

Oct. 19, 1926.

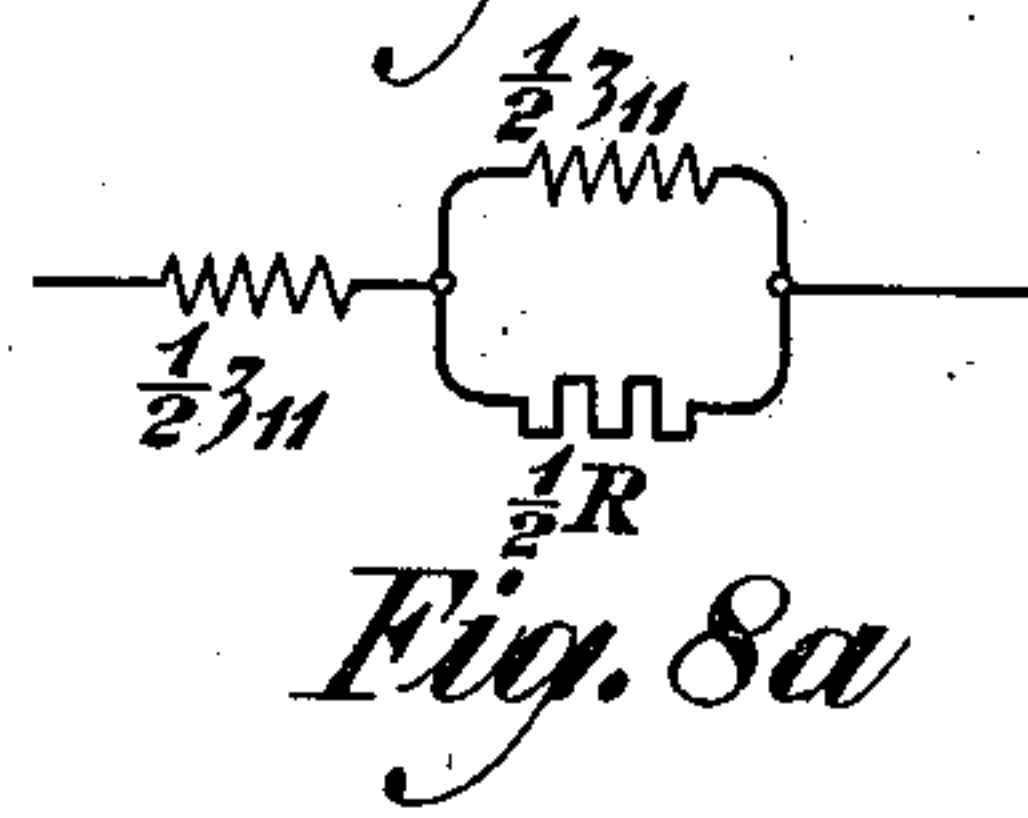
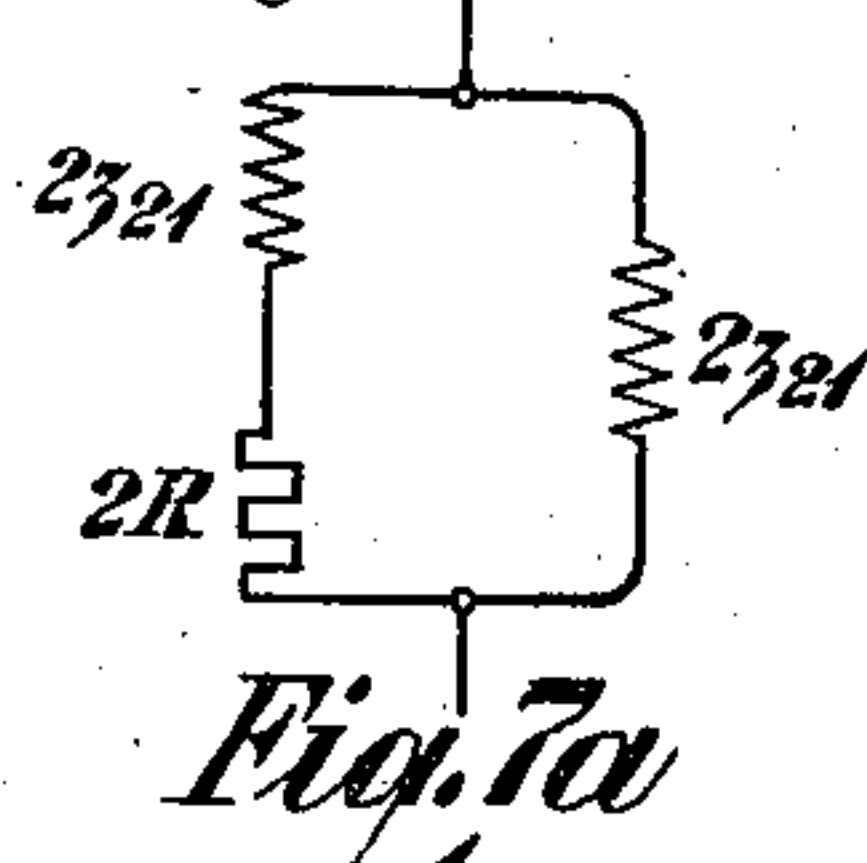
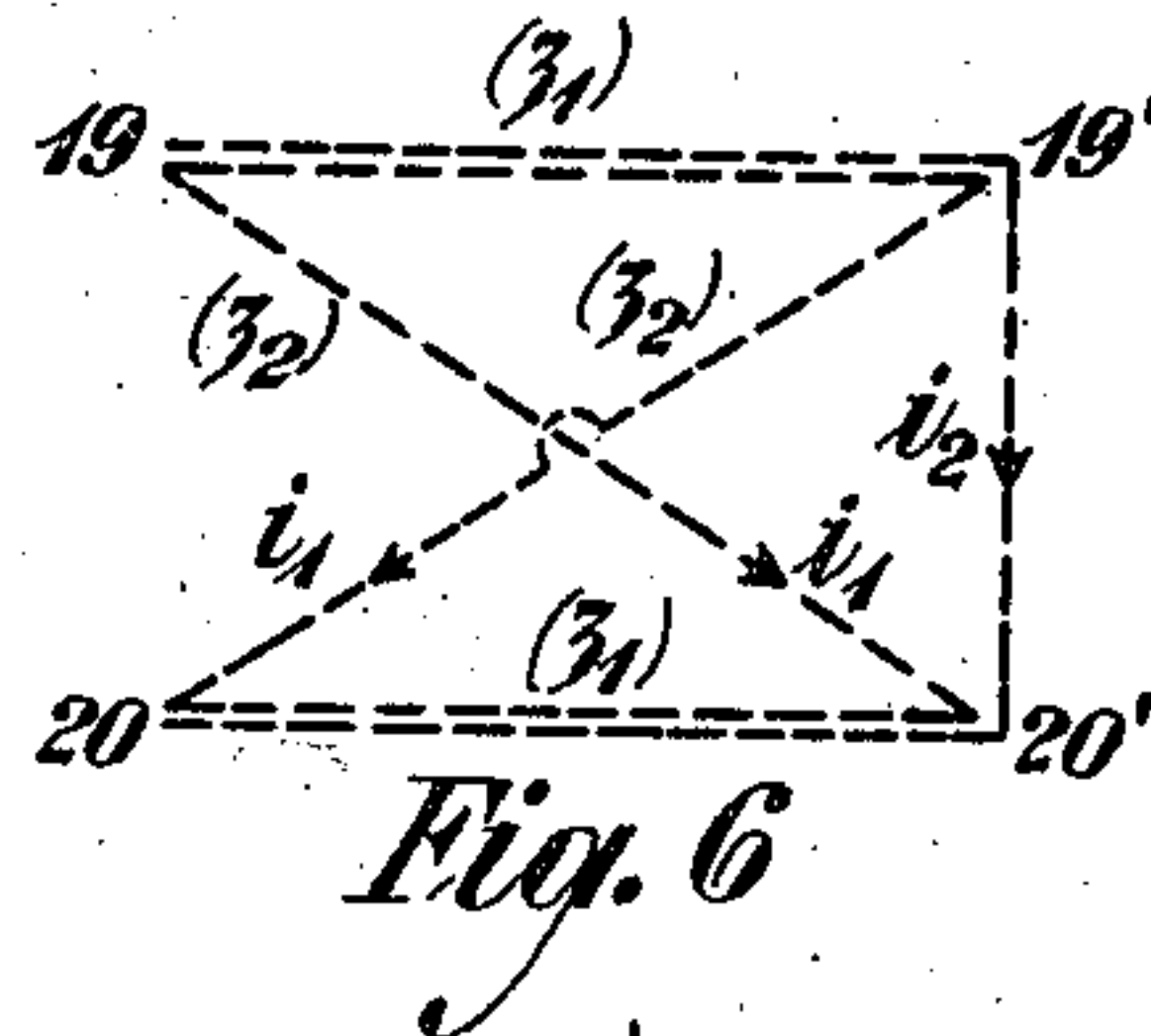
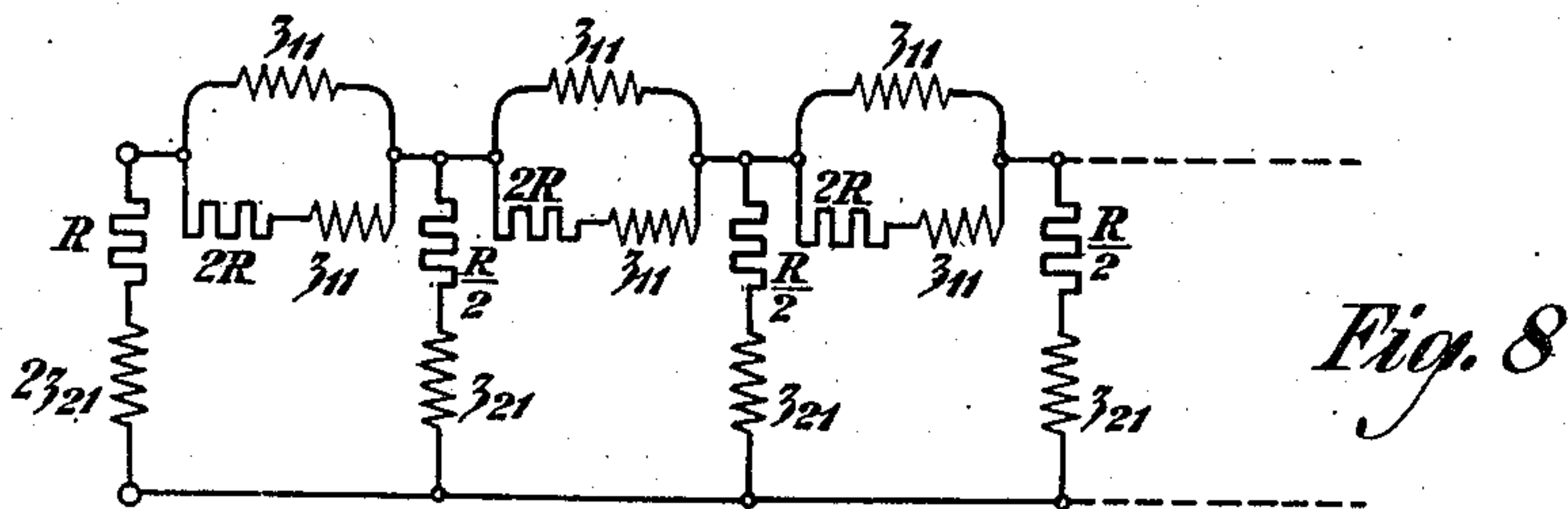
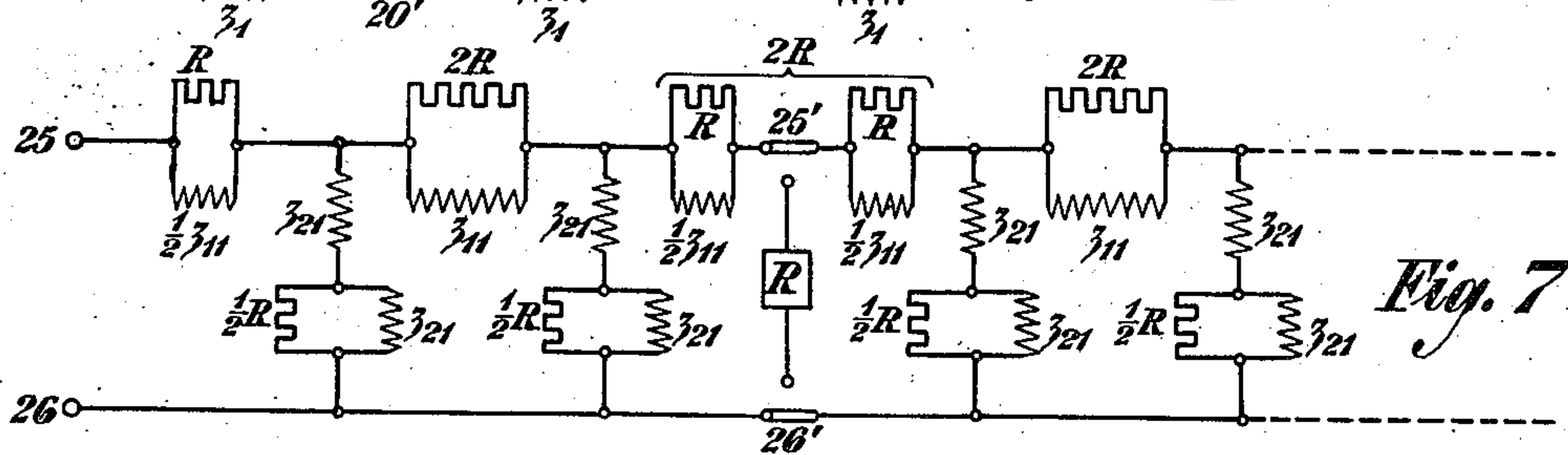
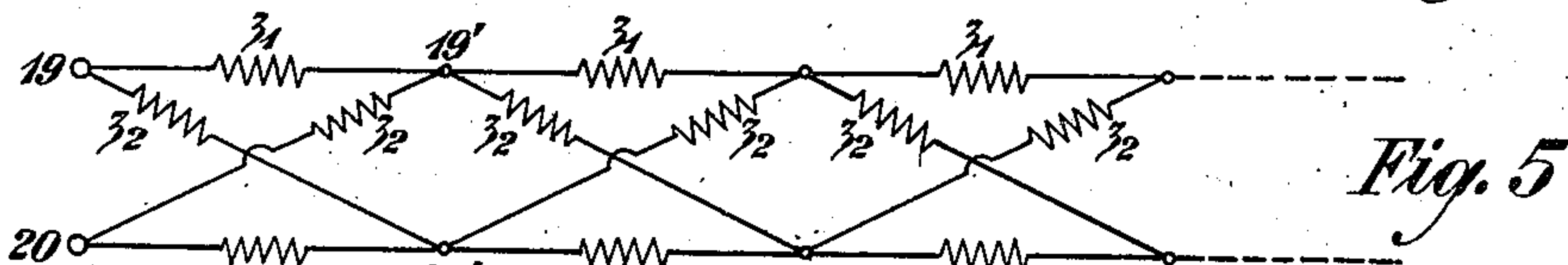
