Jan. 26, 1926.

1,570,771

H. NYQUIST

MEANS FOR AND METHOD OF REDUCING INTERFERENCE

Filed March 20, 1922

4 Sheets-Sheet 1

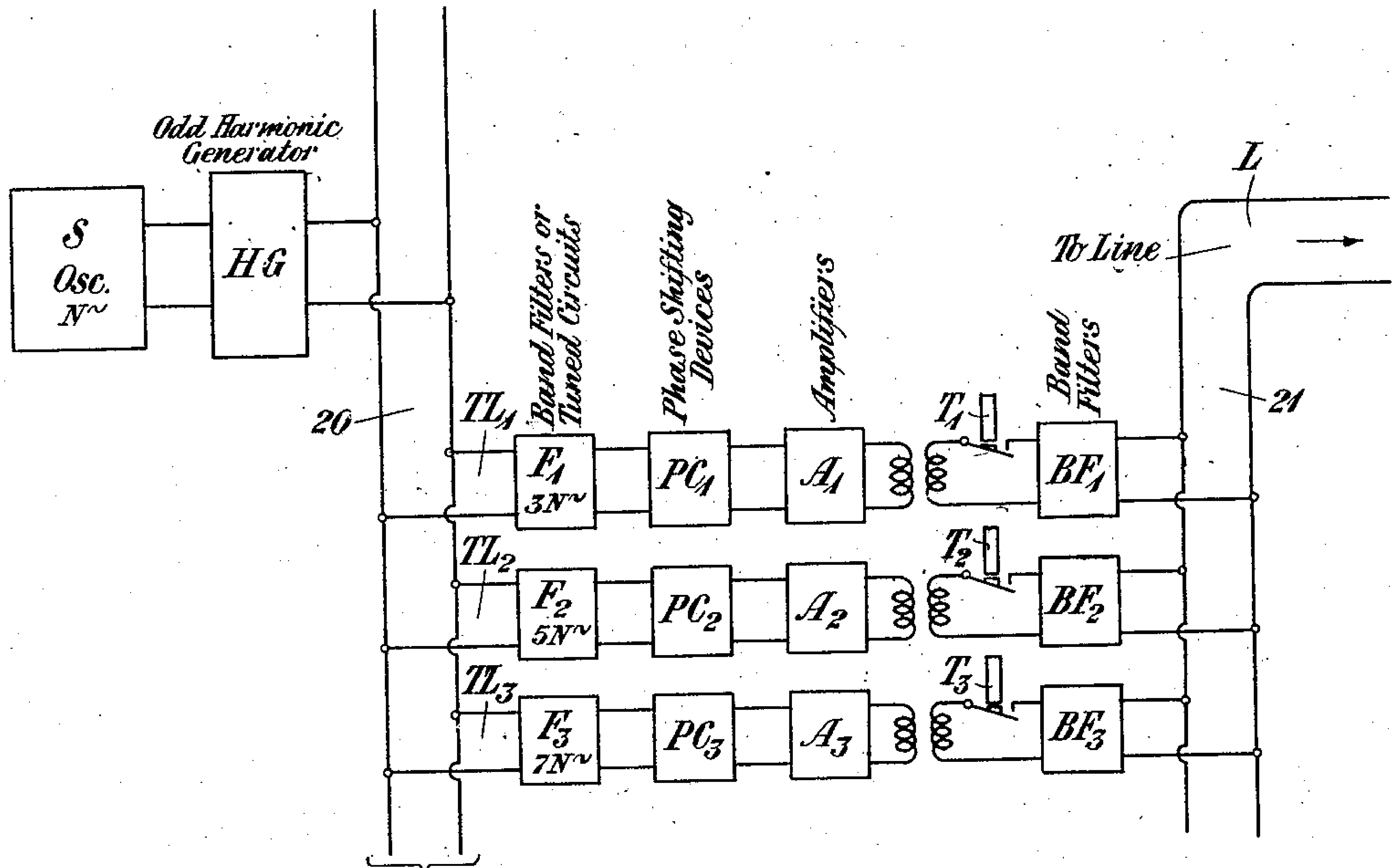

Fig. 1

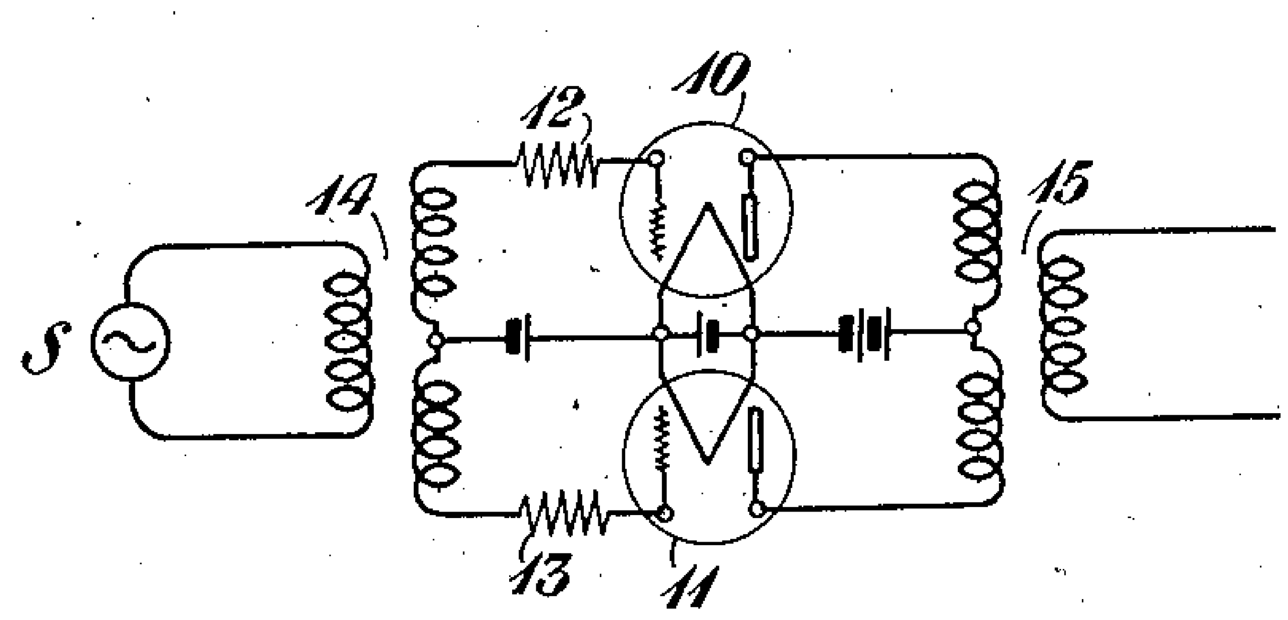

Fig. 2

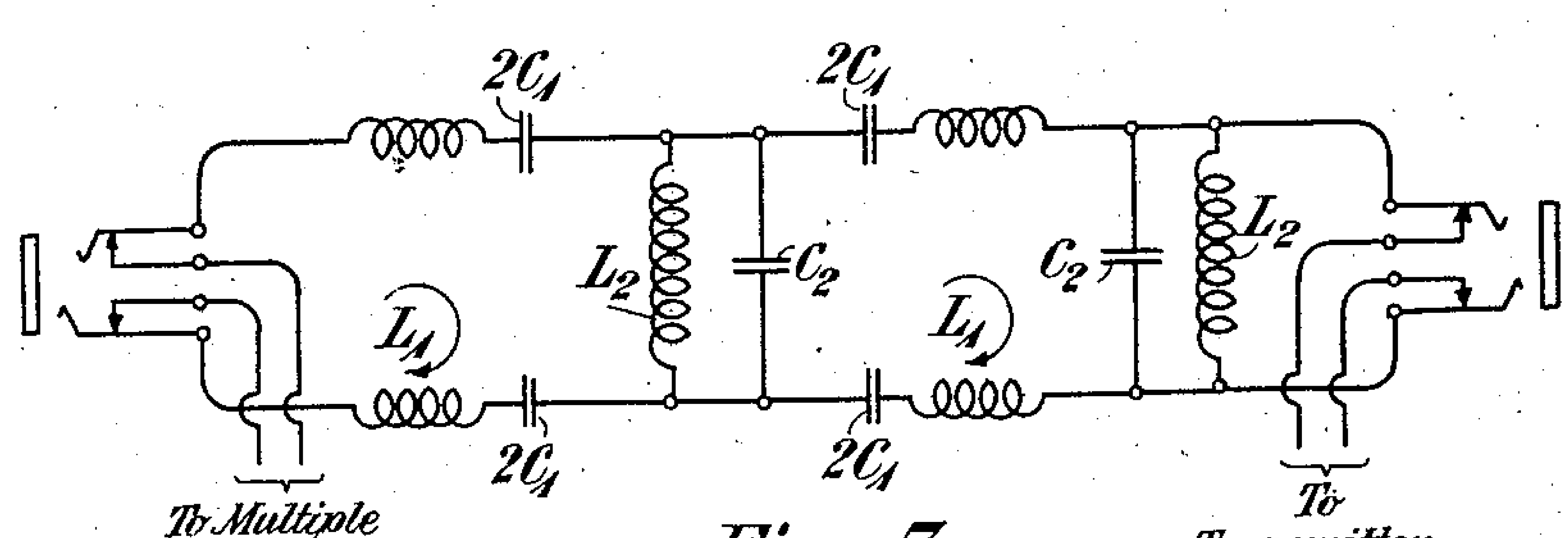

Fig. 3

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4 Sheets-Sheet 2

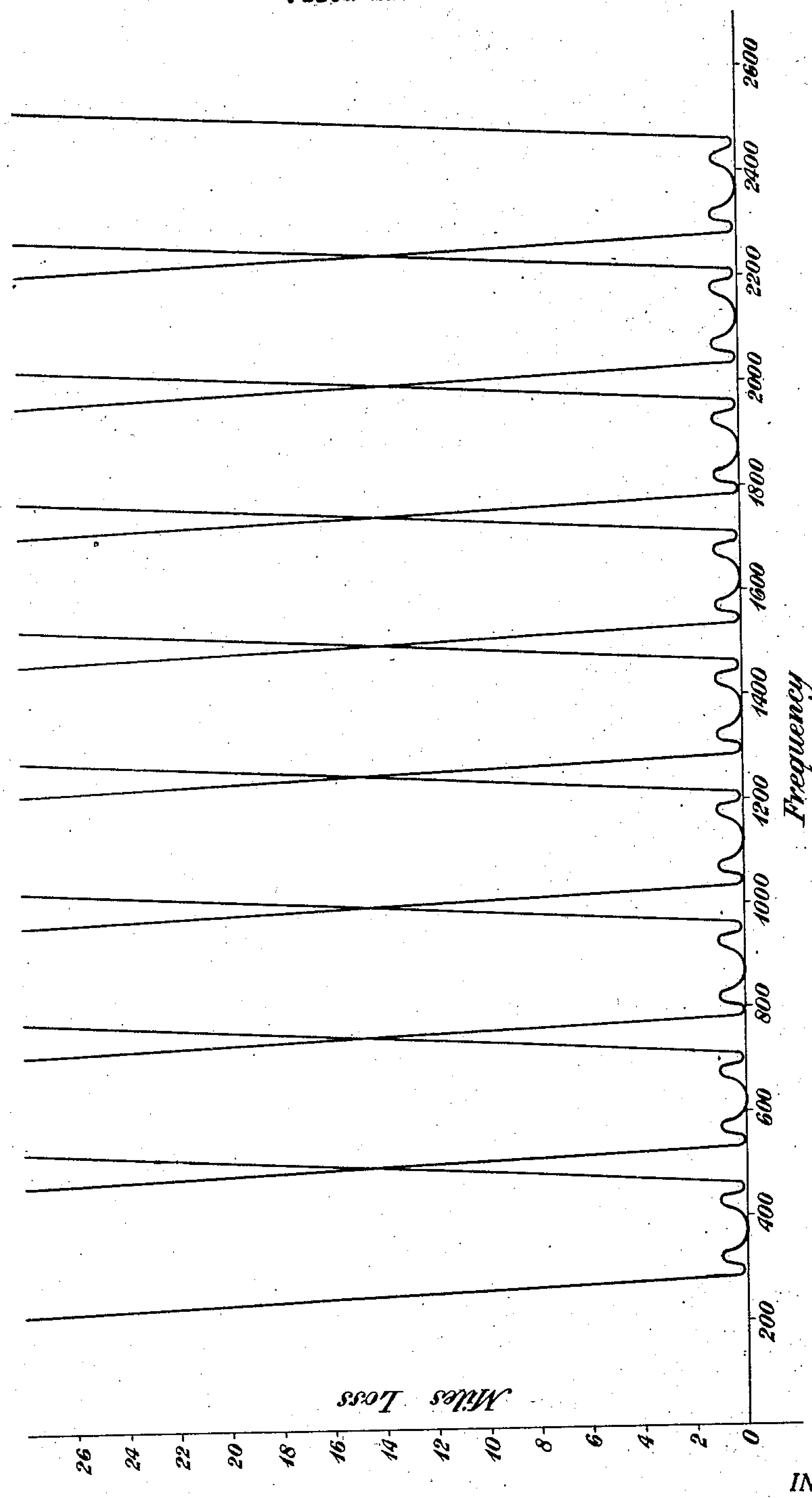

Fig. 4

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MEANS FOR AND METHOD OF REDUCING INTERFERENCE

Filed March 20, 1922 4 Sheets-Sheet 3

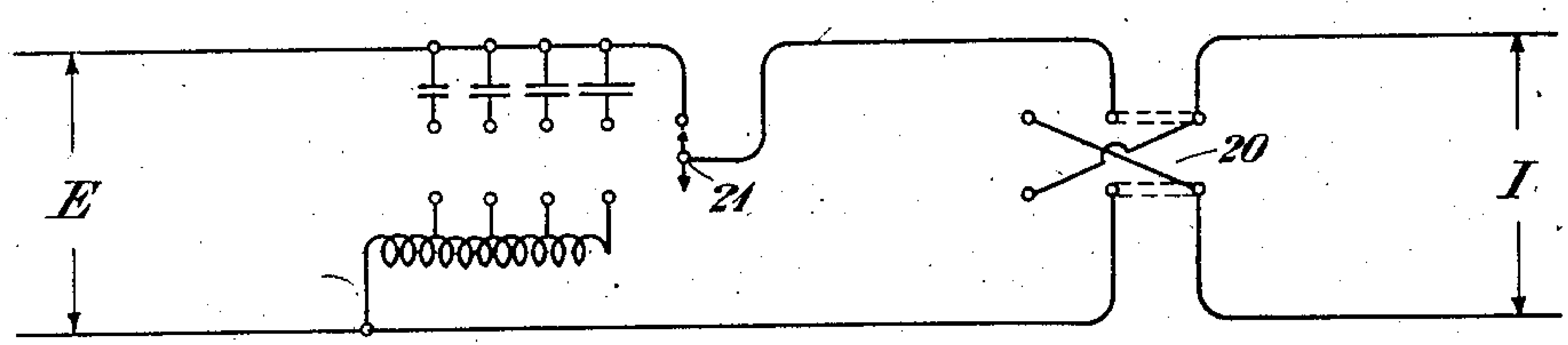

Fig. 5

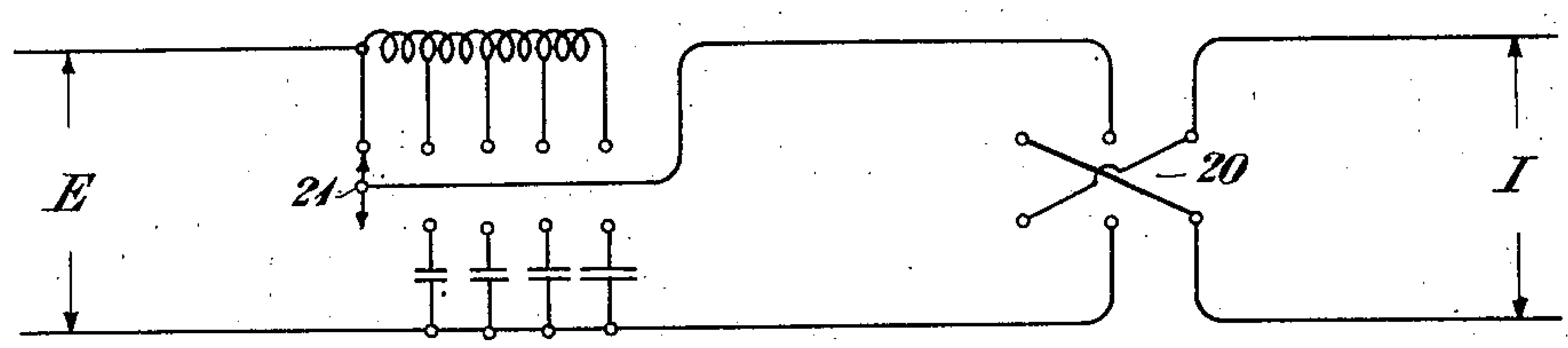

Fig. 6

a	3	5	7	$\overline{9}$	11	$\overline{13}$	15	17	19
3	6	2	4	−6	8	−10	12	14	16
5	8	10	2	−4	6	−8	10	12	14
7	10	12	14	−2	4	−6	8	10	12
−9	−12	−14	−16	+18	−2	+4	−6	−8	−10
11	14	16	18	−20	22	−2	4	6	8
−13	−16	−18	−20				−2	−4	−6
15	18	20						2	4
17	20								2
19									

b

Fig. 8

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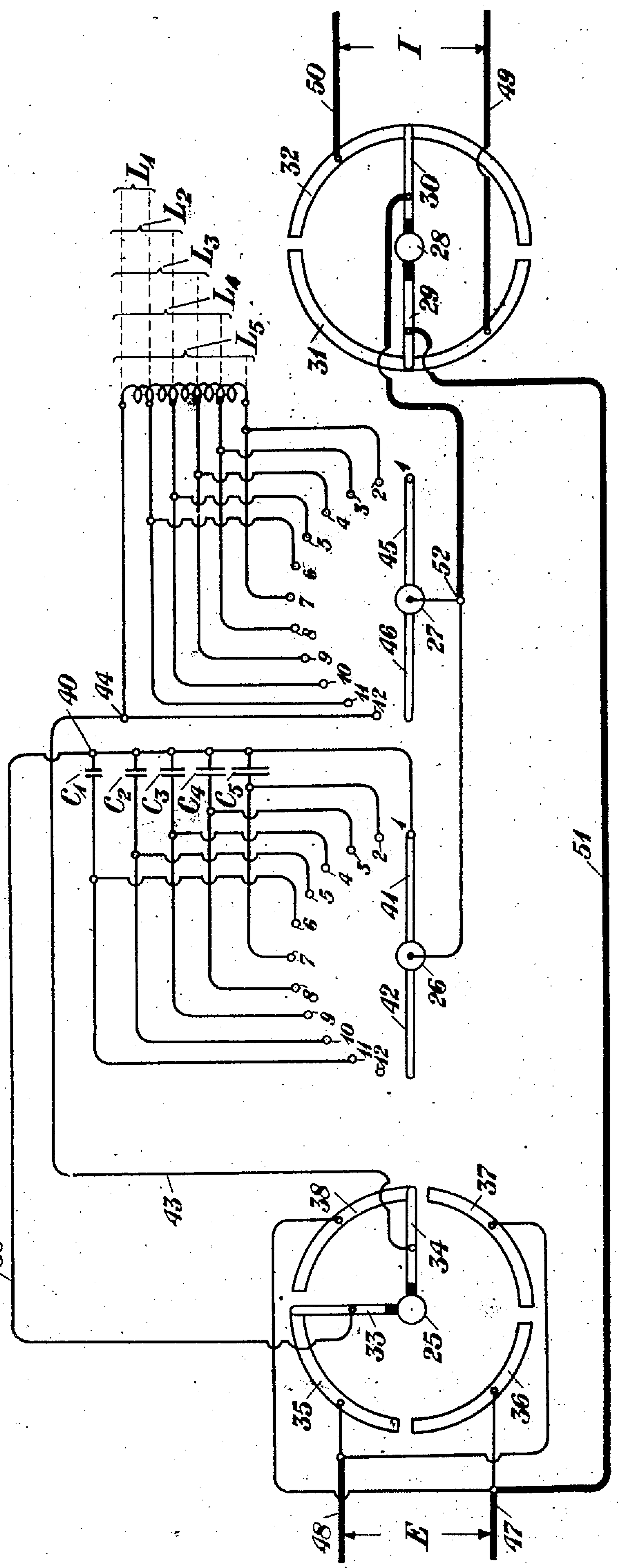

Fig. 7

INVENTOR.
H. Nyquist
BY *[signature]*
ATTORNEY.

Patented Jan. 26, 1926. **1,570,771**

UNITED STATES PATENT OFFICE.

HARRY NYQUIST, OF ELMHURST, NEW YORK, ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

MEANS FOR AND METHOD OF REDUCING INTERFERENCE.

Application filed March 20, 1922. Serial No. 545,296.

To all whom it may concern:

