Feb. 1, 1927. **1,615,911**

H. NYQUIST

APPARATUS AND METHOD FOR REGULATING ATTENUATION AND PHASE ON TRANSMISSION LINES

Filed Feb. 29, 1924 4 Sheets-Sheet 1

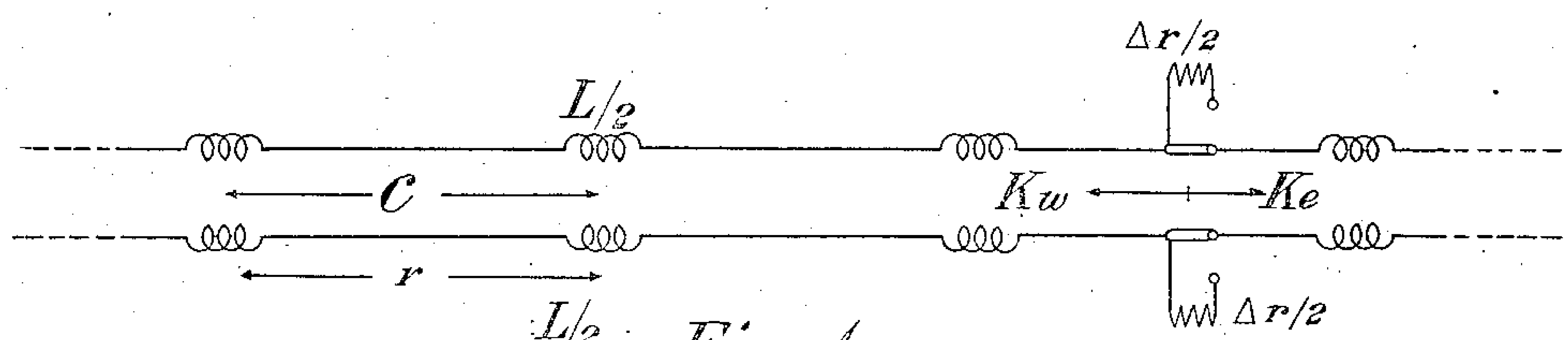

Fig. 1

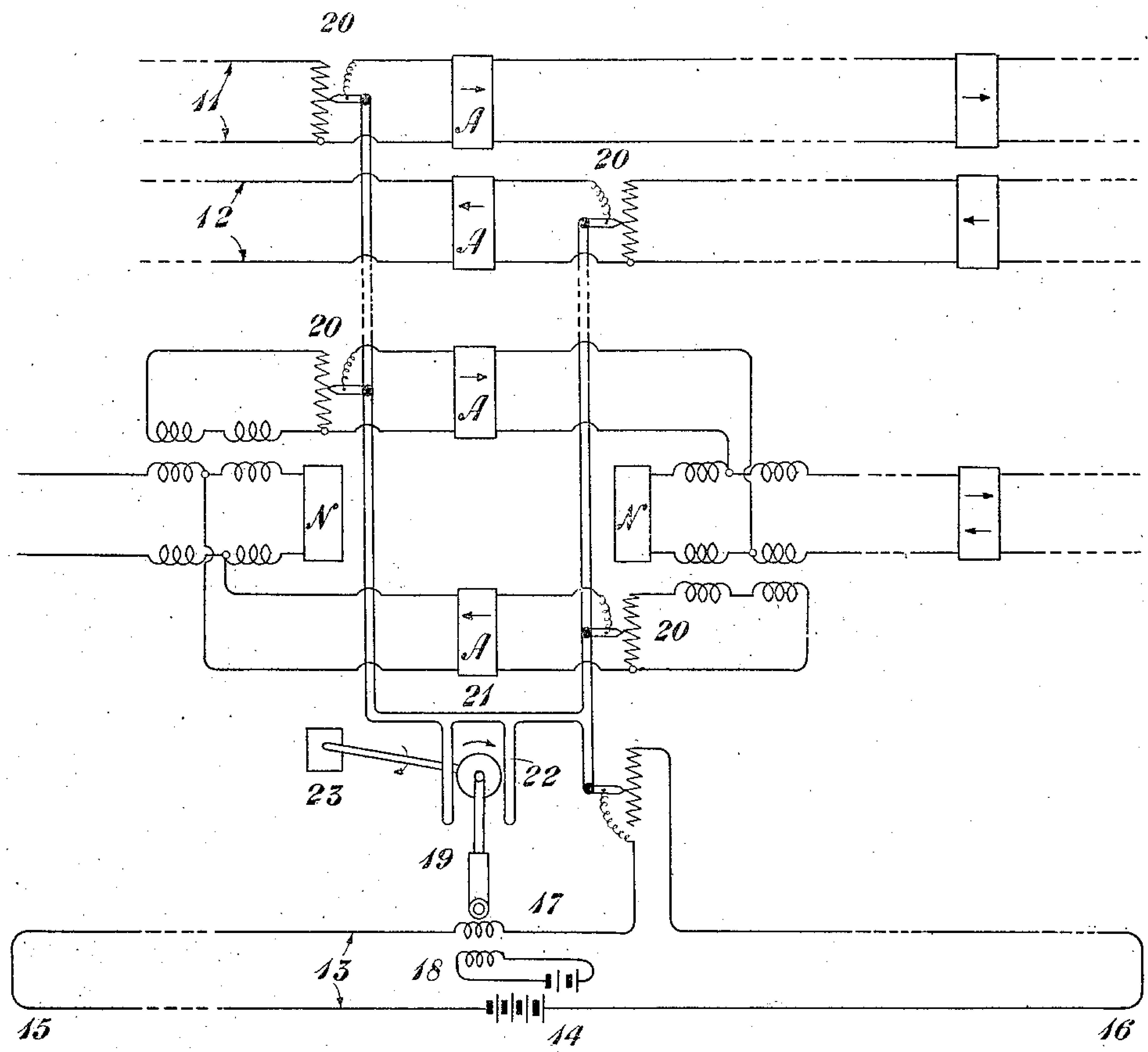

Fig. 2

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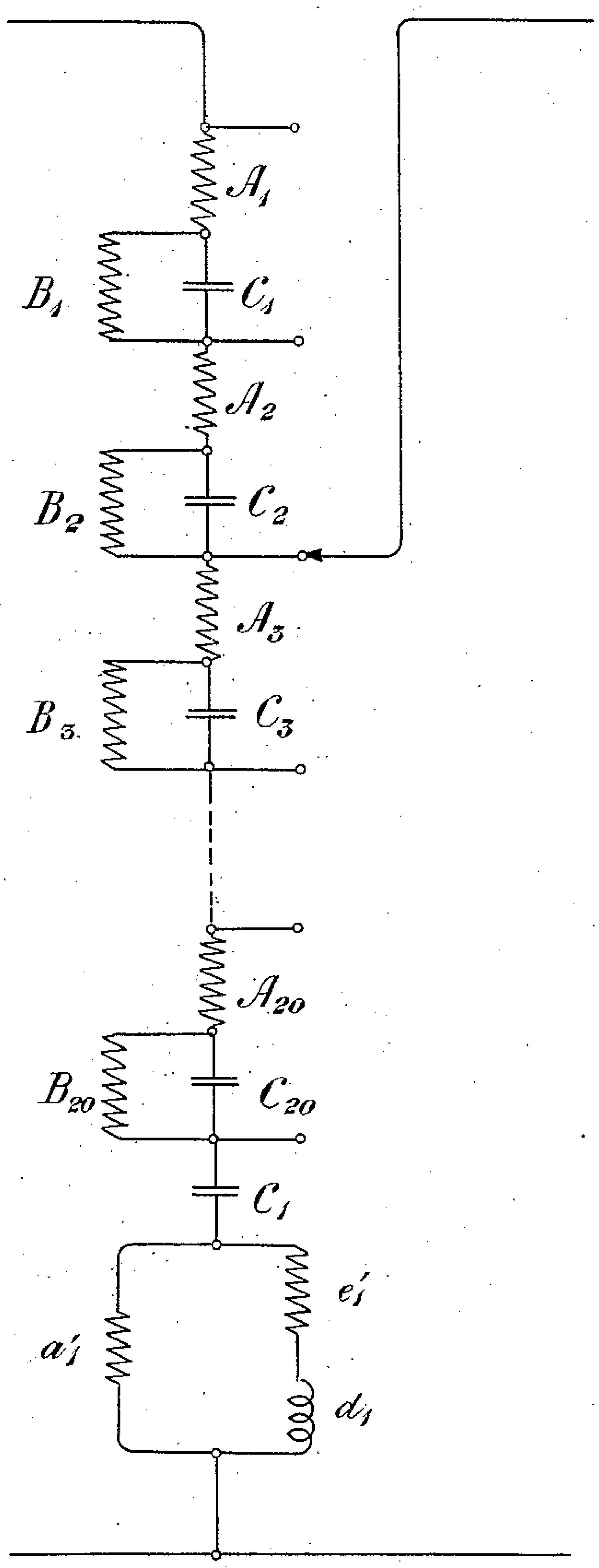

