

June 8, 1926.

O. J. ZOBEL

1,587 574

DOUBLY PERIODIC WAVE FILTER

Filed April 30, 1920

2 Sheets-Sheet 1

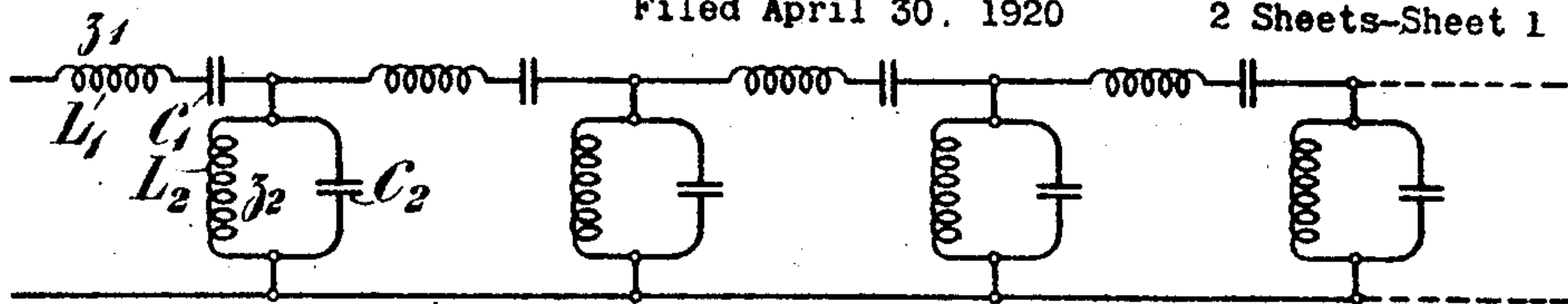


Fig. 1

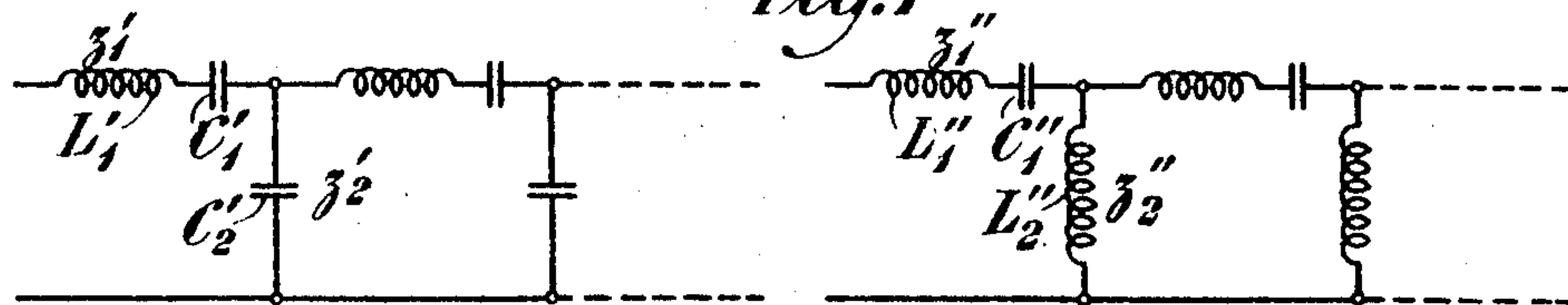


Fig. 2

Fig. 3

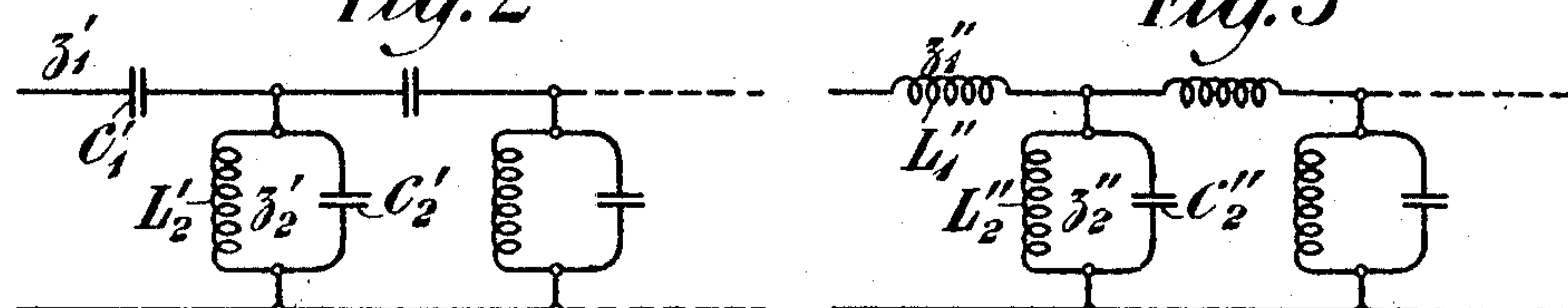


Fig. 4

Fig. 5

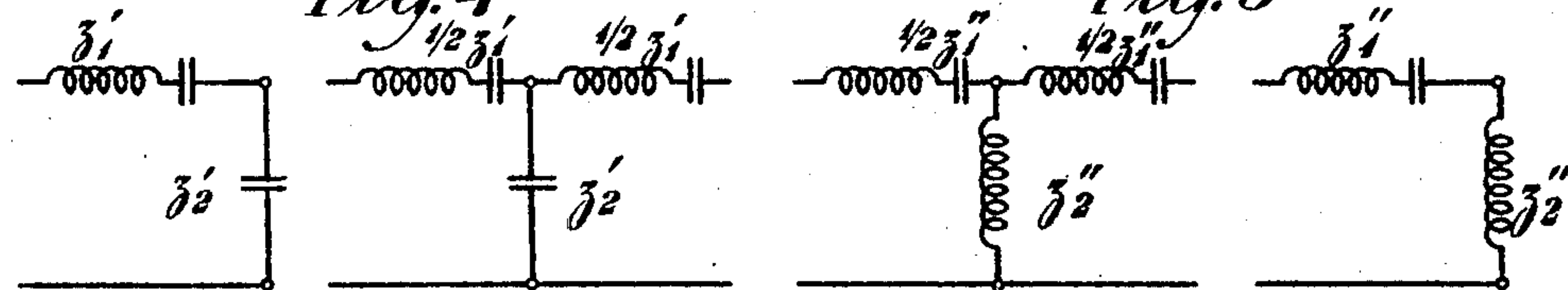


