July 16, 1929.

J. S. STONE

1,720,770

ASSOCIATED RESONANT CIRCUITS

Filed July 11, 1924

2 Sheets-Sheet

Fig. 1

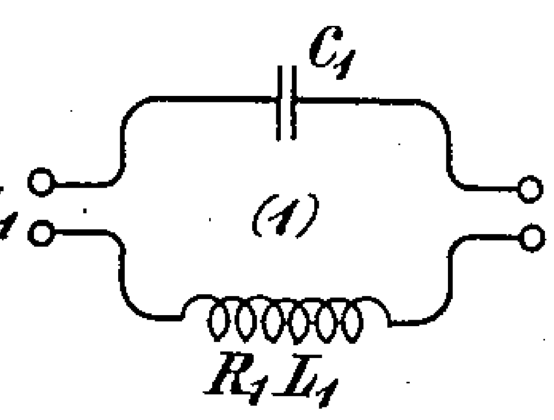


Fig. 2

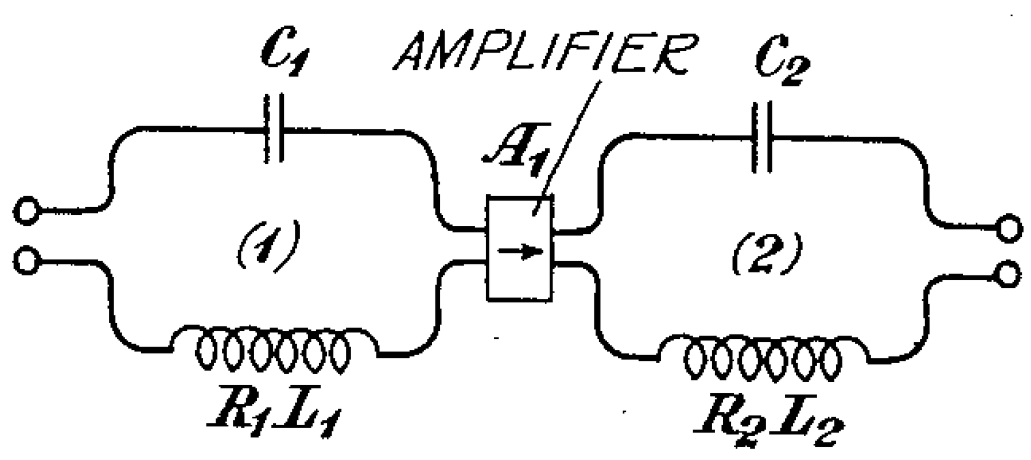


Fig. 3

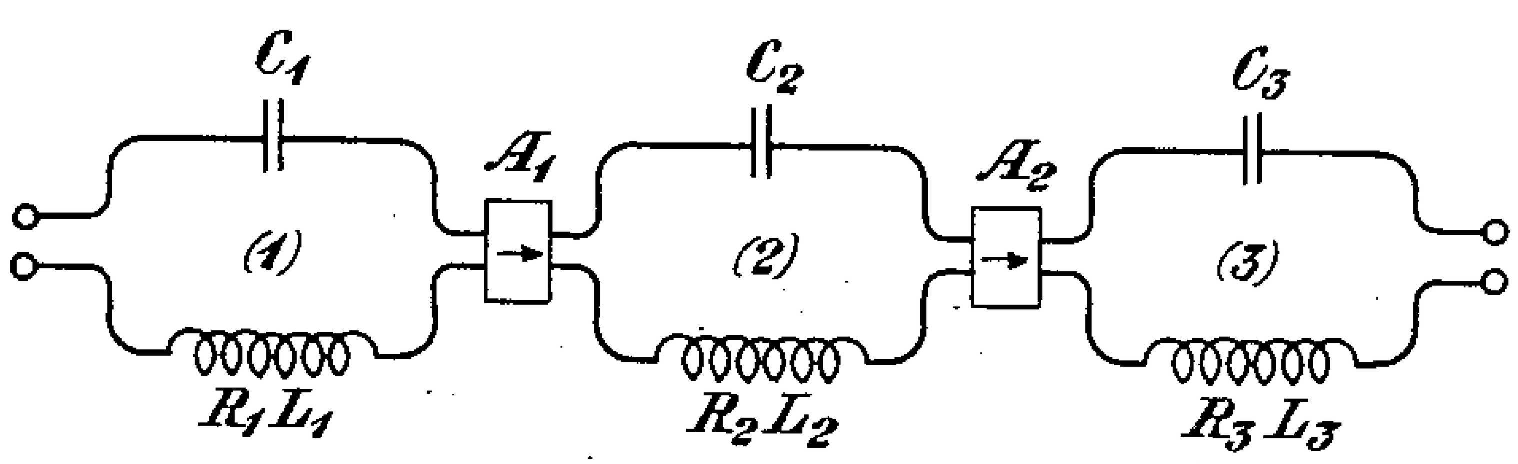


