June 18, 1929. H. NYQUIST 1,717,400

PHASE SHIFTING NETWORK

Filed Aug. 25, 1926

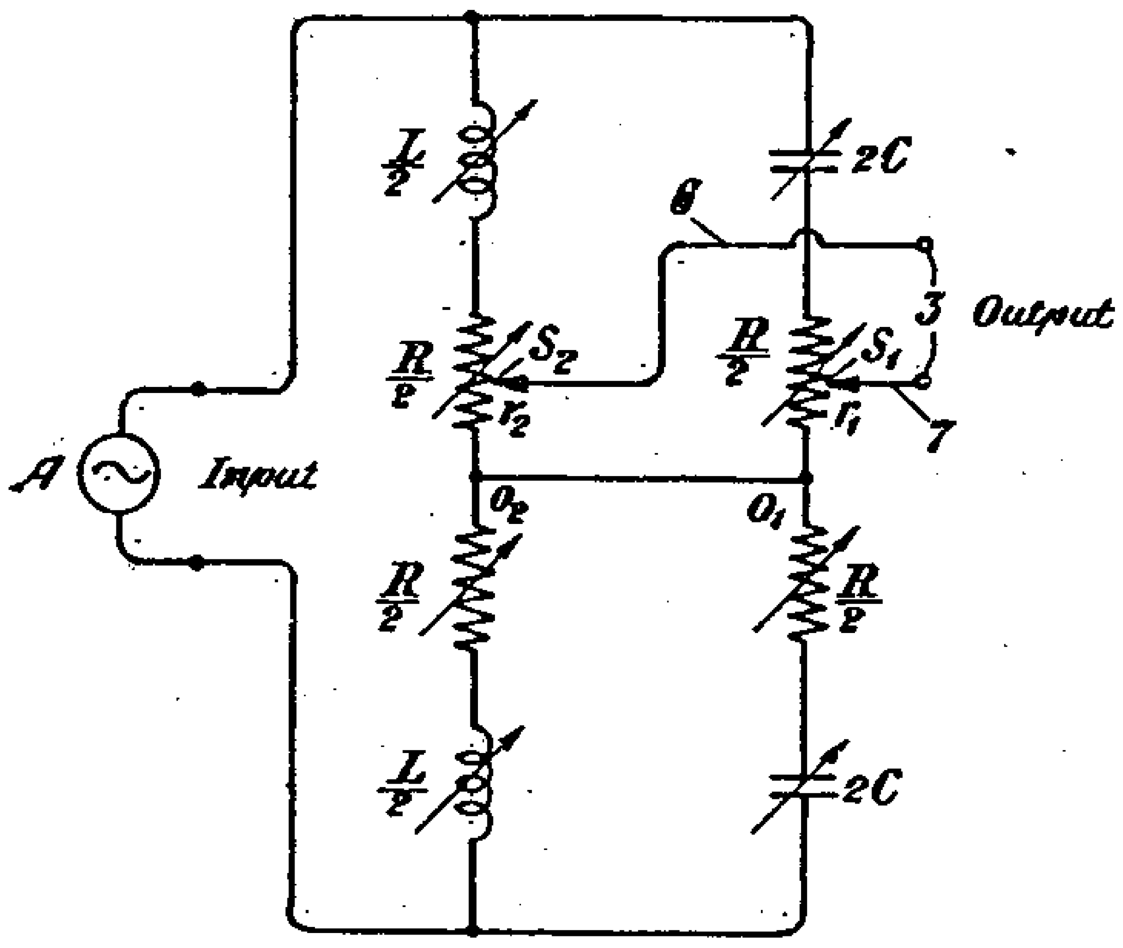

Fig. 1