Fig. 6

Fig. 7

Fig. 8

Fig. 9

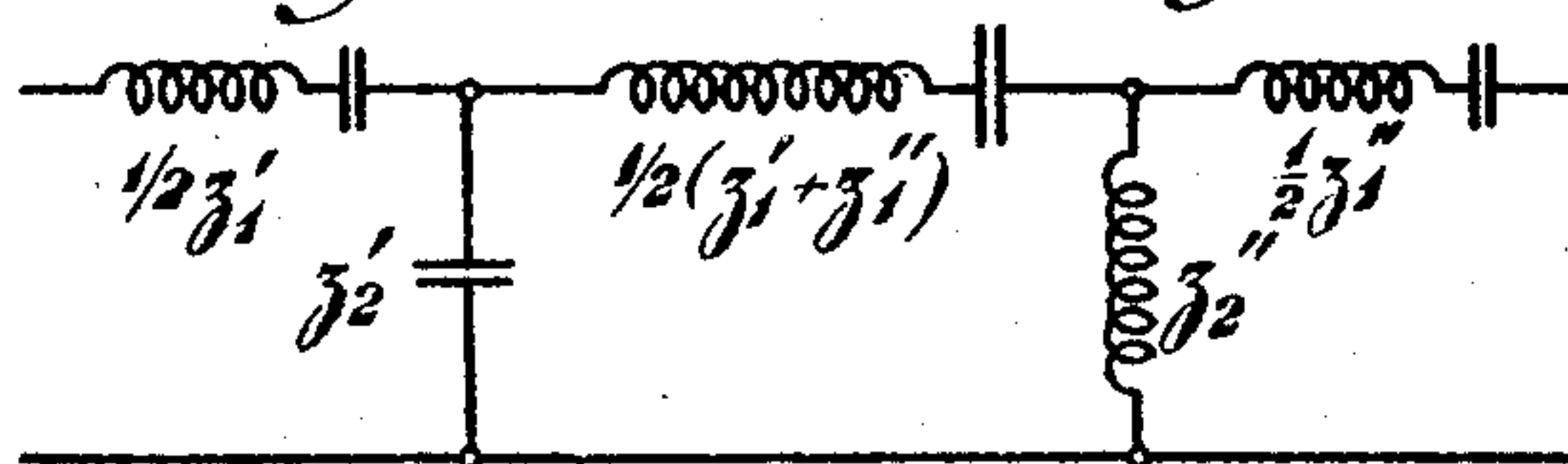


Fig. 10

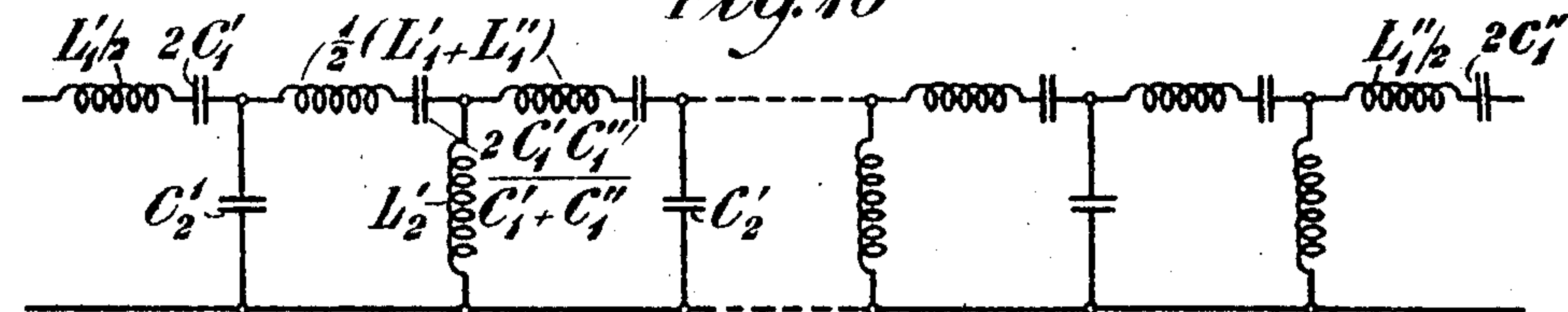


Fig. 11

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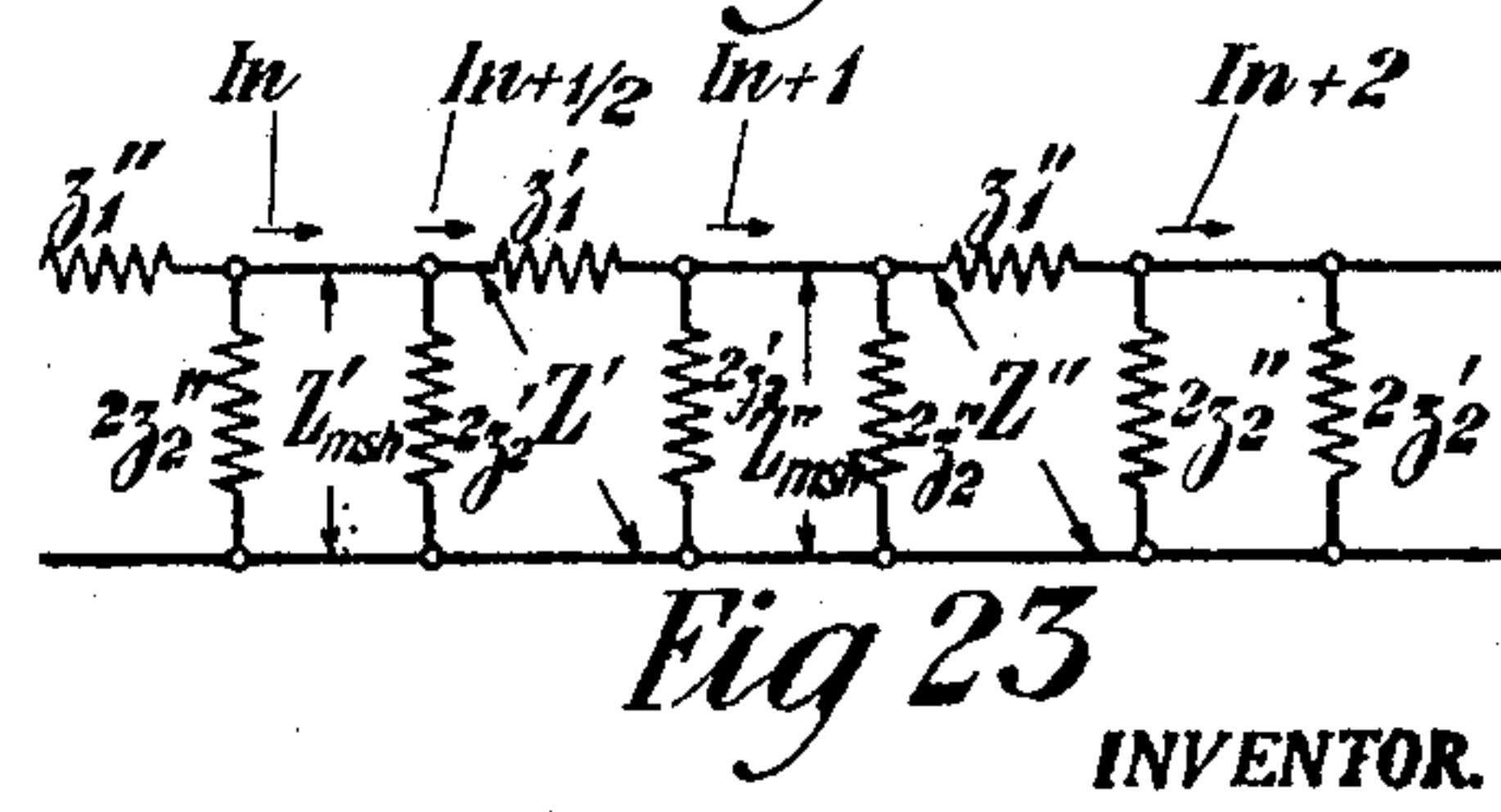
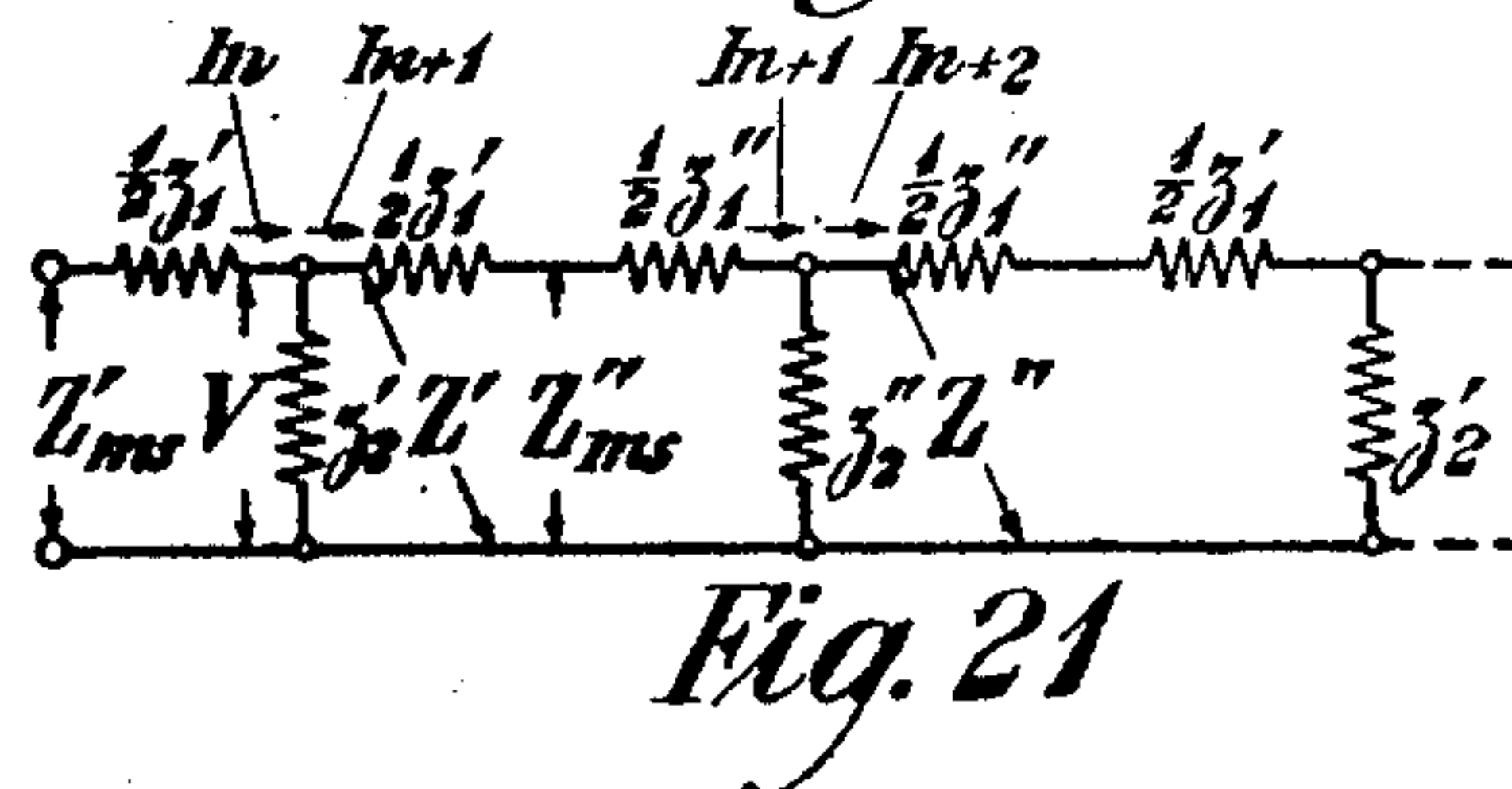
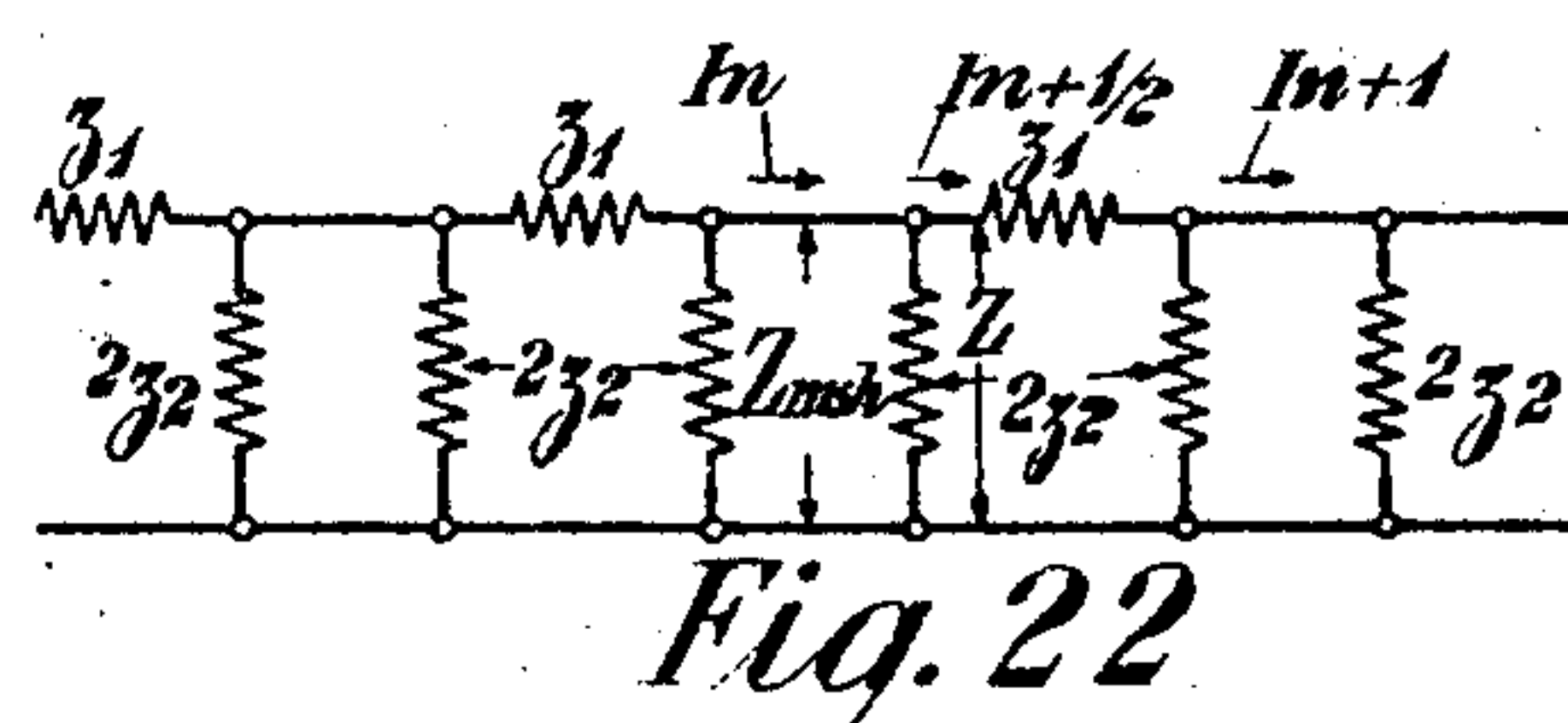