Fig. 4

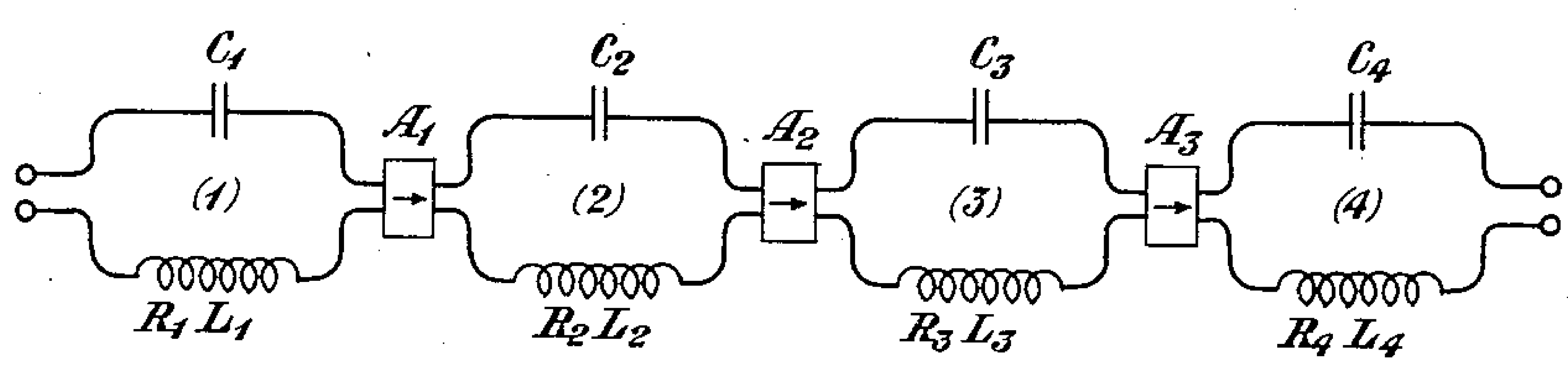


Fig. 5

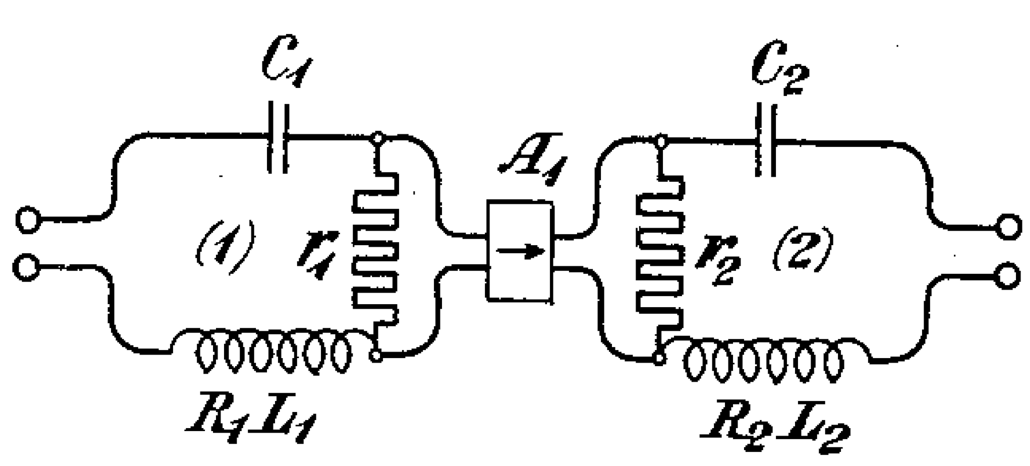


Fig. 6

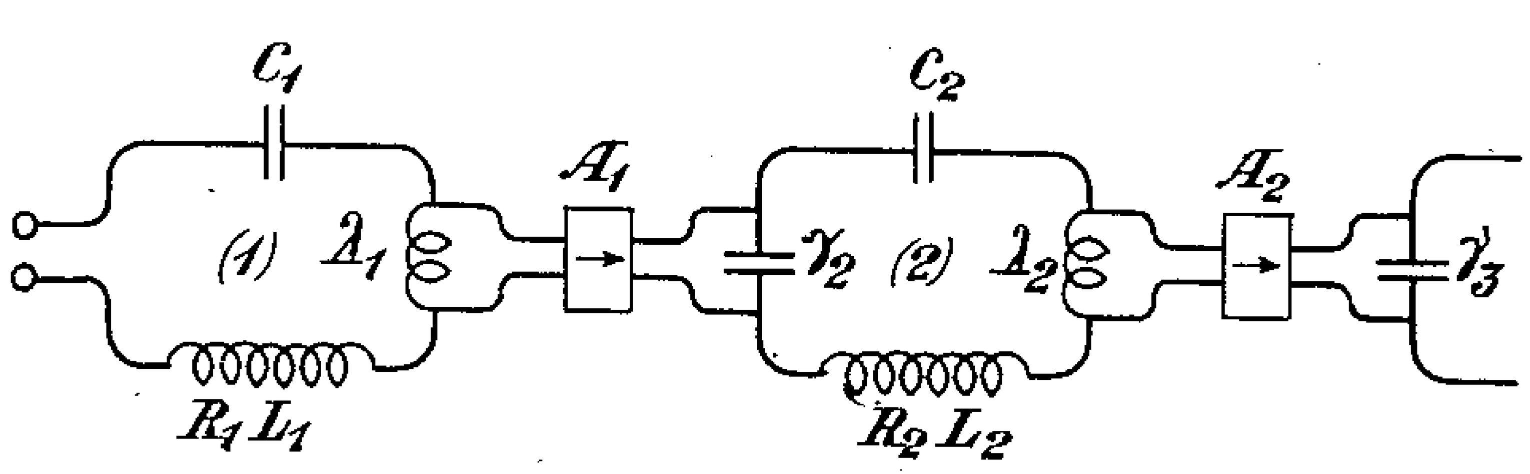
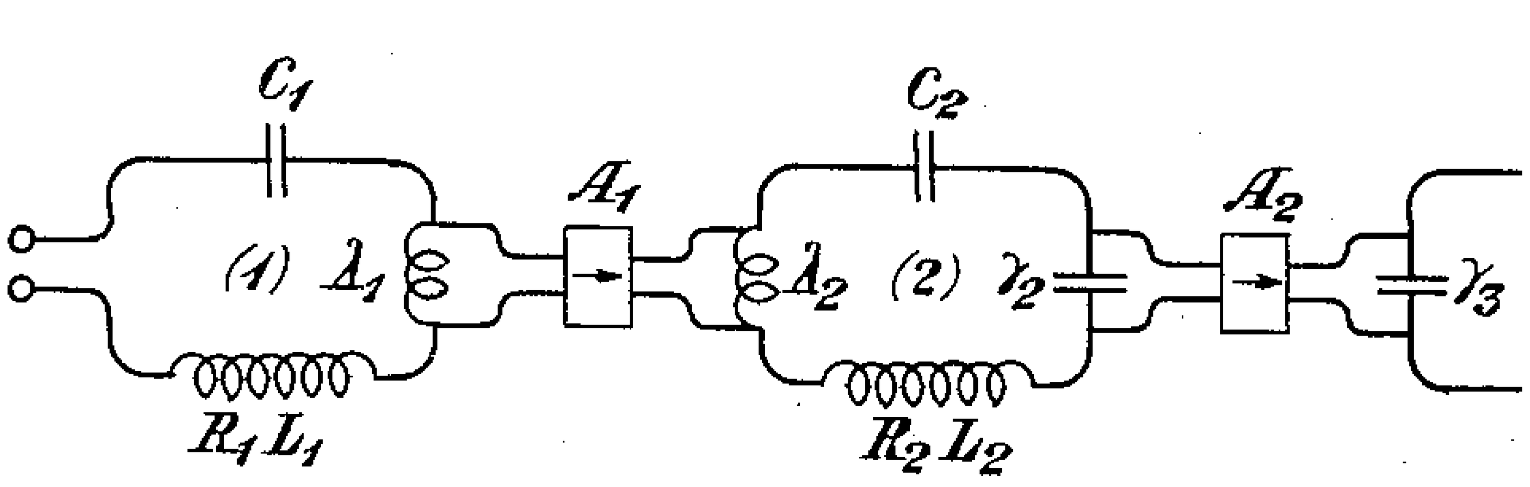


Fig. 7



INVENTOR

John Stone Stone

BY

[illegible]