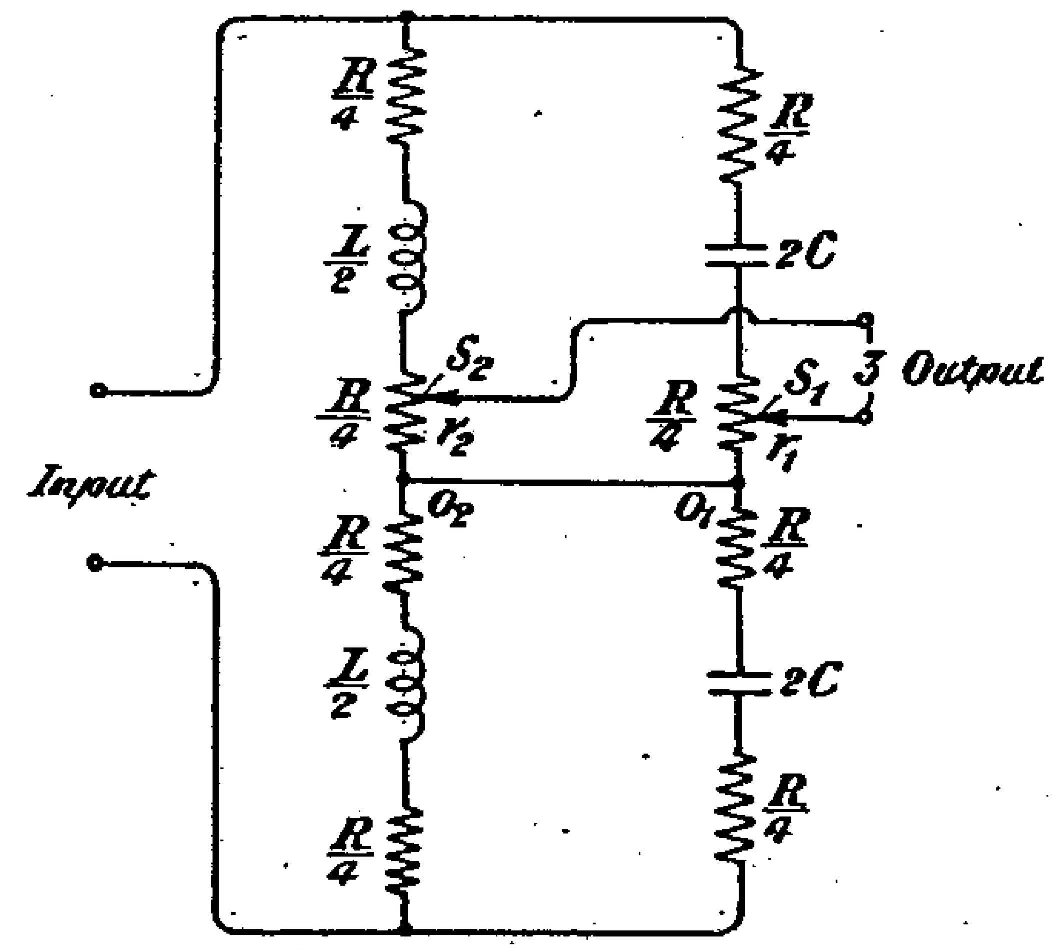

Fig. 2

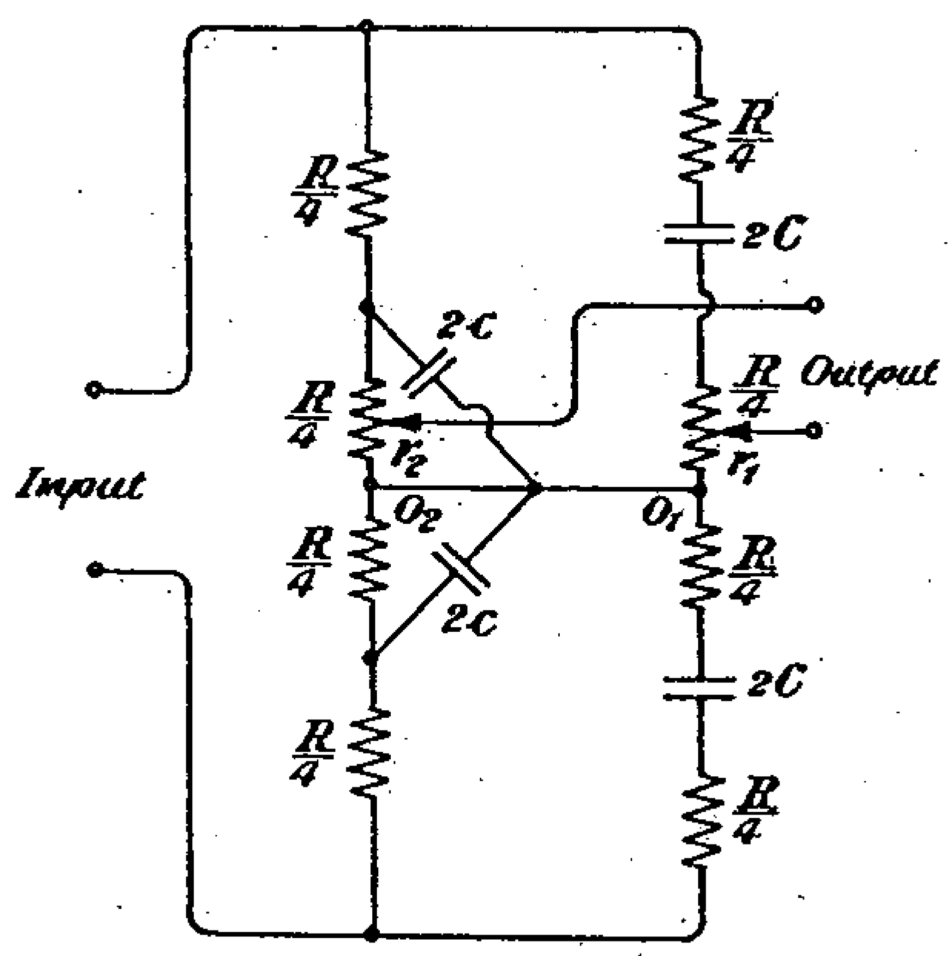

Fig. 3

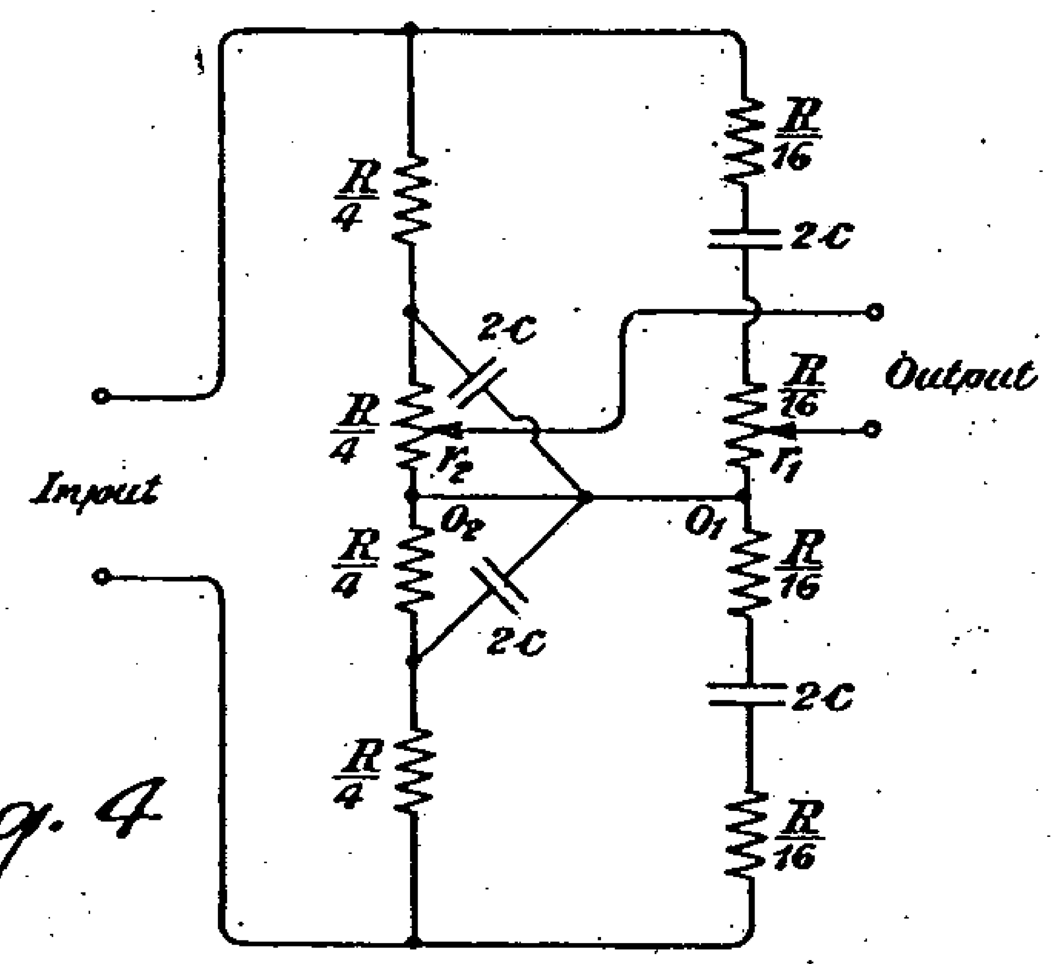