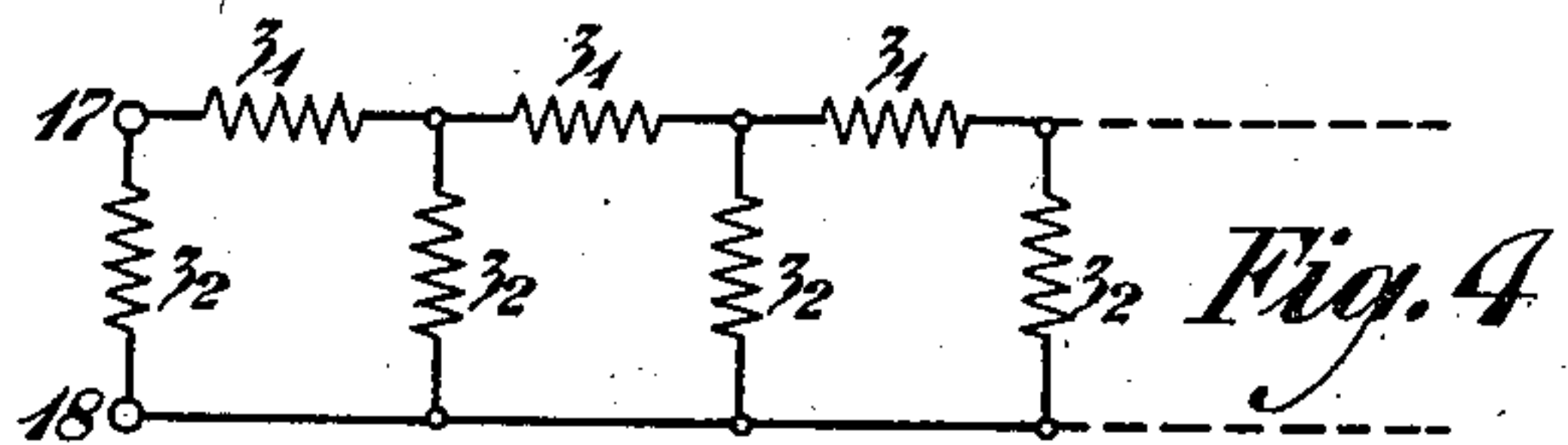
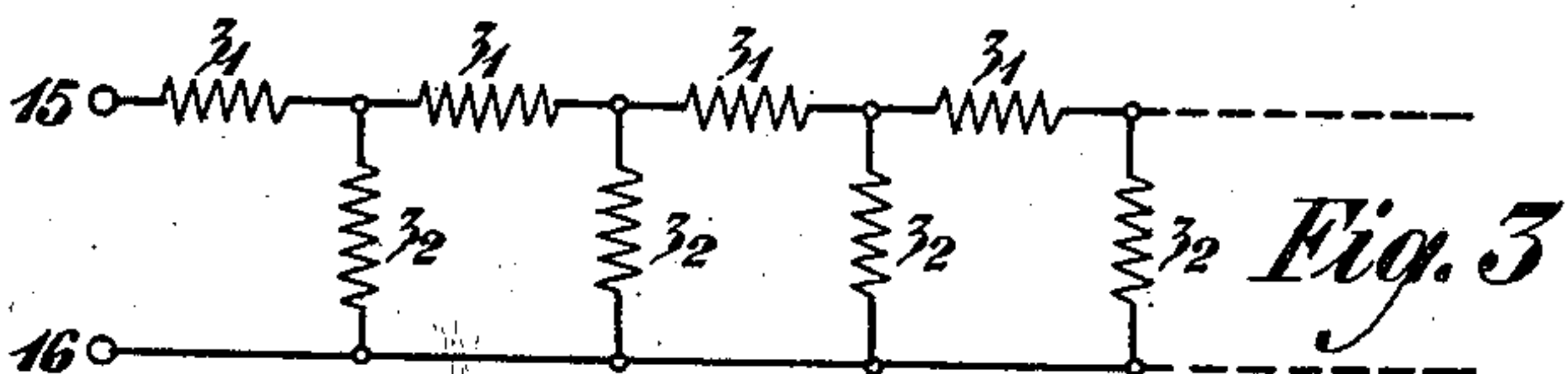
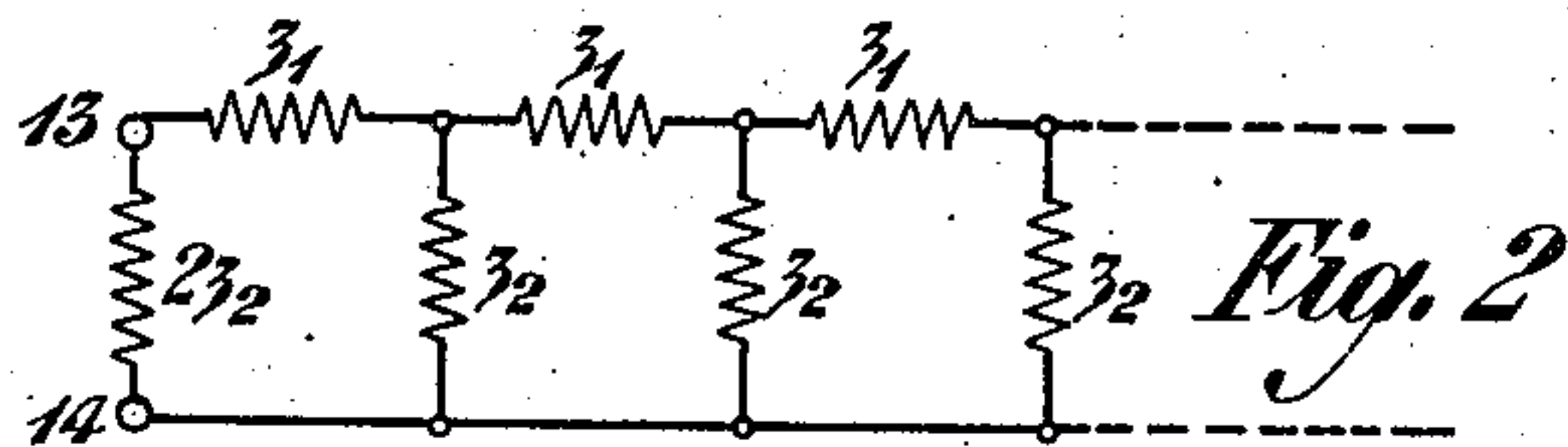
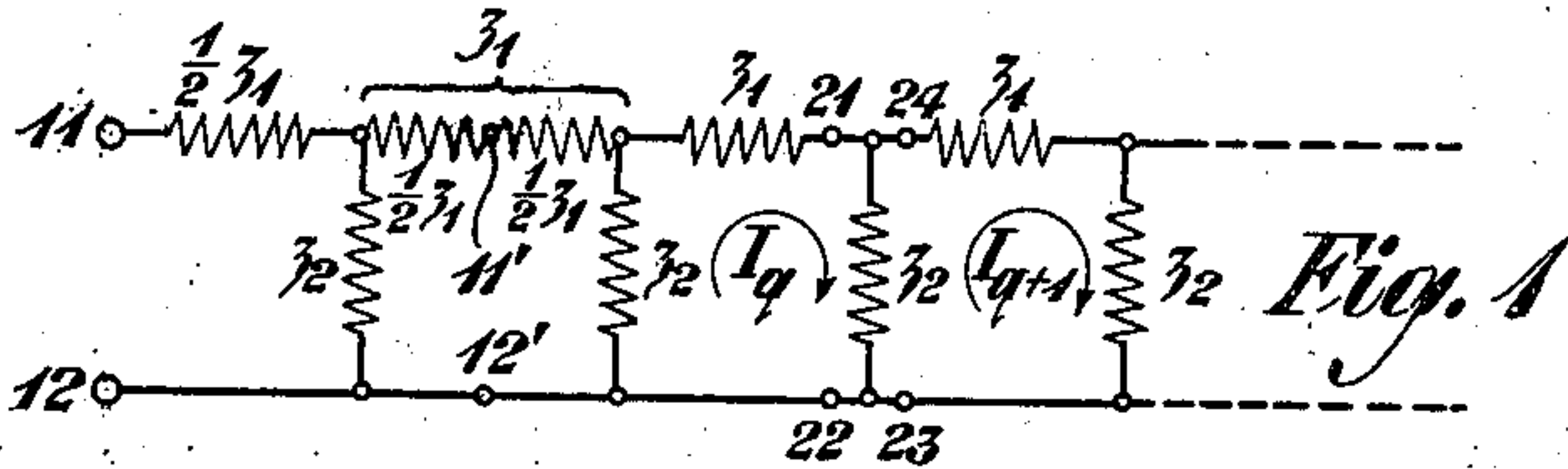
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1,603,305

