

July 31, 1928.

1,678,674

W. KOENIG, JR

METHOD AND APPARATUS FOR COMPUTATION

Filed Dec. 30, 1925

4 Sheets-Sheet 1

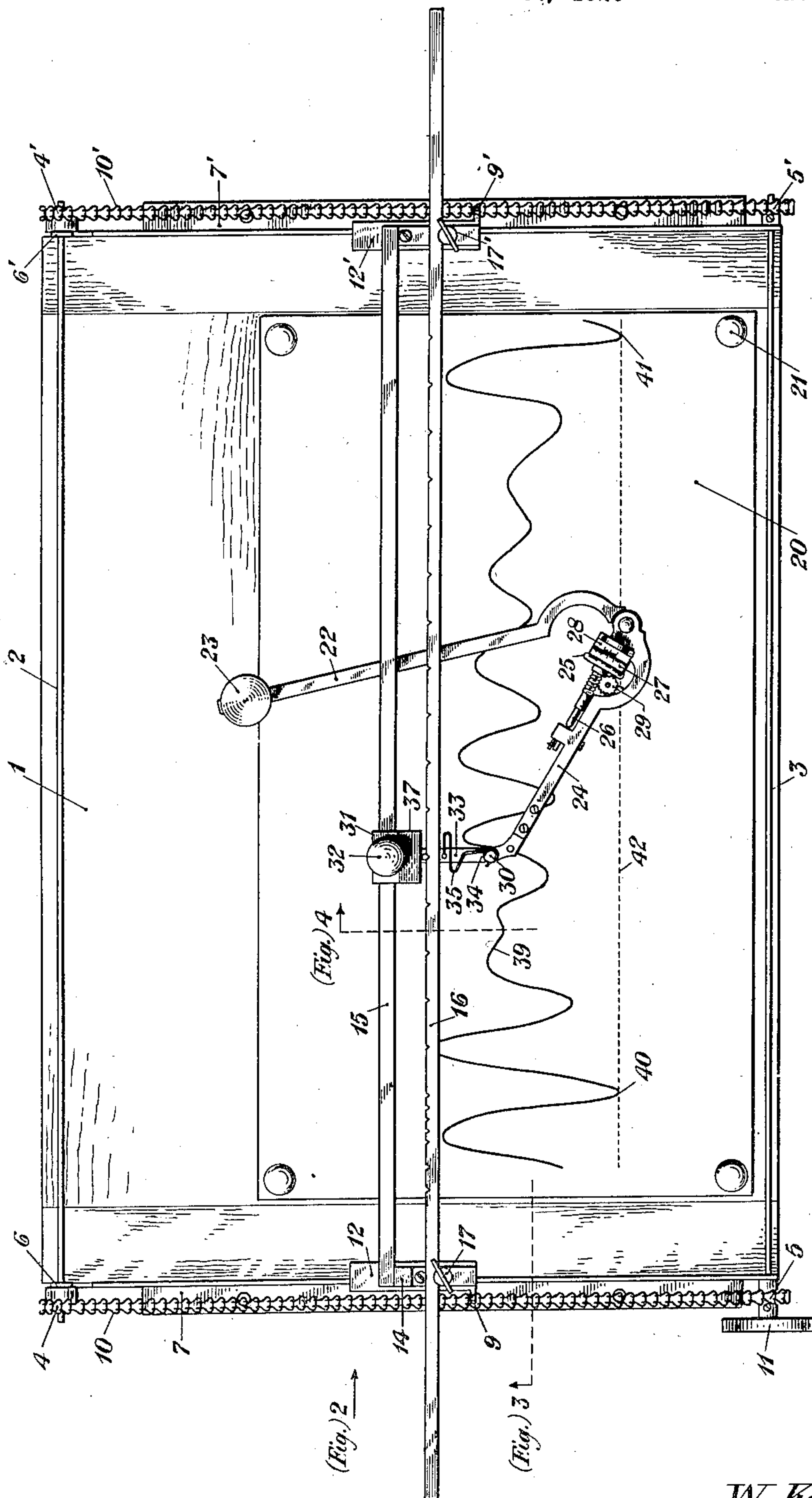


Fig. 1

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4 Sheets-Sheet 2

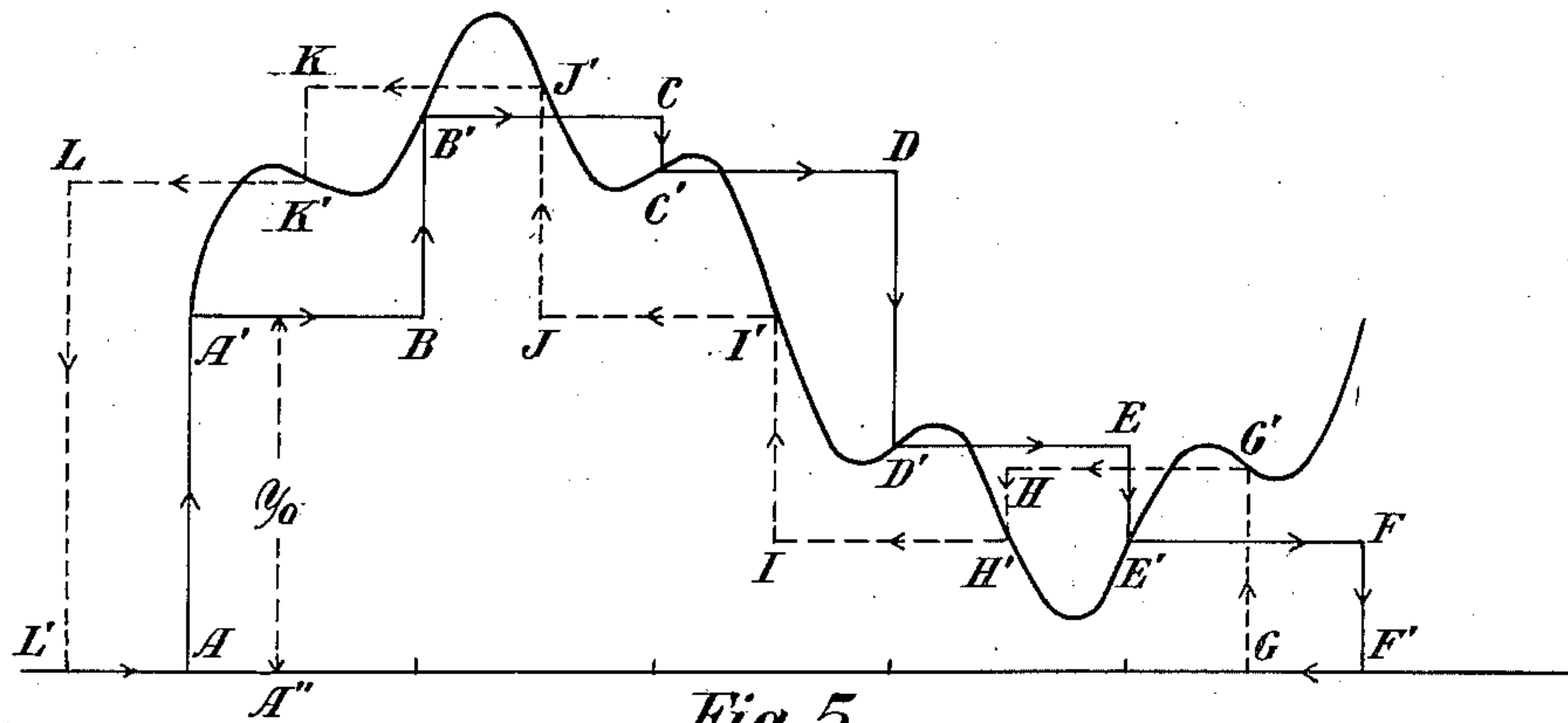


Fig. 5

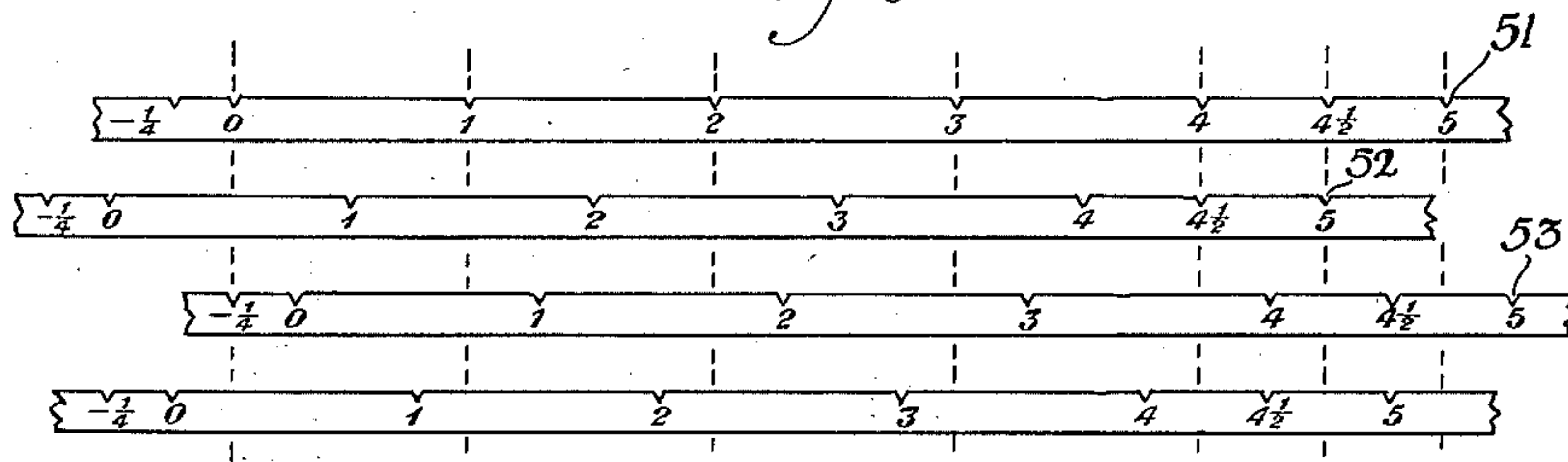


Fig. 6

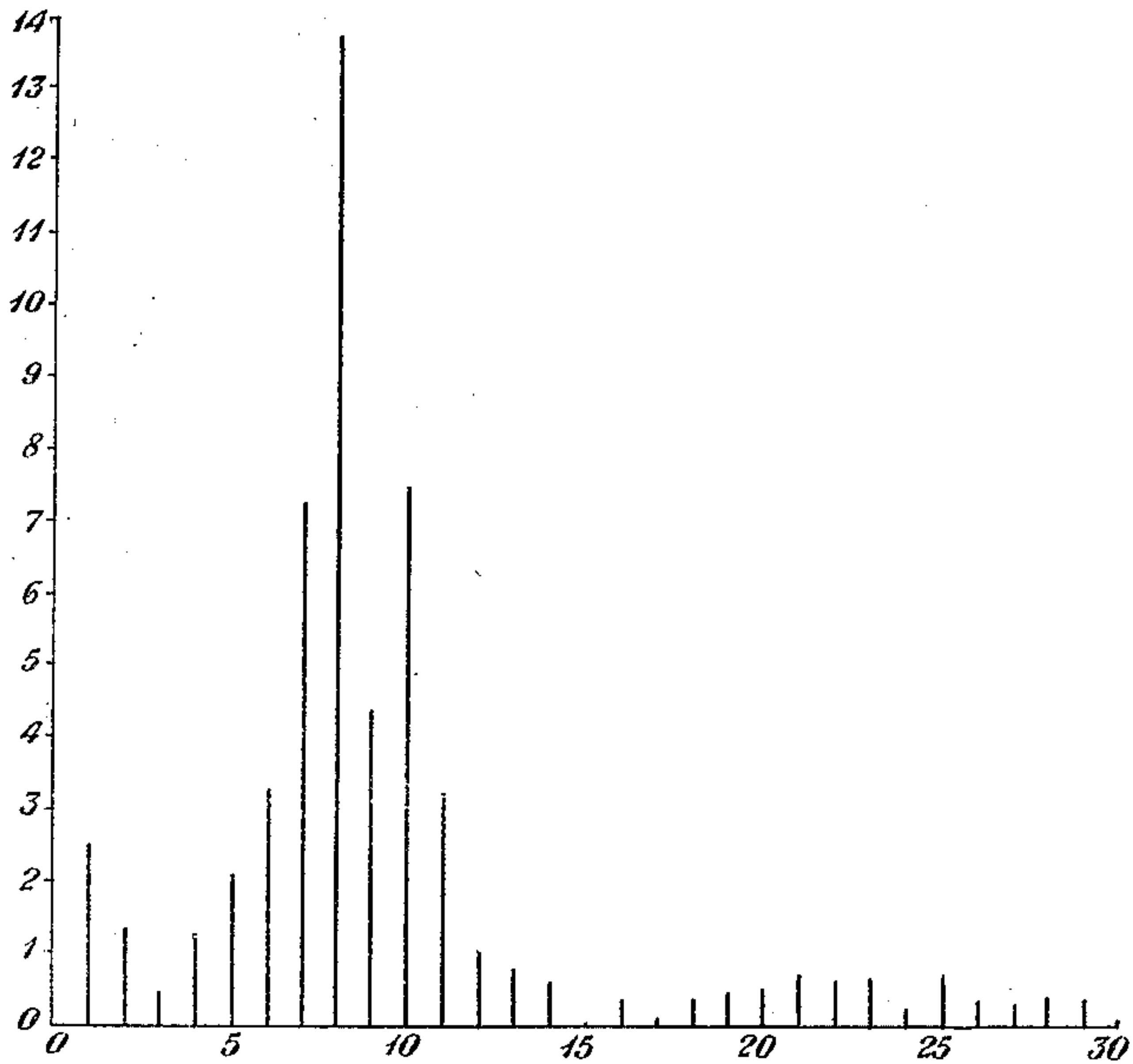


Fig. 8

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a'		a		c		b		b'			
n	PLANIMETER READING	ENTER QUANTITY INDICATED	SUM	a ²	a ² +b ²	√a ² +b ²	b	SUM	ENTER QUANTITY INDICATED	PLANIMETER READING	n
1		⑥							⑩		1
2		⑤							⑪		2
3		④							⑫		3
4		③							⑬		4
5		②							⑭		5
6	-3.50	①+0.23	-3.27					+0.08	⑮-0.27	+0.35	6
7		-a ₂₁							+b ₂₁		7
8		-a ₂₄							+b ₂₄		8
9		-a ₂₇							+b ₂₇		9
10	-7.30	-a ₃₀	-7.37					-0.96	+b ₃₀	-0.98	10
11			-1.85	3.42	10.45	3.23	7.03	-2.65			11
12											12
13											13
14											14
15											15
16											16
17											17
18			-0.30					-0.25			18
19											19
20											20
21											21
22											22
23											23
24											24
25											25
26											26
27											27
28											28
29			+0.34					+0.14			29
30			+0.07					+0.02			30