Fig. 4

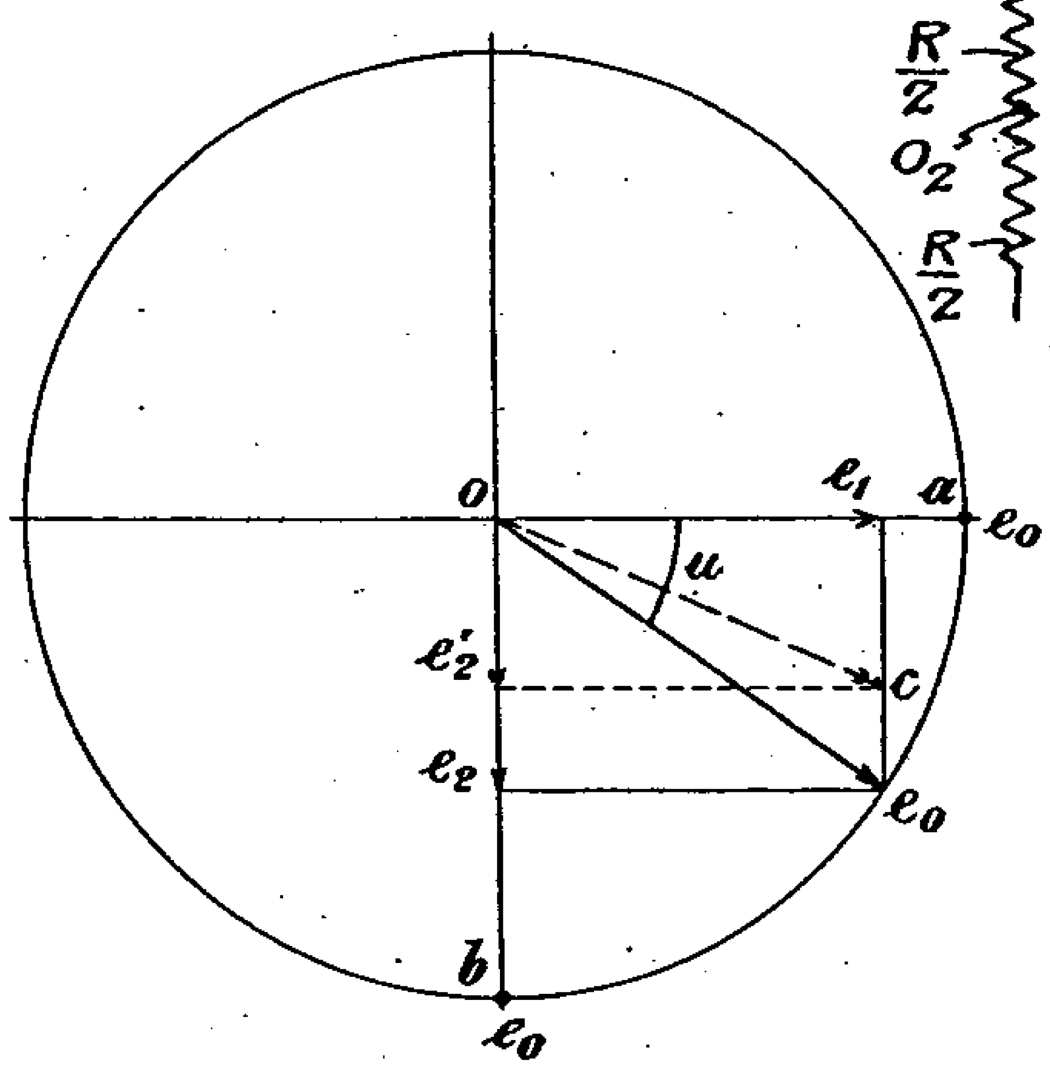

Fig. 5

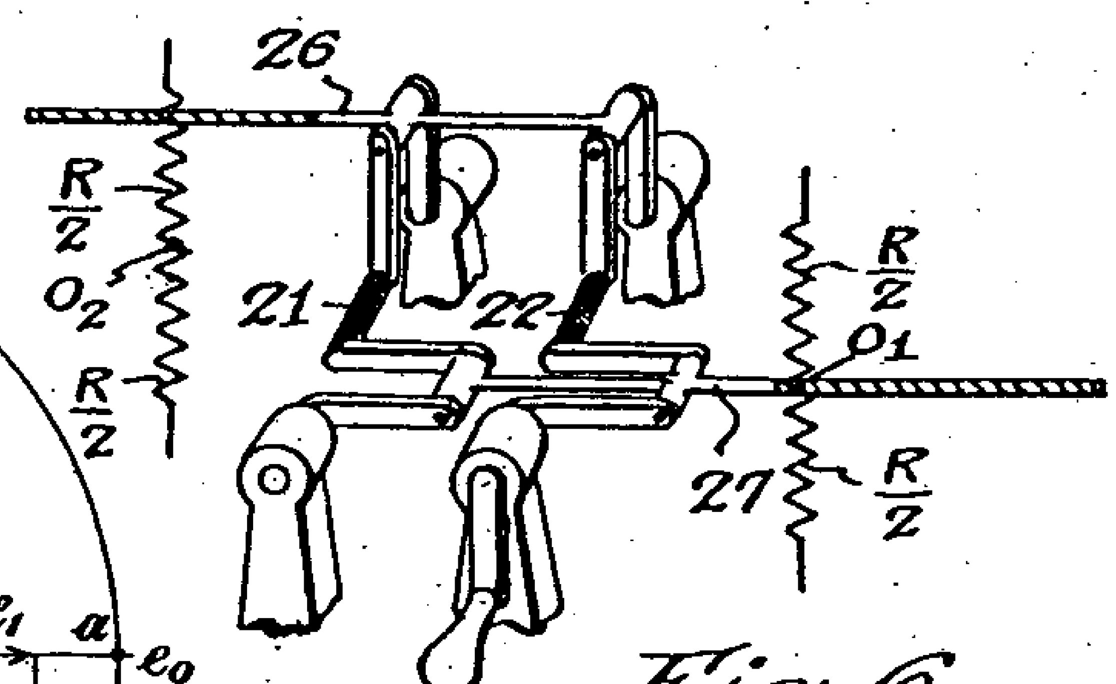

Fig. 6.

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BY [signature]

ATTORNEYS.

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UNITED STATES PATENT OFFICE.

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PHASE-SHIFTING NETWORK.

Application filed August 25, 1926. Serial No. 131,495.