ELECTRICAL NETWORK AND METHOD OF TRANSMITTING ELECTRIC CURRENTS

Filed August 9, 1922

3 Sheets-Sheet 1



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ELECTRICAL NETWORK AND METHOD OF TRANSMITTING ELECTRIC CURRENTS

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3 Sheets-Sheet 2

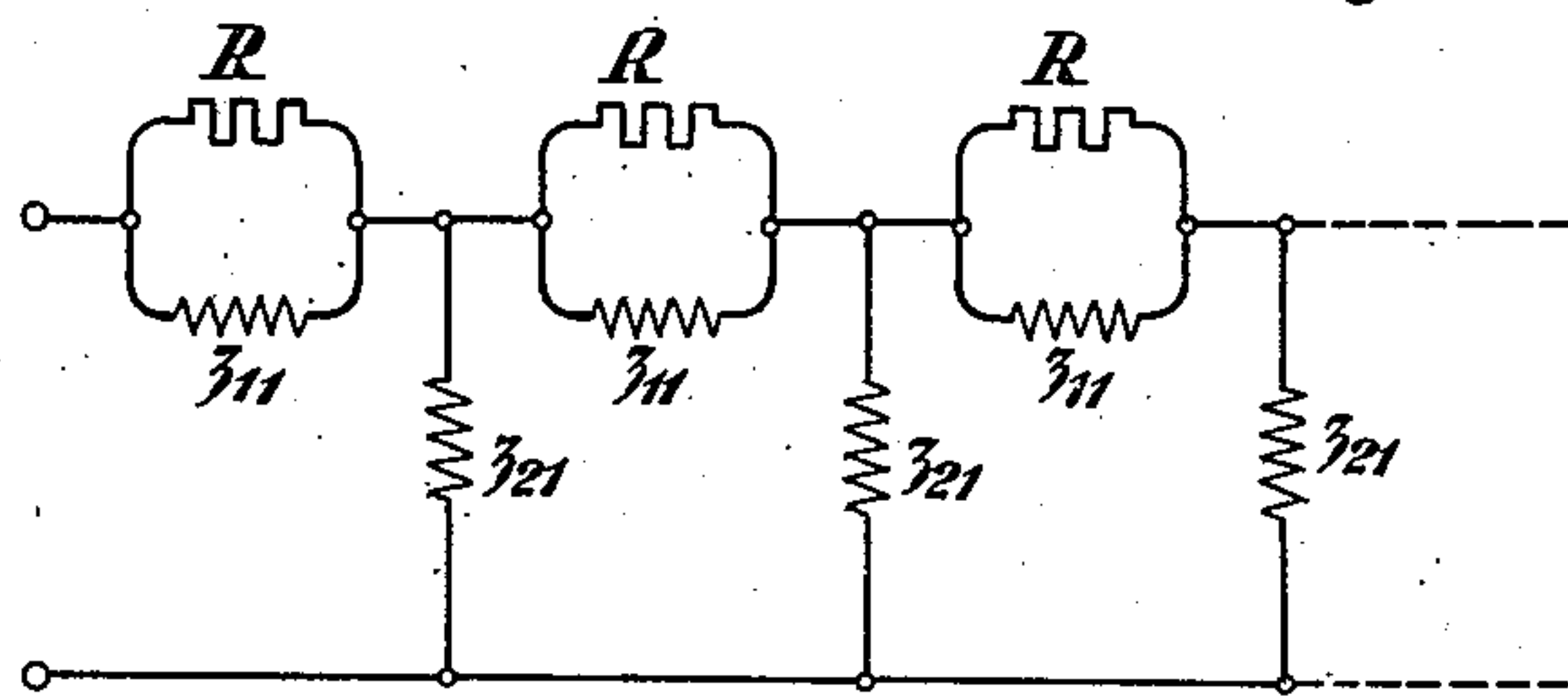


Fig. 9

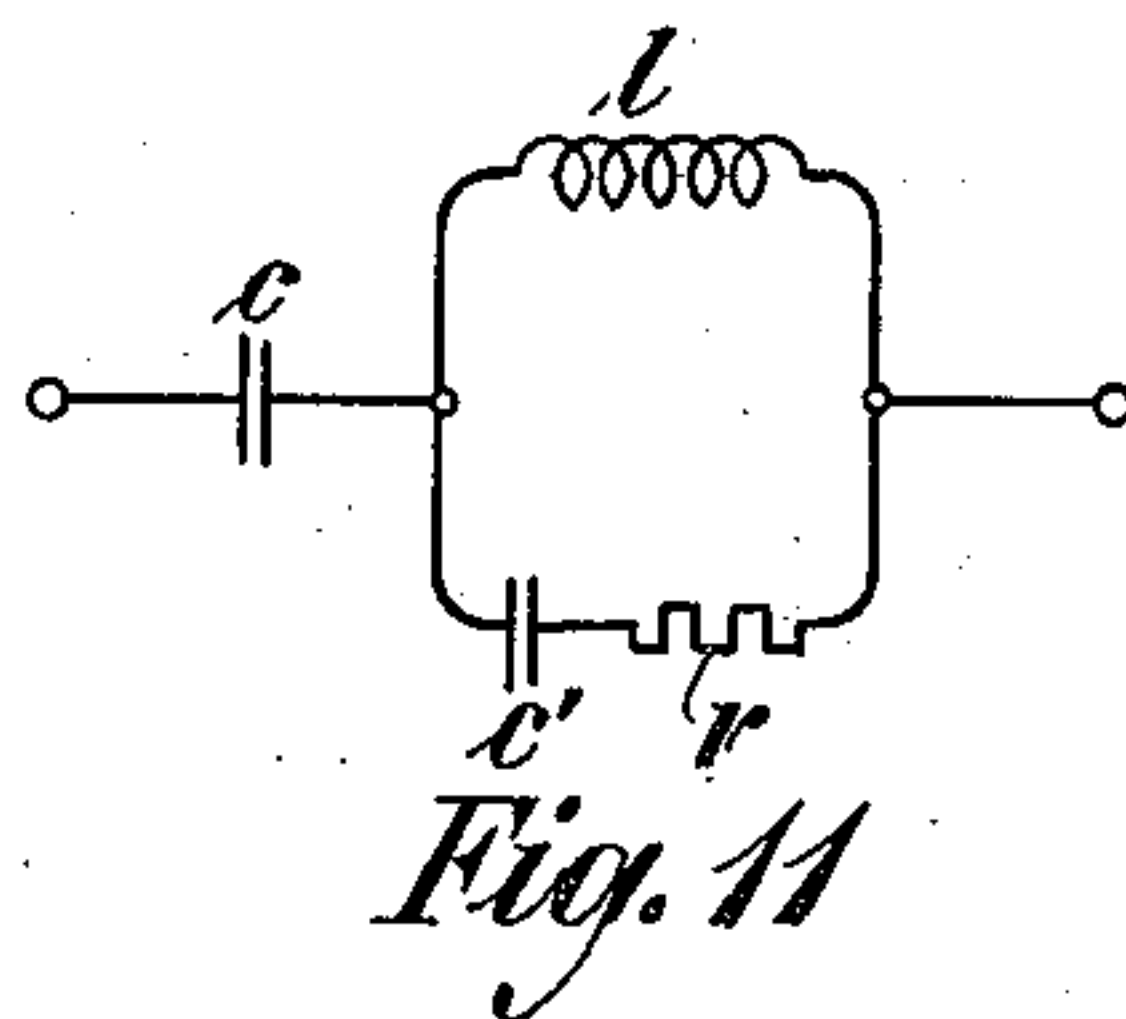


Fig. 11

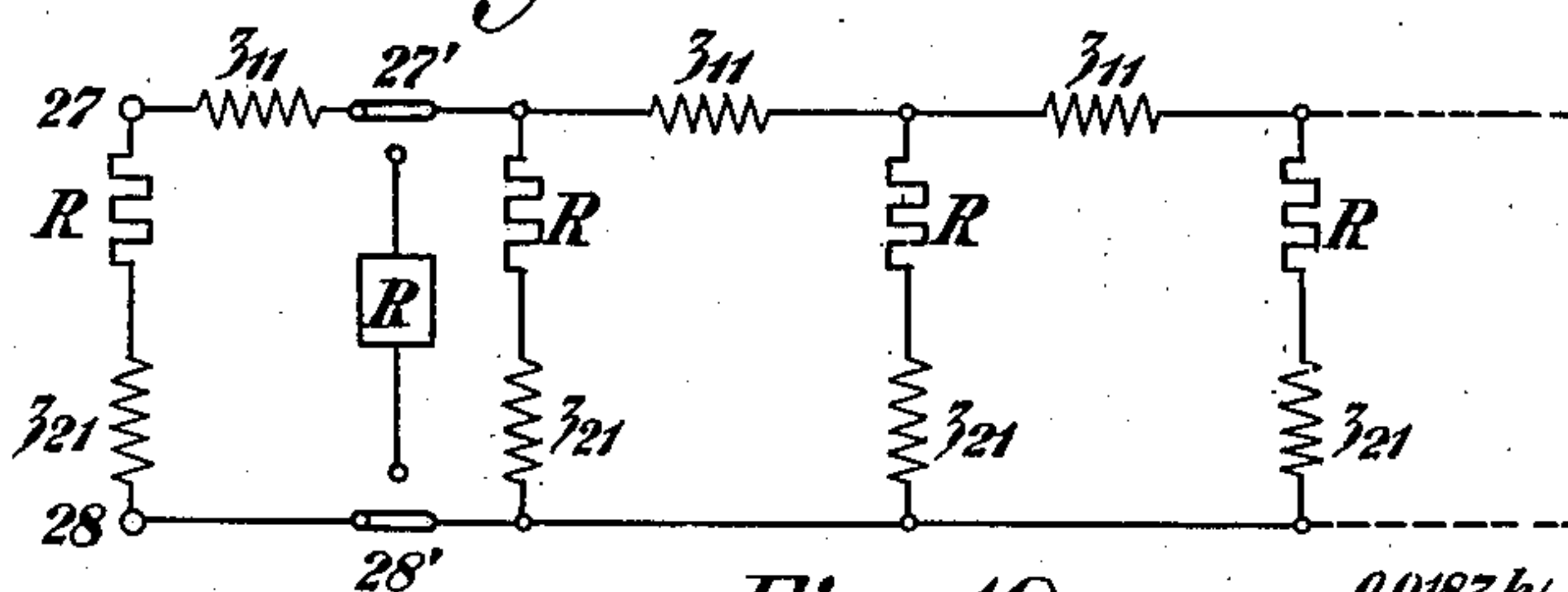


Fig. 10

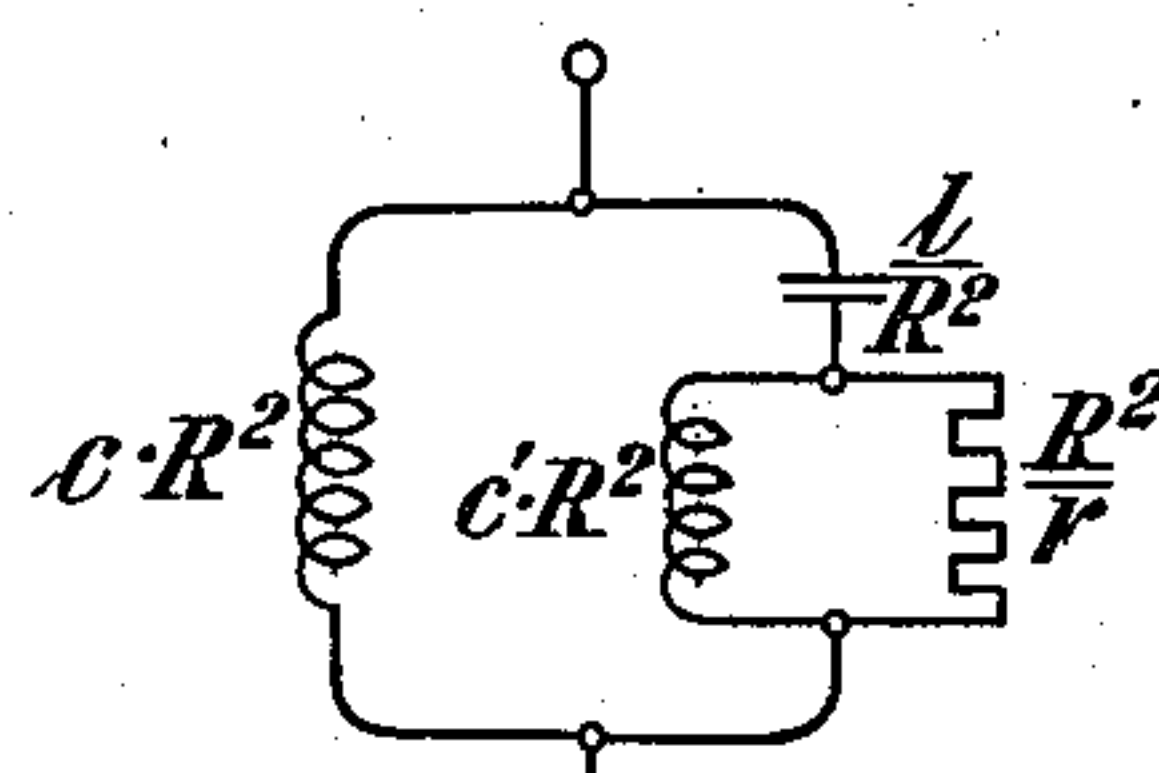


Fig. 12

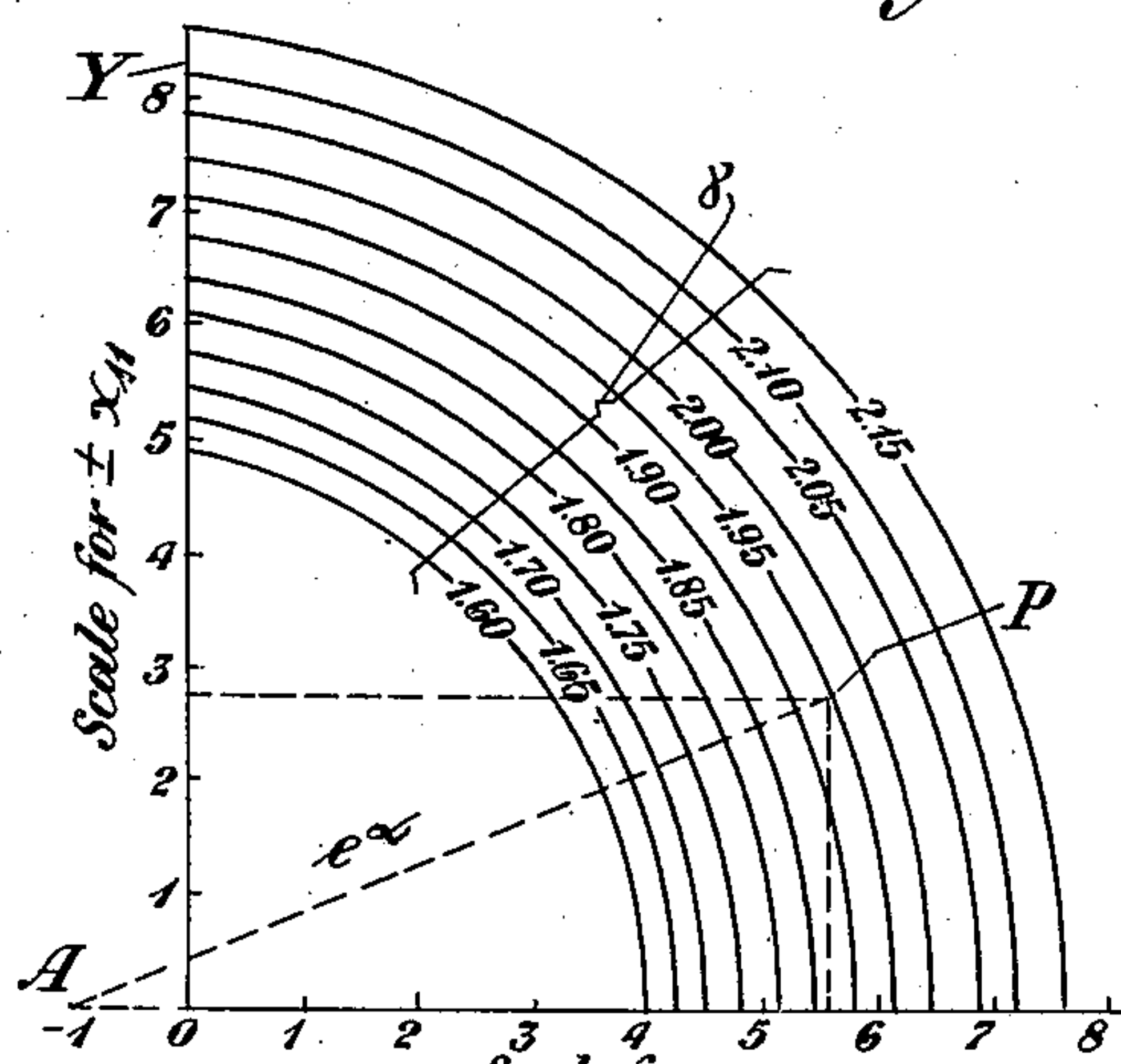


Fig. 13

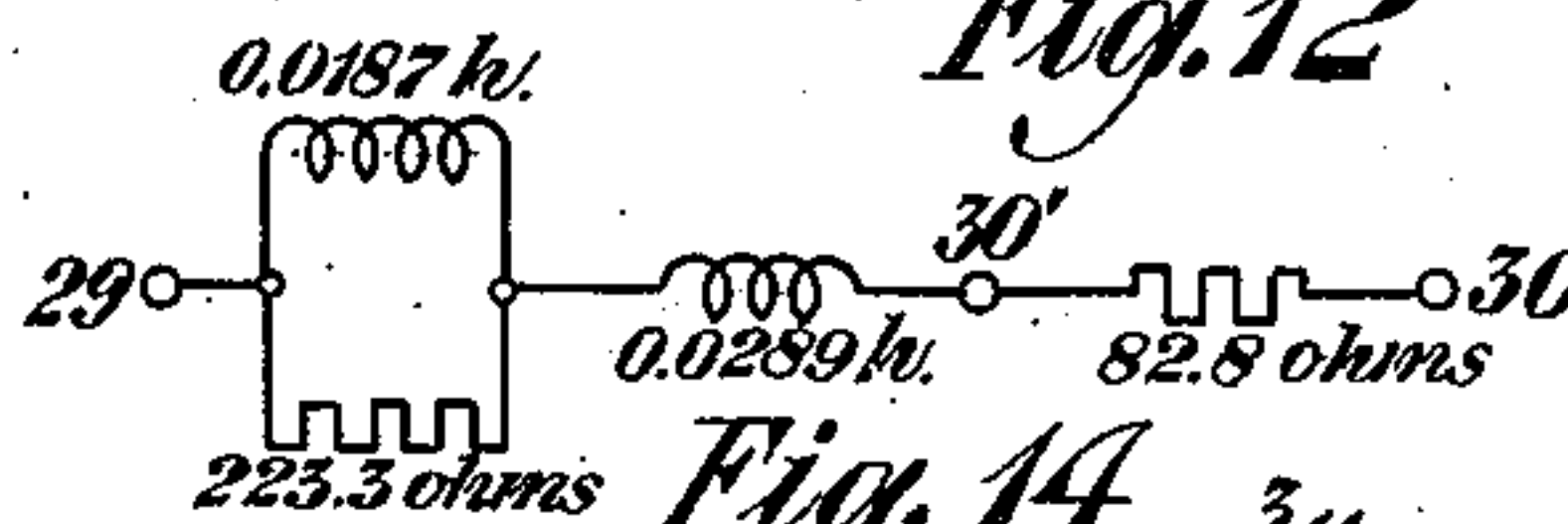


Fig. 14

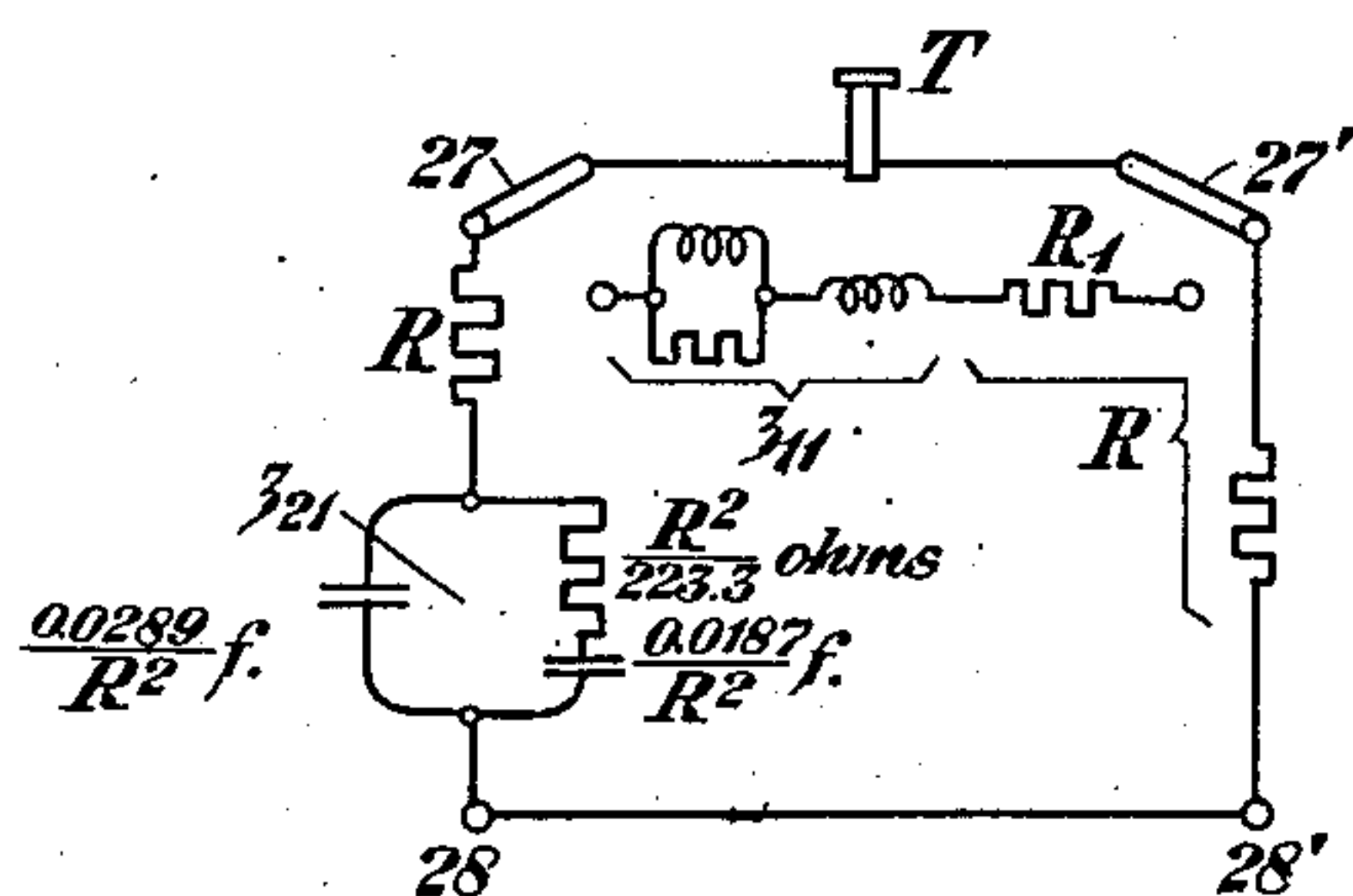


Fig. 15

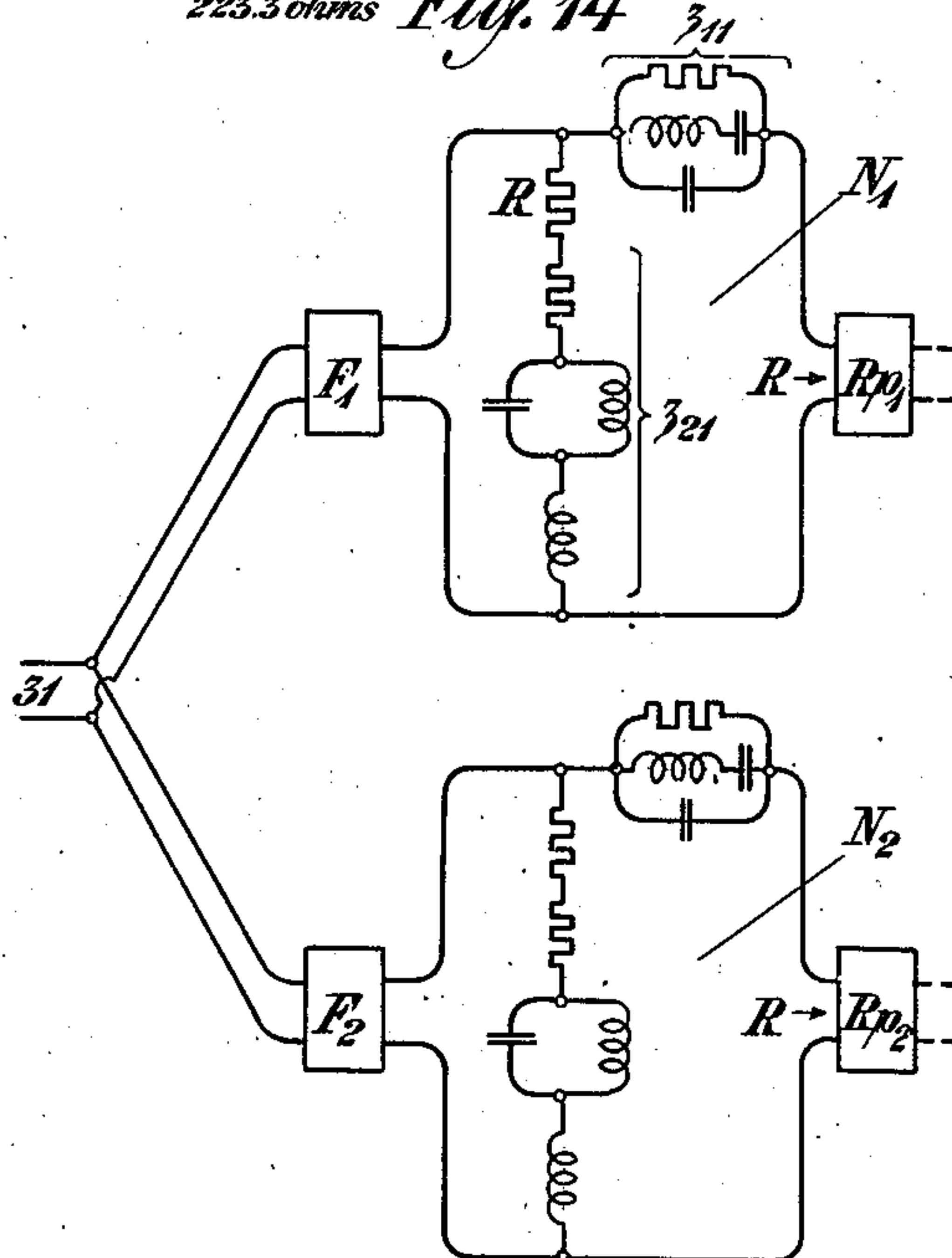


Fig. 16

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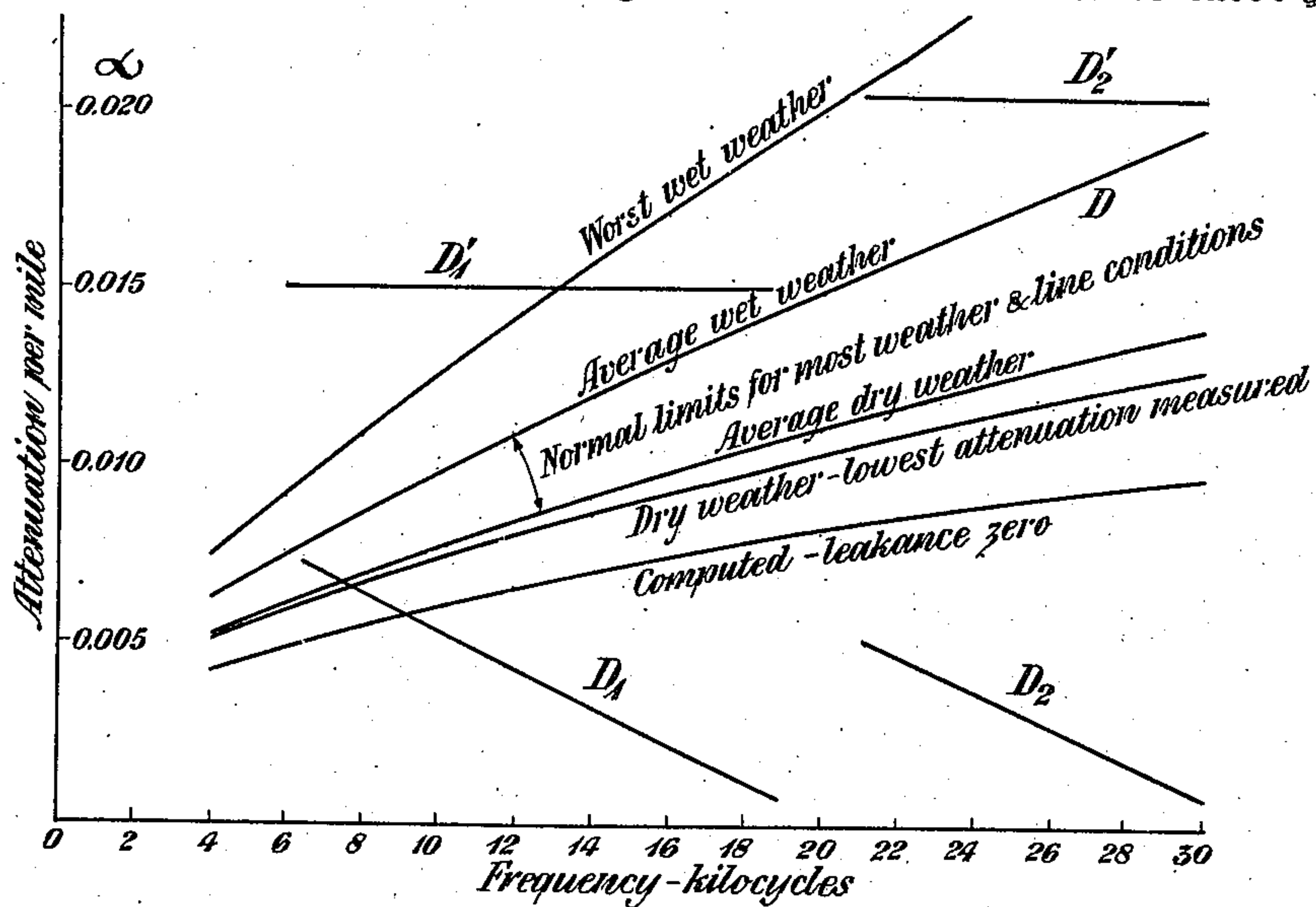


Fig. 17

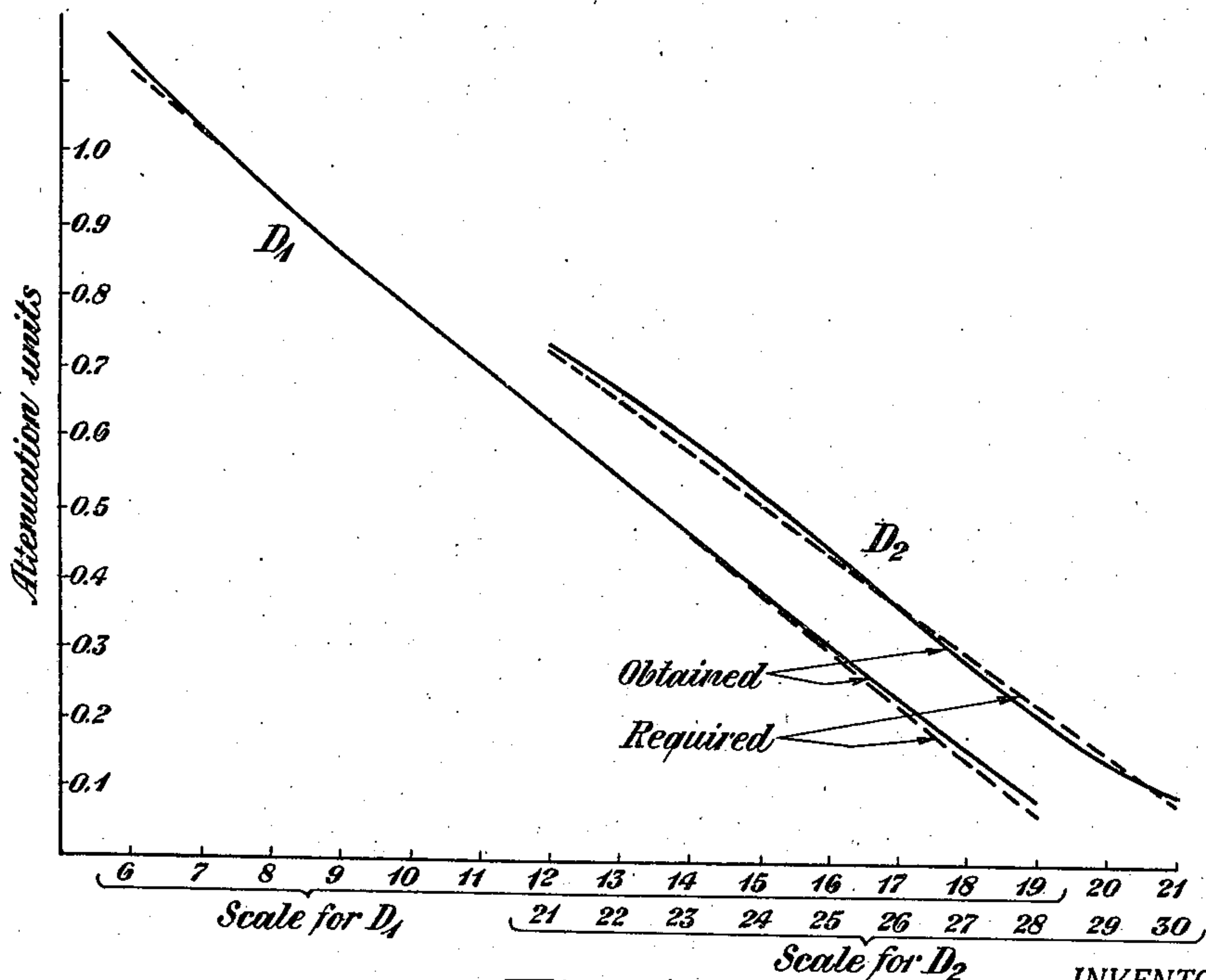


Fig. 18

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Patented Oct. 19, 1926.

1,603,305

UNITED STATES PATENT OFFICE.

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ELECTRICAL NETWORK AND METHOD OF TRANSMITTING ELECTRIC CURRENTS.

Application filed August 9, 1922. Serial No. 580,769.