Be it known that I, Harry Nyquist, residing at Elmhurst, in the county of Queens and State of New York, have invented certain Improvements in Means for and Methods of Reducing Interference, of which the following is a specification.

This invention relates to multiplex signaling, and more particularly to means for and methods of reducing interference between the channels of a multiplex signaling system.

Heretofore, in the operation of multiplex carrier systems, it has been found that the repeater tubes employed at points along the line for transmitting the carrier frequencies, where the same tube is used to transmit the frequency of a number of channels, tended to impose a limit on the number of channels that might be employed over a single circuit. This is for the reason that the tubes do not have a straight line characteristic and a certain amount of distortion occurs, which produces second harmonics of the carrier frequencies and also sum and difference frequencies. These harmonics and sum and difference frequencies, in many instances, assume such amplitudes as to cause interference in the channels whose frequencies are near those of the interfering harmonics, etc. Interference of this character becomes especially serious in a carrier telegraph system involving a large number of channels employing carrier frequencies within the usual voice range, for in such a case it is almost essential to economic working of the system that the channels be narrow and closely spaced, and that all the channels be amplified by the same repeater tube.

In accordance with the present invention, it is proposed to overcome this difficulty by choosing the carrier frequencies so that they will bear a relation to each other such that the harmonics and sum and difference frequencies produced by modulating the several carriers with each other will fall in a position in the frequency spectrum substantially half-way between adjacent carrier frequencies. For example, the carrier frequencies may be made odd harmonics of some fundamental frequency, say 125 cycles per second. In this case, all sum and difference frequencies and all second harmonics of the carrier frequencies will be even multiples of this fundamental frequency. It is therefore possible to design the selecting filters in such a way that they will transmit the major part of the band between two adjacent even multiples of the fundamental frequency without transmitting the even multiples themselves.

As a still further refinement of the invention, it is proposed to adjust the initial phase angles of the carriers of the various channels in such a manner as to reduce the interfering frequencies to relatively small amplitudes.