This invention relates to a network by which a source of alternating current voltage may be obtained, the phase thereof being conveniently controlled and varied. The network comprises a circuit with two input terminals to which an alternating current source may be connected, and two output terminals from which potential difference may be obtained, the phase of the potential difference being completely controllable without altering the condition of the source. The invention may find application in many places such as that illustrated in my application Serial No. 90,653, filed February 25, 1926, in which one form of my invention is shown in use.

The purpose of this invention is to design a network of these general characteristics, and, at the same time, one which will provide a source of alternating electromotive force, which, while variable in phase, shall be constant in amplitude.

Still a further purpose of the invention is to provide such a source which shall be as simple as possible in its adjustments to obtain the desired phase shift and as simple in its adjustments as possible for changes in frequency. The invention will be better understood from the following description taken in connection with the accompanying drawings which embody certain examples of my invention, the invention itself being defined by the appended claims.

In the drawing, Figure 1 represents one form of network for obtaining the desired shift in phase of an alternating current source. Fig. 2 is a modification thereof. Figs. 3 and 4 are still further modifications in which one type of reactance element only is used, Fig. 5 is a vector diagram of E. M. F.'s involved in the circuits and Fig. 6 is a view of a mechanical device for obtaining the desired phase shift.

Referring more particularly to Fig. 1, there is shown a source of alternating E. M. F. A, this source having any desired frequency which we may represent by f. This source is connected to the input terminals of a net work which has two output terminals 3, from which, by suitable adjustment, it will be possible to obtain an alternating current voltage of the frequency given by the generator A, but of controllable phase and magnitude, all of this being determined by the characteristics of the network appearing between the input and output terminals. The network in this figure comprises two branches in parallel, each branch being made up of two halves, and each half comprising a reactance element and a resistance element. More specifically, the one branch is made up of the following elements taken in order: An inductance $\frac{L}{2}$, a resistance $\frac{R}{2}$, a second resistance $\frac{R}{2}$ and an inductance $\frac{L}{2}$. The second branch is similarly made up of the following elements taken in the order of a capacity $2C$, a resistance $\frac{R}{2}$, a resistance $\frac{R}{2}$ and a capacity $2C$. It will appear later that the midpoints O_1 and O_2 of the two branches are at the same potential and therefore there is no objection and in some cases it may be advantageous to connect these directly by a conductor, as shown. For simplicity of design, the resistances are all made of the same value, and, making variable contact with a point on the resistances in each branch, are two conductors 6 and 7 leading to the output terminals 3.

In order to obtain the results desired, the inductances and capacities are made of such value that for the particular frequency given by the generator A there shall exist the relation

$$pL=\frac{1}{pC}=R \quad (1)$$

where $\frac{L}{2}$, $2C$ and $\frac{R}{2}$ represent the inductance and capacity and resistance of the various elements shown, and $p=2\pi x$ frequency. The reason for this relationship will be better understood by the following considerations. The current flowing through the capacity branch will be

$$i_1=\frac{e}{R+\frac{1}{jpC}} \quad (2)$$

where e is the potential difference across the network and $j=\sqrt{-1}$ and where the denominator of this expression represents the impedance of that branch. The current through the inductance branch will similarly be given by

$$i_2=\frac{e}{R+jpL} \quad (3)$$

which shows that the currents in the two branches are not in phase with each other. If we give the inductances and capacities such values that the relations (1) hold, then these two current relations take on the form

$$i_1=\frac{e}{R(1-j)}=\frac{e(1+j)}{2R} \quad (4)$$

and

$$i_2=\frac{e}{R(1+j)}=\frac{e(1-j)}{2R} \quad (5)$$

from which it appears that i_1 and i_2 differ in phase by 90°.