ATTORNEY

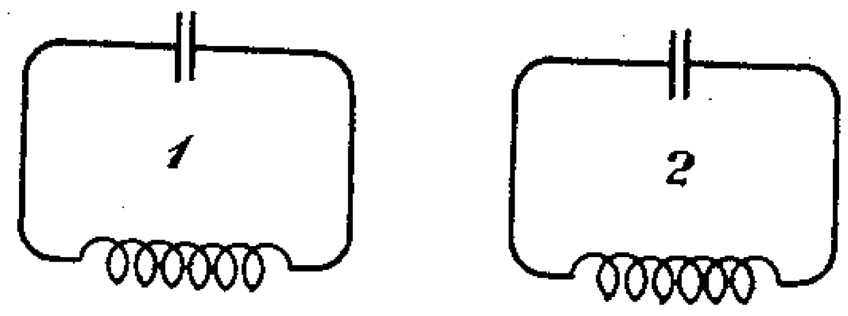


Fig. 9

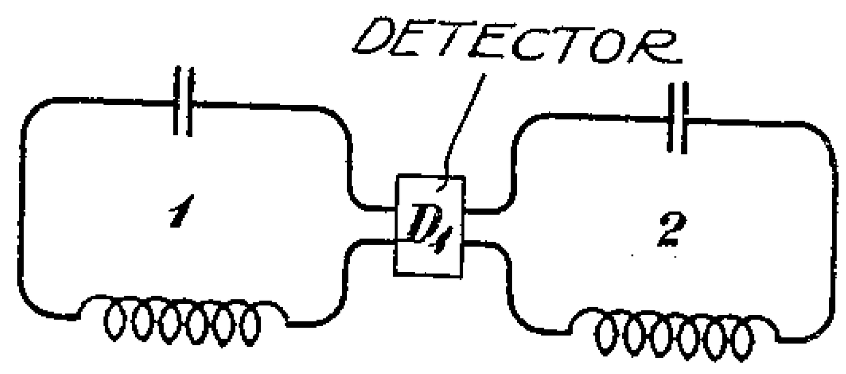


Fig. 10

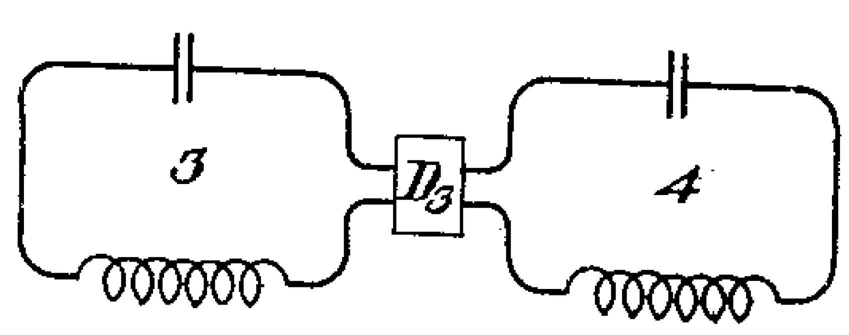


Fig. 11

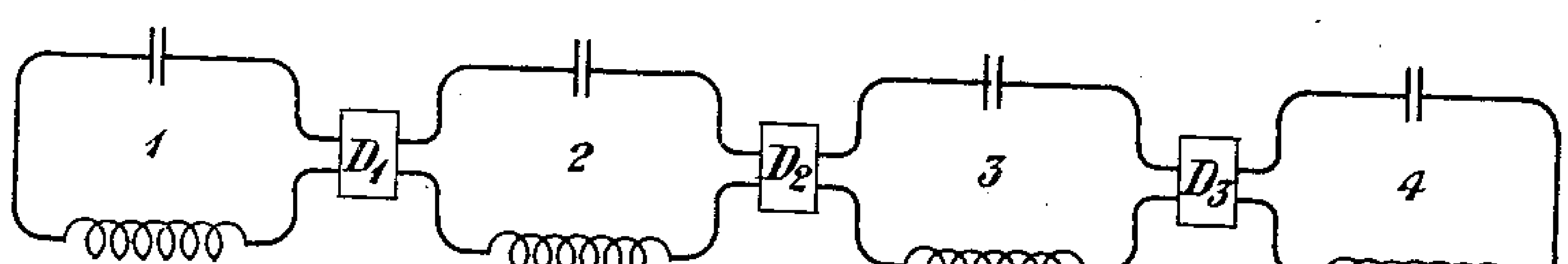


Fig. 12

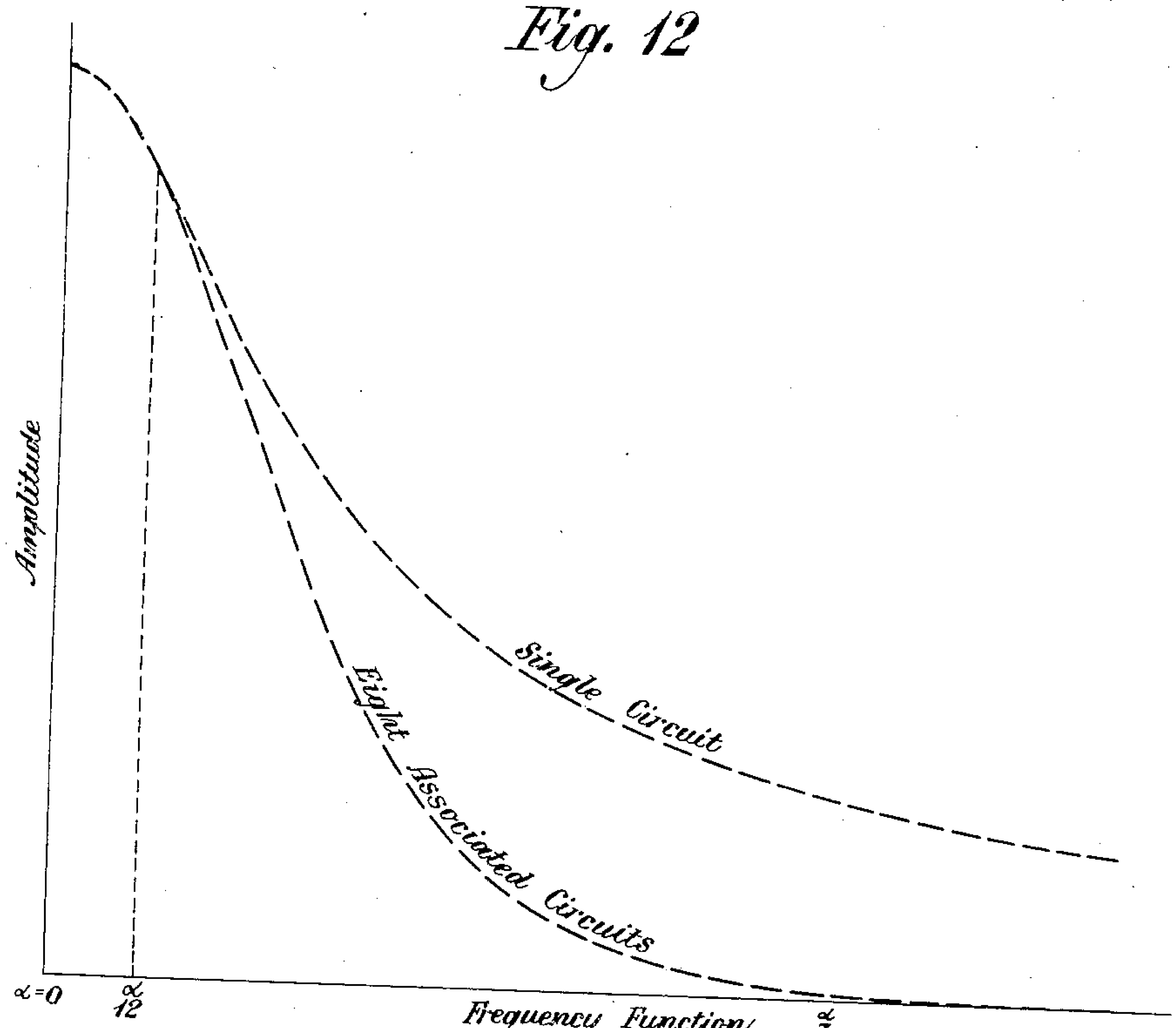


Fig. 8

INVENTOR

John Stone Stone

BY

G E Folk

ATTORNEY

Patented July 16, 1929. **1,720,770**

UNITED STATES PATENT OFFICE.

JOHN STONE STONE, OF SAN DIEGO, CALIFORNIA, ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

ASSOCIATED RESONANT CIRCUITS.

Application filed July 11, 1924. Serial No. 725,488.

An object of my invention is to provide new and improved apparatus and method for the reception of currents of a determinate frequency range or band, with only a determinate amount of distortion within that range and the practical exclusion of currents lying outside that range. Another object of my invention is to combine resonant circuits and amplifiers so as to secure sharp selectivity for a desired frequency band of substantial width with but a small determinate amount of distortion within that band. These and various other objects of my invention will be made apparent in the following specification and claims, taken with the accompanying drawings, in which I have illustrated a limited number of specific embodiments of the invention. It will be understood that the invention is defined in the appended claims and that the following description relates more particularly to these examples of the invention which are shown by way of illustration.

Referring to the drawings, Figure 1 is a diagram of a resonant circuit; Figs. 2, 3, 4, 5, 6 and 7 are respective diagrams of resonant circuits designed and associated in accordance with the principles of my invention; Fig. 8 is a curve diagram illustrating the results obtained with eight associated circuits as compared with a single circuit; and Figs. 9 to 12 are further diagrams of resonant circuits designed and associated according to a modified embodiment of my invention.

Referring to Fig. 1, this represents a simple circuit with capacity C_1, inductance L_1 and resistance R_1. Let a simple harmonic electromotive force of amplitude E_1 and frequency f be applied to this circuit. Then the amplitude of the resulting current is well known to be given by the equation

$$I_1=\frac{E_1}{\sqrt{R_1^2+\left(L_1\omega-\frac{1}{C_1\omega}\right)^2}} \quad (1)$$

where $\omega=2\pi f$. The resonance frequency f_0 is given by

$$(2\pi f_0)^2=\omega_0^2=\frac{1}{C_1L_1}. \quad (2)$$

From the foregoing it can readily be shown that

$$I_1=\frac{E_1}{R_1\sqrt{1+\frac{L_1}{C_1R_1^2}\left(\frac{f^2-f_0^2}{ff_0}\right)^2}}. \quad (3)$$

Define "selectance" S by the equation

$$S^2=\frac{L_1}{C_1R_1^2} \quad (3^a)$$

and define the "frequency function" α by the equation

$$\alpha=\frac{f^2-f_0^2}{ff_0}. \quad (3^b)$$

Then Equation (3) can be expressed

$$I_1=\frac{E_1}{R_1\sqrt{1+\alpha^2S^2}}. \quad (4)$$

and at resonance $I_1=\frac{E_1}{R_1}$.

As will be seen from the discussion that follows, there is an advantage in using the frequency function α instead of the frequency f as the independent variable in connection with problems involving resonant circuits. The frequency function α is obviously zero for resonance, that is for $f=f_0$; and it is $-\infty$ for $f=0$, and $+\infty$ for $f=\infty$. The resonance curve with respect to α is symmetrical about its maximum ordinate, whereas the resonance curve with respect to f is unsymmetrical about its maximum ordinate.

Let the two frequencies f_1 and f_2 be assigned, for which the circuit of Fig. 1 is to be designed to give the same amplitude of current I_1 for the same amplitude of impressed electromotive force E_1. It follows at once that $\alpha_2^2=\alpha_1^2$ or $\alpha_2=\pm\alpha_1$, whence it is readily deduced that

$$f_0=\sqrt{f_1f_2} \quad (4^a)$$

and

$$\alpha_1=-\alpha_2=\alpha_{12}=\frac{f_1-f_2}{\sqrt{f_1f_2}}. \quad (5)$$

Referring to the frequency range from f_1 to f_2 as a frequency band, it will be seen that the resonance frequency is a mean proportional between the terminal frequencies, and it will here be termed the medial frequency of the band. The function α_{12} will here be termed the frequency function of this band.