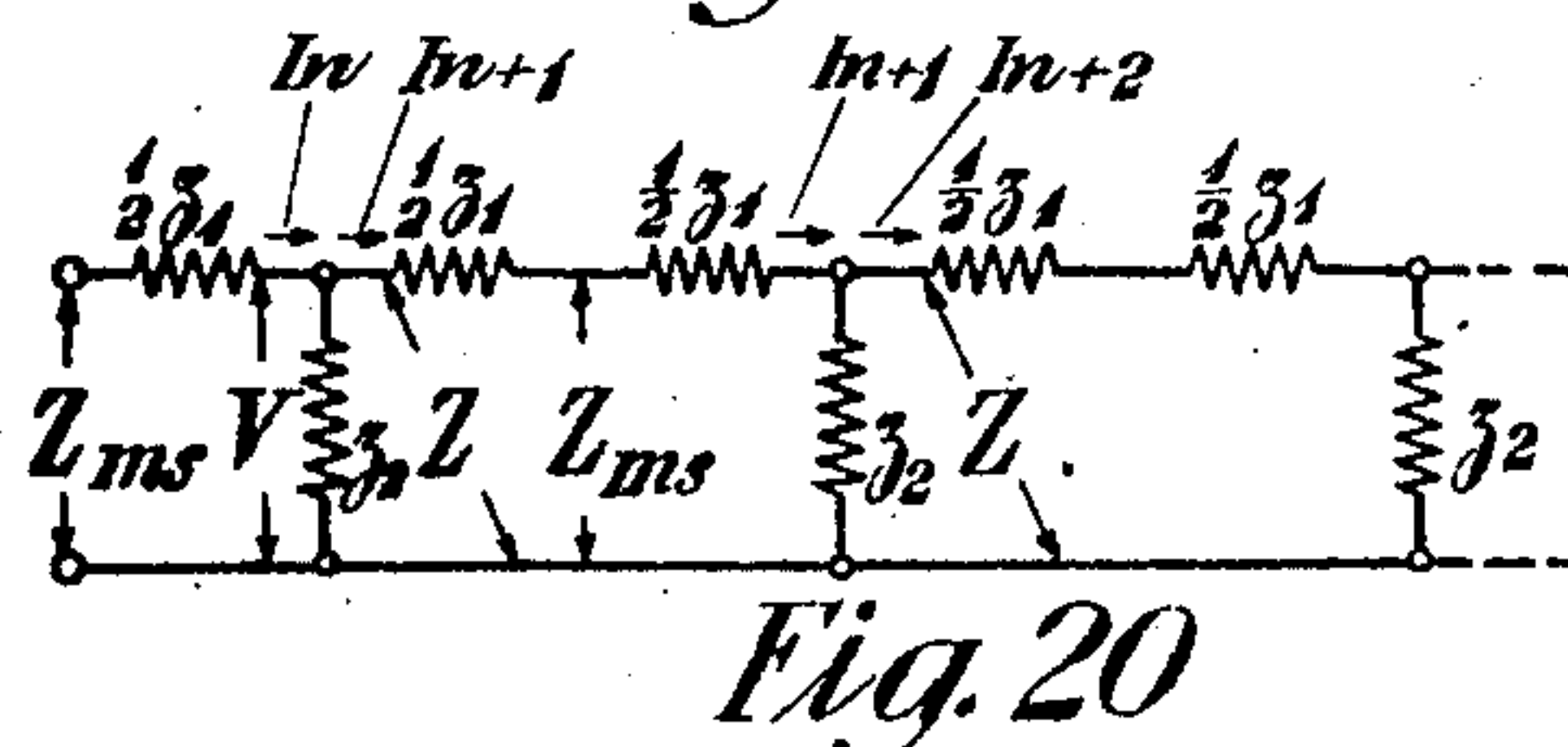
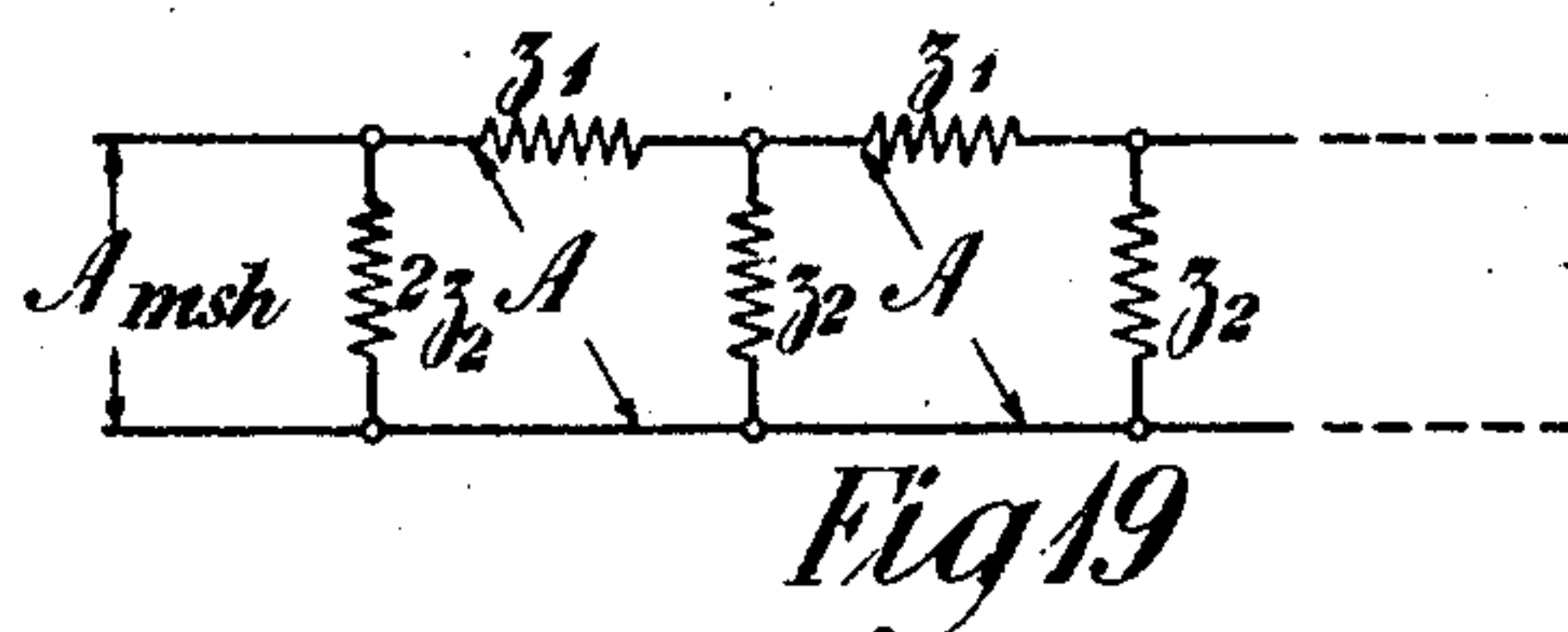
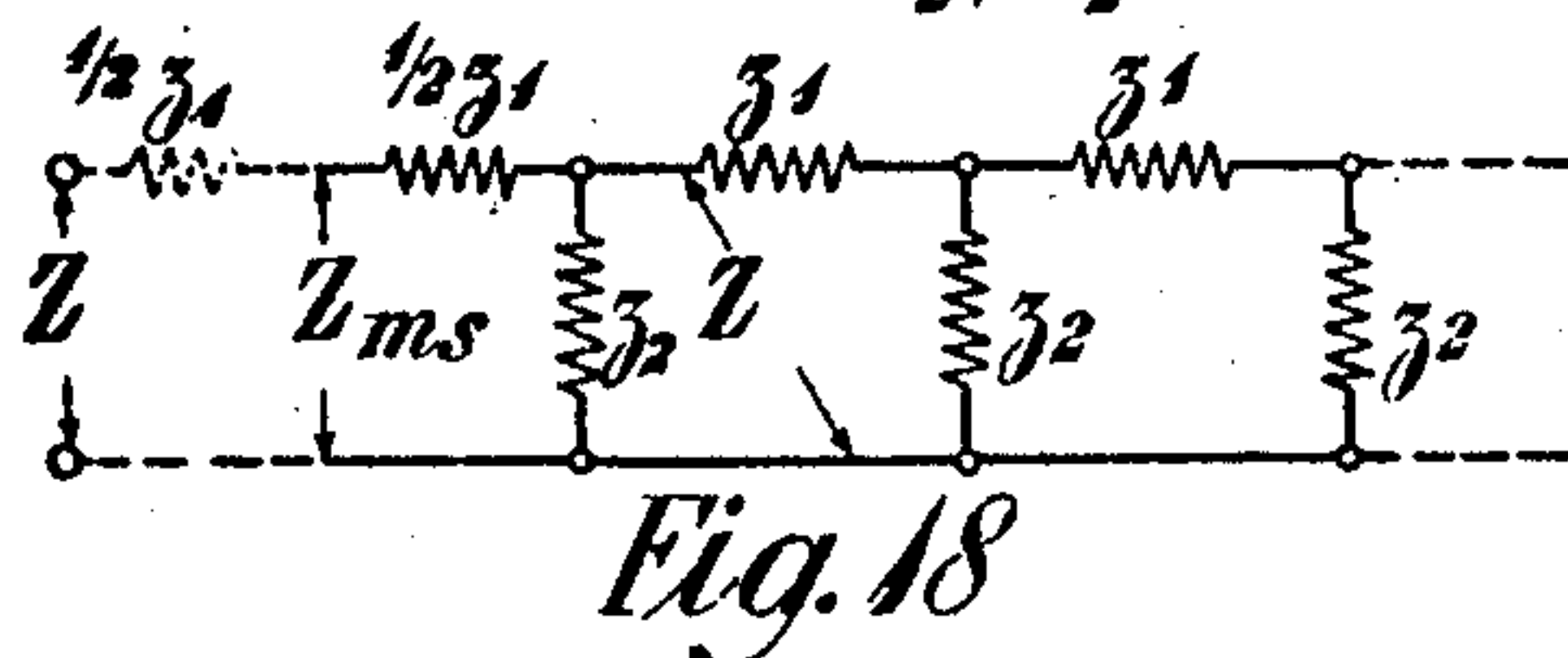
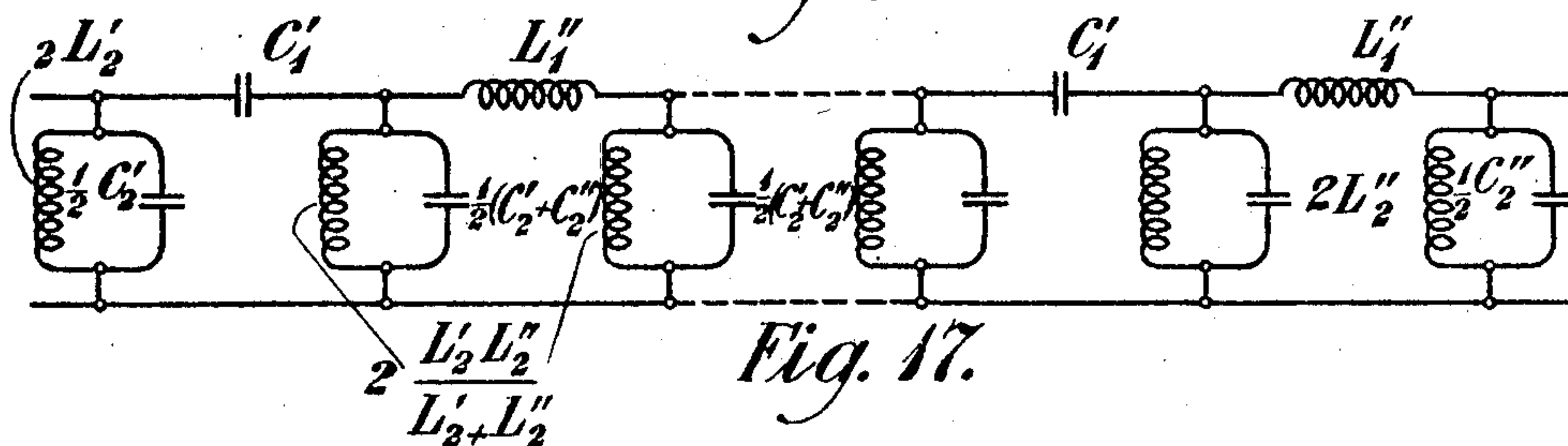
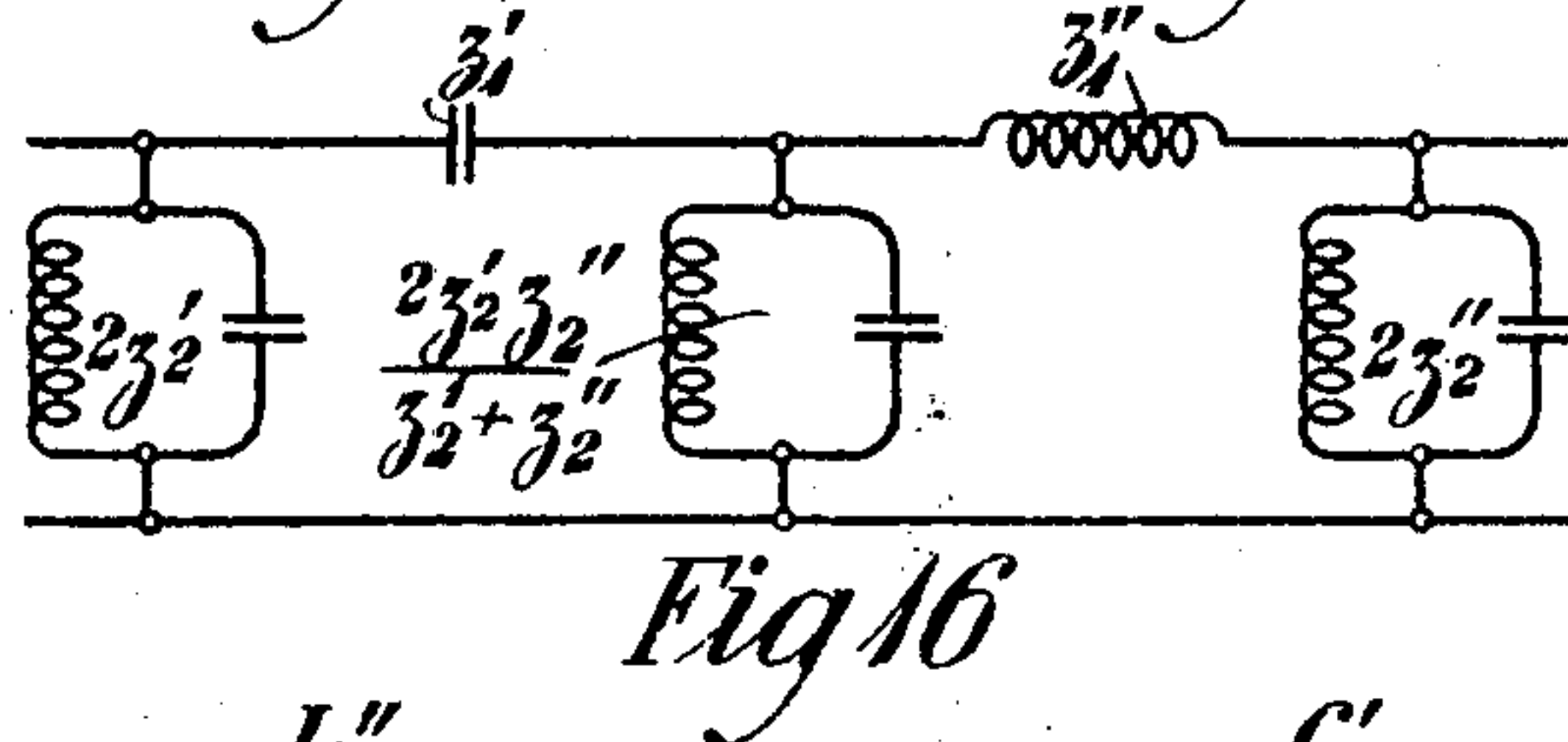
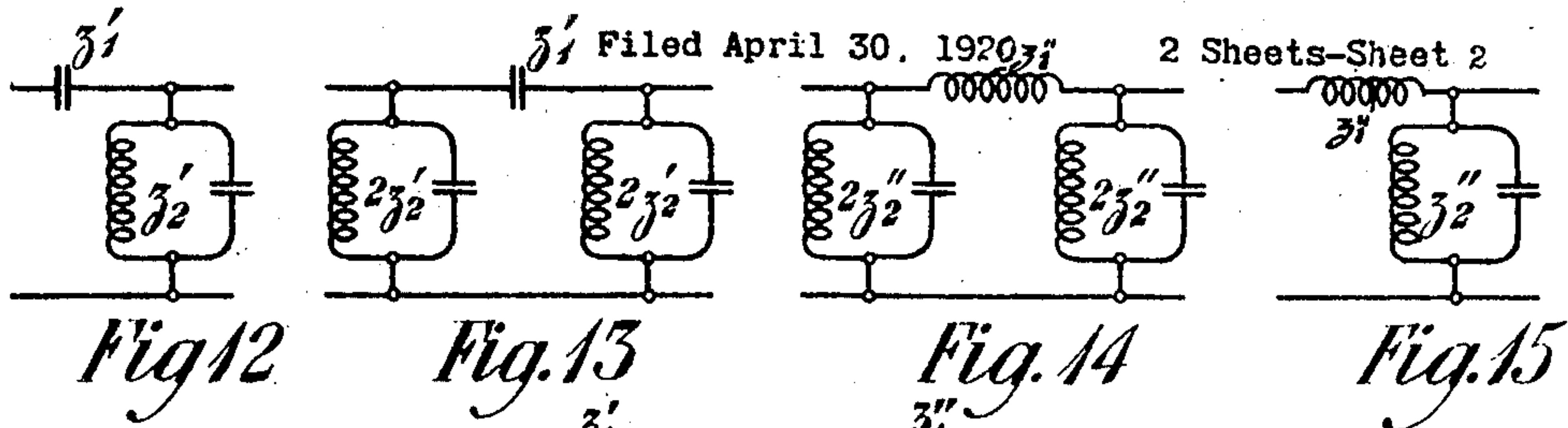
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DOUBLY PERIODIC WAVE FILTER