The various carrier frequencies involved may be generated independently without attempting to maintain them in exact numerical relations. While this will substantially satisfy the first condition by bringing the interfering frequencies substantially midway between the carriers, it will be apparent that even though the individual carriers be maintained of constant and of invariable frequency, the difference frequencies and second harmonics will not all be the same if the carriers are not all exactly evenly spaced. As a result of this, the composite wave made up of the interfering frequencies will vary in amplitude from instant to instant, even though the individual carrier frequencies from which these components are derived are not varying. It is possible, however, to maintain the relation between the various carriers exact by deriving the carriers as harmonics of the same fundamental frequency. In this case, all of the difference frequencies will be the same, and the sum frequencies, difference frequencies and harmonics will, as previously stated, be exact multiples of the fundamental. When the carriers are generated in this manner the phase relations between the different carriers may be controlled or adjusted with respect to each other to determine the amplitude of the disturbing frequencies just mentioned.

If, for example, the odd harmonics of some fundamental are to be used as carrier frequencies it will be seen that the initial phase relations of each of the carriers may be made such that when the fundamental wave is of maximum value and of a given sign all of the carrier waves will be of maximum value at that instant, and of the same sign as the fundamental. In order that this condition may exist at any instant, it will be found that the carrier waves must bear to each other a definite phase relation so that for each complete cycle of the fundamental each of the carrier waves will have a maximum positive value when the fundamental wave has its maximum positive value, and each carrier wave will also have a maximum negative value when the fundamental wave attains its maximum negative value. Also, each of the carrier waves will pass through zero when the fundamental wave passes through zero which will, of course, occur twice for each cycle of the fundamental.

I have discovered by analyzing the interfering frequencies above referred to that the amplitudes of all of them will be greatest when the several carrier frequencies from which they are obtained bear the phase relations with respect to each other which have just been described, and by adjusting the phase relations of the individual carriers in various combinations to avoid these coincidences of phase I am able to reduce the amplitudes of the interfering frequencies to quite small values.

The invention may now be more fully understood by reference to the following detailed description, when read in connection with the accompanying drawings, in which Figure 1 is a schematic diagram of a circuit arrangement embodying the principles of the present invention; Fig. 2 is a circuit arrangement showing the harmonic generator employed in the system of Fig. 1 in more detail; Fig. 3 is a diagram showing the detailed arrangement of the type of filter employed in Fig. 1; Fig. 4 is a series of curves showing the characteristics of the filters of a plurality of channels; Figs. 5 and 6 illustrate in simplified form a phase changer which may be employed in the circuit of Fig. 1; Fig. 7 illustrates the circuit arrangement of the phase changer in greater detail, and Fig. 8 illustrates a method of determining how to change the phase angles of certain carriers in order to produce the best results.

Before discussing in detail the theory underlying the present invention, it will be well to outline the system to which the invention is applied. The invention is of particular utility when applied to a multiplex carrier system having a large number of channels, each employing carrier frequencies within the ordinary voice range, the individual channels being used for the transmission of telegraph signals. Referring to Fig. 1, L designates a transmission line upon which the several carrier channels may be superposed. The carrier frequencies themselves are derived from a fundamental source S, which may be, for example, a vacuum tube oscillator generating a fundamental frequency of say about 125 cycles per second.

The frequency generated by this source may be impressed upon a harmonic generator HG. This harmonic generator may be of any well-known type of distorting arrangement, such as a distorting vacuum tube, which, as is well known, will produce harmonics of the foundamental frequency impressed upon it. Inasmuch, however, as it is desired, in accordance with the principles of the present invention, to use only odd harmonics, a special type of distorting arrangement may be employed, the circuit arrangement of which is shown in greater detail in Fig. 2.

Referring to Fig. 2, two vacuum tubes 10 and 11 are shown, having their input and output circuits connected in parallel. The grid circuit of each tube includes a resistance, as shown at 12 and 13, and the resistance in the grid circuit is so adjusted with respect to the plate and grid potentials as to produce a distortion of the wave impressed upon the tube in a manner which is well understood in the art. The distortion of the wave impressed upon the input circuit of the tube produces harmonics of the impressed wave in the output circuit. By connecting the tubes in parallel as shown, and impressing the fundamental wave from the source S upon the two grids serially through the transformer 14, the even harmonics will be balanced out so far as the secondary of the balanced transformer 15 in the output circuit is concerned.

This action may be better understood from a brief consideration of the theory of a distorting vacuum tube. Assuming that e represents the voltage of the fundamental frequency and i represents the current in the output circuit of the tube, the current may then be expressed as an exponential series in terms of the impressed voltage, so that if the impressed voltage e is positive; the current in the output circuit will be

$$i_1 = a + be + ce^2 + de^3 + fe^4, \text{ etc.} \quad (1)$$

If now the same wave is impressed upon the other tube in opposite phase relation, as is the case in Fig. 2, e will in this case be negative and the current i_2 in the output circuit of the tube 11 may be expressed

$$i_2 = a - be + ce^2 - de^3 + fe^4, \text{ etc.} \quad (2)$$

It will be apparent from the circuit arrangement of Fig. 2 that the output currents in the two primary windings of the transformer 15 oppose each other with regard to the secondary winding. The resultant current may then be obtained by substracting the second equation above given from the first one, so that the resultant output current may be expressed

$$i' = 2be + 2de^3, \text{ etc.} \quad (3)$$

It will be observed that all of the even powers balance each other and only the odd powers remain in the last equation. It is well known that the expression $2be$, which is a first power term, corresponds to the frequency of the fundamental, and that the expression. $2de^3$ corresponds to the third harmonic, and similarly, other odd powers correspond to other odd harmonics. Consequently, the arrangement of Fig. 2 will produce only odd harmonics.

Again referring to Fig. 1, it will be noted that the output side of the harmonic generator HG, whose operation has just been described, is connected to a pair of bus-bars 20, from which branches through channels TL_1, TL_2, TL_3, etc. are led. Each channel includes a suitable filtering arrangement, as indicated at F_1, F_2, F_3, etc., which may be a simple tuned circuit sharply tuned to the particular harmonic which is to be used as the carrier for that frequency. The next element in each channel comprises a phase changer indicated at PC_1, PC_2, PC_3, etc., the function of which is to adjust the phase relations of the carriers applied to the different channels with respect to each other, so as to reduce the amplitudes of the interfering harmonics and sum and difference frequencies in a manner which will be described later. The detailed construction of the phase changer is illustrated in Fig. 7 and will be described hereafter.

The various frequencies assigned to the channels may be amplified by amplifiers such as A_1, A_2, A_3, etc., which may be adjusted to bring the generated carrier frequencies to the same amplitude. These amplifiers may be of any well-known type, such for example, as vacuum tube amplifiers. The amplified frequencies may now be impressed upon a common line L over bus-bars 21 and through band filters such as BF_1, BF_2, BF_3, etc. Suitable transmitting devices T_1, T_2, T_3, etc., may be arranged to interrupt the carrier frequency applied to the band filters, thereby causing signals to be transmitted by means of the carrier currents applied to the line.

The construction of the filters such as BF_1, etc., is illustrated in Fig. 3, which shows a band filter of the Campbell type, comprising two sections each having series resonant elements and shunt anti-resonant elements. In order to give it as great an impedance as possible to currents outside of the range which it is desired to transmit, the section which is connected next to the bus-bars 21 is made a full series section. The other end of the filter is connected to the transmitting side of the circuit.

Fig. 4 shows the transmission characteristics of a number of band filters of this type, arranged to transmit bands having a width of about 200 cycles and corresponding to carriers of 375 cycles, 625 cycles, 875 cycles, etc., these being the successive odd harmonics of a fundamental frequency of 125 cycles. It will be observed that the transmission loss for any one of these filters over a range of about 200 cycles is less than 2 miles, while the loss for frequencies which are even multiples of 125 cycles is about 14 miles. If then the carrier frequencies are so spaced relatively to each other as to produce interfering frequencies falling at points where the attenuation curves of the filters in Fig. 4 cross each other, the interfering frequencies will be so greatly attenuated as to produce but little effect upon the adjacent channels.