The principal object of my invention is to provide a new and improved electrical network whose impedance over a considerable frequency range will be substantially a constant resistance. Another object of my invention is to provide such a network having a given desired attenuation-frequency characteristic. A further object of my invention is to provide an attenuation equalizer for use in carrier current signaling. All these and other objects of my invention will be made apparent in the following specification and claims taken with the accompanying drawings, in which I have disclosed specific examples of practice in accordance with the invention. It will be understood that the invention is defined in the appended claims, and I will now proceed to describe those examples which I have chosen to present by way of illustration.

Referring to the drawings Figure 1 is a diagram of a certain type of network of recurrent like sections with "mid-series" termination; Figs. 2, 3 and 4 are corresponding diagrams for the same network, but respectively with "mid-shunt", "full series" and "full shunt" terminations. Fig. 5 is a diagram for a lattice type recurrent network. Fig. 6 is a diagram showing one way in which the currents may be resolved in a section of Fig. 5. Figs. 7, 8, 9 and 10 are, respectively, special case diagrams corresponding to Figs. 1, 2, 3 and 4, but these special cases all have the property that the characteristic impedance is a constant resistance. Figs. 7^a and 8^a show equivalent designs for the shunt and series elements of Figs. 7 and 8 respectively. Figs. 11 and 12 are diagrams showing how two impedances z_{11} and z_{21} may be embodied physically, subject to the equation $z_{11}z_{21} = R^2$. Fig. 13 is a diagram for graphic computation in connection with Figs. 7 to 10. Fig. 14 shows a structure having its impedance equivalent to the passive impedance of a certain telephone receiver within a range of from 0 to 3000 cycles per second. Fig. 15 shows how that telephone receiver can be connected into a network of constant resistance characteristic impedance. Fig. 16 is a diagram showing an attenuation equalizer made in accordance with my invention. Figs. 17 and

18 give attenuation characteristics involved in connection with the design of Fig. 16.

In explanation of the term "passive impedance" as employed above, this refers to the impedance on the assumption that the receiver diaphragm is held mechanically against vibration so that the reaction due to the vibration of the diaphragm introduces no apparent factor in the impedance across the telephone receiver input terminals.

The type of network shown in Figs. 1 to 4 consists of successive alternate series impedances z_1 and shunt impedances z_2 . For the first stage in the investigation of the theory of such recurrent networks, it will here be assumed, as is usual, that the recurrent structure is continued one way to infinity; this is indicated by the dotted lines to the right. The structure of Fig. 1 begins at the left with the terminals 11, 12 and a series element whose impedance is half the normal series impedance value, that is, $\frac{1}{2}z_1$. This ending is called "mid-series". In Fig. 2 the initial terminals 13 and 14 lead at once to a shunt element whose admittance is one-half the normal shunt admittance, and accordingly whose impedance is $2z_2$. This is a "mid-shunt" termination. Fig. 3 has "full series" termination at 15, 16 and Fig. 4 has "full shunt" termination at 17, 18.