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2 Sheets-Sheet 2



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UNITED STATES PATENT OFFICE.

OTTO J. ZOBEL, OF MAPLEWOOD, NEW JERSEY, ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

DOUBLY-PERIODIC WAVE FILTER.

Application filed April 30, 1920. Serial No. 377,966.

This invention relates to selective circuits, and more particularly to selective circuits of the type known as wave filters.

A number of wave filters are described in certain patents issued to George A. Campbell, 1,227,113 and 1,227,114, dated May 22, 1917. The Campbell patents disclose three types of wave filters, namely: Low pass filters, high pass filters and band filters. A low pass filter is one which transmits with negligible attenuation a band of frequencies extending from zero to some pre-assigned upper limit. A high pass filter is one which transmits with negligible attenuation a band of frequencies extending from a pre-assigned lower limit to infinity. A band filter, however, transmits a band of frequencies extending over a range whose upper and lower limits both lie between zero and infinity.

Five different forms of band filters are described in the Campbell patents, these five forms being illustrated in Figures 1 to 5 inclusive of said patents. The four forms of band filters illustrated in Figs. 2 to 5 inclusive of the Campbell patents are subject to the objection that while the cut-off at one side of the band of free transmission is fairly sharp, the cut-off at the other side of the band is much more gradual. In the case of the band filter illustrated in Fig. 1 of the Campbell patents, while the cut-off is sharp on both sides of the band, very great precision in the design of the elements of the filter is necessary in order to secure uniform transmission in the range of free transmission. This is due to the fact that this particular type of filter is in reality a filter transmitting two bands which are caused to merge or coalesce by making the upper limit of the lower band and the lower limit of the upper band of the same frequency. If these two frequencies are not made identical by careful design of the elements of the filter, an attenuation region may occur about the middle of the band of free transmission of the filter which, unfortunately, is the portion of the filter that in practice is used the most.

It is, therefore, one of the objects of this invention to provide a type of wave filter which will transmit a band of frequencies

having a very sharp cut-off at each side of the transmitted band, said filter being composed of elements which do not require extreme accuracy in their design.

Another object of the invention is to provide a filter whose structure shall be doubly periodic as distinguished from the singly periodic structures of the filters disclosed in the Campbell patents above referred to. Other and further objects of the invention will appear more fully hereinafter.

The objects of this invention are attained by constructing each section of the doubly periodic filter of two parts, one of which may be a section of one of the forms of filters disclosed in Figs. 2 to 5 of the Campbell patents, and the other part of which may be a section of another of the several types of filters disclosed in said figures of the Campbell patents. An analysis of these figures of the Campbell patents shows that there are two filters, each having the same type of series element in each section, but having different shunt elements. In one form of the filter, in accordance with the present invention, each section will comprise a section from each of these two types of filters, so that the section of the doubly periodic structure will consist of two similar series elements with different shunt elements. The other two types of filters in the Campbell patents, it will be noted, each have the same type of shunt element, but different series elements. Another type of filter in accordance with the present invention, may, therefore, be obtained by combining in each of its sections a section from each of these two types of filters, so that the resultant section will have two similar shunt elements, but different series elements.

Of these two types of doubly periodic filters, one has a characteristic impedance at a mid-series termination of a half section of the filter, and the other a characteristic impedance at a mid-shunt termination of a half section of the filter, which in either case is equivalent to the characteristic impedance of the correspondingly terminated Campbell filter of the type illustrated in Fig. 1 of the Campbell patents. For convenience, the first of the above described types of doubly periodic filters is called the

mid-series doubly periodic single band wave filter. This filter is so called from the fact that it is produced by connecting alternately in series single mid-series sections of two different 3-element single band wave filters whose mid-series characteristic impedances are equal to that of the single band type illustrated in Fig. 1 of the Campbell patents. The series element in both of these component sections is an inductance in series with a capacity, while the shunt element is in one a capacity and in the other an inductance.

In the other type herein called the mid-shunt doubly periodic single band wave filter, the double periodic section is produced by alternately connecting in series single mid-shunt sections of two different 3-element single band wave filters whose mid-shunt characteristic impedances are equal to the corresponding impedance of the single band filter illustrated in Fig. 1 of the Campbell patents. The shunt element in both these two component sections is an inductance in parallel with a capacity, while the series element is in one a capacity and in the other an inductance.

The term "mid-series termination" as used in this description has reference to such a termination of the filter that the end section shall terminate in a series element whose impedance is half that of the normal series element. Similarly, the term "mid-shunt termination" means such a termination of a wave filter that the end section of the series shall terminate in a shunt element whose admittance is one-half of the admittance of the normal shunt element. In other words, the impedance of the terminal shunt element is twice that of the normal shunt element.