Let us assume, for example, a fundamental frequency N. The odd harmonics generated by the harmonic generator HG and selected into the various channels will then be 3N, 5N, 7N, etc. If these waves be impressed upon some device having a non-linear characteristic, such for example, as a vacuum tube amplifier in the line circuit, the several carrier frequencies will tend to modulate with each other, thereby producing disturbing frequencies which are principally due to the square term and are consequently made up, as is well known, of second harmonics of the carrier and sum and difference frequencies produced by the several carriers. The second harmonic frequencies, for example, will be 6N, 10N, 14N, etc. The difference frequencies due to the beating action of each pair of adjacent carriers will be 2N; the difference frequencies due to the beating action between every other carrier will be 4N, etc. The sum frequency resulting from the beating action of the first and second carrier will be 8N; that resulting from the beating action of the first and third carrier will be 10N; that resulting from the beating action of the second and third carrier will be 12N, etc. It will thus be seen that the frequencies set up as a result of the distortion are 2N, 4N, 6N, etc., all of which correspond to even multiples of the fundamental frequency N. Therefore, for a fundamental frequency of 125 cycles, as was assumed in the case of the filters illustrated in Fig. 4, the interfering frequencies resulting from the distorting action of amplifiers and the like will all fall at the points of greatest attenuation of the filter arrangement.

It is, of course, not absolutely necessary that the frequencies be produced by the harmonic method, as very good results may be obtained where the spacing of the channels is maintained approximately the same, but the best results will be obtained if the spacing is absolutely accurate, as is the case where the carrier frequencies are produced as harmonics of a fundamental.

Where the spacing of the carrier frequencies is maintained constant and accurate, as is the case where the carriers are harmonics of a fundamental frequency, it is possible to combine the carrier frequencies in such phase relation with respect to each other that the resulting distortion is reduced to a minimum.

In order to understand the manner in which the interfering frequencies may be controlled by controlling the phase relations of the several carriers it will be necessary to analyze the peculiar relations existing between a number of waves which are odd harmonics of a common fundamental. If we let N represent the fundamental frequency the composite wave resulting from the superposition of a number of odd harmonics upon the fundamental wave may be represented as follows:

$$\cos(pt+\theta_1)+\cos(3pt+\theta_3)+\cos(5pt+\theta_5)+\text{etc.} \quad (4)$$

in which p is $2\pi N$ and t is the time. In the above equation the first term represents the fundamental, the second term the third harmonic, the third term the fifth harmonic, etc.

It will be readily apparent that if the fundamental and the several harmonics be so related to each other that the phase angles represented by θ_1, θ_3, etc., become zero all of the harmonic terms will have a maximum positive value whenever the fundamental wave has a maximum positive value, and each will have a maximum negative value whenever the fundamental has a maximum negative value. Also each of the harmonic waves will pass through zero whenever the fundamental wave passes through zero, which will occur twice during each cycle of the fundamental. In other words, there will be a coincidence of the cyclic conditions of all of the waves four times for each cycle of the fundamental. If, however, the initial phase angle of one of the waves be shifted, this relation will be disturbed, and the four coincidences per cycle of the fundamental will not occur. This is for the reason that each time a coincidence occurs for all of the waves except the one whose phase has been shifted, the particular wave which has been shifted will have passed beyond the condition corresponding to the coincidence by an amount determined by the initial phase angle to which the wave was shifted. In order that the four coincidences above referred to may occur, therefore, it is necessary that the fundamental and all of the odd harmonics simultaneously start their cycles under conditions corresponding to one of the points of coincidence.

I have discovered by analyzing the interfering frequencies, such as the sum and difference frequencies and harmonic terms resulting from intermodulation of the individual carriers, that these interfering frequencies will attain their maximum amplitude when the phase relations between the carriers are such that the coincidences above referred to occur. By shifting the initial phase angles of certain of the carriers, the components producing the interfering frequencies will, to some extent, tend to oppose each other, and, consequently by a proper adjustment of the initial phase angles, the components may be reduced to a minimum.

While there is no general rule applicable to all cases, the manner in which the phase relations may be determined may, in general, be best understood by reference to a specific application. Suppose we have a number of carrier waves corresponding to odd multiples of a fundamental wave, which may be represented as $\cos pt$. The composite wave resulting from the nine carriers corresponding to the odd multiples from 3 to 19, inclusive, may be represented as follows:

$$\cos 3pt+\cos 5pt+\cos 7pt+\cos 9pt+\cos 11pt+\cos 13pt+\cos 15pt+\cos 17pt+\cos 19pt. \quad (5)$$

In the above expression, p may be taken to be, for example, 125 times π. Let us now consider the following wave, which is equivalent to it except that a phase shift of 180 degrees has been introduced into some of the component waves.

$$\cos 3pt+\cos 5pt+\cos 7pt-\cos 9pt+\cos 11pt-\cos 13pt+\cos 15pt+\cos 17pt+\cos 19pt. \quad (6)$$

The frequencies in the interfering range due to the square term and their magnitudes in arbitrary units in the two cases, are given in the table below.

Frequency	250	500	750	1,000	1,250	1,500	1,750	2,000	2,250	2,500
Amplitude, first case	8	7	6.5	6	5.5	5	4.5	4	3.5	4
Amplitude, second case	0	3	1.5	2	1.5	3	2.5	0	1.5	0

The manner in which the results set forth in the foregoing table may be obtained will be understood by referring to the chart shown in Fig. 8 of the drawing. In this chart, the frequencies of the nine channels involved in the specific case assumed are arranged along the two sides of a square, as indicated. For the purpose of simplicity, only the number representing the harmonic relation of each carrier to the fundamental is given. A diagonal line *ab* may now be drawn from the upper left-hand corner of the square toward the lower right-hand corner. Upon this diagonal line may be written the second harmonic corresponding to each carrier, this harmonic being written upon the diagonal line at the point where the vertical line drawn through each carrier frequency in the horizontal row intersects the horizontal line drawn through the corresponding frequency in the vertical row. For example, the second harmonic of the carrier designated 3 will be 6 and of the carrier designated 5, will be 10, etc.

Components, such as sum and difference frequencies resulting from the reaction of any two frequencies, may likewise be set down at the points where the vertical and horizontal lines drawn through the two beating frequencies intersect. It will be seen that the sum and difference frequencies will fall either to the right or to the left of the diagonal line *ab*, and we may arbitrarily set all difference frequencies to the right of the diagonal line and all sum frequencies to the left. For example, the action resulting from beating 7 and 9 together produces a difference of 2. 9 in the horizontal row intersects with 7 in the vertical row on the right of the diagonal line *ab*, while 7 in the horizontal row and 9 in the vertical row intersect at the left of the said line. Either of these two points of intersection may be used for writing the difference frequency, and in accordance with the arbitrary convention previously stated, the difference frequency 2 will be written at the intersection on the right of the line, and the corresponding sum frequency 16 will be written at the corresponding point of intersection on the left of the line.

Following this convention, it will be seen that a series of difference frequencies represented by 2 and corresponding to the differences between successive carriers, may be written on the first row of diagonal intersections to the right of the dotted line. Another diagonal row of 4's corresponding to the differences between alternate carrier frequencies will be set down in the next row of diagonal intersections, etc. No frequency higher than 20 need be put down as frequencies higher than this frequency will be so far above the highest carrier, which is 19, as to be of substantially no effect. When all of the sum, difference and harmonic frequencies which are not higher than 20 are set down in the chart in accordance with the plan given, the chart will contain the numbers indicated in Fig. 8.