Fig. 3

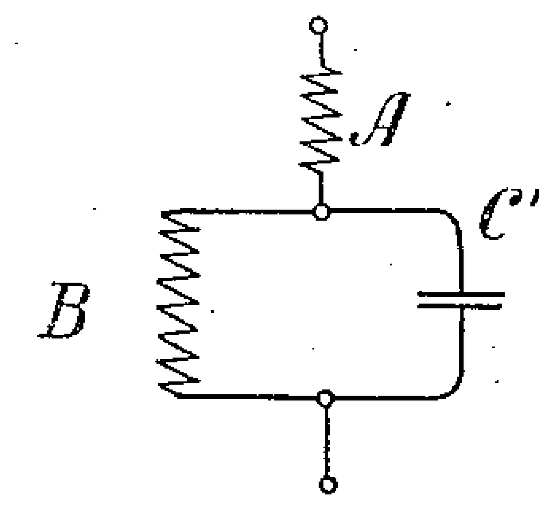

Fig. 4

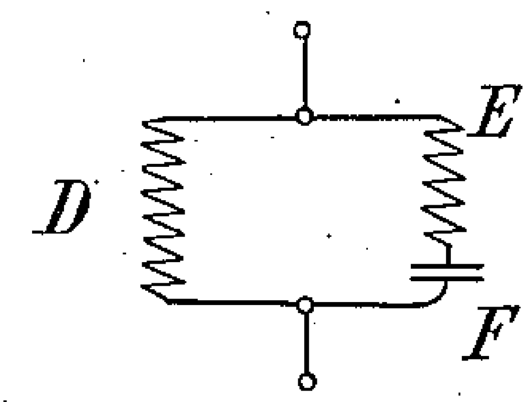

Fig. 5

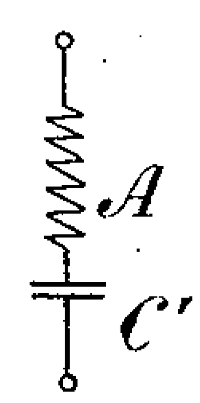

Fig. 7

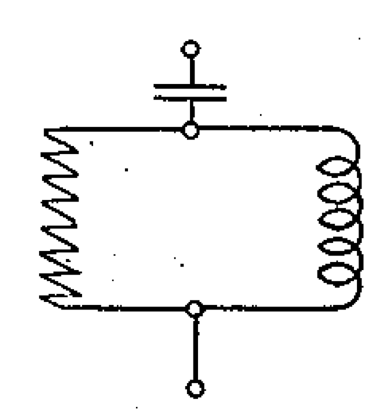

Fig. 9

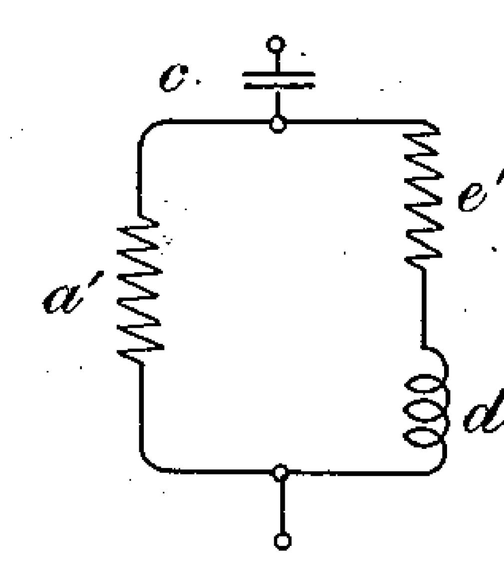

Fig 10

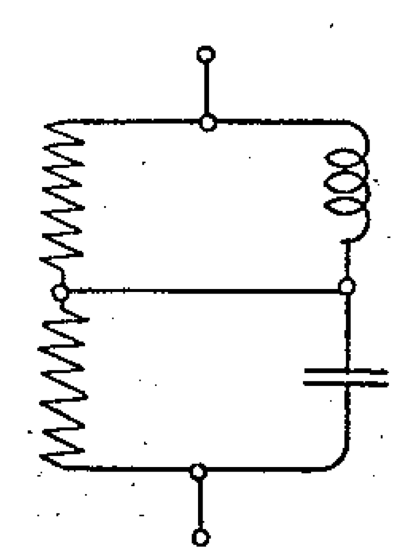

Fig 11

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APPARATUS AND METHOD FOR REGULATING ATTENUATION AND PHASE ON TRANSMISSION LINES

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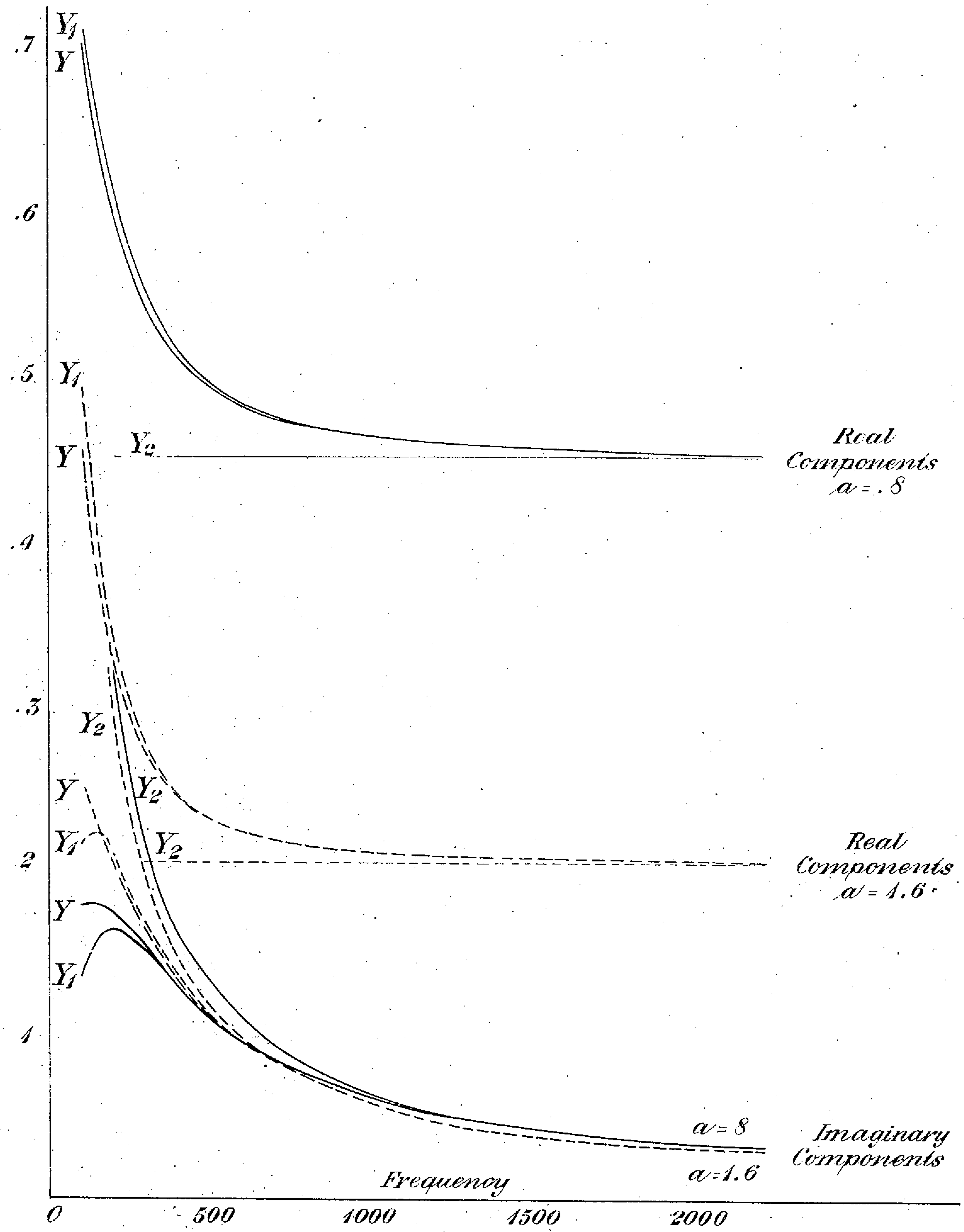