PLANIMETER READING PLANIMETER READING

To determine α_0 ,
 $2 Y_0$ (inches) =
- α_1
+ ⑥
Sum = α_0

Fig. 7

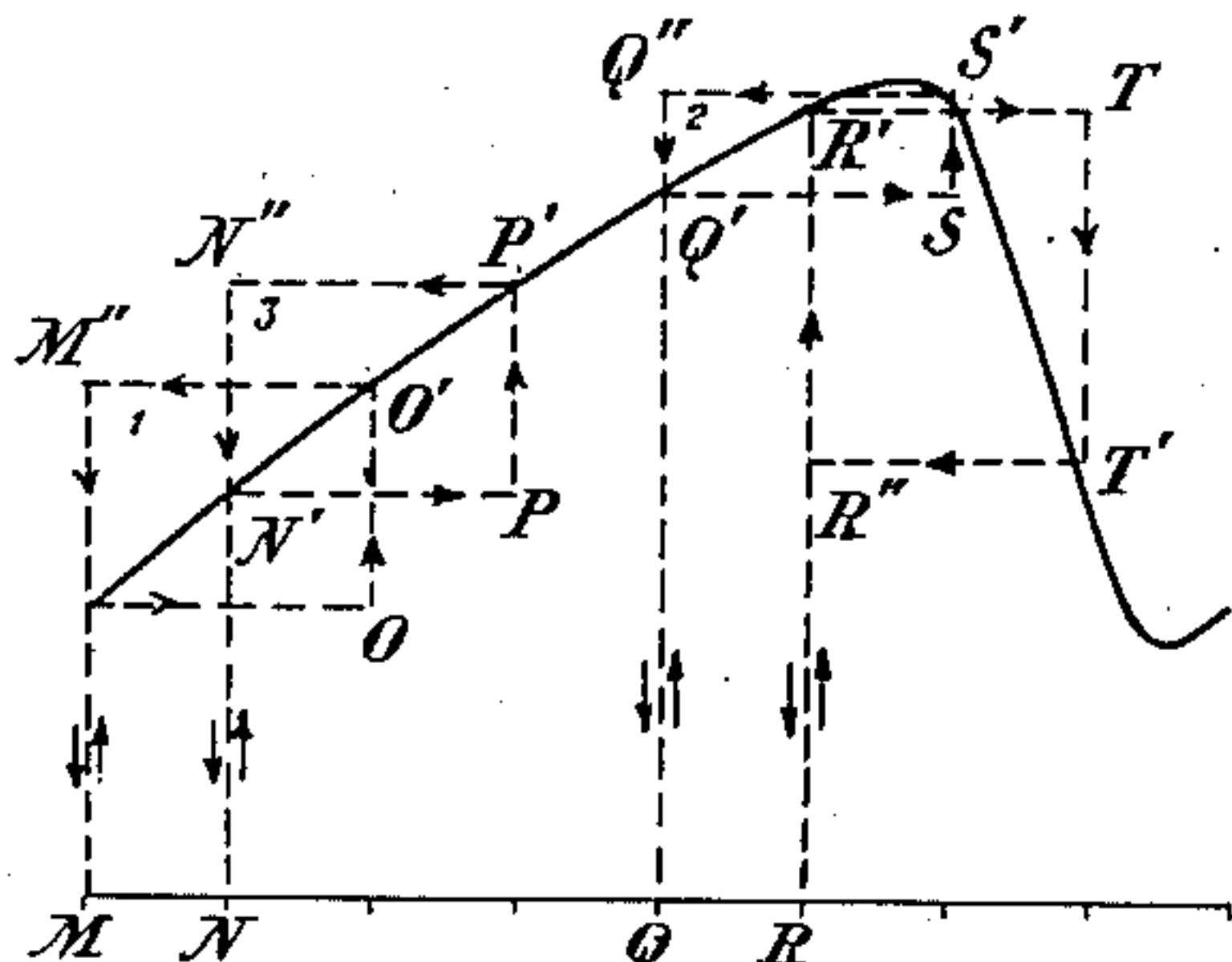