Suppose now all of the carrier frequencies are positive at a given instant, as will be the case under the condition represented by the first algebraic expression above given. All of the numbers in the chart will then be positive. It will be seen that the same number occurs at different places in the chart. For example, all of the numbers corresponding to difference frequencies appear in the diagonal row to the right of the diagonal line *ab*, but the same number may also appear in the diagonal line and also at one or more points to the left of the diagonal line. If we take the number 6, for example, we see that it occurs six times in the diagonal row to the right of the line *ab* and once on the diagonal line *ab*, but it does not occur to the left of the line *ab*. As this number occurs altogether seven times, it would seem that the frequency represented by this number would have an amplitude seven times that of one of the difference components which make up this frequency, assuming, of course, that all of the carriers involved have the same amplitude. It must be noted, however, that while corresponding sum and difference frequencies resulting from combining any two frequencies will each have the same amplitude, the same rule will not apply to those numbers which represent harmonics.

For example, in the case of the number 6 now being discussed, the wave represented by the number 6 at the top of the diagonal column to the right of the dotted line *ab* results from the beating action of two waves, one numbered 9 in the upper horizontal row and the other numbered 3 in the vertical row to the left of the square. The number 6 appearing on the dotted line, however, is a harmonic resulting from the energy of a single wave represented by 3 and not by the combined energy of two waves. Consequently, the amplitude of the harmonic terms appearing on the dotted line will only be one-half of the amplitude of a wave of the same frequency appearing as either a sum or difference term. Thus the total wave made up of all of the 6's in the chart will have an amplitude of 6½ arbitrary units instead of 7, and assuming that 6 represents a single harmonic of a fundamental frequency of 125, or in other words, a frequency of 750, we find that the amplitude of this frequency will be 6.5 as set forth in the chart previously given.

Let us now consider the case of the frequency represented by 8. This frequency occurs five times to the right of the dotted line and once to the left of the dotted line, but does not occur upon the dotted line itself. In other words, the frequency corresponding to 8 will be made up of five difference components and one sum component, and will, consequently, have an amplitude in arbitrary units of 6, as indicated in the chart above given, for the frequency of 1000 cycles. Again, taking the frequency 14, we find that it occurs twice to the right of the dotted line, once upon the dotted line, and twice to the left of the dotted line, so that the corresponding wave will have a total amplitude of 4½ arbitrary units, as indicated under frequency 1750 in the table. Again, in the case of the frequency represented by 16, we find that it occurs once to the right and three times to the left, but does not occur upon the dotted line, so that the total amplitude of this frequency will be 4 arbitrary units, as indicated under the column representing 2000 cycles in the table.

Obviously, these are the greatest amplitudes that any of these disturbing frequences may assume, and by changing the phase angle of certain of the carrier frequencies represented in equation (5) by 180 degrees, some of the individual components making up a given interfering frequency will cancel each other, so that the total amplitude of a given interfering frequency will be reduced. Let us assume that the carrier frequencies represented by 9 and 13 in the chart of Fig. 8 are 180 degrees out of phase with the other carrier frequencies when the other carrier frequencies are in a condition of coincidence, or in other words, are of maximum negative value when the other carrier frequencies are all simultaneously of maximum positive value. It will be seen at once that each time a negative 9 in the chart of Fig. 8, for example, reacts on any positive frequency, the corresponding sum or difference frequency will also be negative. Whenever, the negative 9, however, reacts with another negative wave, the sum or difference frequency will be positive. See, for example, the frequency represented by 4 at the intersection of a vertical line through 13 in the horizontal column and the horizontal line through 9 in the vertical column. The harmonics of 9 and 13, which would appear on the diagonal line *ab*, will also be positive, notwithstanding the fact that the wave from which they are derived is negative. This follows from the fact that the harmonic results from the square term. In short, if the carrier frequencies 9 and 13 are negative, all of the numbers in the vertical columns below 9 and 13 and the horizontal columns to the right of 9 and 13 will be negative, except those numbers occurring at points where the vertical and horizontal columns intersect each other.

Let us now determine the amplitude of the waves represented by the numbers previously considered. The number 6, for example, which occurs 6½ times, has four negative components corresponding to difference frequencies, so that the algebraic sum of the several components is 1½ and the total amplitude of the wave of this frequency will be 1½ arbitrary units, as is indicated in the column below frequency 750 in the table given above. The number 8 occurs four times positive and twice negative, so that the algebraic sum will be 2, and thus we have a positive amplitude of 2 units, as is indicated in the column below 1000 cycles. The frequency 14 occurs 3½ times positive and once negative, so that the total amplitude will be 2½ units, as indicated in the table under 1750 cycles. The frequency 16 occurs four times, twice negative and twice positive, so that its amplitude will be zero, as indicated in the column in the table under 2000 cycles. Comparing the amplitude in the table for the two cases, we find that in the second case, where frequencies 9 and 13 are made negative, some of the interfering frequencies will be eliminated, and all of the others are materially reduced in amplitude.

While a certain amount of cutting and trying is necessary in any given case to determine which waves should be 180 degrees out of the phase relation corresponding to a condition of coincidence with the others, it will be more or less evident from a chart such as is shown in Fig. 8 that by making certain waves negative, a very large number of components of a given frequency will tend to cancel each other. For example, the frequencies 9 and 13 in the chart of Fig. 8 will obviously have the desired effect when made negative, by reason of the fact that they produce in the chart substantially as many negative components as positive components. At any rate, it will be a fairly simple proposition, for any given set of odd harmonic carriers, to prepare a chart such as above given and work out the results for all of the combinations of positives and negatives which might arise, or at least, those which will apparently produce the greatest effect. In this way, the desired phase relations for a given set of carriers may be readily obtained and the phase adjustments made by means of the phase adjusters shown in Fig. 1.

So far we have considered the results which will arise by arranging certain carriers to be 180 degrees out of the phase relation with the others which should exist at a time of coincidence. A desirable reduction in the interfering frequencies will result from other phase relations involving angles other than 180 degrees, and in fact, any phase adjustment which brings one or more of the carriers out of the phase relation with others which produce the conditions of coincidence above mentioned, will result in a diminution of the amplitude of the interfering frequencies. As a practical proposition, the best results in a given case might be obtained, for example, by testing in the line circuit for interfering frequencies and by manually adjusting the phase changing devices in the several channels until such an adjustment is reached as will produce the least interference.

This will, of course, involve the use of a phase changing device which may be readily manipulated by hand, so that by turning a knob, for example, the current corresponding to a given frequency may be set out of phase with the electromotive force producing that frequency by any angle throughout the whole range of 360 degrees. A phase changing device which may be operated in this manner is illustrated in detail in Fig. 7, and in schematic form in Figs. 5 and 6.

Referring to Fig. 5, for example, the

phase of the current at I may be advanced with respect to an electromotive force applied at E by including capacity in series with the line and inductance in shunt therewith. With the switch 20 thrown to the right and the movable switch 21 in its extreme right position, the electromotive force and current will be in phase with each other, as there is no shunt inductance, and the switch 21 being serially connected in the line, the series capacity may be considered infinite. If the switch be moved one step to the left, there will be a large series capacity in the line and a large inductance in shunt with the line, so that the current will be ahead of the electromotive force. As the switch 21 is moved still farther to the left, the series capacity and shunt inductance are decreased step by step with continued advancement of the current with respect to the electromotive force, until at the extreme left position of the switch 21, a small series capacity and a small shunt inductance will be associated with the line, having such values that the current will be just 90 degrees ahead of the electromotive force. By this arrangement, the phase of the current with respect to the electromotive force may be adjusted throughout the first quadrant. By throwing the switch 20 to the left, the connection to the line wires will be reversed, so that for any setting of the switch 21, the current at I will be just 180 degrees out of phase with the current for the corresponding setting of the switch 21 when the switch 20 was thrown to the right. With the switch 20 thrown to the left, therefore, any adjustment of the phase of the current with respect to the electromotive force may be made through the third quadrant.