Fig. 6

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APPARATUS AND METHOD FOR REGULATING ATTENUATION AND PHASE ON TRANSMISSION LINES

Filed Feb. 29, 1924 4 Sheets-Sheet 4

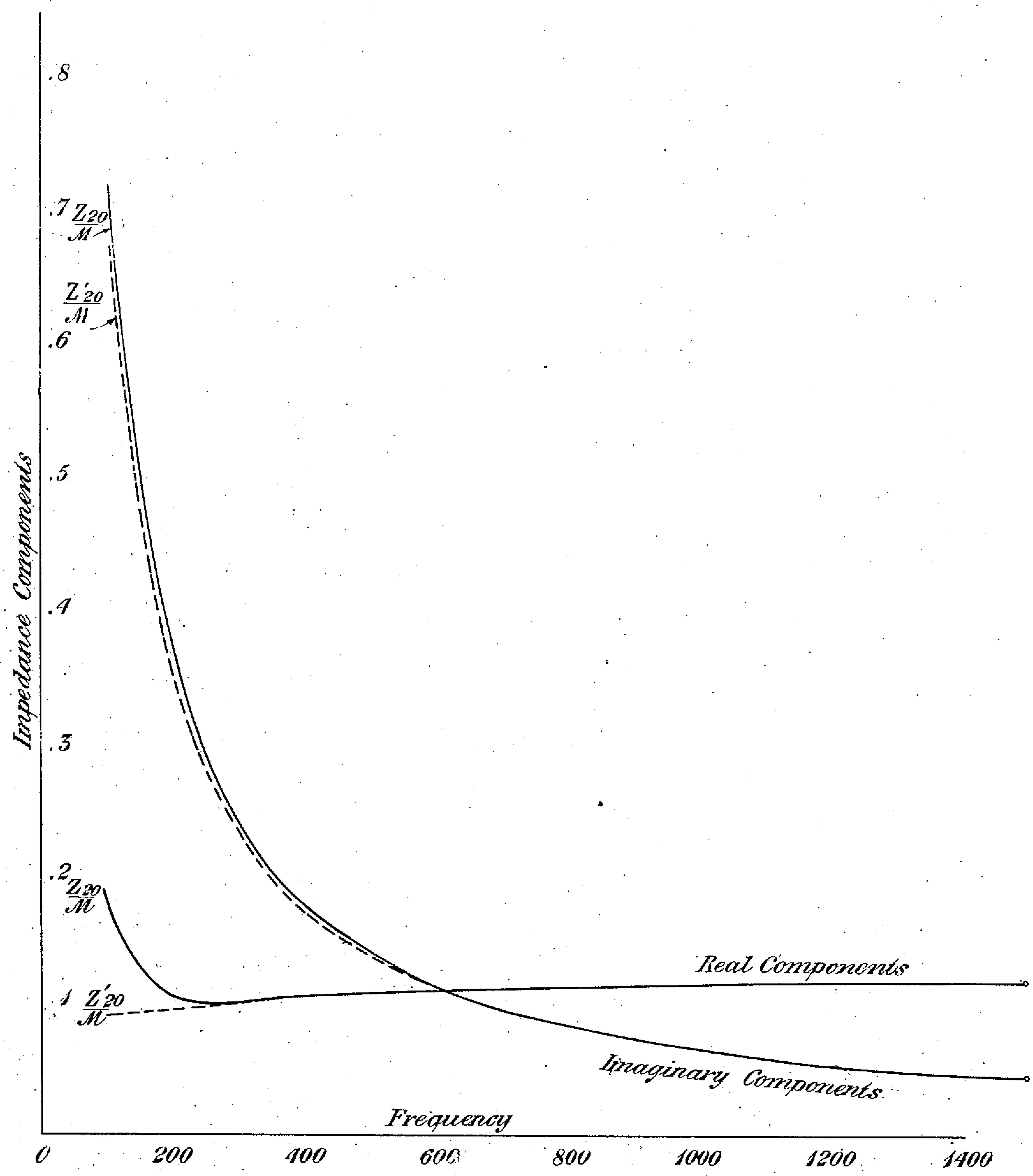

Fig. 8

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Patented Feb. 1, 1927. **1,615,911**

UNITED STATES PATENT OFFICE.

HARRY NYQUIST, OF JACKSON HEIGHTS, NEW YORK, ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

APPARATUS AND METHOD FOR REGULATING ATTENUATION AND PHASE ON TRANSMISSION LINES.

Application filed February 29, 1924. Serial No. 696,049.

It is an object of my invention to provide a new and improved transmission system adapted to maintain the current intensity and the phase relations substantially constant at the receiving station over a considerable frequency range. Another object of my invention is to provide for keeping constant phase relations at the receiving station in a transmission system notwithstanding variations of resistance due to temperature changes or other causes on the line. These and various other objects of my invention will become apparent on consideration of a limited number of examples of the invention which I have chosen for illustration and which I now proceed to disclose in the following specification taken with the accompanying drawings. It will be understood that the following disclosure relates to these examples of the invention and that the scope of the invention will be defined in the appended claims.