Fig. 9

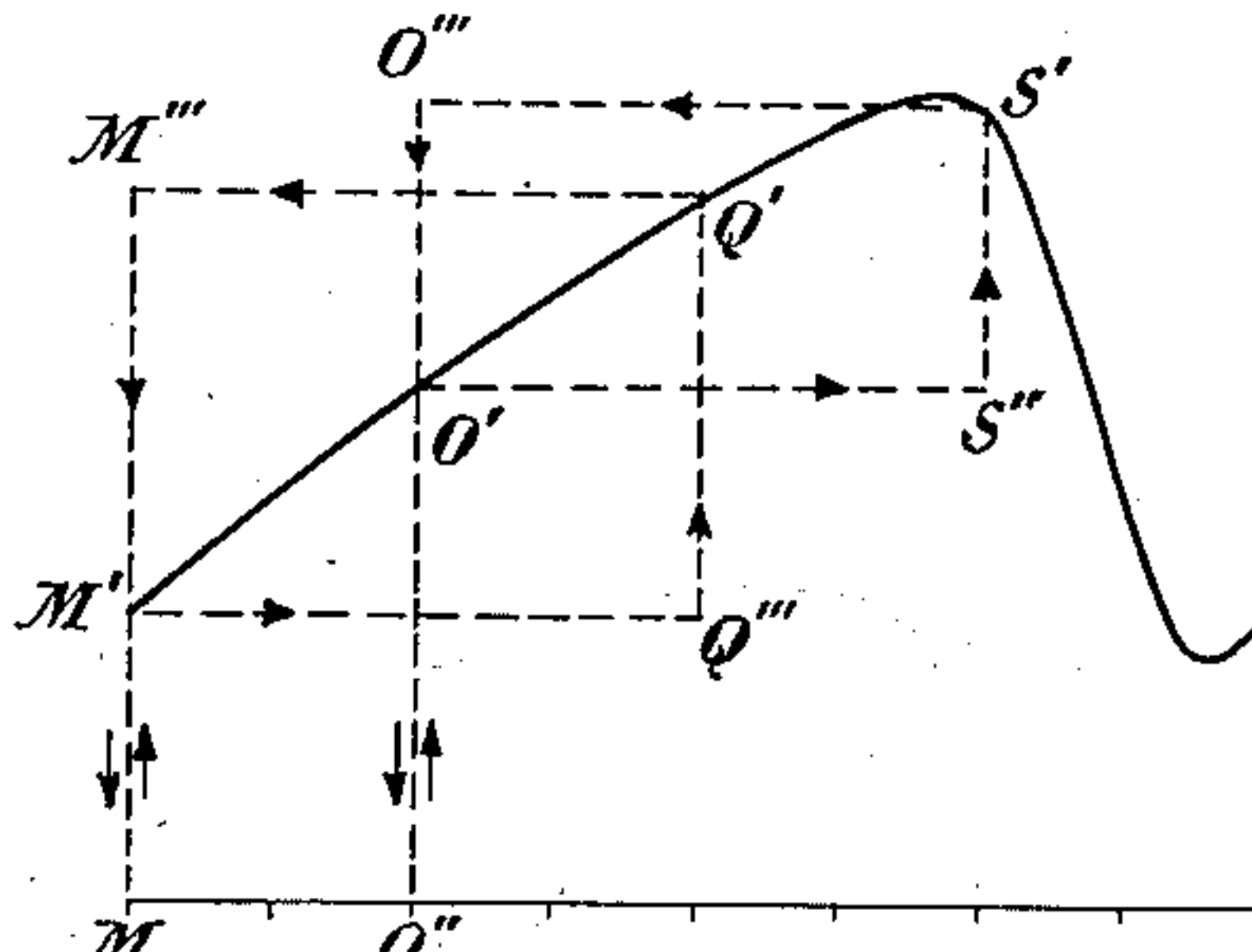


Fig. 10

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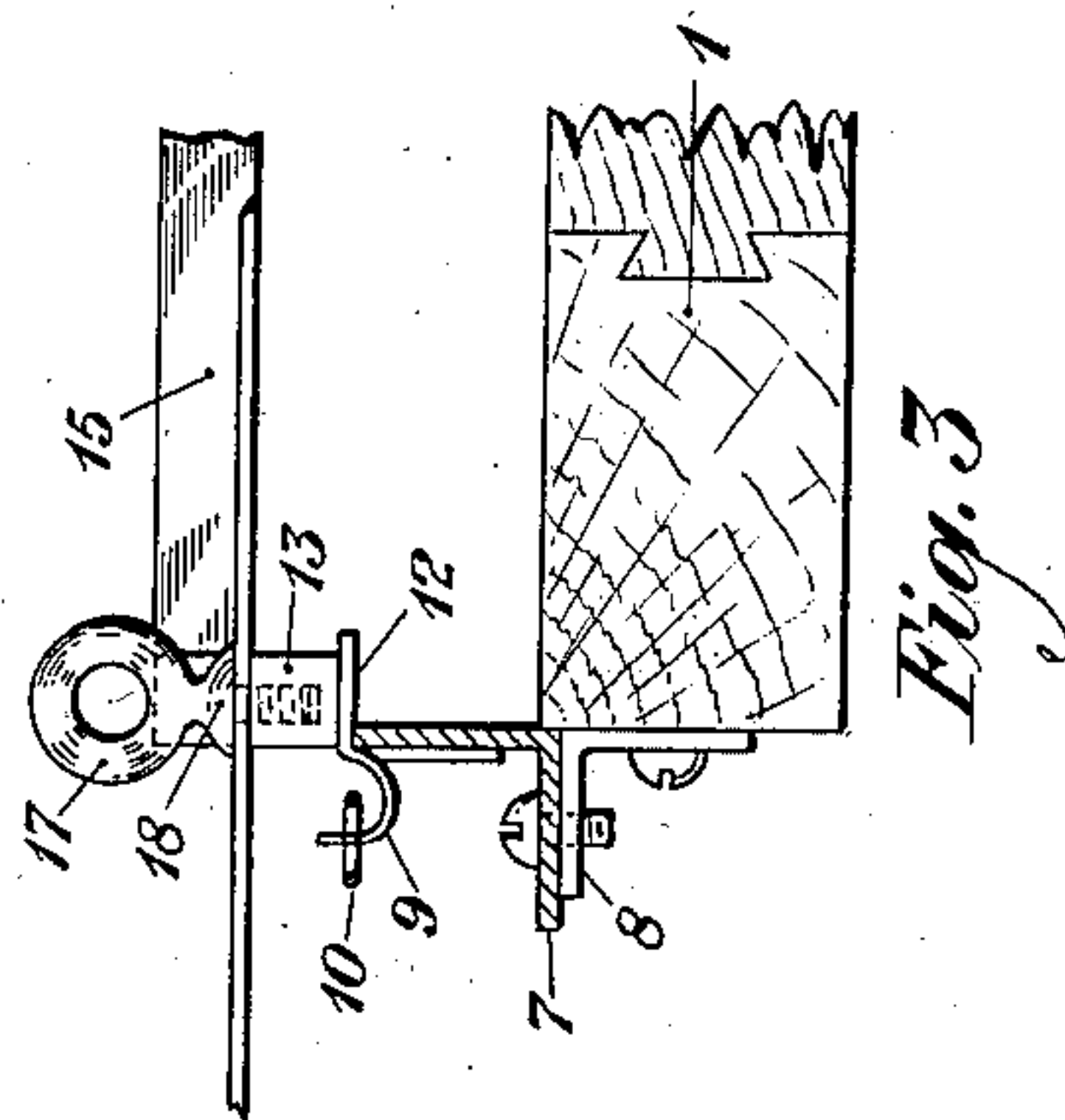