Fig. 6 shows an arrangement for adjusting the phase through the second and fourth quadrants. Considering the switch 20 in this instance thrown to the right, it will be seen that the switch 21 will cut in series inductance and shunt capacity. Such an arrangement, as is well known, operates as a phase retarder, that is, operates to retard the phase of the current with respect to the electromotive force. With the switch 21 in its extreme left-hand position, it will be seen that all of the series inductance is short-circuited and the shunt capacity is open-circuited, so that the current is in phase with the electromotive force, as was the case under the condition first described in connection with Fig. 5. By moving the switch 21 one step to the right, a small series inductance and a small shunt capacity will be connected in circuit, thereby retarding the current with respect to the electromotive force by a small angle. By moving the switch still further to the right, larger series inductance and larger shunt capacity will be connected in circuit, still further retarding the current, until at the extreme right-hand position of the switch 21, large series inductance and a large shunt capacity of such values as will retard the current 90 degrees behind the electromotive force, are connected in circuit. In other words, by setting the switch 21 in its extreme right-hand position and moving it by successive steps to the left, the current may be shifted from 90 degrees behind the electromotive force until it is exactly in phase with the electromotive force, so that the phase angle between the current and the electromotive force may shift throughout the fourth quadrant. Here again, by throwing the switch 20 to the left, a relation will be produced between the current and electromotive force which is just 180 degrees out of phase with the corresponding relation when the switch is thrown to the right. This will enable any phase relation desired to be produced in the second quadrant.

It now remains to combine these four effects so that by one continuous adjustment, the phase angle of the current with respect to the electromotive force may be varied throughout 360 degrees.

Referring to Fig. 7, a switching device is shown for connecting various amounts of inductance and capacity either in series or in shunt with the line. This device comprises four switching elements, 25, 26, 27 and 28, which may be rotated by means of a common shaft. Switching element 28 comprises two switch arms 29 and 30, which may be moved over two segments 31 and 32 connected to two sides of the outgoing line. These two arms 29 and 30 are 180 degrees apart. The switching device 25 comprises two arms 33 and 34, 90 degrees apart, which may wipe over four quarter-segments 35, 36, 37 and 38. One pair of opposite segments of this group of four is connected to one side of the generator or other device for supplying an alternating electromotive force, and the other pair of segments is connected to the other side of the source.

A connection 39 extends from the switch member 33 to a terminal 40, to which a number of different capacities C_1 to C_5, inclusive, are connected. These capacities are connected to a number of commutation points or contacts, which may be engaged by arms 41 or 42 of the switching device 26, these two arms being 180 degrees apart. A connection 43 extends from the arm 34 of the switching device 29 to a terminal 44 of a series of inductances L_1 to L_5, inclusive. These inductances are connected to commutation points or contacts to which connections may be made by switch arms 45 or 46 of a switching device 27, these arms being 180 degrees apart.

The arms 41 and 42 are electrically connected together, and produce the same result, one arm being operative during one half rotation of the switch, and the other arm being operative during the other half rotation. Likewise the arms 45 and 46 are electrically connected together, so that one arm is operative during one-half rotation and the other arm during the other half rotation. The two switches 26 and 27, which comprise these four arms, are electrically connected together and are connected in common to the switch arm 30 of the switch device 28. The switch arm 29 of the same switch is connected to the lower conductor 47 of the pair of conductors 47 and 48 connected to the source.

The arrangement of the switching element 28 is such that when switch arms 29 and 30 rest upon sectors 31 and 32, respectively, line conductors 47 and 48 will be connected to line conductors 49 and 50, respectively. When, however, the switch arms 29 and 30 rest upon sectors 32 and 31, respectively, the line connections are interchanged so that the current I will be 180 degrees out of phase with the current when the switch is in the former condition. The switching element 25, on the other hand, is so arranged that when switch arms 33 and 34 are on segments 35 and 38, respectively, capacity may be connected in series with the line wire 48 under the control of the switch 26, and inductance may be connected in shunt between the line wires 47 and 48 under the control of the switch 27. When switch arms 33 and 34 are arranged on sectors 36 and 35, respectively, inductance may be connected in series with the line and capacity may be connected in shunt therewith. With the switch arms 33 and 34 on sectors 36 and 37, respectively, capacity may again be connected in series with the line and inductance in shunt therewith, while with these switch arms on sectors 37 and 36, inductance may be connected in series with the line and capacity in shunt therewith.

With the various switches in the positions shown, the current I will be in phase with the electromotive force E, as a series circuit may be traced from conductor 48 to conductor 50, over segment 35, switch arm 33, conductor 39, terminal 40, first point of switching device 26, switch arm 41, switch arm 30 and segment 32. Line wire 47 is also directly connected to line wire 49 over conductor 51, switch arm 29 and segment 31. The inductance under the control of the switching element 27 is open-circuited and the capacity controlled by the switch 41 is eliminated.

Suppose we now rotate the switch clockwise 90 degrees, which will correspond to the condition of affairs at the beginning of the fourth quadrant of phase adjustment. Switch arms 46 and 42 of switching devices 27 and 26 will rest upon their seventh contacts, and switch arm 33 of switching device 25 will be upon segment 38 and switch arm 34 upon segment 37. A large inductance L_5 will now be in series with line wires 48 and 47 over a circuit from line wire 48, segment 37, switch arm 34, conductor 43, terminal 44, inductance L_5, seventh contact of switch 27, switch arm 46, switch arm 30, segment 38 and sector 32 to conductor 50. A large capacity C_5 will be shunted across the line from junction point 52 of the switching devices 26 and 27, over the arm 42 of switching device 26, seventh contact thereof, through the capacity C_5, terminal 40, conductor 39, switch arm 33, segment 38 to line conductor 47. Under these conditions the current will be retarded 90 degrees behind the electromotive force.

If the switches now be moved counter-clockwise one point, the condition will be the same, except that switch arms 42 and 46 rest upon their eighth contacts, so that a smaller inductance L_4 is connected in series and a smaller capacity C_4 is connected in shunt. As the switches are advanced step by step in a counter-clockwise direction, the series inductance and shunt capacity are reduced step by step with a consequent reduction of the retardation of the current, until at the end of the fourth quadrant, switch arms 46 and 42 on their twelfth contacts. Under these conditions the shunt capacity will be open-circuited and all of the inductance eliminated. The direct series connection will be extended from conductor 50, over switch arm 30, terminal 52, switch arm 46, twelfth contact of switch 27, terminal 44, condctor 43, switch arm 34, segment 37 and line wire 48. The current will now be in phase with the electromotive force. The conditions above described correspond to various adjustments throughout the fourth quadrant, so that the current will be anywhere from 90 degrees behind the electromotive force to exactly in phase with the electromotive force.

As the switches are stepped one additional step in a counter-clockwise direction, switch arms 46 and 42 rest upon their seventh segments and we have the condition previously described at the beginning of the first quadrant, with the current in phase with the electromotive force. Upon stepping the switches one step further in a counter-clockwise direction, switch arms 45 and 41 rest upon the second contacts of switches 27 and 26, respectively, so that a large series capacity is connected in the line from wire 39, terminal 40, capacity C_5, second contact of switch 26, switch arm 41 to terminal 52. A large inductance L_5 is connected in shunt across the line between terminal 52 and line wire 47, from terminal 52, switch arm 45, second segment of switch 27, through inductance L_5, terminal 44, conductor 43, switch arm 34, and segment 38, to line wire 47. This will advance the current ahead of the electromotive force by a small angle, and for each successive step of the switch in a counter-clockwise direction, a less series capacity and shunt inductance is included in the circuit with a corresponding advance in the phase of the current with respect to the electromotive force, until switch arms 41 and 45 rest upon the sixth contacts. In this case, a small capacity C_1 is connected in series and a small inductance L_1 in shunt, these values being so chosen that the current will be just 90 degrees ahead of the electromotive force.