The impedance of the network of Fig. 1 across the points 11, 12 will be the same as across the points 11', 12', looking to the right in both cases. This iterative impedance is called the mid-series characteristic impedance and is designated K_{ms} . With reference to the section 11, 12, 11', 12', the following equation will serve for the determination of K_{ms} .

$$K_{ms} = \frac{z_1}{2} + \frac{1}{\frac{1}{z_2} + \frac{1}{\frac{z_1}{2} + K_{ms}}}$$

The solution of this equation is:

$$K_{ms} = \sqrt{z_1 z_2 + \frac{1}{4} z_1^2} \quad ()$$

The impedance across the points 13, 14 in Fig. 2 is called the mid-shunt characteristic

impedance and is designated K_{mh} . Comparison with Fig. 1 shows that

$$\sqrt{z_1 z_2 + \frac{1}{4} z_1^2} - \frac{1}{2} z_1 = \frac{K_{mh} \cdot 2z_2}{K_{mh} + 2z_2}$$

The solution is:

$$K_{mh} = \frac{z_1 z_2}{\sqrt{z_1 z_2 + \frac{1}{4} z_1^2}} \quad (2)$$

From equation 1, it follows that the characteristic impedance across 15, 16 in Fig. 3, viz, the full series characteristic impedance, is given by the formula:

$$K_{1s} = \sqrt{z_1 z_2 + \frac{1}{4} z_1^2} + \frac{1}{2} z_1 \quad (3)$$

Also the impedance across 17, 18 in Fig. 4, the full shunt characteristic impedance, is given by

$$K_{1h} = \sqrt{z_1 z_2 + \frac{1}{4} z_1^2} - \frac{1}{2} z_1 \quad (4)$$

Consider the "lattice type" network shown in Fig. 5, this has impedance elements of two different values, z_1 and z_2 , connected and repeated as shown in the figure, and for the present it is assumed that this figure extends to infinity from the terminals 19, 20. The impedance across these terminals 19, 20 will be the same as across the points 19', 20', looking to the right in both cases. Designating this as the "lattice type characteristic impedance" K_1 , we see that the section 19, 20, 19', 20' may be treated as a Wheatstone bridge, the impedance of the arm 19', 20' being K_1 with respect to an electromotive force applied across the terminals 19, 20. Accordingly, by the well known method for the Wheatstone bridge, the solution may be found

$$K_1 = \sqrt{z_1 z_2} \quad (5)$$

The propagation constant Γ of a recurrent network such as shown in Figs. 1 to 5 is defined by the equation:

$$e^{-\Gamma} = \frac{I_q + 1}{I_q} \quad (6)$$

where I_q is the current in the q th mesh or section and I_{q+1} is the current in the $(q+1)$ th mesh or section. Γ is, in general, a complex number, and therefore we have:

$$\Gamma = \alpha + i\beta \quad (7)$$

where α is called the attenuation constant and β is called the phase constant. The propagation constant Γ is evidently the same for Figs. 1, 2, 3 and 4.

Referring to Fig. 1, and expressing the potential drop across the points 21, 22 as

equal to the drop across the points 23, 24, we have the equation:

$$I_q \left(K_{ms} - \frac{1}{2} z_1 \right) = I_{q+1} \left(K_{ms} + \frac{1}{2} z_1 \right)$$

whence

$$e^{-\Gamma} = \frac{I_q + 1}{I_q} = \frac{2K_{ms} - z_1}{2K_{ms} + z_1}$$

It follows that:

$$\cosh \Gamma = \frac{1}{2} (e^{\Gamma} + e^{-\Gamma}) = 1 + \frac{1}{2} \frac{z_1}{z_2} \quad (8)$$

We define $u + iv$ to be such that

$$\cosh \Gamma = 1 + \frac{1}{2} (u + iv) \quad (9)$$

Accordingly for Figs. 1 to 4,

$$\frac{z_1}{z_2} = u + iv \quad (10)$$

Formulas 8, 9 and 10 are for Figs. 1, 2, 3 and 4, but formula 9 is general for any recurrent network. Equations for the propagation constant Γ for the lattice type network of Fig. 5 will now be developed. The currents in a section of Fig. 5 are resolved into components i_1 , i_2 and i_3 as indicated in the diagram Fig. 6. The drop over the path 19, 19', 20 being the same as directly across 19, 20, and in view of equation 5, we have

$$z_1(i_1 + i_2) + z_2 i_1 = \sqrt{z_1 z_2} (2i_1 + i_2)$$

From this equation it is readily deduced that

$$e^{-\Gamma} = \frac{I_q + 1}{I_q} = \frac{i_2}{2i_1 + i_2} = \frac{\sqrt{z_2} - \sqrt{z_1}}{\sqrt{z_2} + \sqrt{z_1}}$$

whence it follows that

$$\cosh \Gamma = \frac{1}{2} (e^{\Gamma} + e^{-\Gamma}) = 1 + \frac{1}{2} \frac{4z_1}{z_2 - z_1} \quad (11)$$

Applying the definition of equation 9, it follows that for the lattice type network of Fig. 5

$$u + iv = \frac{4z_1}{z_2 - z_1} \quad (12)$$

Since equation 9 is general, applying to all the figures, and expressing Γ as a function of u and v , I refer to u and v as the "universal variables."

I shall now proceed to show how, by giving special values to z_1 and z_2 in Figs. 1 to 5, the respective characteristic impedances may be made equal to a constant resistance in each case.

Let the desired constant resistance value be R . Then Figs. 7, 8, 9 and 10 show the respective special case forms assumed by Figs. 1, 2, 3 and 4. The impedance values designated z_{11} and z_{21} in these four figures

are to be understood as subject to the relation

$$z_{11}z_{21}=R^2 \quad (13)$$

Substituting from equations 13 and 14 in equation 10, and reducing, we get for Fig. 7 that

$$u+iv=\frac{\frac{(z_{11})^2}{R}}{1+\frac{z_{11}}{R}} \quad (15) \quad 70$$

Similarly for the cases represented by Figs. 8, 9 and 10, the universal variables u and v are found to be the same and are given by equation 15. 75

For the lattice case of Fig. 5, we substitute from equation 13 in equation 12 and get 80

$$u+iv=\frac{4 \cdot \frac{(z_{11})^2}{R}}{1-\frac{(z_{11})^2}{R}} \quad (16) \quad 85$$

Remembering that the important functions of a recurrent network are its characteristic impedance and its propagation constant, we see that these are all comprised in the foregoing discussion. For each of Figs. 7, 8, 9 and 10 and also Fig. 5, the characteristic impedance is R , provided the elemental impedances z_{11} and z_{21} satisfy equation 13. The propagation constant Γ is given in general by equation 9, which expresses Γ as a function of the universal variables u and v . These variables are given for Figs. 7, 8, 9 and 10 in equation 15 and for Fig. 5 in equation 16. 90 95 100

Thus in Figs. 7, 8, 9, 10 and 5 we have available five different designs from which to choose in setting up a recurrent network of constant resistance characteristic impedance. Moreover, each of these five types is general in that it comprises two impedances z_{11} and z_{21} , one of which may be given any value we please. Thus it will be seen that in the design of networks to suit required conditions, considerable flexibility is afforded. 105 110

It will be recalled that early in this specification the condition was laid down that the recurrent networks under consideration should extend from the initial terminals to infinity. It will now be seen that the impedance and current relations will be unaltered, if, at an appropriate point corresponding to the initial point, the system is closed by a resistance R . Thus, in Fig. 7, which has mid-series initial termination at 25, 26, if at any mid-series points such as 25', 26' the switches are thrown, the effect at the sending points 25, 26 will be unaltered and the currents in the finite sequence of sections will be unaltered. Likewise in Fig. 10, the impedance R across points 27, 28 is unchanged if the switches 27', 28' are thrown. 115 120 125

These networks may be used in any situa- 130

It will be seen that the general series impedance z_1 of Fig. 1 is represented in Fig. 7 by the impedance of the parallel combination having $2R$ in one branch and an impedance z_{11} in the other branch. This impedance z_{11} may have any arbitrary design. It may be represented by resistance, inductance or capacity or any combination of these. However, when a value has been assigned for z_{11} , the value of z_{21} is determined by equation 13. The shunt element z_2 of Fig. 1 is seen to be represented in Fig. 7 by the impedance z_{21} in series with the parallel combination of $\frac{R}{2}$ and z_{21} . An equivalent structure for z_2 is shown in Fig. 7a. According to the resistance and reactance values involved, one design or the other may be the more convenient.

Whatever the physical structure that gives the impedance z_{11} , provided it is composed of lumped elements, it is possible in practical cases to design a physical structure of lumped elements for z_{21} , so that equation 13 will be satisfied. For example, suppose that z_{11} is embodied in the structure shown in Fig. 11. Then z_{21} will be given by the structure shown in Fig. 12. It will be seen that there is a complete homology, a series inductance in z_{11} , corresponding to a shunt condenser in z_{21} and vice versa, these respective inductance and capacity elements being related by the equation $\frac{L}{C}=R^2$; also a shunt or series resistance z_{11} corresponds respectively to a series or shunt resistance of reciprocal value times R^2 in z_{21} . 25 30 35 40

I shall now proceed to show that the characteristic impedance for Fig. 7 reduces to R . Comparison between Fig. 1 and Fig. 7 shows that 45

$$z_1=\frac{2R \cdot z_{11}}{2R+z_{11}} \text{ and } z_2=\frac{\frac{1}{2}R \cdot z_{21}}{\frac{1}{2}R+z_{21}}+z_{21} \quad (14) \quad 50$$

Substituting these values in equation 1 and expressing z_{21} in terms of z_{11} by means of equation 13, we get $K_{ms}=R$. 55

By a similar procedure it may be shown that the respective characteristic impedances for Figs. 8, 9 and 10 each reduce to R .

Referring to Fig. 5, let z_1 and z_2 take the respective values z_{11} and z_{21} , specially related as in equation 13. Then by equation 5 we get at once that $K_1=R$, thus putting Fig. 5 in the same class with Figs. 7, 8, 9 and 10 in that they all have a characteristic impedance which is the constant resistance R , independent of frequency. 60 65

tion where the termination is a constant resistance. In the absence of reasons for making the choice otherwise, Figs. 9 and 10 have some advantage over Figs. 7 and 8 in that they are more simple in structure and therefore more economical to build and maintain, but where symmetry in opposite directions is desired, the mid-series and mid-shunt endings of Figs. 7 and 8 respectively and the symmetrical structure of Fig. 5 all have an advantage over Figs. 9 and 10.

It has been shown in my application, Serial No. 437,527, filed January 15, 1921, that in a recurrent network or wave filter of the kind shown in Figs. 1 to 5, interior sections can be replaced by sections of altered design (subject to certain conditions) without affecting the characteristic impedance. With respect to a given design of sections as in Figs. 1 to 4, I have used the term "M-types" to designate such substituted sections which do not alter the characteristic impedance, but which do change the attenuation properties. Such M-type substitutions may be made in any one of the networks of Figs. 7, 8, 9 and 10, and corresponding substitutions of what I call an "L-type" may be made in the sections of Fig. 5.