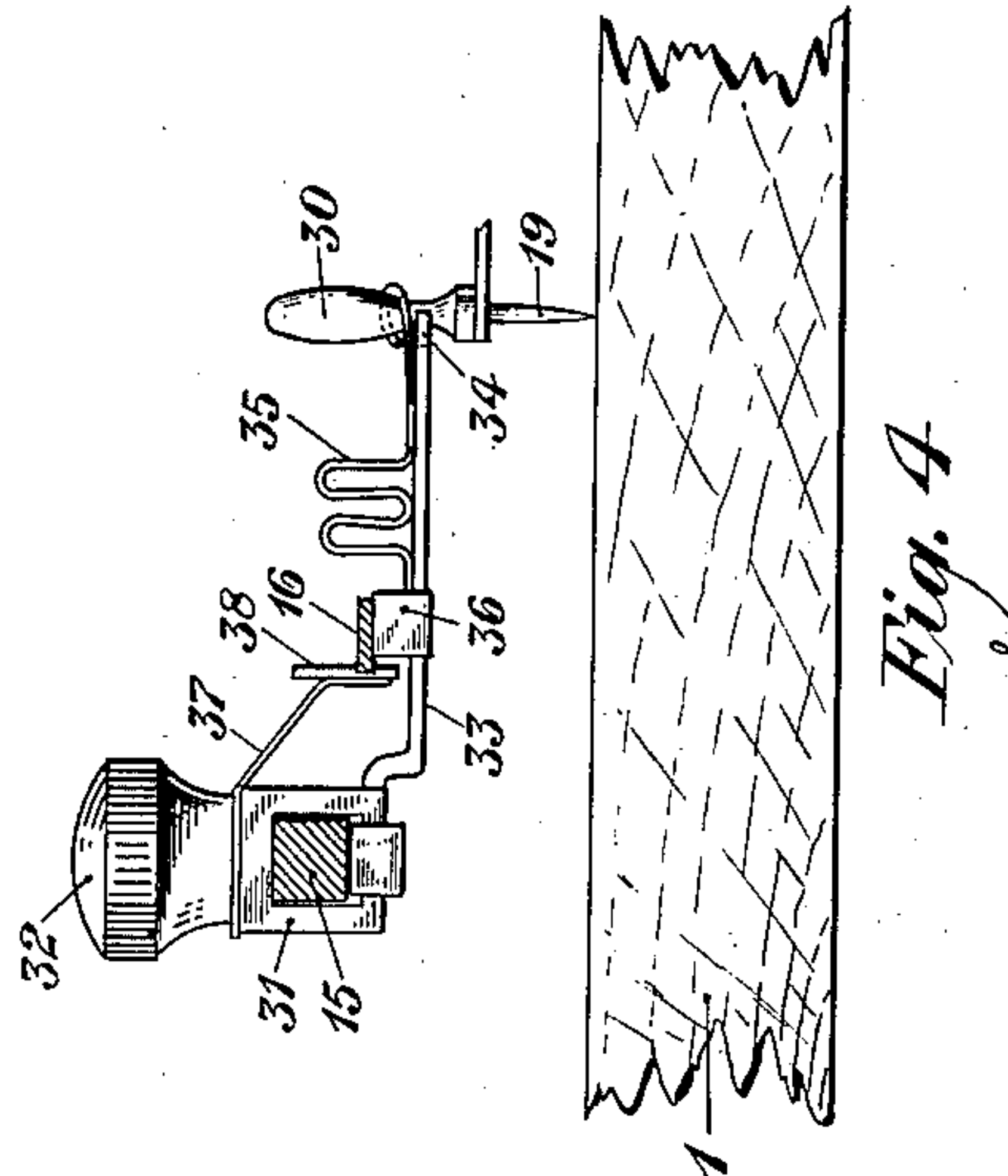
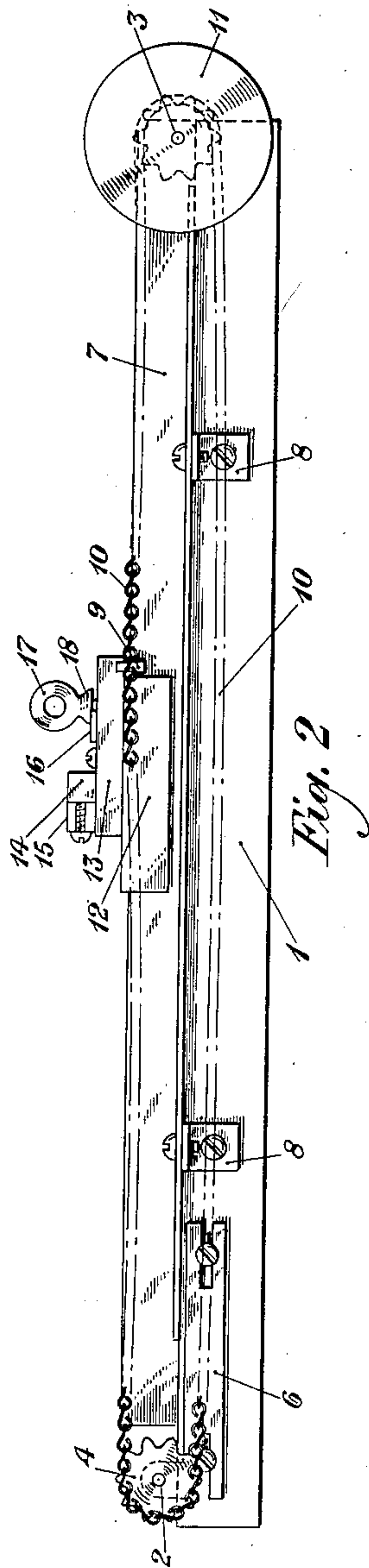
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4 Sheets-Sheet 4