Let it be required that for a constant amplitude of impressed electromotive force E_1, the amplitude of the current I_1 at the terminal frequencies f_1 and f_2 shall be $\frac{1}{m}$ of the amplitude at resonance. Accordingly, it follows from Equation (4) that

$$\frac{E_1}{R_1}\cdot\frac{1}{\sqrt{1+\alpha_{12}{}^2S^2}}=\frac{1}{m}\frac{E_1}{R_1}. \quad (6)$$

From this it follows that

$$S^2=\frac{m^2-1}{\alpha_{12}{}^2}=\frac{(m^2-1)f_1f_2}{(f_1-f_2)^2}=\frac{(m^2-1)f_0{}^2}{(f_1-f_2)^2} \quad (7)$$

and

$$S=\sqrt{m^2-1}/\alpha_{12}. \quad (8)$$

Substituting from Equations (2) and (3^a) in Equation (7) and solving, the result is obtained that

$$L_1=\frac{\sqrt{m^2-1}}{2\pi(f_1-f_2)}R_1. \quad (9)$$

Substituting this value of L_1 in Equation (2) and, also substituting from Equation (4^a),

$$C_1=\frac{f_1-f_2}{2\pi f_1f_2\sqrt{m_1{}^2-1}}\,\frac{1}{R_1}. \quad (10)$$

It will be seen that C_1 and L_1 of Fig. 1 are determined by the foregoing Equations (9) and (10) in terms of R_1 and the terminal frequencies f_1 and f_2 of the frequency band for which the currents are equal at the ends of the band, also in terms of m, the assigned factor giving the ratio of the current at resonance frequency f_0 to the currents at the terminal frequencies f_1 or f_2.

We now proceed to inquire what are the frequencies f_3 (greater than f_1 and f'_3 (less than f_2) at which, for a given amplitude of impressed electromotive force E_1, the amplitude of the currents will be equal and each will be $\frac{1}{p}$ times the amplitude of the current at resonance for the same electromotive force E_1. The condition is expressed thus:

$$I'_3=\frac{E_1}{R_1}\,\frac{1}{\sqrt{1+\alpha_3{}^2S^2}}=\frac{1}{p}\frac{E_1}{R_1}.$$

Solving this equation, the result is readily obtained that

$$\alpha_3=\pm\frac{\sqrt{p^2-1}}{S} \quad (11)$$

but from Equation (5)

$$\alpha_3=\frac{f_3{}^2-f_0{}^2}{f_3f_0}\text{ and }-\alpha_3=\frac{f'_3{}^2-f_0{}^2}{f'_3f_0}.$$

From these equations and from Equation (11) the solutions for f_3 and f'_3 are obtained as follows:

$$\left.\begin{aligned}f_3&=\frac{\sqrt{p^2-1}}{2S}\left(\sqrt{1+\frac{4S^2}{p^2-1}}+1\right)f_0\\ f'_3&=\frac{\sqrt{p^2-1}}{2S}\left(\sqrt{1+\frac{4S^2}{p^2-1}}-1\right)f_0\end{aligned}\right\} \quad (12)$$

As a numerical example to illustrate the foregoing, consider a band of frequencies from $f_1=102{,}000$ to $f_2=98{,}000$. We may consider that we are dealing with a carrier current whose frequency is 100,000 and that it is modulated by current of frequency from zero to 2,000. The resulting frequencies will lie within the band range that has been assigned. The frequency f_0, the medial frequency of the band, to which the circuit must be made resonant in order that it shall be equally responsive at the terminal frequencies f_1 and f_2 is $\sqrt{f_1f_2}=99{,}980$. In accordance with Equation (5) the value of α_{12} for this band is 0.040,008.

Let it be required that this band of frequencies be received with so little distortion that for equal impressed electromotive forces, the amplitude of the current at the terminal frequencies f_1 and f_2 shall be $\frac{9}{10}$ of the amplitude at the resonance frequency f_0. Then $m=\frac{10}{9}$ and from Equations (8), (9) and (10), $S=12.106$, $L_1=1.927\times10^{-5}R_1$ and

$$C_1=1.315\times10^{-7}\frac{1}{R_1}.$$

With the foregoing illustration it will be seen that we have now provided an analysis which gives the conditions for the reception by a simple resonant circuit of a band of frequencies with a determinate amount of distortion within the band. We have also provided (in connection with the values f_3 and f'_3) for determining the degree of exclusion of frequencies differing by a determinate amount from the medial frequency of the band.

Let $p=100$. This means that f_3 and f'_3 of Equations (12) will give us the frequencies for which, with the same impressed electromotive force, the amplitude of the current will be but 1% of that at the resonance frequency f_0. Substituting in Equations (12) and evaluating, the results are obtained that $f_3=838{,}000$ and $f'_3=11{,}900$.

With the single circuit of Fig. 1, the requirement for distortion to be restricted within the ratio $1:\frac{10}{9}$ results in small selectivity. The example that has just been worked out corresponds to a simple telephone receiving circuit when the carrier frequency is 100,000, when both side bands are transmitted and when the distortion is limited to a 10% discrimination against the higher voice frequencies within the useful range of 0 to 2,000 cycles. The corresponding low selectivity is evidenced by the wide distance along the frequency range of the values of f_3 and f'_3. The circuit considered may be said to be practically unscreened from interference.

Consider next the case of Fig. 2. Here are two resonant circuits 1 and 2, each having the same selectance S and each tuned to the frequency f_0. It should be noted that the condition of equal selectance S for the two circuits does not imply that R_2, L_2 and C_2 are respectively equal to R_1, L_1 and C_1. A_1 is a relay or amplifier of such character and so connected between the two circuits that the amplitude of the impressed electromotive force E_2 in circuit 2 is proportional only to the amplitude of the current I_1 in circuit 1. That is, $E_2=K_1 I_1$, where K_1 is a constant independent of frequency. Accordingly, when a simple harmonic electromotive force of amplitude E_1 and frequency f is impressed on the first circuit, the amplitude of the current in the second circuit will be (by Equation (4))

$$I_2=\frac{K_1 I_1}{R_2}\frac{1}{\sqrt{1+\alpha^2 S^2}}=\frac{K_1 E_1}{R_1 R_2}\frac{1}{1+\alpha^2 S^2} \quad (13)$$

where R_2 is the resistance of the second circuit.

The idea of Fig. 3 will readily be apparent. Circuits 1 and 2 and their connection are the same as for Fig. 2. The third circuit 3 has the same selectance S and is tuned to the same frequency f_0 and the relay A_2 is of such character and so connected between the circuits 2 and 3 that the amplitude of the electromotive force E_3 in circuit 3 is proportional only to the amplitude of the current flowing in circuit 2, that is

$$E_3=K_2 I_2$$

and

$$I_3=\frac{K_2 I_2}{R_3}\frac{1}{\sqrt{1+\alpha^2 S^2}}=\frac{K_1 K_2 E_1}{R_1 R_2 R_3}\frac{1}{(1+\alpha^2 S^2)^{3/2}} \quad (14)$$

where R_3 is the resistance of circuit 3. Similarly for Fig. 4,

$$I_4=\frac{K_3 I_3}{R_4}\frac{1}{\sqrt{1+\alpha^2 S^2}}=\frac{K_1 K_2 K_3 E_1}{R_1 R_2 R_3 R_4}\frac{1}{(1+\alpha^2 S^2)^2} \quad (15)$$

where R_4 is the resistance of circuit 4.

In general, let there be n circuits each tuned to the common frequency f_0 and each having the selectance S. Let each of these circuits except the first be so coupled to its antecedent circuit in order through a relay that the amplitude of the electromotive force impressed upon it shall be proportional only to the amplitude of the current flowing in its antecedent circuit. Then the amplitude of the current in the nth or last circuit resulting from an impressed simple harmonic electromotive force of amplitude E_1 and frequency f in the first circuit will be

$$I_n=\frac{K_1 K_2 K_3 \ldots\ldots K_{n-1} E_1}{R_1 R_2 R_3 \ldots\ldots\ldots R_n}\frac{1}{(1+\alpha^2 S^2)^{n/2}} \quad (16)$$

where R_j is the resistance of the jth circuit. The number n may be the number of any intermediate circuit or it may be the number of the last circuit of the sequence. Whatever value is given to n within the total number of circuits in the sequence, the foregoing Equation (16) gives the current in the correspondingly numbered circuit of the sequence.