In connection with any one of the networks of Figs. 7, 8, 9 and 10, it may be convenient to employ a graphic method of computation, which I will outline briefly with reference to Fig. 13.

From equations 8, 13 and 14, or directly from Figs. 7 to 10, it may readily be deduced that

$$e^r = 1 + \frac{z_{11}}{R} \quad (17)$$

Let

$$\frac{z_{11}}{R} = r_{11} + ix_{11} \quad (18)$$

Substituting from equation 18 in equation 17 and also relying on equation 7, the result is obtained that

$$e\alpha + i\beta = e\alpha \cdot e^{i\beta} = 1 + r_{11} + ix_{11} \quad (19)$$

In Fig. 13, the point A is on the axis of abscissæ at unit distance from origin O and from this origin O a scale of values is laid off along the axis of abscissæ for r_{11} . The line OY is the axis of ordinates and on it a scale of values for x_{11} is laid off. Since r_{11} represents resistance, it is usually positive, and in absolute value the right-hand member of equation 19 will be the same whether x_{11} is positive or negative. Hence, the quadrant shown in Fig. 13 will be sufficient for all cases. Draw AP to the point P, determined by particular values of r_{11} and x_{11} . The length AP represents $e\alpha$ and the angle OAP in circular measure gives the value of β , its sign corresponding to the sign of x_{11} .

A set of circles may be drawn from O as center, and each one labelled with the corresponding value for α , then, by means of a diagram such as Fig. 13, the attenuation constant α can be immediately determined when the components of $\frac{z_{11}}{R}$ are known.

Also the phase constant can be quickly determined as the circular measure of the angle OAP.

Corresponding formulas and charts for the lattice type network of Fig. 5 can also be deduced and a chart prepared for graphic solution.

To illustrate a use for my improved network, suppose that we have a piece of apparatus whose impedance comprises reactance and varies over a certain frequency range, whereas it is desired that the introduction of the apparatus in a reactanceless circuit shall not introduce reactance nor introduce any variable impedance. By the use of my invention this piece of apparatus may be introduced as an element of one or more sections of a network such as shown in Figs. 7, 8, 9, 10 or 5. The design of the network comprising this piece of apparatus will be such as to make the characteristic impedance a constant resistance, then the network so designed can be put in the circuit without introducing any reactance or any impedance that varies with frequency.

For example, the passive impedance of a certain telephone receiver, on being measured over the voice frequency range, was found to have the following values:

Frequency (cycles).	Impedance (ohms).
0	80 + i0
200	85 + i60
600	103 + i173
1000	123 + i267
1500	160 + i375
2000	200 + i475
3000	280 + i640

By trial it was found that this impedance could be represented approximately by the network shown in Fig. 14. The constants of this network were chosen to give it the exact impedance of the telephone receiver T at 600 cycles and at 2000 cycles and it was then found to differ by less than 3.5% at all frequencies up to 2,500 cycles and by less than 5.5% at 3,000 cycles.

Let the impedance of the structure between the points 29 and 30' of Fig. 14 be taken as z_{11} of the section 27, 28, 27' 28' of Fig. 10, the switches 27', 28' being thrown to close the section on R, and let the resistance R_1 of Fig. 14 be part of the terminal R of that section of Fig. 10. This gives Fig. 15 on the assumption that the switches 27 and 27' of Fig. 15 are thrown down. But when they are thrown up, as shown in Fig. 15, the equivalent network for the tele-

phone receiver is substituted by the receiver itself. The structure for z_{21} of Fig. 10 is shown at the left between the points 27, 28 of Fig. 15. This structure between the points 27 and 28 is obtained to satisfy equation 13.

Accordingly, the result is obtained that, although the impedance of receiver T has a very considerable reactance component and both components of this impedance are widely variable over the voice frequency range, nevertheless across the terminals 27, 28 of the network of Fig. 15, of which network the receiver T is an element, the impedance is substantially a constant resistance.

For another illustration of a use for my invention, I will explain the design of an attenuation equalizer to be interposed between a line and a repeater. Each repeater Rp_1 and Rp_2 in Fig. 16 is assumed to have constant resistance R , and it is desired to interpose attenuation equalizing networks N_1 and N_2 between these repeaters and the line 31, so that the attenuation to the repeaters shall be constant and the impedance of the networks combined with the respective repeaters shall be the same as of the repeaters alone.

The line 31 with which connection is made is assumed for the purpose of this example to be a certain line 150 miles long for which the attenuation characteristics have been determined experimentally, as shown in Fig. 17. It will be seen that the attenuation increases with frequency, but at a rate depending largely on weather conditions. The curve D is chosen as the basis for design, this curve giving the average attenuation for this 150 mile line under wet weather conditions.

Wave filters F_1 and F_2 are connected to the line 31 as shown in Fig. 16 to separate into respective channels the two frequency ranges, one extending from 6.6 to 19 kilocycles and the other extending from 21 to 30 kilocycles. The line 31 attenuates the currents as shown by the characteristic D in Fig. 17. It is desired that the attenuation up to the repeater input terminals shall be constant, as shown, for example, by the characteristics D_1' , D_2' , therefore the networks N_1 and N_2 should have attenuation characteristics like D_1 and D_2 , so that at all frequencies within the respective ranges the ordinates of D_1' and D_2' shall be obtained by summing the ordinates of D and D_1 or D_2 , as the case may be. All the attenuations of Fig. 17 are expressed per mile of the line, hence the ordinates of D_1 and D_2 must be multiplied by 150 to give the desired attenuations in the networks N_1 and N_2 .

Having determined in the characteristics D_1 and D_2 how the networks N_1 and N_2 are

desired to function, we assume that the network will be a single section of Fig. 10, namely the section 27, 28, 27', 28'. The given constant resistance characteristic impedance of the repeater becomes element R of the network of Fig. 10, on which the switches 27', 28' are assumed to be closed.

To embody z_{11} in an appropriate physical structure to realize the attenuation characteristic D_1 or D_2 , we are guided by equation 17, (taking the absolute values of its members) and by trial the network designated z_{11} at the top of Fig. 16 is found to be appropriate. Then by the aid of equation 13 the combination of elements designated z_{21} in Fig. 16 is obtained to represent the like designated element in Fig. 10.

For greater accuracy account is taken of the dissipation in the inductance coils, and this is treated as a small equivalent resistance.

As to the particular values to be assigned to the resistance and reactance elements that make up the impedances z_{11} and z_{21} of Fig. 16, these may be determined by assuming a number of points on the characteristic D_1 or D_2 of Fig. 17 corresponding to the number of available parameters and then making the attenuation characteristic for the network N_1 or N_2 pass through those points. In this way a very close approximation may be obtained, as shown by the curves in Fig. 18.

It will be seen that although the currents come in over the line 31 variously attenuated according to the frequency, nevertheless the currents at the repeater input terminals are of constant attenuation for all frequencies within their respective ranges. The attenuation equalizing networks N_1 and N_2 give high attenuation when the line attenuation is low and vice versa, so that as combined with the line they give a constant attenuation. At the same time this result is secured without introducing impedance irregularities and consequent reflection effects. The impedance looking from the line 31 into the network N_1 has the same constant resistance value at all frequencies as when looking into the repeater at its input terminals.

The same z_{11} and z_{21} might be used in a single section of the networks of Figs. 7, 8 and 9 to get N_1 and N_2 . The same attenuations and constant resistance impedances would result. Fig. 10 gives an economical design however.

I claim:

1. A network comprising a series network in which there is an impedance of general value z_{11} where z_{11} is any function of frequency, and also comprising a shunt network in which there is another impedance z_{21} related by the equation $z_{11}z_{21}=R^2$ where

R is a constant, said network as a whole having constant resistance R over a range of frequencies at its input terminals.

2. A network whose characteristic impedance over a range of frequencies is the constant resistance R, said network comprising at least one section having a series impedance element z_{11} and a shunt impedance element $R+z_{21}$ subject to the equation $z_{11}z_{21}=R^2$, said network having a resistance R across the output terminals of the last section.

3. A network having a pair of input terminals and a pair of output terminals, and comprising a resistance R, a general series impedance z_{11} and a shunt impedance z_{21} subject to the relation $z_{11}z_{21}=R^2$, said elements being so combined and related that, with resistance R across the output terminals the characteristic impedance across the input terminals will be R over a range of frequencies.

4. A network having a pair of input terminals and a pair of output terminals, said network comprising at least one section with a series impedance element z_{11} and a shunt impedance element $R+z_{21}$ subject to the equation $z_{11}z_{21}=R^2$, said network also having a constant resistance R across its output terminals and its elements being so proportioned and arranged that the characteristic impedance at the input terminals over a range of frequency is R.

5. A network having a pair of input terminals and a pair of output terminals, said network comprising at least one of several like sections between said pairs of terminals with a constant resistance R across the output terminals, each such section comprising a series network in which there is a general impedance z_{11} and also comprising a shunt network in which there is another impedance z_{21} related by the equation $z_{11}z_{21}=R^2$, the input impedance of the network with the said constant resistance R across its output terminals being a constant resistance of the same value.

6. An attenuation equalizer between a

source of electromotive force and a constant resistance R consisting of an interposed network comprising two impedances z_{11} and z_{21} such that $z_{11}z_{21}=R^2$, one impedance z_{11} being designed to satisfy the equation

$$e\Gamma = 1 + \frac{z_{11}}{R}$$

where Γ is a propagation constant whose attenuation component is complementary to the attenuation to be equalized, one of said impedances being in series in relation to said source and said constant resistance R, and the other of said two impedances being in shunt to said source and resistance.

7. The method of combining a device of general impedance value in a network that shall have a constant resistance R over a desired frequency range which consists in determining by trial an impedance structure that shall have the same characteristic impedance z_{11} as said device over said frequency range, then forming a structure having impedance z_{21} such that $z_{11}z_{21}=R^2$ and combining said device and said structure in a network with z_{11} as a series component and z_{21} as a shunt component.

8. The method of equalizing the attenuation of a current from a source of E. M. F. to a constant resistance R without altering the impedance presented to said source, which consists in interposing a suitable network comprising a general impedance z_{11} designed by trial to satisfy the equation

$$e\Gamma = 1 + \frac{z_{11}}{R}$$

where Γ is the propagation constant whose attenuation component is complementary to the attenuation to be equalized, said network also comprising another impedance z_{21} related by the equation $z_{11}z_{21}=R^2$.

In testimony whereof, I have signed my name to this specification this 8th day of August, 1922.

OTTO J. ZOBEL.