Referring to the drawings, Figure 1 is a diagram of a loaded line to which reference will be made in discussing the principles involved in my invention; Fig. 2 is a diagram of a transmission system embodying my invention; Fig. 3 is a diagram of a potentiometer employed in the system of Fig. 2; Figs. 4 and 5 are diagrams of impedance combinations that may be considered for the steps of potentiometer; Fig. 6 shows impedance as a function of frequency for the combinations of Figs. 4 and 5; Fig. 7 is another impedance combination alternative to Figs. 4 and 5; Fig. 8 shows impedance as a function of frequency for the main part of the potentiometer and Figs. 9, 10 and 11 show special alternative designs for this main part.

On an alternating current transmission line, as the resistance changes due to change of temperature, it is well known that the attenuation also changes, and moreover, if there are various frequency components in the current, their phase relation at the receiving end will be different for different resistances. It is an object of my invention to provide means for adjustment, so that whatever the changes in resistance on the line within normal limits, the received currents will always come in with a certain intensity and with unchanging phase relations among their components. It is important to maintain local sources of current at the receiving end that shall be accurately in step with the respective components of the received carrier current, and to this end it is important that those components shall not vary in relative phase with changes in resistance on the line.

Fig. 2 shows a transmission system with a plurality of repeaters at a station. A 22-type repeater is shown with the balancing networks N. In all of these repeaters the amplifying elements are designated A. The pair of conductors 11 is one pair of a four-wire line on which the transmission is from west to east, and the corresponding pair for transmission from east to west is designated 12. On each of these pairs there may be a plurality of carrier currents of respective different frequencies and on each of these frequencies there may be several message channels, with phase discrimination and magnitude discrimination.

The foregoing example indicates the importance of transmitting with constancy of current intensity and constancy in the phase relations at all frequencies at the receiving end. To provide an adjustment by which this constancy may be secured, I appropriate one conductor pair 13 in the cable in which the conductors pairs 11 and 12 are found. At the repeater station shown in Fig. 2, a direct current battery 14 is introduced in one member of the pair 13 and at a suitable distance each way, as at 15 and 16, the conductors 13 are interrupted and their ends joined together so as to form a complete circuit on the battery 14 through the winding 17 of a relay at the same station with the battery 14 as shown in the drawing.

The points 15 and 16 may be at any suitable distance, which may be several times the interval between two consecutive repeater stations on the line.

Whatever the change in the resistance of the conductors 11 and 12, there will be a corresponding change in the resistance of the conductors 13, in the same cable with 11 and 12. The change of resistance in the conductors 13 will produce a change of the current in the circuit of the battery 14. By means of the biasing winding 18, a normal neutral position will be established for the relay armature 19.

On the input side of each repeater amplifier at the station is a potentiometer 20, and the movable contacts of all the potentiometers 20 are mechanically connected to be adjusted by displacement of the crosshead 21. The wheel 22 is rotated continuously by the motor 23 and is adapted to be displaced by the armature of the relay 19, so that one way it engages to move the crosshead 21 one way, and for displacement of the armature 19 the other way, the wheel 22 engages to move the crosshead 21 the other way. Thus it will be seen that any changes in the resistance of the pilot conductors 13, such as due to temperature changes, lead to a change in the current flowing through the relay having the winding 17, and this relay adjusts the potentiometers so as to bring the current back to the normal value in the conductors 13, and at the same time the relay adjusts all the input potentiometers 20 for the repeater amplifiers at the station.

These potentiometers 20 are designed in such a way that at all adjustments they keep the attenuation constant over the length of line from 15 to 16, and they keep the phase displacements constant for all essential frequencies in the transmission between the points 15 and 16.

Similar adjustment is provided on other lengths of line in succession with the stretch 15—16, so that the constant attenuation and unvarying phase relation are preserved over the whole length of the line.

A suitable potentiometer is indicated in Fig. 3 and I will now show how I arrive at this design and how I determine the resistance and reactance values to be assigned to the various elements of this potentiometer.

Consider the loaded line shown in Fig. 1, in which the capacity per loading section is C, the inductance per loading section is L and the resistance per loading section is r. Assume that in one of the loading sections a small additional resistance Δr is inserted and let us investigate the effect on the received current. Assume that the impedance looking one way is K_E and looking the other way is K_W. The total impedance before inserting the resistance r is K_E+K_W and the total impedance after inserting the resistance is

$$K_E+K_W+\Delta r.$$

Hence the resultant effect on the received current will be given by the complex ratio R, where

$$R=\frac{K_E+K_W}{K_E+K_W+\Delta r} \quad (1)$$

To a close approximation, since Δr is relatively small, it follows that

$$\log R=-\frac{\Delta r}{K_E+K_W} \quad (2)$$

The valve of K_E+K_W varies along the loading section, being a minimum at the loading coil and a maximum at the midpoint of the section. Assume that the increment Δr, instead of being lumped as in Fig. 1, is distributed uniformly over the loading section; then instead of equation (2), we shall have

$$\log R=-\int_0^{\Delta r}\frac{dr}{K_E+K_W} \quad (3)$$

To a fairly close approximation, this may be written

$$\log R=-\frac{\Delta r}{2\sqrt{\frac{L}{C}\left(1+\frac{r}{ipL}\right)}} \quad (4)$$

and if there is an increment of resistance Δr in each of n loading sections, the corresponding equation is

$$\log R=-\frac{\Delta nr}{2\sqrt{\frac{L}{C}\left(1+\frac{r}{ipL}\right)}} \quad (5)$$

wherein Δnr is the total increment of resistance in all the sections. The real part of equation (5) gives the total attenuation in napiers and the imaginary part gives the total phase change in radians.