From these definitions it follows, of course, that the mid-series impedance is the impedance measured at a mid-series termination, while the mid-shunt impedance is the impedance measured at a mid-shunt termination.

The invention may now be more fully understood from the following description, when read in connection with the accompanying drawing in which:—

Fig. 1 illustrates a confluent band filter of the type illustrated in Fig. 1 of the Campbell patents above referred to.

Fig. 2 illustrates a filter having the characteristics of the filter of Fig. 5 of the Campbell patents.

Fig. 3 illustrates a filter corresponding to the filter of Fig. 3 of the Campbell patent.

Fig. 4 illustrates a filter similar to that shown in Fig. 2 of the Campbell patent.

Fig. 5 illustrates a filter corresponding to that of Fig. 4 of the Campbell patent.

Figs. 6 to 10 inclusive illustrate how sec-

tions of the filters of Figs. 2 and 3 may be transformed into mid-series terminations and combined to form a single section of a doubly periodic filter.

Fig. 11 is a diagram illustrating a plurality of sections of the resultant mid-series doubly periodic single band filter.

Figs. 12 to 16 inclusive are diagrams showing how sections of the filters of Figs. 4 and 5 may be transformed into mid-shunt terminated sections and combined to form one section of a doubly periodic filter.

Fig. 17 is a diagram illustrating a number of sections of the resultant mid-shunt doubly periodic single band filter.

Figs. 18 to 23 inclusive are schematic diagrams used in connection with the development of the mathematical theory underlying the wave filter of this invention.

The mid-series double periodic type will be considered first, and in order to understand the theory underlying this filter it will first be necessary to develop the design formula of the confluent band type of filter illustrated in Fig. 1.

Pre-determination of inductances and capacities in the confluent band "constant k" wave filter.

The confluent band filter illustrated in Fig. 1 has a characteristic such that the product of the impedances of its series and shunt elements is equal to the square of a constant, called for convenience, " k ." Hence, the term "constant k " filter. The filters illustrated in Figs. 2 to 5 inclusive do not have this characteristic, although doubly periodic filters are herein developed from these types which will have a characteristic impedance equal to that of the "constant k " filter just referred to.

While design formulæ are given in the Campbell patents above referred to for filters of the type illustrated in Figure 1, it will be more convenient for the purpose of this specification to derive these design formulæ in terms of the limiting frequencies and the constant k .

It is well known that a smooth line having uniform series impedance distributions z_1 , and uniform shunt impedance distributions z_2 per unit length has the characteristic impedance

$$k = \sqrt{z_1 \cdot z_2} \quad (1)$$

and a propagation constant

$$\gamma = \sqrt{\frac{z_1}{z_2}} \quad (2)$$

A wave filter such as illustrated in Fig. 1, having series elements z_1 and shunt elements z_2 , has a characteristic impedance at any termination which is a function of both the

product and the ratio of z_1 and z_2 and has a propagation constant which is a function of their ratio, hence it has been found convenient to express both the characteristic impedance and propagation constant of the wave filter in terms of k and γ , the parameters of the corresponding smooth line. As previously stated, the filter of Fig. 1 is of the constant k type, and in this type of filter it is necessary that the product of the series impedance z_1 and the shunt impedance z_2 shall be a constant independent of frequency, that is,

$$z_1 \cdot z_2 = k^2 = \text{a constant.} \quad (3)$$

As pointed out in the above mentioned patents to Campbell, free transmission takes place through such a network for frequencies which give

$$\gamma^2 = \frac{z_1}{z_2} \quad (4)$$

within the range whose limits are defined by

$$\gamma^2 = 0 \text{ to } \gamma^2 = -4. \quad (5)$$

By combining equation 5 with equation 3, it is apparent that the filter transmits within a range of frequencies in which z_1 lies between

$$z_1 = 0 \text{ and } z_1 = \pm i2k \quad (6)$$

In Fig. 1, the series element z_1 consists of an inductance L_1 in series with a capacity C_1 , and the shunt element z_2 consists of an inductance L_2 in parallel with a capacity C_2 . Hence

$$z_1 = iL_1 2\pi f - \frac{i}{C_1 2\pi f} = -i \frac{(1 - L_1 C_1 4\pi^2 f^2)}{C_1 2\pi f} \quad (7)$$

and

$$z_2 = \frac{iL_2 2\pi f \frac{(-i)}{C_2 2\pi f}}{iL_2 2\pi f - \frac{i}{C_2 2\pi f}} = \frac{iL_2 2\pi f}{(1 - L_2 C_2 4\pi^2 f^2)} \quad (8)$$

If now, we put

$$a = 4\pi^2 L_1 C_1 \quad (9)$$

$$\frac{b}{k} = 2\pi C_1 \quad (10)$$

we have, by substituting these values in equation 7

$$z_1 = -i \frac{(1 - a f^2)}{b f} \cdot k \quad (11)$$

If, now, we let f_1 and f_2 represent the known critical frequencies at the cut-off points of the filter and assume the constant k to be known, it follows that z_1 must equal $-i2k$ at f_1 and $+i2k$ at f_2 .

By substituting these two values of z_1 in equation 11, we get

$$\begin{aligned} f_1^2 \cdot a + 2f_1 \cdot b &= 1 \\ f_2^2 \cdot a - 2f_2 \cdot b &= 1 \end{aligned} \quad (12)$$

The solution of equations 12 gives

$$\begin{aligned} a &= \frac{1}{f_1 f_2} \\ b &= \frac{(f_2 - f_1)}{2f_1 f_2} \end{aligned} \quad (13)$$

Substituting these values of a and b in equations 9 and 10, we have as the design formulæ for the series inductance and capacity

$$\begin{aligned} L_1 &= \frac{1}{\pi (f_2 - f_1)} \cdot k \\ C_1 &= \frac{(f_2 - f_1)}{4\pi f_1 f_2} \cdot \frac{1}{k} \end{aligned} \quad (14)$$

For a constant k type of filter, a relation exists between each element of the series impedance z_1 and the corresponding element of the shunt impedance z_2 similar to the relation between z_1 and z_2 as expressed by equation (3). Hence, in the filter of Fig. 1, the product of the impedance of the inductance L_1 and the impedance of the capacity C_2 will be equal to k^2 . Hence, we have

$$iL_1 2\pi f \cdot \frac{-i}{C_2 2\pi f} = k^2 \quad (15)$$

Similarly, the product of the impedance of the capacity C_1 and the impedance of the inductance L_2 will be equal to k^2 . Hence,

$$\frac{-i}{C_1 2\pi f} \cdot iL_2 2\pi f = k^2 \quad (16)$$

Solving these equations we at once have the design formulæ for the shunt inductance L_2 and shunt capacity C_2 as follows:

$$\begin{aligned} L_2 &= C_1 k^2 = \frac{(f_2 - f_1)}{4\pi f_1 f_2} \cdot k \\ C_2 &= \frac{L_1}{k^2} = \frac{1}{\pi (f_2 - f_1)} \cdot \frac{1}{k} \end{aligned} \quad (17)$$

Predetermination of the constants of the single band filters of Figures 2 and 3.

The desired mid-series doubly periodic type of filter in accordance with this invention is constructed by making up each complete section of alternate mid-series sections of the filters of Figs. 2 and 3. As shown, the impedance z_1' in Fig. 2 comprises an inductance L_1' and capacity C_1' in series, while the shunt impedance z_2' consists of a capacity C_2' . Likewise, in Fig. 3, the series element z_1'' of each section consists of an inductance L_1'' and capacity C_1'' in series, while the

shunt element z_2'' consists of an inductance L_2'' .

In order that the individual sections of the filters of Figs. 2 and 3 may be combined alternately in series and without impedance irregularity to form doubly periodic sections of the mid-series doubly periodic type illustrated in Fig. 11, the mid-series characteristic impedance of each of these types of filters must be equal to that of the constant k type illustrated in Fig. 1. The manner in which alternate sections are to be combined to form the doubly periodic type is illustrated in Figs. 6 to 10 inclusive. Figs. 6 and 9 illustrate single sections of the filters of Figs. 2 and 3 respectively. As indicated in Fig. 7, a filter section equivalent to that of Fig. 6 may be constructed by connecting the shunt impedance z_2' at the mid-point of a series impedance made up of two series elements each having a value equal to half of the impedances z_1' . It will be seen that if a filter of the type illustrated in Fig. 2 be made up of a number of sections such as illustrated in Fig. 7, the filter will obviously terminate in mid-series, since the terminal element of the filter will have an impedance equal to one-half of the full series element.

In a similar manner the filter section of Fig. 9 may be transformed into an equivalent structure illustrated in Fig. 8 by connecting the shunt impedances z_2' at the mid-point of a series impedance made up of two elements each having one-half the impedance of a full series element. If, now, the two sections illustrated in Figs. 7 and 8 have the same mid-series characteristic impedances, there will be no reflection losses if the sections be connected in series as illustrated in Fig. 10, in which the adjacent elements

$$\frac{1}{2} z_1' \text{ and } \frac{1}{2} z_1''$$

are combined to form the series impedance element

$$\frac{1}{2} (z_1' + z_1'').$$

The network of Fig. 10 comprises a complete section of a mid-series doubly periodic filter. By combining a plurality of such sections, a complete wave filter of the mid-series doubly periodic type will be constructed as illustrated in Fig. 11. In this

$$Z_{ms}^2 = z_1 z_2 + \frac{1}{4} z_1^2 = z_1' z_2' + \frac{1}{4} z_1'^2 = z_1'' z_2'' + \frac{1}{4} z_1''^2 \quad (24)$$

The above relation is sufficient to obtain all of the inductances and capacities of the filters of Figs. 2 and 3 in terms of f_1 , f_2 and k . The values of these inductances and capacities will now be obtained.

figure the relations of the various inductances and capacities to the corresponding inductances and capacities of Figs. 2 and 3 are illustrated. It will be noted that the filter terminates at the left in a mid-series section whose series element has a value equal to one-half of the full series element of Fig. 2. Similarly the filter of Fig. 11 terminates at its right hand end in a mid-series section whose series element has one-half the value of a full series element of Fig. 3.