Let the band of frequencies to be received extend from f_1 to f_2 as in the case already considered for the single circuit of Fig. 1. For the same amplitude of impressed electromotive force in circuit 1, let the amplitude of the current in circuit n be the same for the extreme frequencies f_1 and f_2. The medial frequency of the band, the resonance frequency, will be

$$f_0=\sqrt{f_1 f_2}$$

and the frequency function for the band is

$$\alpha_{12}=\frac{f_1-f_2}{\sqrt{f_1 f_2}} \quad (16^a)$$

Also as before, the carrier frequency is assumed to be

$$f_c=\frac{f_1+f_2}{2}.$$

Represent the expression

$$\frac{K_1 K_2 K_3 \ldots\ldots K_{n-1} E_1}{R_1 R_2 R_3 \ldots\ldots\ldots R_n}$$

in Equation (16) by F. When the frequency is f_0, $\alpha=0$ and Equation (16) becomes $I_n(f_0)=F$. As before, let the current at f_1 and f_2 be $\frac{1}{m}$ of the current at f_0. Then

$$I_n(f_1 \text{ or } f_2)=F\frac{1}{(1+\alpha^2 S^2)^{n/2}}=\frac{1}{m}F$$

whence $m=(1+\alpha^2 S^2)^{n/2}$

From Equations (3^a), (16^a), (2) and (4^a), the result is obtained that

$$L_n=\frac{\sqrt{m^{2/n}-1}}{2\pi(f_1-f_2)}R_n \quad (17)$$

and from this by Equation (2)

$$C_n = \frac{f_1 - f_2}{2\pi f_1 f_2 \sqrt{m^{2l^n}-1} R_n}. \quad (18)$$

From (17), (18) and (3^a)

$$S_n^2 = \frac{m^{2l^n}-1}{\alpha_{12}^2}. \quad (18^a)$$

The resistances R of the circuits in sequence are the resistances which reside in the inductances L as indicated in the drawings, but the inductances L and the capacities C have progressively different values as given by Equations (17) and (18). That is, an inspection of these equations shows that in the successive circuits, the inductance L is given by giving successive integer values to n in Equation (17). Similarly, the capacity C in each circuit is found by giving a proper value to the number n in Equation (18). Thus it will be seen that the inductances and capacities of the successive circuits are progressively different from circuit to circuit.

By a procedure parallel to that for Equations (12) it is deduced that

$$f_3 = \frac{\sqrt{p^{2l^n}-1}}{2S}\left(\sqrt{1+\frac{4S^2}{p^{2l^n}-1}}+1\right)f_0 \quad (19)$$

$$f'_3 = \frac{\sqrt{p^{2l^n}-1}}{2S}\left(\sqrt{1+\frac{4S^2}{p^{2l^n}-1}}-1\right)f_0 \quad (20)$$

To illustrate the principles that have been worked out in the foregoing discussion, consider the construction of a system of eight connected circuits each tuned to the common frequency f_0, each having the common selectance S, and between each pair of successive members a relay so designed and so connected that the amplitude of the electromotive force impressed upon the output circuit shall be proportional only to the amplitude of the current in the input circuit. Let this system of associated resonant circuits be required to pass a band of frequencies between $f_1=102{,}000$ and $f_2=98{,}000$, with a limit of 10% distortion as for the case of the single circuit of Fig. 1 already considered. As in that case, the carrier frequency is $f_c=100{,}000$, the medial or resonance frequency is $f_0=99{,}980$ and the frequency function for the band is $\alpha_{12}=0{,}040{,}008$. However, in this case the ultimate value of S is no longer the same as for Fig. 1 but is now given by substituting in Equation (18^a) with result as follows:

$$S = \frac{\sqrt{\sqrt[4]{\frac{10}{9}}-1}}{0.040{,}008} = 4.08$$

Also from Equations (17) and (18)

$$L_8 = 6.50 \times 10^{-6} R_8$$

and

$$C_8 = \frac{3.89 \times 10^{-7}}{R_8}$$

Similarly, the values for L_1, C_1, L_2, C_2, etc. in terms of the respective resistances R_1, R_2, etc. can be determined by application of Equation (18^a).

For the eight associated resonant circuits now under consideration we have determined a design and fixed values for the inductances and capacities in terms of the respective resistances. We proceed to inquire what are the frequencies f_3 and f'_3 for which the amplitude of the current I_8 is 1% of the resonance amplitude. From Equations (19) and (20) the following values are obtained:

$$f_3 = \left(\sqrt{1+\frac{\alpha_3^2}{4}}+\frac{\alpha_3}{2}\right)f_0 = 121{,}000 \quad (21)$$

$$f'_3 = \left(\sqrt{1+\frac{\alpha_3^2}{4}}-\frac{\alpha_3}{2}\right)f_0 = 85{,}000 \quad (22)$$

A comparison of the values of f_3 and f'_3 that were obtained for the single circuit of Fig. 1 with the values for the eight associated circuits now considered is given below, the band width being the same in both cases.

Eight circuit filter.	Band.	Single circuit.
$f_3=121{,}000$	f_1 102,000	$f_3=838{,}000$
	f_c 100,000	
$f'_3=$ 85,000	f_2 98,000	$f'_3=$ 11,900

This comparison is also exhibited in Fig. 8.

In all the illustrations that have been considered heretofore, we have assumed a carrier frequency of 100,000. It will be instructive now to assume a carrier frequency of 1,000,000 with the same band width as before, that is, the band extends from $f_1=1{,}002{,}000$ to $f_2=998{,}000$. As before, a distortion of 10% only is permitted and a system of eight associated resonant circuits is compared with a single circuit. The results are given in the following table:

Eight circuit filter.	Band.	Single circuit.
$f_3=1{,}026{,}000$	f_1 1,002,000	$f_3=1{,}494{,}000$
	f_c 1,000,000	
$f'_3=$ 974,500	f_2 998,000	$f'_3=$ 669,000

Comparing the two tables, it will be seen that the ability of the system of associated resonant circuits to exclude frequencies lying outside the band is greater, the higher the carrier frequency; for this reason a larger number of elements would need to be employed for low carrier frequencies than for high carrier frequencies.

The formula for the number of simple circuits required to reduce the amplitude of the current to the fraction $\frac{1}{p}$ of its resonance value for any particular frequency is deduced from Equation (16); this gives

$$(1+\alpha_3^2S^2)^{n/2}=p,$$

whence

$$n=\frac{1}{\log_p\sqrt{1+\alpha_3^2S^2}}=\frac{1}{\log_{10}\sqrt{1+\alpha_3^2S^2}\log_p 10} \quad (23)$$

In practice the next larger whole number than n of the foregoing equation will be taken.

It will be seen that for a preassigned band of frequencies and a preassigned degree of permissible distortion, we have (*a*) tuned all the component circuits to the same frequency, namely the medial frequency of the band, (*b*) made the selectance the same in all the component circuits, namely S, but while S is a function of capacity (C), inductance (L) and resistance (R), these values are not the same in all the component circuits, and (*c*) we have so associated each component circuit with its antecedent circuit that the amplitude of the electromotive force impressed upon it is proportional only to the amplitude of the current flowing in said antecedent circuit. In a system of associated resonant circuits in which the band and the degree of distortion have been rendered determinate as aforesaid, we have secured a determinate degree of exclusion of all frequencies greater than f_3 and less than f'_3 by (*d*) employing a determinate number (n) of such elements.

It now remains to show how the required coupling between the successive circuits can be effected.

One method is illustrated in Fig. 5 in which each resonant circuit is coupled with its antecedent circuit through a relay whose input is shunted across a resistance in the antecedent circuit and whose output is shunted across a resistance in the consequent circuit.

Another method of coupling the circuits is illustrated in Fig. 6. Each resonant circuit is coupled with its antecedent circuit through a relay whose input is shunted across a portion of the reactance of the antecedent circuit and whose output is shunted across a portion of the reactance of the consequent circuit. The reactances shunted in this way are different from each other for the reason that otherwise the amplitude of the electromotive force impressed upon any given circuit of the combination would not be solely proportional to the amplitude flowing in the antecedent circuit but would be a function of the frequency of the current in that antecedent circuit as well.