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UNITED STATES PATENT OFFICE.

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METHOD AND APPARATUS FOR COMPUTATION.

Application filed December 30, 1925. Serial No. 78,470.

An object of my invention is to provide new and improved apparatus and a corresponding method to facilitate the computation of the amplitudes of various harmonics in a composite periodic curve. Another object of my invention is to provide apparatus and method for computing the coefficients of a Fourier series representing the variation of a given magnitude with time. Another object of my invention is to provide for utilizing a planimeter for rapid summation of lengths on a diagram in a certain way. These objects and various other objects of my invention will become apparent on consideration of an example of practice according to the invention. In the following specification I will disclose certain apparatus and a certain method in illustration of my invention. It will be understood that the following specification will relate to this particular example of the invention and that the invention will be defined in the appended claims.

Referring to the drawings, Figure 1 is a top plan view of apparatus that may be employed in the practice of my invention; Fig. 2 is an end elevation looking in the direction of the arrow 2 of Fig. 1; Fig. 3 is a detail sectional elevation taken on the line 3 of Fig. 1; Fig. 4 is another detail sectional elevation taken on the line 4 of Fig. 1; Fig. 5 is a diagram showing a comparatively simple curve and illustrating a step of my method in relation thereto; Fig. 6 is a scale in different positions to be considered relatively to Fig. 5; Fig. 7 is a form in which the steps of computation may be entered for the analysis of a curve such as that shown in Fig. 1; Fig. 8 is a diagram showing the amplitudes of various harmonics of said curve of Fig. 1 as determined by the practice of my invention; and Figs. 9 and 10 are special diagrams to illustrate certain steps in the performance of the method as disclosed.

Referring to Figs. 1 to 4, the drawing board 1 has guides 7 and 7' fixed on its ends by brackets such as 8. At their front ends these guides have holes forming bearings for the shaft 3, carrying the sprocket wheels 5 and 5' at its ends. Adjustable supports

6 and 6' on the ends of the board 1 also carry bearings for the parallel shaft 2 with the sprocket wheels 4 and 4' at its ends. The chains 10 and 10' connect the sprocket wheels 4, 5 and 4', 5', respectively. On one end of the front shaft 3 is a handwheel by which the shaft may be rotated, thus driving the chains 10 and 10'.

The shoe 12—13—14 slides on the guide 7, and there is a similar shoe on the guide 7'. These shoes are rigidly connected with the ends of a cross-bar 15. Each shoe has a lug 9 (or 9') engaging a link of the chain 10.

Each shoe member 13 carries a clamping screw with a handle 17 and a shoulder 18 beneath which is clamped the notched scale 16.

The curve to be analyzed is photographically enlarged so that its period length will be definite, say one foot. Such a curve 39 is shown on the sheet 20, fastened by thumb tacks 21 on the board 1.

The planimeter shown in Fig. 1 is of familiar design comprising the jointed arms 22 and 24, and with its fixed pivot under the weight 23. The planimeter wheel adapted to slide and rotate is shown at 25 on the shaft 26 with the scale 27 reading against the vernier 28; 29 is a worm wheel engaging a worm on the shaft 26 and adapted to count complete rotations of the wheel 25. The planimeter tracing stylus 19 is directly under the handle 30.