Let us now determine the mid-series characteristic impedance Z_{ms} of the constant k type of wave filter illustrated in Fig. 1. A wave filter of this type is schematically indicated in Fig. 18. From this figure it is apparent that

$$Z_{ms} = Z - \frac{1}{2} z_1 \quad (18)$$

Also, from this figure we may express the characteristic impedance Z measured across the terminals of a full section, as,

$$Z = z_1 + \frac{Z z_2}{Z + z_2} \quad (19)$$

which may be written

$$Z - \frac{z_1}{2} = \sqrt{z_1 z_2 + \left(\frac{z_1}{2}\right)^2} \quad (20)$$

Solving equations 18 and 19, we have as the expression of the mid-series impedance of the filter of Fig. 1

$$Z_{ms} = \sqrt{z_1 z_2 + \frac{1}{4} z_1^2} \quad (21)$$

The mid-series impedances of the types of filters illustrated in Figs. 2 and 3 may obviously be determined in a similar manner and may be expressed as follows:

$$Z_{ms}' = \sqrt{z_1' z_2' + \frac{1}{4} (z_1')^2} \quad (22)$$

$$Z_{ms}'' = \sqrt{z_1'' z_2'' + \frac{1}{4} (z_1'')^2} \quad (23)$$

Now, since, as above stated, the mid-series characteristic impedances of the filters of Figs. 2 and 3 are to be made equal to that of the filter of Fig. 1, we have

$$Z_{ms} = Z_{ms}' = Z_{ms}'' \quad (24)$$

and hence

For the constant k type of filter illustrated in Fig. 1, we have

$$(Z_{ms})^2 = z_1 z_2 + \frac{1}{4} z_1^2 = k^2 + \frac{1}{4} z_1^2 \quad (25)$$

Substituting in equation 25 the value of z_1 as given by equation 7, we have

$$(Z_{ms})^2 = k^2 + \frac{1}{4} \left(-i \frac{1 - L_1 C_1 4\pi^2 f^2}{C_1 2\pi f} \right)^2 \quad (26)$$

By simplifying, this equation reduces to

$$(Z_{ms})^2 = k^2 + \frac{1}{2} \frac{L_1}{C_1} - \frac{1}{16\pi^2 C_1^2 f^2} - \pi^2 L_1^2 f^2 \quad (27)$$

In a similar manner, for the filter of Fig. 2, we have

$$(Z_{ms}')^2 = z_1' z_2' + \frac{1}{4} (z_1')^2 \quad (28)$$

By analogy to equations 7 and 8 we have as the values of the series element z_1' and the shunt element z_2' of the filter of Fig. 2 the following:

$$z_1' = -i \frac{(1 - L_1' C_1' 4\pi^2 f^2)}{C_1' 2\pi f} \quad (29)$$

$$z_2' = \frac{-i}{C_2' 2\pi f} \quad (30)$$

Substituting these values in equation 28, we have

$$(Z_{ms}')^2 = L_1' \left(\frac{1}{C_2'} + \frac{1}{2C_1'} \right) - \frac{1}{4\pi^2 C_1'} \left(\frac{1}{C_2'} + \frac{1}{4C_1'} \right) \frac{1}{f^2} - \pi^2 L_1'^2 f^2 \quad (31)$$

So, also, for the filter of Fig. 3 we have

$$(Z_{ms}'')^2 = z_1'' z_2'' + \frac{1}{4} (z_1'')^2 \quad (32)$$

$$z_1'' = -i \frac{(1 - L_1'' C_1'' 4\pi^2 f^2)}{C_1'' 2\pi f} \quad (33)$$

$$z_2'' = i L_2'' 2\pi f \quad (34)$$

Again, by analogy to equations 7 and 8 we have

Substituting these values in equation 32 we get

$$(Z_{ms}'')^2 = \frac{1}{C_1''} \left(L_2'' + \frac{1}{2} L_1'' \right) - \frac{1}{16\pi^2 (C_1'')^2 f^2} - \pi^2 [(L_1'')^2 + 4L_1'' L_2''] f^2 \quad (35)$$

A comparison of equations 27, 31 and 35 shows that they are all of the same form as regards the factor frequency f , and since, by equation 23 the left-hand terms of each of these three equations are equal to each other, it follows that the right-hand terms are also equal. If, therefore, the right-hand term of equation 31 is equal to the right-hand term of equation 27, the corresponding co-efficients of the frequency factor must be equal, whence we have

equal to the right-hand term of equation 27. Again equating co-efficients we have

$$\begin{aligned} \frac{1}{C_1''} \left(L_2'' + \frac{1}{2} L_1'' \right) &= k^2 + \frac{1}{2} \frac{L_1}{C_1} \\ C_1'' &= C_1 \\ L_1''^2 + 4L_1'' L_2'' &= L_1^2 \end{aligned} \quad (38)$$

Combining the above equations with equation 14, we have as the design formulæ for the filter of Fig. 3 the following:

$$L_1' = L_1$$

$$\frac{C_1'^2 C_2'}{4C_1' + C_2'} = C_1^2 \quad (36)$$

$$L_1' \frac{(2C_1' + C_2')}{2C_1' C_2'} = k^2 + \frac{L_1}{2C_1}$$

$$L_1'' = \frac{f_1}{\pi f_2 (f_2 - f_1)} \cdot k$$

$$C_1'' = \frac{(f_2 - f_1)}{4\pi f_1 f_2} \cdot \frac{1}{k} \quad (39)$$

$$L_2'' = \frac{(f_1 + f_2)}{4\pi f_1 f_2} \cdot k$$

Mid-series doubly periodic single band wave filter.

If a mid-series doubly periodic wave filter is made up of alternating mid-series sections of the two types of wave filters illustrated in Figs. 2 and 3, the values of the inductances and capacities as indicated in Fig. 11 may be at once obtained from the design formulæ 37 and 39 of the filters of Figs. 2 and 3. Also, since

$$Z_{ms} = Z_{ms}' = Z_{ms}'' \quad (40)$$

there will be no reflection losses at the junction points of the components.

It now remains to be shown that the prop-

Equation 36 in connection with equation 14 gives the following as the design formula of the filter of Fig. 2:

$$L_1' = \frac{1}{\pi(f_2 - f_1)} \cdot k$$

$$C_1' = \frac{(f_2 - f_1)}{4\pi f_1^2} \cdot \frac{1}{k} \quad (37)$$

$$C_2' = \frac{1}{\pi(f_1 + f_2)} \cdot \frac{l}{k}$$

It will be observed that the above design formulæ are obtained from the design formulæ of the constant k filter of Fig. 1.

In a similar manner the design formulæ for the filter of Fig. 3 may be obtained by setting the right-hand term of equation 35

agation constant of the doubly periodic filter thus constructed will be the same as the propagation constant of the filter of Fig. 1.

Referring to Fig. 21 which illustrates schematically a mid-series doubly periodic wave filter, it is apparent from the relations between applied voltage, current and impedance that the current in the n th section of the filter flows through an impedance equivalent to the impedance of a shunt element z_2 in parallel with the impedance Z' of an infinite number of sections. Hence,

$$\frac{V}{I_n} = \frac{Z'z_2'}{Z' + z_2'} \quad (40)$$

where I_n designates the current in the n th component section of the filter. Similarly, from Fig. 21, since the current in the next half section flows through an impedance Z' , we have

$$\frac{V}{I_{n+1}} = Z' \quad (41)$$

where I_{n+1} represents the current flowing in the $(n+1)$ st component section of the filter. From Fig. 21 it is also apparent that the impedance Z' is equivalent to an impedance $\frac{1}{2}z_1'$ in series with the mid-series impedance Z_{ms}' . But since from equation 23, Z_{ms}'' is equal to Z_{ms}' we have

$$Z' = \frac{z_1'}{2} + Z_{ms}' = \frac{V}{I_{n+1}} \quad (42)$$

which may be written

$$Z_{ms}' = \frac{z_1'}{2} + \frac{Z'z_2'}{Z' + z_2'} \quad (43)$$

Whence

$$\frac{Z'z_2'}{Z' + z_2'} = Z_{ms}' - \frac{z_1'}{2} = \frac{V}{I_n} \quad (44)$$

Combining equations 42 and 44, we have

$$\frac{I_{n+1}}{I_n} = \frac{Z_{ms}' - \frac{z_1'}{2}}{Z_{ms}' + \frac{z_1'}{2}} = e^{-\Gamma'} \quad (45)$$

where e denotes the base of Napierian logarithms and Γ' denotes the propagation constant of a wave filter of the type illustrated in Fig. 21. The relation between the first and last members of equation 45 is established by equation 1 of the Campbell patent above referred to.

In a manner similar to the derivation of equation 45 the following relation may also be obtained from Fig. 21:

$$\frac{I_{n+2}}{I_{n+1}} = \frac{Z_{ms}'' - \frac{z_1''}{2}}{Z_{ms}'' + \frac{z_1''}{2}} = e^{-\Gamma''} \quad (46)$$

where Γ'' is the propagation constant of a filter of the type illustrated in Fig. 3.

In an analogous manner the following equation may be derived from Fig. 20.

$$\frac{I_{n+1}}{I_n} = \frac{I_{n+2}}{I_{n+1}} = \frac{Z_{ms} - \frac{z_1}{2}}{Z_{ms} + \frac{z_1}{2}} = e^{-\Gamma} \quad (47)$$

where Γ is the propagation constant of a filter of the type illustrated in Fig. 1, it being noted in this connection that Fig. 20 represents schematically a filter of this type.

If, now, we desire to obtain the ratio between the currents flowing in adjacent full sections of the doubly periodic type of filter shown in Fig. 21, it will be apparent from Fig. 20 that the currents flowing in two complete adjacent sections may be taken respectively as I_n and I_{n+2} . Therefore, we have from equations 45 and 46

$$\frac{I_{n+2}}{I_n} = e^{-\Gamma'} \cdot e^{-\Gamma''} = e^{-P_s} \quad (48)$$

where P_s designates the propagation constant of the mid-series doubly periodic wave filter. Since this propagation constant is to be the same as that of the constant k type of filter illustrated in Fig. 1, we have

$$e^{-\Gamma} = e^{-\Gamma'} \cdot e^{-\Gamma''} = e^{-P_s} \quad (49)$$

This is the relation to be proven, and in order to show that this relation holds, it is merely necessary to show that the equivalent relation obtained from equation 45, 46 and 47 holds, namely:

$$\frac{Z_{ms} - \frac{1}{2}z_1}{Z_{ms} + \frac{1}{2}z_1} = \frac{Z_{ms}' - \frac{1}{2}z_1'}{Z_{ms}' + \frac{1}{2}z_1'} \cdot \frac{Z_{ms}'' - \frac{1}{2}z_1''}{Z_{ms}'' + \frac{1}{2}z_1''} \quad (50)$$

where we have already chosen the constants of the impedance elements Z_1' and Z_1'' so as to make $Z_{ms} = Z_{ms}' = Z_{ms}''$, as indicated by design formulæ 14, 17, 37 and 39.