The potential difference which exists between the point O_1 and a point S_1 between which there exists the resistance r_1 will then be

$$e_1=\frac{e}{2R}r_1(1+j) \quad (6)$$

and that between the point O_2 and a point S_2 between which there exists the resistance r_2 will be

$$e_2=\frac{e}{2R}r_2(1-j) \quad (7)$$

The potential difference, then, between the points S_1 and S_2 as measured from output terminals 3 will be the difference between these values, that is,

$$e'=\frac{e}{2R}[r_1(1+j)-r)_2 1-j)] \quad (8)$$

The interpretation of this last relation will be made evident by reference to Fig. 5, where the various elements are plotted graphically. In that figure, the potential drop through the resistance of the capacity branch is indicated by *oa* and is of magnitude e_o. Similarly, the potential difference over the corresponding resistance in the inductance branch, also of the magnitude e_o, is given by *ob*, which is 90° in phase behind *oa*. If the contact 7 includes between itself and the point o_1 an amount of resistance r_1, the potential difference over that resistance will be e_1 and is indicated as such in Fig. 5. Similarly, for a resistance r_2 in opposite branch there will be potential difference e_2'. The resultant of these two components, represented by their vector sum, is shown on the diagram as *oc* and it will be noted that the magnitude and the phase of this resultant will depend upon the values r_1 and r_2. It is also evident that if the proper relationship is maintained between these two resistances, that their vector sum will, in magnitude, always be equal to e_o and such a relationship on the diagram is indicated by the two components e_1 and e_2. In order that this relationship shall be maintained, it will be evident that the following shall hold

$$r_1=\frac{R}{2}\sin u \qquad r_2=\frac{R}{2}\cos u \quad (9)$$

where "u" gives the phase angle of the resultant, using the phase of *oa* as the reference. Under these conditions, it will be observed that while the phase of the resultant E. M. F. may be shifted through the angle 360° or any part thereof, the magnitude remains constant and this feature of a constant magnitude with any desired change in the phase contitutes an important feature of this invention. In order to accomplish this result the contacts S_1 and S_2 should be varied together in such manner as to maintain the relationship given above and any suitable mechanism may be used for this. One means for accomplishing this is shown in Fig. 6, in which there are two crank-shafts 21 and 22 in parallel with each other and each having two throws insulated from one another and 90° apart. The connecting rods 26 and 27 make contact respectively with and travel over the resistances between the inductances of one branch and between the capacities of the other branch of the network. In view of the circular motion and the 90° phase relationship of the two arms, it is seen that the resistances included at any moment across the output terminals 3 of Fig. 1 are in accordance with the relations of Equation (9).

It is apparent that if the frequency of the generator A is changed, corresponding changes must be made in the inductances and in the capacities or in the resistances in order to restore or maintain the conditions of relations (1). For this reason, it will be desirable to make these variable or to make the resistances variable, and this is indicated in Fig. 1 by the arrows passing through the elements.

Various modifications may be made in the network in order to obtain somewhat different results. In Fig. 2, for example, the number of resistance elements of each branch have been increased and so located that only half of the resistance of each branch occurs adjacent to the bridge O_1 O_2. The mathematical analysis in this case is similar to that for Fig. 1, and differs only in that the maximum voltage at the output terminals is one-half as great.

In some cases, it may be desirable to reduce the number of types of elements included in the network. This is particularly true if the frequency of the generator A is to be changed in which case the adjustment which must be made may be brought about by changes in the inductances or in the capacities. In general, however, it is more convenient to make use of variable capacities than to make use of variable inductances, and Fig. 3 shows a circuit in which the inductances have been dispensed with. As shown in that figure, the inductances of Fig. 2 have been removed, and the resistances in that branch which are adjacent to the bridging point O_2 are shunted by condensers of capacity $2c$. In this case, it may be shown that the current flowing through these resistances adjacent O_2 is given by

$$i_3=\frac{4e}{R(4+jpcR)} \quad (10)$$

and the current through the branch which has not been altered is

$$i_1=\frac{ejpC}{1+jpCR} \quad (11)$$

If, now, the condition is set that

$$R=\frac{1}{pC}=\frac{4}{pc}$$

these current relations simplify to

$$i_3=\frac{e}{2R}(1-j)$$

$$i_1=\frac{e}{2R}(1+j)$$

from which it will be seen that a relationship similar to that of Fig. 1 and described by means of Fig. 5 again exists. And if the same relationship regarding the values of r_1 and r_2 be maintained, then the output voltage will remain constant in amplitude without regard to the total change in phase.