The cross-bar 15 carries a slider 31, which may be moved by means of the handle 32. Projecting from the slider is a bar 33 notched at its end 34, and the planimeter handle 30 lies in this notch and is held snugly by the spring 35. 36 is a spring on the bar 33, lying under the scale 16, and adapted to support it if it should tend to sag. Projecting from the slider 31 is a leaf spring 37 carrying a pin 38 adapted to engage a notch of the scale 16. The notches in the scale 16 are shallow and bevelled so that while the engagement of the pin 38 with one of them tends to hold the slider 31 at the corresponding position on the bar 15, nevertheless a push on the handle 32 along the bar 15 will disengage the pin 38 from a notch.

40 and 41 are two points of the curve 39.

just a period apart, and the sheet 20 is pinned on the board 1 so that the line joining these points is parallel with the bar 15; this may easily be determined by turning the handwheel 11 and moving the slider handle 32 to bring the planimeter tracing stylus 19 to the point 40; then motion of the slider 31 should carry the tracing stylus 19 to the point 41.

In Fig. 5 is shown a certain curve which may be taken as one period of a periodic curve. I will illustrate the use of the foregoing described apparatus by explaining how the coefficients for the fifth harmonic of this curve may be obtained. When I refer to a harmonic by number, I count the

fundamental as the first harmonic, for example, what I call the fifth harmonic is the fourth above the fundamental.

Fourier's series may be written in the familiar forms

$$y = A_0 + A_1 \cos x + A_2 \cos 2x + \dots + A_n \cos nx + \dots + B_1 \sin x + B_2 \sin 2x + \dots + B_n \sin nx + \dots \quad (1)$$

and

$$y = A_0 + \sqrt{A_1^2 + B_1^2} \sin \left(x + \tan^{-1} \frac{A_1}{B_1} \right)$$

$$+ \sqrt{A_2^2 + B_2^2} \sin \left(x + \tan^{-1} \frac{A_2}{B_2} \right) + \dots$$

$$\dots + \sqrt{A_n^2 + B_n^2} \sin \left(x + \tan^{-1} \frac{A_n}{B_n} \right) + \dots \quad (2)$$

and we seek the coefficients A_5 and B_5 in equation (1) for Fig. 5.

AF' is any line parallel to the x -axis of the curve corresponding to the line 40—41 of Fig. 1. There are various scales 16 for the various harmonics. The scale for the fifth harmonic as shown at 51 in Fig. 6 is placed in the machine as indicated at 16 in Fig. 1. The planimeter tracing stylus 19 is set at A, and the scale is moved lengthwise till its zero notch engages the pin 38. Then the scale is clamped at its ends by means of the screws 17. The planimeter is set at zero and the handwheel 11 is rotated, thus moving the planimeter stylus 19 up till it meets the curve at A'. Then the handwheel 11 is held fixed and the slider 31 is moved until its catch 38 engages the next notch 1 (in Fig. 5). Thus the planimeter tracing stylus 19 is carried along the line A' B in Fig. 5. Next the handwheel 11 is turned until the planimeter stylus 19 meets the curve as at B', the stylus 19 moving along the path B B' in Fig. 5. Then by a movement of the slider to notch 2 of the scale, the tracing stylus 19 goes along the path B' C. Similarly, the movement is continued along the full line path C C' D D' E E' F F', the point F' being on the line A F' parallel to the curve-axis.

Then the planimeter tracing stylus 19 is moved along the line A F' from F' to G as determined by notch 4½, half way between notches 4 and 5. Then the slider 31 is held and the scale is loosened and displaced to the left till notch 5 reengages the pin 38 on the slider, just as they previously engaged at position G. The new position of the scale is indicated at 52 in Fig. 6. Having clamped the scale in this new position, the tracing stylus 19 is moved up from G to G' and the dotted line path is traced back in the manner that will be well understood from what

has been said, G G' H H' K K' L L' A.

If it should be known that harmonics of order higher than the fourteenth are absent or negligible, then the final reading of the planimeter at A divided by twice the length of the base line AF' will be the value of the coefficient A_5 . If harmonics higher than the fourteenth are present and not negligible, then the reading of the planimeter will be subject to a "correction" that will be explained presently.

To get the coefficient B_5 the stylus 19 is held at A and the scale is again loosened and shifted to the right of its original position a quarter of the distance between the usual scale divisions, that is, to the position shown at 53 in Fig. 6, and a new start is made from A, going first to the initial notch now at A' and thereafter the procedure is as described above from this point until the planimeter tracer is brought back to A, and (subject to the same possible need for correction) the planimeter reading divided by twice the length of the base line AF' will be the value of B_5 .

Specifically to illustrate the practice of my invention, I show in Fig. 7 a form used in the analysis of oscillograph records of voice curves. In these cases the harmonics are of considerable magnitude and importance, and Fig. 7 is designed to deal with all the harmonics as high as the thirtieth. The procedure is to get the thirtieth harmonic first and the lower harmonics successively in decreasing order. Hence the scale 16 for the thirtieth harmonic is used first. The procedure is similar to that already described for the fifth harmonic. It is assumed that harmonics higher than the thirtieth are negligible. Therefore the value +0.07, which is the final planimeter reading at A, is entered directly in Fig. 7. Repre-

sent this planimeter reading by a_{30} ; it must be divided by twice the length l of the base 40—41 (in Fig. 1) to give A_{30} . In general

$$a_n = A_n \times 2l \quad (3)$$

and similarly

$$b_n = B_n \times 2l \quad (4)$$

With l equal to 12 inches and the planimeter reading in inches

$$a_n = 24A_n \text{ and } b_n = 24B_n \quad (5)$$

Since we are concerned with the relative amplitudes of the harmonics, the direct planimeter readings a are entered in the table Figure 7, and the value $a_{30} = +0.07$ will be found in the column headed " a " and on the line marked "30" at the left. Similarly for the value $b_{30} = +0.02$.