Substituting Z_{ms} for Z_{ms}' and Z_{ms}'' in equation 50, clearing of fractions and simplifying, we have

$$4_{ms}^2(z_1' + z_1'' - z_1) - z_1 z_1' z_1'' = 0 \quad (51)$$

In order to prove equation 51, it is necessary to determine the relationship existing between z_1 , z_1' and z_1'' . From equations 7 and 14 we get

$$z_1 = -i \frac{(1 - L_1 C_1 4\pi^2 f^2)}{C_1 2\pi f} = -i 2k \frac{(f_1 f_2 - f^2)}{(f_2 - f_1) f} \quad (52)$$

From equations 29 and 37 we have

$$z_1' = -i \frac{(1 - L_1' C_1' 4\pi^2 f^2)}{C_1' 2\pi f} = -i 2k \frac{(f_1'^2 - f^2)}{(f_2 - f_1) f} \quad (53)$$

and from equations 33 and 39 we have

$$z_1'' = -i \frac{(1 - L_1'' C_1'' 4\pi^2 f^2)}{C_1'' 2\pi f} = -i 2k \frac{(f_1 f_2 - \frac{f_1 f^2}{f_2})}{(f_2 - f_1) f} \quad (54)$$

Substituting k^2 for $z_1 z_2$ in equation 24, we have

$$Z_{ms}^2 = z_1 z_2 + \frac{1}{4} z_1^2 = k^2 + \frac{1}{4} z_1^2 \quad (55)$$

Substituting the value of z_1 as given in equation 52 in equation 55, we have

$$4Z_{ms}^2 = 4k^2 - 4k^2 \cdot \frac{(f_1 f_2 - f^2)^2}{(f_2 - f_1)^2 f^2} = 4k^2 \frac{(f_1^2 f^2 + f_2^2 f^2 - f_1^2 f_2^2 - f^4)}{(f_2 - f_1)^2 f^2} \quad (56)$$

Also, from equations 52, 53 and 54 the expression $z_1' + z_1'' - z_1$ may be written as follows

$$z_1' + z_1'' - z_1 = \frac{-i 2k}{(f_2 - f_1) f} \left[f_1'^2 - \frac{f_1 f^2}{f_2} \right] \quad (57)$$

Also, from these same equations, the expression $z_1 z_1' z_1''$, may be written

$$z_1 z_1' z_1'' = \left[\frac{-i 2k}{(f_2 - f_1) f} \right]^3 (f_1 f_2 - f^2) (f_1'^2 - f^2) \left(f_1 f_2 - \frac{f_1 f^2}{f_2} \right) \quad (58)$$

Substituting the values given by equations 56, 57 and 58 in equation 52, and cancelling the factor

$$\left[\frac{-i 2k}{(f_2 - f_1) f} \right]^3$$

which occurs in both terms, we have, by expansion

$$\begin{aligned} f_1^4 f^2 + f_1^2 f_2^2 f^2 - f_1^4 f_2^2 - f_1^2 f^4 - \frac{f_1^3 f^4}{f_2} - f_1 f_2 f^4 + f_1^3 f_2 f^2 + \frac{f_1 f^6}{f_2} + \\ f_1^4 f_2^2 - f_1^3 f_2 f^2 - f_1^2 f_2^2 f^2 + f_1 f_2 f^4 - f_1^4 f^2 + \frac{f_1^3 f^4}{f_2} + f_1^2 f^4 - \frac{f_1 f^6}{f_2} = 0 \end{aligned} \quad (59)$$

It will be noted that the terms in equation 59 all cancel out, which proves the relation established by equation 50. Hence the propagation constant of the mid-series doubly periodic wave filter of this invention is the same as the propagation constant of the confluent band filter of Fig. 1.

Predetermination of inductances and capacities in mid-shunt doubly periodic single band wave filter.

This type of wave filter is illustrated in Fig. 17 and is produced by combining alternately in series sections of the filters of Fig. 4 and Fig. 5 respectively. The manner in which these sections are combined is illustrated in Figs. 12 to 16 inclusive. Fig. 12 indicates a single element of the filter of Fig. 4, while Fig. 15 illustrates a single element of the filter of Fig. 5. By dividing the shunt element z_2' of Fig. 12 into two similar impedance elements, having twice the value of z_2' and arranged in parallel, the equivalent mid-shunt structure of Fig. 13 may be attained. In a similar manner the structure illustrated in Fig. 14 is equivalent to the filter structure illustrated in Fig. 15. By joining together the structures of Figs. 13 and 14, and combining the adjacent shunt elements $2z_2'$ and $2z_2''$ as one unit we have

the network shown in Fig. 16, which illustrates one section of the mid-shunt doubly periodic filter.

Fig. 17 illustrates a filter of this type composed of a plurality of sections terminating at the ends in a mid-shunt termination. The left-hand termination is a mid-shunt termination for a filter of the type of Fig. 4, while the right end of the filter has a mid-shunt termination corresponding to the filter of Fig. 5. The values of the various inductances and capacities in the filter of Fig. 17 are given in terms of the corresponding inductances and capacities of the filters of Figs. 4 and 5. As shown in Fig. 17, each full section of the filter comprises two half sections having shunt elements of inductances and capacity in parallel and series elements which are for the first half section a capacity, and for the next half section an inductance. The series capacity will be the same as that of Fig. 4, and the series inductance the same as that of Fig. 5. The inductance of the shunt element will have a value of

$$\frac{2L_2' L_2''}{L_2' + L_2''}$$

and the capacity of the shunt element will have a value

$$\frac{1}{2} (C_2' + C_2'')$$

In order that the two half sections may be combined to give a full section of the filter of Fig. 17 each of these half sections which are in fact full sections of the filter of Figs. 4 and 5 must have a mid-shunt characteristic impedance equal to that of the constant k type illustrated in Fig. 1.

Let us now determine the mid-shunt characteristic impedance of the filter of Fig. 1. Fig. 19 illustrates schematically a filter of this type terminating in mid-shunt. From the diagram of Fig. 19 it is apparent that the mid-shunt admittance A_{msh} measured across the terminals of the filter is equal to the admittance of the mid-shunt element $2z_2$

in parallel with the admittance A measured across a full section of the filter. Hence, we have

$$A_{msh} = \frac{1}{2z_2} + A = \frac{1}{2z_2} + \frac{1}{Z} \quad (60)$$

where A_{msh} denotes the mid-shunt admittance. Also, since the admittance A is the reciprocal of the impedance Z of a full sectional termination we have from Fig. 19

$$\frac{1}{A} = Z = z_1 + \frac{z_2 Z}{z_2 + Z} = z_1 + \frac{l}{\frac{l}{z_2} + \frac{l}{Z}} \quad (61)$$

Substituting A for $\frac{l}{Z}$ equation 60 may be expressed

$$A + \frac{1}{2z_2} = \sqrt{\frac{1}{z_1 z_2} + \left(\frac{1}{2z_2}\right)^2} = A_{msh} \quad (62)$$

Since the mid-shunt impedance is the reciprocal of the mid-shunt admittance, we have, from equation 62

$$Z_{msh} = \frac{1}{\sqrt{\frac{1}{z_1 z_2} + \left(\frac{1}{2z_2}\right)^2}} = \sqrt{\frac{z_1 z_2}{z_1 z_2 + \frac{1}{4} z_1^2}} \quad (63)$$

where Z_{msh} denotes the mid-shunt impedance.

In a similar manner it may be shown that for the filter of Fig. 4 the mid-shunt admittance of said filter may be expressed as follows:

$$Z_{msh}' = \frac{z_1' z_2'}{\sqrt{z_1' z_2' + \frac{1}{4} (z_1')^2}} \quad (64)$$

Likewise for the filter of Fig. 5, the mid-shunt impedance may be expressed

$$Z_{msh}'' = \frac{z_1'' z_2''}{\sqrt{z_1'' z_2'' + \frac{1}{4} (z_1'')^2}} \quad (65)$$

As the mid-shunt admittance expressed in equations 63, 64 and 65 must be equal, we have from these three equations

$$Z_{msh}^2 = \frac{1}{\frac{1}{z_1 z_2} + \frac{1}{4z_2^2}} = \frac{1}{\frac{1}{z_1' z_2'} + \frac{1}{4z_2'^2}} = \frac{1}{\frac{1}{z_1'' z_2''} + \frac{1}{4z_2''^2}} \quad (66)$$

If, now in equation 63 we substitute k^2 for $z_1 z_2$, and for z_1 substitute its value as given in equation 7, we have for the constant k type of filter illustrated in Fig. 1

$$Z_{msh}^2 = \frac{1}{k^2 \left(1 + \frac{L_1}{2C_1 k^2}\right) - \frac{1}{16\pi^2 C_1^2 k^4 \cdot f^2} - \frac{\pi^2 L_1^2 \cdot f^2}{k^4}} \quad (67)$$

In the case of the filter of Fig. 4 the series and shunt impedance may be expressed as follows:

$$z_1' = -\frac{i}{C_1' 2\pi f} \text{ and } z_2' = i \frac{L_2' 2\pi f}{1 - L_2' C_2' 4\pi^2 f^2} \quad (68)$$

Substituting these values in equation 64, we have

$$Z_{msh}' = \frac{1}{\frac{1}{L_2'} \left(C_1' + \frac{C_2'}{2} \right) - \frac{1}{16\pi^2 L_2'^2} \cdot f^2 - \pi^2 (4C_1' C_2' + C_2'^2) \cdot f^2} \quad (69)$$

Likewise in the case of the filter of Fig. 5 the series and shunt impedances may be expressed as follows:

$$z_1'' = iL_1'' 2\pi f \text{ and } z_2'' = \frac{iL_2'' 2\pi f}{1 - L_2'' C_2'' 4\pi^2 f^2} \quad (70)$$

Substituting these values in equation 65, we have

$$Z_{msh}'' = \frac{1}{C_2'' \left(\frac{1}{L_1''} + \frac{1}{2L_2''} \right) - \frac{1}{16\pi^2 \frac{L_1'' L_2''^2}{L_1'' + 4L_2''}} \cdot f^2 - \pi^2 C_2''^2 \cdot f^2} \quad (71)$$

Comparing equations 67, 69 and 71, it will be seen that since the left-hand terms of these equations are equal the right-hand terms must also be equal, and since the left-hand terms of these equations are all of the same form as regards the frequency f , the co-efficients may be equated. Hence, by equating the co-efficients of the right-hand terms of equations 67 and 69 we get

$$\frac{1}{L_2'} \left(C_1' + \frac{C_2'}{2} \right) = \frac{1}{k^2} \left(1 + \frac{L_1}{2C_1 k^2} \right) \quad (72)$$

$$L_2'^2 = C_1^2 K^4$$

$$4C_1' C_2' + C_2'^2 = \frac{L_1^2}{k^4}$$

from which, in connection with equations 14, we get the expressions for the inductances and capacities of the filter of Fig. 4 as follows:

$$C_1' = \frac{(f_1 + f_2)}{4\pi f_1 f_2} \cdot \frac{1}{k}$$

$$L_2' = \frac{(f_2 - f_1)}{4\pi f_1 f_2} \cdot k \quad (73)$$

$$C_2' = \frac{f_1}{\pi f_2 (f_2 - f_1)} \cdot \frac{1}{k}$$

Similarly, by equating the co-efficients of 67 and 71 we get

$$C_2'' \left(\frac{1}{L_1''} + \frac{1}{2L_2''} \right) = \frac{1}{k^2} \left(1 + \frac{L_1}{2C_1 k^2} \right)$$

$$\frac{L_1'' L_2''^2}{L_1'' + 4L_2''} = C_1^2 k^4 \quad (74)$$

$$C_2''^2 = \frac{L_1^2}{k^4}$$

from which, in connection with equations 14, the expressions for the inductances and

capacities of the filter of Fig. 5 may be obtained as follows:

$$L_1'' = \frac{1}{\pi(f_1 + f_2)} \cdot k \quad (75)$$

$$L_2'' = \frac{(f_2 - f_1)}{4\pi f_1^2} \cdot k$$

$$C_2'' = \frac{1}{\pi(f_2 - f_1)} \cdot \frac{1}{k}$$

The inductances and capacities for the filter of Fig. 17 may at once be obtained from the values given by formulæ 73 and 75.

It now remains to be shown that the propagation constant of the filter of Fig. 17 is the same as that of the filter of Fig. 1. Referring to Fig. 23, which is a schematic diagram of a filter of the type of that of Fig. 17, it will be seen that a complete section of the doubly periodic filter may be considered as being made up by directly connecting together the elements of Figs. 12 and 15.