Fig. 7 shows still another way of coupling the circuits through the relays. In this case the amplitude of the electromotive force impressed on any given resonant circuit is not strictly proportional to the amplitude of the current in the antecedent circuit but the departure is compensated by alternating the character of the reactances which are shunted so that the amplitude of the impressed electromotive force in the last resonant circuit will be proportional to the amplitude of the current in the first circuit.

I will now disclose another embodiment of my invention, and for this purpose I will make reference to Figs. 9, 10, 11 and 12.

Let the resistance, inductance, capacity and selectance of circuit 1 be R_1, L_1, C_1 and S_1, respectively, and let the corresponding constants of circuit 2 be R_2, L_2, C_2 and S_2 respectively. Let circuits 1 and 2 be resonant to frequencies f_{01} and f_{02} respectively. On circuit 1 let there be impressed a simple harmonic E. M. F. of amplitude E_1 and frequency f, while on circuit 2, a simple harmonic E. M. F. of amplitude E_2 and frequency f is impressed.

The amplitudes of the resulting currents in these two circuits will be

$$I_1=\frac{E_1}{R_1\sqrt{1+\alpha_1^2S_1^2}} \quad (24)$$

and

$$I_2=\frac{E_2}{R_2\sqrt{1+\alpha_2^2S_2^2}} \quad (25)$$

respectively, where α_1 and α_2 are the frequency functions of circuits 1 and 2 respectively, these functions being defined as

$$\alpha_1=\frac{f_1^2-f_{01}^2}{f_1f_{01}} \quad (26)$$

and

$$\alpha_2=\frac{f_2^2-f_{02}^2}{f_2f_{02}} \quad (27)$$

Let it be required that, for equal amplitudes of impressed force E_1, the amplitude I_1 of the current in circuit 1 shall be the same for frequencies $f=f_{c1}\pm f_t$, and that for equal amplitudes of impressed force E_2, the amplitude I_2 of the current in circuit 2 shall be the same for frequencies $f=f_{c2}\pm f_t$. Let

$$\alpha_{c1\pm t}=\frac{(f_{c1}\pm f_t)^2-f_{01}^2}{(f_{c1}\pm f_t)f_{01}} \quad (28)$$

and

$$\alpha_{c2\pm t}=\frac{(f_{c2}\pm f_t)^2-f_{02}^2}{(f_{c2}\pm f_t)f_{02}} \quad (29)$$

Then clearly from (24) and (25), we have

$$-\frac{(f_{c1}+f_t)^2-f_{01}^2}{(f_{c1}+f_t)f_{01}}=\frac{(f_{c1}-f_t)^2-f_{01}^2}{(f_{c1}-f_t)f_{01}} \quad (30)$$

and

$$-\frac{(f_{c2}+f_t)^2-f_{02}^2}{(f_{c2}+f_t)f_{02}}=\frac{(f_{c2}-f_t)^2-f_{02}^2}{(f_{c2}-f_t)f_{02}} \quad (31)$$

hence

$$f_{01}=\sqrt{(f_{c1}+f_t)(f_{c1}-f_t)}=\sqrt{f_{c1}^2-f_t^2} \quad (32)$$

$$f_{02}=\sqrt{(f_{c2}+f_t)(f_{c2}-f_t)}=\sqrt{f_{c2}^2-f_t^2} \quad (33)$$

$$\alpha_{c1\pm t}=\mp 2\frac{f_t}{f_{01}} \quad (34)$$

and

$$\alpha_{c2\pm t}=\mp 2\frac{f_t}{f_{02}} \quad (35)$$

If $f_{c_1}\pm f_t$ are the extreme frequencies of a radio telephone band, and $f_{c_2}\pm f_t$ are the extreme frequencies of a second telephone band, then f_{c_1} and f_{c_2} are the carrier frequencies of the bands and f_{01} and f_{02} are the medial frequencies of these bands. From the foregoing we see that when a circuit is tuned to the medial frequency of a band, then the circuit is equally responsive to the extreme frequencies of the band.

Let is be required that, for equal amplitudes E_1 of impressed E. M. F., the amplitude of the current I_1 in circuit 1, for frequencies $f_{c_1}\pm f_t$, shall be $\frac{1}{m}$th of the resonance amplitude $\frac{E_1}{R_1}$. Then

$$\frac{1}{\sqrt{1+\alpha_{c1\pm t}^2S_1^2}}=\frac{1}{m} \quad (36)$$

and

$$\frac{1}{\sqrt{1+\alpha_{c2\pm t}^2S_2^2}}=\frac{1}{m} \quad (37)$$

hence

$$S_1^2=\frac{m^2-1}{\alpha_{c1\pm t}^2} \quad (38)$$

$$S_2^2=\frac{m^2-1}{\alpha_{c2\pm t}^2} \quad (39)$$

and

$$S_2^2 : S_1^2 :: \alpha_{c1\pm t}^2 : \alpha_{c2\pm t}^2 :: f_{02}^2 : f_{01}^2 \quad (40)$$

Under these conditions we have,

$$I_1=\frac{E_1}{R_1\sqrt{1+(m^2-1)\frac{\alpha_1^2}{\alpha_{c1\pm t}^2}}} \quad (41)$$

and

$$I_2=\frac{E_2}{R_2\sqrt{1+(m^2-1)\frac{\alpha_2^2}{\alpha_{c2\pm t}^2}}} \quad (42)$$

Each of the circuits is now tuned to the medial frequency of its radio frequency telephone band, and each is $\frac{1}{m}$th as responsive to the extreme frequencies of its band as it is to its resonance frequency.

If now we associate these two circuits together through a translating device D_1, as illustrated in Fig. 10, and this association be such that the amplitude E_2 of the impressed E M. F. in circuit 2 is equal to K_1, I_1, where K_1 is a numerical constant, then, for an impressed simple harmonic E. M. F. of amplitude E_1 and frequency f in circuit 1, the current in circuit 2 will be

$$I_2=\frac{K_1E_1}{R_1R_2}\,\frac{1}{\sqrt{1+(m^2-1)\frac{\alpha_1^2}{\alpha_{c1\pm t}^2}}}\,\frac{1}{\sqrt{1+(m^2-1)\frac{\alpha_2^2}{\alpha_{c2\pm t}^2}}} \quad (43)$$

When the frequencies of the force impressed upon the first circuit are $f_{01}\pm f_t$, and those of the force impressed upon the second circuit are $f_{02}\pm f_t$, (43) reduces to

$$I_2=K_1E_1/(m^2R_1R_2) \quad (44)$$

The difference between these frequencies is $f_{b_1}=f_{02}-f_{01}$.

In the case of complete resonance, i. e. when the frequency impressed upon circuit 1 is f_{01} and that on circuit 2 is f_{02}, then (43) reduces to

$$I_2=K_1E_1/R_1R_2 \quad (45)$$

The difference between these medial frequencies is $f_{02}-f_{01}$. But by (32) and (33)

$$f_{02}-f_{01}=\sqrt{f_{c2}^2-f_t^2}-\sqrt{f_{c1}^2-f_t^2}=f_{c2}\sqrt{1-\frac{f_t^2}{f_{c2}^2}}-f_{c1}\sqrt{1-\frac{f_t^2}{f_{c1}^2}} \quad (46)$$

For radio telephony $\frac{f_t}{f_{c2}^2}$ and $\frac{f_t}{f_{c1}^2}$ are always negligible compared to unity, so that we have in effect,

$$f_{02}-f_{01}=f_{c2}-f_{c1}=f_{b1} \quad (47)$$

It follows that if D_1 in Fig. 10 is a beat receiver in which the beat frequency is f_{b_1}, conditions (44) and (45) can be effectively **realized.**

For frequencies to be excluded, (41) and (42) reduce in effect to

$$I_1=\frac{E_1\alpha_{c1\pm t}}{R_1\sqrt{m^2-1}\,\alpha_1} \quad (48)$$

and

$$I_2=\frac{E_2\alpha_{c2\pm t}}{R_2\sqrt{m^2-1}\,\alpha_2} \quad (49)$$

For the coupled system of Fig. 10 we therefore have substantially

$$I_2 = K_1 E_1 \alpha_{c1\pm t}\alpha_{c2\pm t}/\{R_1R_2(m^2-1)\alpha_1\alpha_2\} \quad (50)$$

in the case of frequencies to be excluded.