To get a practical idea of the actual changes involved in such a case as that under consideration, consider a medium heavy loaded 19-gauge side circuit with 1,000 loading sections and assume that the resistance increment is 1 ohm per loading section, that is, assume that $\Delta r=1$. Reasonable values under the foregoing assumptions will be that $L/C=1540^2$; $r=100$ (approximately); and $L=0.175$. Substituting these values, equation (5) becomes

$$\log R=-\frac{1000}{3080\sqrt{1+\frac{100}{ip0.175}}} \quad (6)$$

Assigning to p values corresponding to frequencies of 500, 1,000 and 1,500, we get for the respective phase shift 1° 39′, 0° 51′, and 0° 34′.

From these values it is apparent that in the case of medium heavy loaded circuits the phase displacement is probably small enough so that it can be taken care of by manual adjustments. But for an extra light loaded 19-gauge side circuit the case is different; here $L/C=780^2$, $r=100$, and $L=0.045$. Proceeding in the same way as for the case of the medium heavy loaded 19-gauge side circuit, we find that for frequencies of 500, 1,000 and 1,500, the phase shifts are, respectively, 10° 3′, 6° 2′, and 4° 11′. These values are considerably greater than those obtained for medium heavy loading, and so great as to make manual adjustments quite impracticable.

Referring to the potentiometers 20 of Fig. 2, it will be seen that these face amplifiers are of the audion type, and have very high input impedances; hence the current drawn from a potentiometer to its amplifier is negligible and the voltage impressed on the amplifier is proportional to the impedance of the part of the potentiometer within the amplifier input terminals. Let this impedance at any adjustment be denoted by Z. The effect on the input voltage of the amplifier due to a change ΔZ may be given by the complex ratio

$$R'=\frac{Z+\Delta Z}{Z} \quad (7)$$

and since ΔZ is small compared to Z, it is approximately true that

$$\log R'=\frac{\Delta Z}{Z} \quad (8)$$

An increment of resistance Δnr on the line would decrease the received current in a certain rato R as expressed in equation (5). It is desired to increase the amplifier input voltage in the reciprocal ratio so as to compensate and keep the current constant. The condition for this is that $R=1/R'$ or $\log R=-\log R'$ and accordingly from equations (5) and (8) it follows that

$$\frac{\Delta Z}{Z}=\frac{\Delta nr}{2\sqrt{\frac{L}{C}\left(1+\frac{r}{ipL}\right)}} \quad (9)$$

Further, to facilitate design and within the freedom of choice afforded by the foregoing difference equation, we impose the condition that the top steps of the potentiometer 20 shall approach pure resistances,—in other words, that the last few increments ΔZ shall approach a real value. This gives for the value of the impedance of the potentiometer at its top tap

$$Z_0=M\left(1+\frac{r}{ipL}\right)^{1/2} \quad (10)$$

where M is an arbitrary real constant. Putting equation (9) in the form of a differential equation and solving, and making use of the boundary condition afforded by equation (10), the result is obtained that

$$\frac{Z}{M}=\left(1+\frac{r}{ipL}\right)^{1/2} e^{-(1/2)\Delta nr(L/C)^{-1/2}\left(1+\frac{r}{ipL}\right)^{-1/2}} \quad (11)$$

The assumption was made that the top few steps of the potentiometer are real. But having passed to a differential equation, these become infinitesimal, and the top finite step in Fig. 3 is seen to be other than a pure resistance, though its reactance component is small as will appear presently.

If Z is chosen according to equation (11) for all the steps, the regulating network will compensate for attenuation changes and phase changes at all frequencies. In equation (11) the increments Δnr of resistance on the pilot circuit are counted from the condition corresponding to the uppermost tap of the potentiometer 20. From equation (10) it is apparent that M is the limiting value approached by the impedance of the whole potentiometer at high frequencies. In what follows we shall assume that Δnr is the difference between the resistance of the whole regulator circuit 13 at maximum temperature and at the temperature under consideration.

By putting p equal to a large value in equation (5), it will be apparent that the expression

$$\frac{\Delta nr}{2\sqrt{L/C}}$$

will equal the whole difference in attenuation for a high frequency; accordingly, this expression will be denoted by the symbol a. Also, for convenence, the expression r/L will be denoted by the symbol b. Equation (5) then takes the form

$$\log R=-a\left(1+\frac{b}{ip}\right)^{-1/2} \quad (12)$$

While b may be determined from the measurements of r and L by taking their ratio, it may be preferable to determine b otherwise as outlined below and thereby compensate for approximations involved in deriving equation (12). To do this, measure the attenuation and phase change for two temperatures and for a number of frequencies, including some rather high values of frequency, where b is small in comparison with p. From these measurements compute the values for the complex ratio R. The value of the logarithm of R is obviously equal to $-a$ for large values of p. Substituting the values for R and a in equation (12) and solving for b, the result is obtained.

$$b=ip\left[\left(\frac{-a}{\log R}\right)^2-1\right] \quad (13)$$

If these values for b are computed and plotted as a function of p, they will give a substantially horizontal line.

Owing to the manner in which b enters equation (12), it will be apparent that an error in the choice of b has the same effect as a corresponding error in p. For this reason, it follows that if the computations are carried through, the final networks computed can be corrected by multiplying all the condensers and inductances by a suitable factor and leaving the resistances unchanged.