Having determined the thirtieth harmonic, the scale 16 therefor is replaced by the scale for the twenty-ninth harmonic and the values $a_{29} = +0.34$ and $b_{29} = +0.14$ are obtained and entered as shown in Fig. 7. This procedure is repeated for the lower harmonics in descending order and eventually the values -1.85 and -2.65 are obtained and entered for a_{11} and b_{11} , respectively.

For harmonics from the thirtieth to the sixteenth inclusive, I employ 15 respective scales. For the fifteenth harmonic, I use every other notch of the scale for the thirtieth, for the fourteenth every other notch of the twenty-eighth, and so on.

Proceeding in the same way using the proper scale for the tenth harmonic, the planimeter readings are -7.30 and -0.98 but I designate these a_{10}' and b_{10}' because they are subject to correction as compared with a_{10} and b_{10} . In general, any planimeter readings a_n and b_n are subject to correction according to the equations

$$a_n = a_n' - a_{10} - a_{11} - a_{12} - a_{13} - a_{14} - a_{15} \quad (6)$$

$$b_n = b_n' + b_{10} - b_{11} - b_{12} - b_{13} - b_{14} - b_{15} \quad (7)$$

It will be seen that each planimeter reading is to be corrected by higher harmonic amplitudes, the lowest of which has an order three times the one for which the planimeter readings are taken. Accordingly the lowest order correction for the thirtieth harmonic is the ninetieth, for the twenty-ninth it is the eighty-seventh and so on, and for the eleventh it is the thirty-third. Having assumed that all harmonics higher than the thirtieth are negligible, no corrections have been made for the planimeter readings hitherto. In other words, Equations (6) and (7) reduce to $a_n = a_n'$ and $b_n = b_n'$, nearly enough for practical purposes for $n = 11, 12, \dots, 29, 30$. But for $n = 10$, the Equations (6) and (7) become $a_{10} = a_{10}' - a_{30}$ and $b_{10} = b_{10}' + b_{30}$, for practical purposes. Accordingly the planimeter readings -7.30 and -0.89 obtained for a_{10}' and b_{10}' , respectively, are entered on line 10 under the respective column headings a' and b' . The

proper "corrections" are copied in the adjoining spaces and the "corrected" values are entered in the columns headed a and b .

Proceeding similarly, the columns a' and b' and a and b are filled on up through for the seventh harmonic. For the practical corrections for the sixth harmonic Equations (6) and (7) reduce to

$$\begin{aligned} a_6 &= a_6' - a_{18} - a_{24} \\ b_6 &= b_6' + b_{18} - b_{24} \end{aligned}$$

nearly enough for practical purposes. The resultant corrections $+0.23$ and -0.27 are computed as indicated in the upper right corner of Fig. 7 and carried over to the indicated places adjacent the planimeter readings with which they are combined to give the corrected values -3.27 and $+0.08$ in the columns a and b . Similarly, the columns a' , b' , a and b of the table of Fig. 7 are filled on up through the values for the third harmonic, using the corrections computed as indicated at the right of Fig. 7. For the third harmonic I use the scale for the twenty-fourth and move the slider 8 notches at a time.

For the second and first harmonics, a similar procedure could be used, but this would place the point L' of Fig. 6 at an inconvenient distance. Accordingly I follow a special procedure, using intermediate notches on the scale for the sixteenth harmonic as shown in Figs. 9 and 10, which will be readily understood from the following equations on Fig. 9. Planimeter readings $M M' O O' M'' M + Q Q' S S' Q'' Q = \frac{1}{2} a_2'$, planimeter readings $N N' P P' N'' N + R R' T T' R'' R = \frac{1}{2} b_2'$, and on Fig. 10, planimeter reading $M M' Q''' Q' M''' M = \frac{1}{2} a_1'$, planimeter reading $O'' O' S'' S' O''' O'' = \frac{1}{2} b_1'$.

Having filled the columns for a and b , the intermediate columns for a^2 , b^2 , $a^2 + b^2$ and $\sqrt{a^2 + b^2}$ are computed by the aid of a slide rule. This gives us a set of numbers proportional to the coefficients in Equation (2), that is proportional to the amplitudes of the various harmonics. One such number, 3.23, is shown, for example, in Fig. 7 for the eleventh harmonic. All these resultant numbers are plotted as ordinates in Fig. 8.

The absolute term A_0 , if desired, may be obtained by measuring y_0 (Fig. 5) in inches, doubling to get $2y_0$, subtracting a_1 , and adding the correction for a_1 (indicated by the encircled "6" at the bottom of Fig. 7). The result, $a_0 = 24A_0$.

The method and apparatus herein disclosed will serve to obtain the harmonics of a composite periodic function with much less labor than by direct numerical computation. By my invention a practiced operator can get all the harmonics to the thirtieth in about three hours whereas by numerical computation several days would be required. The results by my method are accurate to 1

percent or better, depending on the accuracy and sharpness of the enlarged curve to be analyzed.

I claim:

1. A harmonic analyzer comprising a board adapted to receive an impression of the curve to be analyzed, a planimeter on the board, a bar across the board parallel with the curve axis, means to move the bar transversely of its length, and a slider on the bar engaging the planimeter tracing stylus.

2. A harmonic analyzer comprising a board adapted to receive an impression of the curve to be analyzed, a planimeter on the board, a bar across the board parallel with the curve axis, means to move the bar transversely of its length, and interchange-

able scales with notches to be engaged by the slider to determine steps of movement thereof along the bar.

3. A harmonic analyzer comprising a board adapted to receive an impression of the curve to be analyzed, a planimeter on the board, a bar across the board parallel with the curve axis, two rotatable shafts on respectively opposite edges of the board parallel with the said bar and carrying sprocket wheels on their ends, two chains on said sprocket wheels, and a slider on the bar engaging the planimeter tracing stylus.

In testimony whereof, I have signed my name to this specification this 29th day of December, 1925.

WALTER KOENIG, JR.