Let the amplitude I_1 be $\frac{1}{p}$th of its resonance value $\frac{E_1}{R_1}$ when $\alpha_1=\alpha_{p1}$ and the amplitude I_2 be $\frac{1}{p}$th its resonance value $\frac{E_2}{R_2}$, when $\alpha_2=\alpha_{p2}$, then

$$I_2=\frac{K_1E_1}{R_1R_2}\frac{1}{p^2} \quad (51)$$

$$\frac{1}{\sqrt{1+(m^2-1)\frac{\alpha_{p1}^2}{\alpha_{c1\pm t}^2}}}=\frac{1}{p} \quad (52)$$

and

$$\frac{1}{\sqrt{1+(m^2-1)\frac{\alpha_{p2}^2}{\alpha_{c2\pm t}^2}}}=\frac{1}{p} \quad (53)$$

whence

$$\alpha_{p1}=\sqrt{\frac{p^2-1}{m^2-1}}\alpha_{c1\pm t} \quad (54)$$

and

$$\alpha_{p2}=\sqrt{\frac{p^2-1}{m^2-1}}\alpha_{c2\pm t} \quad (55)$$

If

$$\alpha_{p1}=\frac{f_{p1}^2-f_{01}^2}{f_{01}f_{p1}} \quad (56)$$

and

$$\alpha_{p2}=\frac{f_{p2}^2-f_{02}^2}{f_{02}f_{p2}} \quad (57)$$

$$f_{p1}=\frac{af_{01}}{2}\alpha_{(c1\pm t)}\left\{\pm1+\sqrt{1+\frac{4}{a^2\alpha_{(c1\pm t)}^2}}\right\} \quad (58)$$

$$f_{p2}=\frac{af_{02}}{2}\alpha_{(c2\pm t)}\left\{\pm1+\sqrt{1+\frac{4}{a^2\alpha_{(c2\pm t)}^2}}\right\} \quad (59)$$

where

$$a=\sqrt{\frac{p^2-1}{m^2-1}} \quad (60)$$

As a numerical example, let the band of frequencies received extend from $f'_1=102{,}000$ to $f''_1=98{,}000$. The carrier frequency of the received band is $f_{c1}=\frac{f'_1+f''_1}{2}=100{,}000$, $f_t=\frac{f'_1-f''_1}{2}$ which equals 2,000 and the medial frequency of the received band is $f_{01}=\sqrt{f_{c1}^2-f_t^2}=99{,}980$.

Let the beat frequency $f_{b1}=900{,}000$, then $f_{c2}=f_{c1}+f_t=1{,}000{,}000$, and

$$f_{02}=\sqrt{f_{c2}^2-f_t^2}=1{,}000{,}000.$$

The band of frequencies impressed upon circuit 2 extends from $f''_2=998{,}000$ to $f'_2=1{,}002{,}000$.

$$\alpha_{c1\pm t}=\mp\frac{2f_t}{f_{01}}=\mp0.04001,\ \alpha_{c2\pm t}=\mp\frac{2f_t}{f_{02}}=\mp0.004000.$$

Let $\frac{1}{m_2}=\frac{9}{10}$ or $m^2=\frac{10}{9}$, then

$$S_1^2=\frac{10-9}{9(0.04001)^2}=69.40,S_1=8.331,$$

$$S_2^2=\frac{10-9}{9(0.004000)^2}=6944,S_2=83.33.$$

Let $\frac{1}{p^2}=\frac{1}{100}$, $p^2=100$, then $a=29.83$

$$\alpha_{p1}=\pm\sqrt{\frac{100-1}{\frac{10}{9}-1}}\ 0.04001=\pm1.193$$

$$\alpha_{p2}=\pm\sqrt{\frac{100-1}{\frac{10}{9}-1}}\ 0.004000=0.1193$$

$$f_{p1}=\frac{1}{2}29.83\times99{,}980\times0.04001\left\{\sqrt{1+\frac{4}{890\times0.001601}}\pm1\right\}$$

$$f_{p1}=\begin{cases}176{,}000\\56{,}740\end{cases}$$

$$f_{p2}=\begin{cases}1{,}070{,}000\\940{,}200\end{cases}$$

From the foregoing we see that the orginization of Fig. 10 permits of the reception of a telephone band extending from $f=102{,}000$ to $f=98{,}000$ with a discrimination no greater than 10% against any frequency of the band and that this system has a receptivity of less than 1% for all frequencies greater than 176,000 or less than 56,740.

In the case of a single circuit receiving such a band with a similar limitation of the distortion, the frequencies for which the circuit would show a receptivity of less than 1% would be $f > 838{,}000$ and $f < 11{,}900$.

The advantage gained by the use of the coupled system of Fig. 10 with the conditions imposed above is thus seen to be very marked.

Suppose we have another pair of circuits, 3 and 4 of Fig. 11 associated together by means of a beat receiver D_3, in such a manner that the amplitude of the force E_4 impressed on circuit 4 is $K_3 I_3$, where K_3 is a numerical constant and I_3 is the amplitude of the current in circuit 3. If circuits 3 and 4 moreover be made resonant respectively to frequencies f_{03} and f_{04}, where $f_{04}-f_{03}$ equals f_{b3}, f_{b3} being the beat frequency of D_3, then we have, because of the similarity of the system to that of Fig. 10:

$$I_3 = E_3/\left\{R_3\sqrt{1+\alpha_3^2 S_3^2}\right\} = E_3/\left\{R_3\sqrt{1+(m^2-1)\frac{\alpha_3^2}{\alpha_{(c3\pm t)}^2}}\right\} \quad (61)$$

$$I_4 = E_4/\left\{R_4\sqrt{1+\alpha_4^2 S^2}\right\} = E_4/\left\{R_4\sqrt{1+(m^2-1)\frac{\alpha_4^2}{\alpha_{(c4\pm t)}^2}}\right\} \quad (62)$$

$$\alpha_3 = \frac{f^2-f_{03}^2}{f f_{03}} \quad (63)$$

$$\alpha_4 = \frac{f^2-f_{04}^2}{f f_{04}} \quad (64)$$

$$\alpha_{c3\pm t} = \frac{(f_{c3}\pm f_t)^2-f_{03}^2}{(f_{c3}-f_t)f_{03}} = \mp 2\frac{f_t}{f_{03}} \quad (65)$$

$$\alpha_{(c4\pm t)} = \mp 2\frac{f_t}{f_{04}} \quad (66)$$

$$S_3 = \sqrt{m^2-1}/\alpha_{c3\pm t} \quad (67)$$

$$S_4 = \sqrt{m^2-1}/\alpha_{c4\pm t} \quad (68)$$

$$\frac{S_4}{S_3} = \frac{\alpha_{c3\pm t}}{\alpha_{c4\pm t}} = \frac{f_{04}}{f_{03}} \quad (69)$$

$$f_{p3} = \frac{1}{2}a f_{03}\alpha_{c3\pm t}\left\{\sqrt{1+\frac{4}{a^2\alpha_{c3\pm t}^2}}\pm 1\right\} \quad (70)$$

$$f_{p4} = \frac{1}{2}a f_{04}\alpha_{c4\pm t}\left\{\sqrt{1+\frac{4}{a^2\alpha_{c4\pm t}^2}}\pm 1\right\} \quad (71)$$

$$I_4 = \frac{K_3 E_3}{R_3 R_4}\frac{1}{\sqrt{1+(m^2-1)\frac{\alpha_3^2}{\alpha_{c3\pm t}^2}}}\frac{1}{\sqrt{1+(m^2-1)\frac{\alpha_4^2}{\alpha_{c4\pm t}^2}}} \quad (72)$$

Let this pair of circuits, 3 and 4 of Fig. 11, be associated with the pair of circuits 1 and 2 of Fig. 10 by means of a beat receiver D_2, as illustrated in Fig. 12, and let us have the relation $f_3-f_2=f_{b2}$, f_{b2} being the beat frequency of D_2, then if $E_3=K_2 E_2$, where K_2 is a numerical constant, we shall have

$$I_4 = \frac{K_1K_2K_3E_1}{R_1R_2R_3R_4}\frac{1}{\sqrt{1+(m^2-1)\frac{\alpha_1^2}{\alpha_{c1\pm t}^2}}}\frac{1}{\sqrt{1+(m^2-1)\frac{\alpha_2^2}{\alpha_{c2\pm t}^2}}}\frac{1}{\sqrt{1+(m^2-1)\frac{\alpha_3^2}{\alpha_{c3\pm t}^2}}}\frac{1}{\sqrt{1+(m^2-1)\frac{\alpha_4^2}{\alpha_{(c4\pm t)}^2}}} \quad (73)$$