Substituting the values of a and b in equation (11), it becomes

$$\frac{Z}{M}=\left(1+\frac{b}{ip}\right)^{\frac{1}{2}}e^{-a(1+b/p)^{-1/2}} \quad (14)$$

This gives the total impedance for the potentiometer on any tap. At the top tap a should be zero. Assuming that the potentiometer steps are to be in so-called standard miles at high frequencies, and that we are dealing with the extra light loaded 19-gauge side circuit heretofore specified, a should have the value 0.109 at the second tap, 0.218 at the third tap, 0.327 at the fourth tap, etc.

An obvious procedure to compute the individual impedance steps would be to compute the values of Z from equation (14) for any two adjacent steps and take the difference of the two results. It will make the computation simpler and lead to substantially the same result to obtain the first derivative of equation (14) with respect to a at the midpoint of the step, and multiply that derivative by the increment of a, which is 0.109. This gives for the increment corresponding to any one step

$$\frac{\Delta Z}{0.109M}=e^{-a(1+b/ip)^{-1/2}} \quad (15)$$

Here a is to be given the values 0.109 x 0.5, 0.109 x 1.5, 0.109 x 2.5, etc.

To design a simple network to simulate the impedance ΔZ represented by equation (15), we proceed to develop that equation as a power series in $\frac{1}{ip}$. This can be done in the usual manner by taking successive derivatives starting with the expression given in equation (15), and the result is obtained

$$\frac{\Delta Z}{0.109M}=e^{-a}\left(1+\frac{ab}{2}\cdot\frac{1}{ip}+\frac{ab^2}{8}(3-a)\frac{1}{p^2}+\cdots\cdots\right) \quad (16)$$

The simplest networks that will reproduce this impedance correctly to three terms are two, and each of them consists of one condenser and two resistances. The first of these is made up of a resistance A in series with a parallel combination comprising another resistance B and a capacity C′, as shown in Fig. 4. The other net work will consist of a resistance E and capacity F in series, this combination being connected in parallel with another resistance D, as shown in Fig. 5. The impedance expressions for these net works can each be written out in a power series corresponding to that of equation (16), and the first three terms of each series can then be respectively equated to the corresponding terms in equation (16); the resulting equations may be solved to obtain the values for A, B and C′ or D, E and F, as the case may be. Carrying this work through, the following results are obtained:

$$A=e^{-a} \quad (17)$$

$$B=\frac{2a}{3-a}e^{-a} \quad (18)$$

$$C'=\frac{2}{ab}e^{a} \quad (19)$$

$$D=\frac{3+a}{3-a}e^{-a} \quad (20)$$

$$E=\frac{3+a}{2a}e^{-a} \quad (21)$$

$$F=\frac{8a}{b(3+a)}2e^{a} \quad (22)$$

Substituting the values for a and the value for b in these expressions, it becomes possible to compute the corresponding network elements as they are designated in Figs. 4 and 5. To get an idea as to how closely the resulting networks obtained from equations (17) to (22) would simulate the ideal networks specified by equation (16), computations have been carried out for two values of a, namely 0.8 and 1.6, using the value b=100/0.045, which is approximately correct for extra light loaded 19-gauge side circuits. The results are shown in Fig. 6. The curves marked Y are the ideal curves required by equation (16). The curves marked Y_1 are obtained by using the three element networks of Fig. 4 or Fig. 5 as defined by equations (17) to (22). The curves marked Y_2 are obtained by making use of a two element network as shown in Fig. 7, whose elements are determined by equations (17) and (19). It will be apparent from Fig. 6 that for the important part of the frequency range, the three element network of Fig. 4 or Fig. 5 gives very close approximation to the ideal of equation (16).

Of the two designs shown in Figs. 4 and 5, that of Fig. 4 gives more constant values for the capacity C′, and accordingly it is adopted and embodied in Fig. 3, and equations (17) to (19) are utilized to compute the elements A, B and C′ of each section. A convenient value of the arbitrary constant M is 20640. This value has been used with results as follows:

Step.	A. Ohms.	B. Ohms.	C′. M. F.
1	2130	78.8	7.75
2	1910	220	2.88
3	1710	342	1.93
4	1540	448	1.54
5	1380	538	1.33
6	1240	617	1.22
7	1110	685	1.15
8	994	745	1.11
9	890	797	1.09
10	799	842	1.09
11	716	884	1.10
12	642	922	1.12
13	576	958	1.15
14	517	995	1.18
15	463	1030	1.23
16	415	1070	1.28
17	372	1110	1.34
18	334	1170	1.41
19	300	1230	1.49
20	269	1310	1.58

From the expression $3-a$, which occurs in some of the equations (17) to (22), it will be seen that these equations break down where $a=3$ and beyond this point, for the reason that the expressions will then call for negative values for either the resistance or the capacity. However, this is not a serious objection because the value $a=3$ corresponds to $27\frac{1}{2}$ standard miles, which is a greater range than it will be necessary to cover.

Having determined the design of the steps of the potentiometer between the taps, it remains to determine the design of the main portion that is always included in the output circuit. We have assumed that the number of steps beyond this main part is 20. We turn to equation (14) and substitute for a the value 0.109 x 20, or 2.18. Equation (14) then becomes

$$\frac{Z_{20}}{M}=\left(1+\frac{b}{ip}\right)^{1/2}e^{-2.18(1+b/ip)^{-1/2}} \qquad (23)$$

The same value of b heretofore mentioned, namely 100/0.045 has been used to evaluate equation (23) and the result is plotted in full lines in Fig. 8. From these full line curves it is inferred that networks of the type shown in one of Figs. 9, 10 and 11 could be made to simulate this impedance closely. They have all been investigated carefully and Fig. 10 has been adopted as most satisfactory. The procedure for Fig. 10 will be given and may be taken as an example for all three Figs. 9, 10 and 11.