In some cases, it may be desirable to use condenser units of the same type or size in the two parallel branches. Such an arrangement is shown in Fig. 4, the capacities 2C being replaced by capacities $2c$ of the same size as in the parallel branch. In order to maintain the proper relationships, it is necessary in this case to also alter the value of the resistances in the branch in which the condensers have been changed, these resistances now taking on the value $\frac{R}{16}$. In this case, the current through the resistance r_1 is four times as great as that in r_2, but if the adjustments are so controlled that

$$r_1=\frac{R}{16}\sin b$$

$$r_2=\frac{R}{4}\cos b$$

then the output voltage will be of constant magnitude and the same as that for Fig. 3.

It may be mentioned that each of the circuits can be shown in a slightly different form in which they take on the appearance of the conventional Wheatstone bridge network. Also it may be pointed out here that the input impedance of the network of Fig. 1 and of Fig. 2 is R, in both cases being pure resistances. On the other hand, the input impedances of Figs. 3 and 4 can be shown to be equal, respectively, to

$$\frac{R}{5}(2-j) \text{ and } \frac{R}{37}(7-5j).$$

What is claimed is:

1. A source of single phase alternating E. M. F. and a network associated therewith adapted to serve as a source of E. M. F. of variable phase, said network comprising two parallel branches across the terminals of said source, each possessing lumped reactance and resistance, and means for keeping the delivered E. M. F. of constant amplitude.

2. In a variable phase shift network, two branches in parallel, each possessing capacitance and resistance, each branch as seen from its midpoint being made up of two identical portions with the elements arranged in the same order, movable terminals attached to the resistances of said branches and adapted to deliver E. M. F. of variable phase and constant amplitude.

3. A variable phase shift network comprising two branches in parallel, each possessing reactance and resistance, each branch as seen from its midpoint being made up of two identical portions with the elements arranged in the same order, terminals making movable contact with the resistances and means to so relate the movements of the contacts as to deliver E. M. F. of variable phase and constant amplitude.

4. In a variable phase shift network, two branches in parallel, each possessing capacities and resistances, each branch as seen from its midpoint being made up of two identical portions with the elements arranged in the same order, and with resistance elements adjacent the midpoints, terminals making movable contact with the resistances, and means to so relate the movements of the contacts as to deliver E. M. F. of variable phase and constant amplitude.

5. A variable phase shift network for delivering E. M. F. of variable phase, comprising two branches in parallel adapted to be supplied from an AC source, the one branch comprising four equal resistance elements in series and a condenser bridged across each of the two inner resistance elements, the second comprising four resistance elements in series each equal to the resistance elements of the first branch and further comprising a condenser between each pair of outer resistance elements, and variable contact leads from points on the inner resistance elements.

6. A variable phase shift network for delivering E. M. F. of variable phase, comprising two branches in parallel adapted to be supplied from an AC source, the one branch comprising four equal resistance elements in

series and a condenser bridged across each of the two inner resistance elements, the second comprising four resistance elements in series each equal to the resistance elements of the first branch and further comprising a series condenser adjacent each outer resistance element, and variable contact leads from points on the inner resistance elements, the impedance of each of the shunting condensers in the first branch being one-half that of each of the resistance elements, and the impedance of each of the condensers in the second branch being twice that of each resistance element, the contact leads being so related in their movement as to yield E. M. F. of constant amplitude.

7. In a variable phase shift network, two branches in parallel, each possessing capacitance and resistance, each branch as seen from its midpoint being made up of two identical portions with the elements arranged in the same order, movable terminals attached to the resistances of said branches, and means for so moving the terminals with respect to each other that the potential difference across said terminals shall be of constant amplitude and variable phase.

In testimony whereof, I have signed my name to this specification this 24th day of August, 1926.

HARRY NYQUIST.