal conductor system at $n+1$ different pre-assigned frequencies, said conductor system having an impedance-frequency characteristic similar to that of a smooth line, and said network having a close approximation to the same impedance-frequency characteristic over the frequency range within the extreme values of said $n+1$ different frequencies. 30

8. A network of three parallel branches between a pair of terminals, one branch consisting of a resistance R_1 , another branch consisting of a resistance R_2 and capacity C_4 in series, and the remaining branch consisting of a resistance R_3 and a capacity C_5 in series, the values of said R's and C's being in accordance with Equations (15) expressed in terms of g, h, m, n and s and these being in accordance with Equations (10) expressed in terms of sets of quantities f, r and x , where each f represents a given frequency and each r represents a corresponding resistance and each x represents a corresponding reactance, these f 's, r 's and x 's being values for a two-terminal conductor system similar to a smooth line. 35 40

7. A network of three parallel branches between a pair of terminals, one branch consisting of a resistance R_1 , another branch consisting of a resistance R_2 and capacity C_4 in series, and the remaining branch consisting of a resistance R_3 and a capacity C_5 in 20

In testimony whereof, I have signed my name to this specification this 7th day of September, 1926.

OTTO J. ZOBEL.