From Fig. 23 it will be noted that the current I_n flowing into the mid-shunt impedance Z_{msh}' of the doubly periodic filter divides, part flowing through the impedance $2z_2'$ and the remainder,

$$I_n + \frac{1}{2}$$

through the parallel full series impedance, Z' . Since for the same voltage drop across Z_{msh}' and Z' the currents through them are inversely as their impedances, we have

$$\frac{I_n + \frac{1}{2}}{I_n} = \frac{Z_{msh}'}{Z'} \quad (76)$$

It is also evident from Fig. 23 that the mid-shunt impedance Z_{msh}' is equivalent to the

impedance $2z_2'$ in parallel with the impedance Z' , hence,

$$\frac{l}{Z_{msh}'} = \frac{l}{2z_2'} + \frac{l}{Z'} \quad (77)$$

This equation may be rewritten:

$$\frac{l}{Z'} = \frac{2z_2' - Z_{msh}'}{Z_{msh}' \cdot 2z_2'} \quad (78)$$

Combining equations 76 and 78 we obtain

$$\frac{I_n + \frac{1}{2}}{I_n} = \frac{2z_2' - Z_{msh}'}{2z_2'} \quad (79)$$

In a similar manner from Fig. 23

$$\frac{I_{n+1}}{I_n + \frac{1}{2}} = \frac{2z_2'}{2z_2' + Z_{msh}''} \quad (80)$$

From equations 79 and 80 and since $Z_{msh}' = Z_{msh}''$ we have, as the expression for the ratio of currents in the n th and $(n+1)$ st half section of the doubly periodic filter the following

$$\frac{I_{n+1}}{I_n} = \frac{2z_2' - Z_{msh}'}{2z_2' + Z_{msh}'} = e^{-\Gamma'} \quad (81)$$

In the same manner it may be shown from Fig. 23 that the expression for the ratio between currents in the $(n+2)$ nd and $(n+1)$ st half sections is

$$\frac{I_{n+2}}{I_{n+1}} = \frac{2z_2'' - Z_{msh}''}{2z_2'' + Z_{msh}''} = e^{-\Gamma''} \quad (82)$$

Fig. 22 is a diagram similar to that of Fig. 23 showing in a similar schematic manner a coalescent band constant k filter of the type shown in Fig. 1. From this figure it may be shown in a manner similar to the manner in which equation 81 was derived, that the ratio between current in the n th and $(n+1)$ st sections of the constant k filter of Fig. 22 may be expressed as follows:

$$\frac{I_{n+1}}{I_n} = \frac{2z_2 - Z_{msh}}{2z_2 + Z_{msh}} = e^{-\Gamma} \quad (83)$$

Now, the ratio between the currents flowing in two adjacent full sections of the filter of Fig. 23 is the ratio of the current I_{n+2} to

$$Z_{msh}^2 = \frac{z_1^2 z_2^2}{Z_{ms}^2} = \frac{k^4}{Z_{ms}^2} = \frac{k^2 (f_2 - f_1)^2 f^2}{[f_1^2 f^2 + f_2^2 f^2 - f_1^2 f_2^2 - f^4]} \quad (90)$$

From equations 89 it is apparent that

$$4(z_2' z_2'' - z_2 z_2'' - z_2 z_2') = -k^2 (f_2 - f_1)^2 f^2 \left[\frac{1}{\left(\frac{f_1 f_2 - f_1 f^2}{f_2} \right)} \cdot \frac{1}{(f_1^2 - f^2)} - \frac{1}{(f_1 f_2 - f^2)} \cdot \frac{1}{(f_1^2 - f^2)} - \frac{1}{(f_1 f_2 - f^2)} \cdot \frac{1}{\left(\frac{f_1 f_2 - f_1 f^2}{f_2} \right)} \right] \quad (91)$$

the current I_n . Hence, from equations 81 and 82 we have where P_s is the propagation constant of the doubly periodic filter.

$$\frac{I_{n+2}}{I_n} = e^{-\Gamma'} \cdot e^{-\Gamma''} = e^{-P_s} \quad (84)$$

Now, in order that the doubly periodic filter of Fig. 17 be equivalent in its transmitting properties to the coalescent band filter of Fig. 1, it is necessary that both filters have the same propagation constant; hence the following relation must hold true.

$$e^{-\Gamma} = e^{-\Gamma'} \cdot e^{-\Gamma''} = e^{-P_s} \quad (85)$$

In order to show that the relation of equation 85 holds, it is only necessary to prove that the following equivalent relation holds:

$$\frac{2z_2 - Z_{msh}}{2z_2 + Z_{msh}} = \frac{2z_2' - Z_{msh}'}{2z_2' + Z_{msh}'} \cdot \frac{2z_2'' - Z_{msh}''}{2z_2'' + Z_{msh}''} \quad (86)$$

Since

$$Z_{msh} = Z_{msh}' = Z_{msh}'' \quad (87)$$

this reduces to

$$Z_{msh}^2 + 4(z_2' z_2'' - z_2 z_2'' - z_2 z_2') = 0 \quad (88)$$

It is now necessary to determine the relation between the impedances z_2 , z_2' and z_2'' . These impedances may be expressed as follows:

$$\begin{aligned} z_2 &= \frac{iL_2 2\pi f}{(1 - L_2 C_2 4\pi^2 f^2)} = \frac{ik(f_2 - f_1)f}{2(f_1 f_2 - f^2)} \\ z_2' &= \frac{iL_2' 2\pi f}{(1 - L_2' C_2' 4\pi^2 f^2)} = \frac{ik(f_2 - f_1)f}{2\left(f_1 f_2 - \frac{f_1 f^2}{f_2}\right)} \\ z_2'' &= \frac{iL_2'' 2\pi f}{(1 - L_2'' C_2'' 4\pi^2 f^2)} = \frac{ik(f_2 - f_1)f}{2(f_1^2 - f^2)} \end{aligned} \quad (89)$$

Now, since in a constant k filter the product of the series and shunt impedances z_1 , z_2 must be equal to k^2 , it follows that the products of the mid-shunt impedance and mid-series impedance (which in a singly periodic filter always equals the product z_1 , z_2) must also be equal to k^2 . From this relation, taken in connection with equation 56, we have

Substituting the values given by equations 90 and 91 in equation 88, and omitting the common factor, we get

$$\frac{1}{(f_1^2 f_2^2 + f_2^2 f^2 - f_1^2 f_2^2 - f^4)} - \left[\left(\frac{1}{f_1 f_2 - \frac{f_1 f^2}{f_2}} \right) \cdot \frac{1}{(f_1^2 - f^2)} - \frac{1}{(f_1 f_2 - f^2)} \cdot \frac{1}{(f_1^2 - f^2)} - \frac{1}{(f_1 f_2 - f^2)} \right. \\ \left. \cdot \frac{1}{\left(f_1 f_2 - \frac{f_1 f^2}{f_2} \right)} \right] = 0 \quad (92)$$

Clearing of fractions and expanding this equation reduces to

$$f_1^4 f_2^2 - f_1^3 f_2 f^2 - f_1^2 f_2^2 f^2 + f_1 f_2 f^4 - f_1^4 f^2 + \frac{f_1^3 f^4}{f_2} + f_1^2 f^4 - \frac{f_1 f^6}{f_2} - \frac{f_1^3 f^4}{f_2} - f_1 f_2 f^4 + f_1^3 f_2 f^2 \\ + \frac{f_1 f^6}{f_2} + f_1^4 f^2 + f_1^2 f_2^2 f^2 - f_1^4 f_2^2 - f_1^2 f^4 = 0$$

It will be seen that all of the terms in equation 93 cancel out, which proves the assumed relation of equation 85. Consequently the propagation constant of the mid-shunt doubly periodic wave filter is the same as that of the constant k type of Fig. 1.

From the above mathematical exposition of the properties of the filters of Figs. 11 and 17, it is apparent that either of these doubly periodic filters may be designed to have the same critical frequencies, and the same sharp cut-off as the constant k filter of Fig. 1. Furthermore, since these filters are inherently single band filters, their elements need not be so carefully proportioned in order to avoid attenuations at the middle of the transmitted band, thereby avoiding the difficulty occurring in the filter of Fig. 1 where the single band is formed by causing two bands to coalesce.

It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated, without departing from the spirit of the invention as defined in the following claims.

What is claimed is:—

1. A selective circuit for selectively transmitting a band of frequencies comprising a plurality of similar sections, each section consisting of two part sections and each part section being made up of two impedance elements, one in series and one in shunt, one of said impedances being a combination of inductance and capacity in both part sections, the other impedance element being a capacity in the case of one part section and an inductance in the case of the other part section, the said part sections of each kind being so designed that a complete selective circuit of only one such kind will have the same critical frequencies as a complete selective circuit of the other kind.

2. A selective circuit for selectively transmitting a band of frequencies comprising a plurality of similar sections, each section consisting of two part sections, said part

sections being made up of two impedance elements, one in series and one in shunt, one of said impedances being the same for both part sections and the other impedance element being in the one case a capacity and in the other case an inductance, the said part sections of each kind being so designed that a complete selective circuit of only one such kind will have the same critical frequencies as a complete selective circuit of the other kind.

3. A selective circuit for selectively transmitting a band of frequencies comprising a plurality of elements connected successively in series, each element consisting of two impedances, one in series and one in shunt, one of said impedances being the same for each element, the other impedance being of two types arranged alternately with regard to successive elements, the said impedances being so proportioned that a selective circuit made up wholly of alternate midseries or midshunt sections will have the same critical frequency.

4. A selective circuit for selectively transmitting a band of frequencies comprising a plurality of elements, each element including a pair of impedances, one in series and one in shunt, one of said impedances being composed of a similar combination of inductance and capacity for each element, the other impedance being a capacity in the case of every other element, and an inductance in the case of the remaining elements, the said impedances being so proportioned that a selective circuit made up wholly of alternate midseries or midshunt sections will have the same critical frequency.

5. A composite single transmission band wave filter comprising alternately arranged part sections taken from two other single transmission band filters having periodically recurring sections and having sharp cut-off respectively at opposite ends of the transmission band whereby the composite filter has sharp cut-off at both ends of its transmission band.

6. A composite single transmission band

filter having alternate elements drawn respectively from each of two different simple single transmission band filters giving sharp cut-off respectively at opposite ends of the transmission band whereby the composite filter is given a sharp cut off at both ends.

7. A composite single transmission band filter having the same characteristic impedance and the same propagation constant as a certain confluent band filter, and comprising alternately disposed part sections drawn respectively from two single band filters of the same characteristic impedance and with sharp cut-off respectively at opposite ends of their transmission bands, where-

by the composite filter is given a sharp cut-off at both ends thereof.

8. A composite single transmission band wave filter comprising two kinds of part sections alternately arranged, the part sections of one kind being adapted in combination to give a sharp cut-off at one end of the transmission band and the part sections of the other kind being adapted in combination to give a sharp cut-off at the opposite end of the transmission band.

In testimony whereof, I have signed my name to this specification this 28th day of April 1920.

OTTO J. ZOBEL.