In this system, when the frequencies of the force impressed upon circuit 1 are $f_{c1}\pm f_t$, we have

$$I_4 = K_1K_2K_3E_1/(R_1R_2R_3R_4m^4) \quad (74)$$

and when the frequency impressed upon circuit 1 is f_{01} we have the condition of resonance of the system and

$$I_4 = K_1K_2K_3E_1/(R_1R_2R_3R_4) \quad (75)$$

In general, if there be n circuits associated in the manner indicated, we have for impressed frequencies $f_{c1}\pm f_t$

$$I_n = K_1 \ldots\ldots K_{n-1}E_1/(R_1 \ldots\ldots R_n m^n) \quad (76)$$

and for the condition of resonance

$$I_n = K_1 \ldots\ldots K_{n-1}E_1/(R_1 \ldots\ldots R_n) \quad (77)$$

If the responsiveness of the nth circuit, when the transmitted frequencies are $f_{c1}\pm f_t$, is to be $\frac{1}{r}$th of that when the transmitted frequency f is such as to produce resonance of the system, then $m = r^{\frac{1}{n}}$. Thus if there are 8 circuits, $n=8$, and if $\frac{1}{m^n}=\frac{9}{10}$, then $m^8=\frac{10}{9}$ and $m^2=\left(\frac{10}{9}\right)^{1/4}=(1.11)^{1/4}=1.02669$, $m^2-1=0.02669$.

$$\therefore S_1^2=\frac{0.02669}{0.001601}=16.67,\ S_1=4.083$$

$$p^8=100,\ p^2=(100)^{1/4}=(10)^{1/2}=3.16228$$

$$p^2-1=2.16228$$

$$a^2=\frac{2.16228}{0.02669}=81.01$$

$$\frac{a^2}{4}=20.25$$

$$a=(81.01)^{1/2}=9.001$$

$$\frac{a}{2}=4.500$$

$$f_{p1}=4.500\times 0.04001\times 99{,}980\left\{\left(\frac{1}{20.25\times 0.01601}+1\right)^{1/2}\pm 1\right\}$$

$$f_{p1}=\begin{cases}119{,}600\\ 83{,}580\end{cases}$$

A comparison of these values of f_{p1} with those of f_{p1} given in the foregoing text between Equations (60) and (61) shows the advantage gained by the successive association of the circuits contemplated in this memorandum.

The organization may be described as a sequence of circuits, associated through frequency changing translating devices, in which each circuit is resonant to the frequency impressed upon it, in which the amplitude of the E. M. F. impressed upon each circuit is proportional to the amplitude of the current in its antecedent circuit, in which the selectivities of the circuits are severally proportional to their frequencies and in which the selectivity of the first circuit is given by the expression

$$S_1 = \sqrt{r^{2l^n} - 1}/\alpha_{(c1\pm t)} \quad (78)$$

The transmitted frequencies at which the responsiveness of the nth circuit will be $\frac{1}{q}$th that which it exhibits when the transmitted frequency is such as to produce a resonant response throughout the system is

$$f_{t1} = \frac{a}{2}\alpha_{c1\pm t}\left(\sqrt{1+\frac{4}{a^2\alpha_{c1\pm t}^{\ 2}}} \pm 1\right) \quad (79)$$

where

$$a = \sqrt{\frac{q^{2l^n}-1}{r^{2l^n}-1}} \quad (80)$$

In these expressions r is the specified ratio of the amplitude of the current in the nth circuit, when the transmitted frequency is such as to produce resonance throughout the system to the amplitude of the current in this circuit when the transmitted frequencies are $f_{c1} \pm f_t$, that is to say, when they are the extreme frequencies of the telephone band transmitted.

In the following claims the term "beat receiver" refers to a receiver that receives current of a certain frequency and also a locally generated current of a different frequency, and modulates them together and gives an output current which is the sum or difference (or both sum and difference) of the two input frequencies.

I claim:

1. A plurality of resonant circuits and a plurality of translating devices alternating with them in succession, each intermediate translating device having its input connected with the antecedent resonant circuit and its output connected with the consequent resonant circuit, said resonant circuits having their inductances and capacities progressively different from circuit to circuit and proportioned so that in the output of the last of the series there shall be only a small determinate variation of current intensity over a determinate frequency range and so that beyond said range said intensity shall diminish abruptly, the connections of the intermediate translating devices as aforesaid being such that the output electromotive force from each such device is proportional to the current in the corresponding antecedent circuit.

2. A plurality of resonant circuits and a plurality of translating devices alternating with them in succession, each intermediate translating device having its input connected with the antecedent resonant circuit and its output connected with the consequent resonant circuit, said resonant circuits having their inductances and capacities progressively different from circuit to circuit and proportioned so that in the output of the last of the series there shall be only a small determinate variation of current intensity over a determinate frequency range and so that beyond said range said intensity shall diminish abruptly, each said translating device constituting means to apply an electromotive force in the consequent circuit proportional to the current intensity in the antecedent circuit.

3. A plurality of resonant circuits having a plurality of translating devices arranged alternately in succession, each intermediate translating device having its input connected in shunt to a substantial part of the impedance of the antecedent resonant circuit and its output connected in series in the consequent resonant circuit, whereby the output electromotive force from such translating device will be proportional to the current in the circuit connected with the corresponding input to such device, said resonant circuits having their inductances and capacities progressively different from circuit to circuit and designed so that the ultimate output current of the system varies only a small permissible amount over a considerable desired frequency range and falls off abruptly outside that range.

4. A plurality of resonant circuits and a plurality of translating devices alternating with them in succession, each intermediate translating device comprising means to give an output electromotive force proportional to the input current and having its input connected wth the antecedent resonant circuit and its output connected with the consequent resonant circuit, said resonant circuits having respective selectance values proportional to frequency and the elements of the combination being of progressively different values from circuit to circuit and so proportioned and designed that there shall be only a small determinate variation of current intensity over a determinate frequency range and so that beyond said range said intensity shall diminish abruptly.

5. In combination, a plurality of resonant circuits having a plurality of translating devices arranged alternately in succession, said resonant circuits having selectances propor-

tional to frequency but having their inductances and capacities progressively different from circuit to circuit, each intermediate translating device having its input connected with the antecedent resonant circuit and its output connected with the consequent resonant circuit and comprising means to give an output electromotive force proportional to the input current so that the ultimate output current of the system varies only a small permissible amount over a considerable desired frequency range and falls off abruptly outside that range.

6. A plurality of resonant circuits and a plurality of beat receivers alternating with them in succession, each intermediate beat receiver having its input connected with the antecedent resonant circuit and its output connected with the consequent resonant circuit, said resonant circuits having their inductances and capacities progressively different from circuit to circuit and proportioned so that in the output of the last of the series there shall be only a small determinate variation of current intensity over a determinate frequency range and so that beyond said range said intensity shall diminish abruptly.

7. A plurality of resonant circuits and a plurality of frequency changing translating devices alternating with them in succession, each intermediate translating device having its input connected with the antecedent resonant circuit and its output connected with the consequent resonant circuit, said resonant circuits being tuned to different frequencies appropriate to said translating devices and having their resistance and reactance components proportioned so that in the output of the last of the series there shall be only a small determinate variation of current intensity over a determinate frequency range and so that beyond said range said intensity shall diminish abruptly.

8. A plurality of resonant circuits and a plurality of frequency changing translating devices alternating with them in succession, each said circuit being resonant to the frequency impressed upon it from the respective antecedent translating device, the amplitude of the electromotive force impressed upon each circuit being proportional to the amplitude of the current in the respective antecedent circuit and the selectances of the circuits being respectively proportional to their resonant frequencies.

9. A plurality of resonant circuits and a plurality of frequency changing translating devices alternating with them in succession, each intermediate translating device having its input connected with the antecedent resonant circuit and its output connected with the consequent resonant circuit, each said circuit being resonant to the frequency impressed upon it from the respective antecedent translating device and the selectances of the said circuits being respectively proportional to their resonant frequencies.

In testimony whereof, I have signed my name to this specification this 3d day of July 1924.

JOHN STONE STONE.