To obtain the numerical values for the elements of the network of Fig. 10, equation (23) is first utilized to express $\frac{Z_{20}}{M}$ as a power series in $\frac{1}{ip}$ with the following result:

$$\frac{Z_{20}}{M}=0.113+339\frac{1}{ip}+110{,}000\left(\frac{1}{ip}\right)^2-222\cdot10^6\left(\frac{1}{ip}\right)^3+\cdots\cdots \qquad (24)$$

Then the expression for the impedance of the circuit of Fig. 10 is expanded, giving the following:

$$\left(\frac{Z_{20}}{M}\right)_{II}=a+\left(\frac{1}{c}-\frac{a^2}{d}\right)\frac{1}{ip}+\frac{a^2(a+e)}{d^2}\left(\frac{1}{ip}\right)^2-\frac{a^2(a+e)^2}{d^3}\left(\frac{1}{ip}\right)^3+\cdots\cdots \qquad (25)$$

Equating the first four coefficients of equations (24) and (25), and solving, the following values are obtained for the elements of the network of Fig. 10.

$a'=0.1130$;
$c=0.00221$;
$d=0.000236$;
$e'=0.3651$.

These numerical values were next substituted in the expression for the impedance of the network of Fig. 10 and the resulting values of impedance were plotted on Fig. 8. As indicated in the drawing, it will be seen that the agreement is fairly close within the essential frequency range. Beyond a frequency of 600 cycles per second, the curves are indistinguishable on the drawing.

The value for the arbitrary constant M which has been used above is 20640. The foregoing values of a', d and e' are multiplied by 20640, and the value of c is divided by 20640, and thus are obtained the following values for the elements as indicated on Fig. 3,

$a'_1=2330$ ohms.
$c_1=0.107$ mf.
$d_1=4.87$ h.
$e'_1=7540$ ohms.

I claim:

1. A conductor system for transmitting currents of various frequencies, said system being subject to changes of condition tending to change the phase relations among the various frequencies, a pilot circuit comprised in said system, and automatically adjustable compensating means controlled thereby to keep the said phase relations constant.

2. A conductor system for transmitting currents of various frequencies, said system being subject to changes of condition tending to change the phase relations among the various frequencies, a pilot circuit comprised in said system, and compensating means con-

trolled thereby to keep the said phase relations constant.

3. A conductor system for transmitting an alternating current, said system being subject to changes of condition tending to change the attenuation and tending to produce a different phase displacement at the receiving end at different times, means responsive to such changes, and compensating means controlled thereby to keep the attenuation substantially constant and to keep the phase displacement substantially constant.

4. A conductor system for transmitting an alternating current of several components, said system being subject to changes of condition tending to change the phase relations of said components at the receiving end, a pilot circuit comprised in said system, and compensating means controlled thereby to keep the said phase relations constant.

5. In a multiplex carrier current system subject to changes of condition tending to change the phase relations among the elementary currents of the system, a pilot circuit comprised in said system, and automatically adjustable compensating means controlled thereby to maintain the phase relations unchanged at the receiving end.

6. In a conductor system for transmitting alternating currents, a potentiometer, a pilot circuit subject to the same changes of condition as the other circuits of the system, and means governed by a current in said pilot circuit to adjust said potentiometer, the steps of said potentiometer being formed to compensate for a variation in phase displacement due to the said changes of condition.

7. A cable comprising a plurality of conductor pairs, potentiometers interposed in these pairs respectively, one of the said conductor pairs being appropriated to a pilot circuit, and means responsive to a variation of current in the pilot circuit to adjust the said potentiometers, the steps of these potentiometers being formed to keep the phase displacement constant in the circuits comprising the said pairs of conductors.

8. A circuit comprising a potentiometer consisting of impedance steps, each step comprising a resistance element and a reactance element so proportioned that at all adjustments the phase relations among current components of different frequency in the output of the circuit are kept substantially constant for a constant relation among these components in the input of the circuit.

9. A potentiometer consisting of impedance steps, each step comprising a resistance element and a reactance element, a circuit comprising said potentiometer and other apparatus, said other apparatus being subject to changes of condition tending to change the current through said circuit, and the steps of said potentiometer being constructed so that its adjustment will compensate for phase displacements produced by such change of condition.

10. In combination, a transmission line subject to a change of resistance due to temperature changes along its length, a pilot line exposed like the transmission line to the same temperature influences, a potentiometer in the transmission line having steps adapted to change the phase relations of currents of different frequency in that line, and means controlled by current in the said pilot circuit to adjust said potentiometer to compensate for phase displacements occasioned by the temperature changes.

11. The method of maintaining the phase relations among a plurality of alternating currents subject to varying conditions at a certain part of their circuit or circuits, which consists in passing those currents through a potentiometer adapted to give various phase displacements at various adjustments and automatically adjusting said potentiometer in accordance with the change of the conditions referred to.

12. In combination, a transmission line subject to attenuation changes and phase displacement changes from time to time, a potentiometer comprising resistance elements and reactance elements in its steps and adapted at its various adjustments to compensate for such changes, and an associated pilot circuit controlling said potentiometer.

13. The method of maintaining definite phase relations at the receiving end in a multiplex carrier current system, which consists in testing temperature conditions at points along the transmission line and adjusting for amplitude and phase relations as determined by these conditions in order to keep the amplitude and phase relations substantially unchanged at the receiving end.

14. The method of maintaining definite phase relations at the receiving end in a multiplex carrier current system, which consists in testing temperature conditions at points in a pilot circuit along the transmission line and adjusting phase relations in other circuits along said line as determined by the test in the pilot circuit.

In testimony whereof, I have signed my name to this specification this 27th day of February 1924.

HARRY NYQUIST.