When the switch arms 41 and 45 rest upon the seventh contacts, we have the same condition, so far as capacity and inductance are concerned, as at the beginning of the fourth quadrant, namely, a large series inductance L_5 serially connected between wires 48 and 50, and a large shunt capacity C_5 bridged across the line from terminal 52 to wire 47 through switch arm 33 and segment 36. Switch arms 41 and 45 will now be effective to wipe over the contacts instead of switch arms 46 and 42 as described in connection with the fourth quadrant, but the electrical result is the same. The circuit as a whole differs from the circuit condition existing as described in connection with the fourth quadrant, however, in that switch arm 30 of switch 28 now rests on sector 31 and switch arm 29 upon sector 32, thereby reversing the connections from the phase changer to the outgoing circuit 49—50. This throws the current 180 degrees out of phase with the current existing under the similar conditions described at the beginning of the fourth quadrant, and consequently, the current, instead of being 90 degrees behind the electromotive force, is, in effect, 90 degrees ahead of the electromotive force. In other words, no change in the phase of the current with respect to the electromotive force takes place in moving from the sixth to the seventh points of switches 26 and 27. As the switch arms 41 and 45 move over on the eighth, ninth, tenth and eleventh contacts, the current is advanced over the quadrant between 90 and 180 degrees ahead of the electromotive force, until said switch arms rest on their twelfth contacts. Under these circumstances, the shunt capacity is open-circuited and the series inductance eliminated, so that, remembering that wires 49 and 50 are now transposed, the current is 180 degrees out of phase with the electromotive force.

The next movement of the switch will bring switch arms 42 and 46 upon the first contacts of switches 26 and 27, and switch arm 33 of switch 25 will rest upon segment 37 and switch arm 34 upon segment 36. We now again revert to the conditions of series capacity and shunt inductance, but the series capacity is eliminated and the shunt inductance open-circuited. This would bring the current in phase with the electromotive force, except for the fact that switch 28 is still in such position that switch arm 30 rests upon sector 31 and switch arm 29 upon sector 32, reversing the outgoing connections and thereby bringing the current 180 degrees out of phase with the electromotive force, as just described with the wipers of switches 26 and 27 resting upon the twelfth contacts. As the switch is advanced, the same conditions are repeated as described in connection with the first quadrant, except that a phase difference of 180 degrees comes in, because of the reversal of the connections of wires 49 and 50. When wipers 42 and 45 rest upon the seventh contacts, we will have a small series capacity C_1 and a small shunt inductance L_1. This brings the current 90 degrees plus 180 degrees, or 270 degrees, ahead of the electromotive force, which is merely another way of saying that the current is 90 degrees behind the electromotive force, the condition which obtained at the beginning of the fourth quadrant. When switches 42 and 46 rest upon the seventh contacts, the same phase conditions will still exist, and the connections are identical with those described at the beginning of the fourth quadrant.

It will be seen that by means of the arrangement of Fig. 7, it is possible to continuously adjust the phase angle of the current with respect to the electromotive force through 360 degrees. Consequently, by using phase shifters of this character, or equivalent arrangements, in the circuit of Fig. 1, the phase relations of the various carriers applied to the channels may be varied until the interfering second harmonics and sum and difference frequencies are reduced to the lowest magnitudes.

It will be understood that in some cases it will be sufficient to merely employ the principle of so choosing the carrier frequencies as to cause the interfering frequencies to fall midway between the adjacent carriers, as set forth in the description of the first feature of the invention, without having recourse to the additional advantages obtained by regulation of the phase relations of the individual carriers. In other cases it may be advantageous to employ both principles in combination, so that either or both of these expedients are within the scope of the present invention.

It will also be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated, without departing from the spirit of the invention as defined in the following claims.

What is claimed is:

1. In a multiplex signaling system employing different carrier frequencies for a

plurality of channels, the method of reducing interference between the channels due to interaction between frequencies of different channels, which consists in generating carrier frequencies for the various channels so related to each other that the second harmonics of the carrier frequencies will fall substantially midway between two adjacent carrier frequencies.

2. In a multiplex signaling system employing different carrier frequencies for a plurality of channels, the method of reducing interference between the channels due to interaction between frequencies of different channels, which consists in generating carrier frequencies for the various channels so related to each other that the sum and difference frequencies resulting from intermodulation of the carrier frequencies will fall substantially midway between adjacent carrier frequencies.

3. In a multiplex signaling system employing different carrier frequencies for a plurality of channels, the method of reducing interference between the channels due to interaction between frequencies of different channels, which consists in generating carrier frequencies for the various channels so related to each other that the second harmonics and sum and difference frequencies resulting from intermodulation of the carrier frequencies will fall substantially midway between adjacent carrier frequencies.

4. In a multiplex signaling system employing different carrier frequencies for a plurality of channels, the method of reducing interference between the channels due to interaction between frequencies of different channels, which consists in generating carrier frequencies for the different channels so related to each other that they will be odd harmonics of some one fundamental frequency.

5. In a multiplex signaling system, a plurality of transmission channels, means for supplying different carrier frequencies to each channel, and means to so relate the frequencies of the channels that the second harmonics of the various carrier frequencies will fall substantially midway between adjacent carrier frequencies, thereby reducing interference due to interaction between the frequencies of different channels.

6. In a multiplex signaling system, a plurality of transmission channels, means for supplying different carrier frequencies to each channel, and means to so relate the frequencies of the channels that the sum and difference frequencies obtained by intermodulation of the various carrier frequencies will fall substantially midway between adjacent carrier frequencies, thereby reducing interference due to interaction between the frequencies of different channels.

7. In a multiplex signaling system, a plurality of transmission channels, means for supplying different carrier frequencies to each channel, and means to so relate the frequencies of the channels that the second harmonics of the carriers and sum and difference frequencies obtained by intermodulation of the carrier frequencies will fall substantially midway between adjacent carrier frequencies, thereby reducing interference due to interaction between the frequencies of different channels.

8. In a multiplex signaling system, a plurality of transmission channels, means for supplying different carrier frequencies to each channel, and means to so relate the frequencies of the channels that they will bear the relation to each other of odd harmonics of one common fundamental frequency, thereby reducing interference due to interaction between the frequencies of different channels.

9. In a multiplex signaling system employing carrier frequencies bearing the relation of harmonics of a common fundamental for a plurality of channels, the method of reducing interference due to the square term components resulting from intermodulation of the individual carrier frequencies, which consists in impressing the several carriers upon the transmission circuit in such phase relation with each other that a coincidence of phase conditions will occur in less than all of the channels.

10. In a multiplex signaling system employing carrier frequencies bearing the relation of harmonics of a common fundamental for a plurality of channels, the method of reducing interference due to the square term components resulting from intermodulation of the individual carrier frequencies, which consists in impressing certain of the carrier frequences upon a common transmission circuit 180 degrees out of phase with the phase relation resulting in phase coincidence of all of the carrier frequencies.

11. In a multiplex signaling system employing carrier frequencies bearing the relation of harmonics of a common fundamental for a plurality of channels, the method of reducing interference due to the square term components resulting from intermodulation of the individual carrier frequencies, which consists in individually adjusting the initial phase angles of the carrier frequencies relative to each other, so that the component electromotive forces making up the different interfering frequencies will at least partially oppose each other.

12. In a multiplex signaling system having a plurality of transmission channels, means for supplying carrier frequencies to each channel, said carrier frequencies bearing the relation of harmonics of a common fundamental, and means for impressing the carrier frequencies of the different channels

upon a common transmission circuit in such phase relation with each other that a coincidence of phase conditions will occur in less than all of the channels, whereby interfering components due to the square term resulting from intermodulation of the individual carriers will be reduced in magnitude.

13. In a multiplex signaling system having a plurality of transmission channels, means for supplying carrier frequencies to each channel, said carrier frequencies bearing the relation of harmonics of a common fundamental, and means for impressing the carrier frequency from each channel upon a common transmission circuit in such phase relation that certain of the carrier frequencies will be 180 degrees out of phase with the phase relation resulting in phase coincidence of all of the carrier frequencies, whereby the interfering frequencies resulting from the square term component due to intermodulation of the individual carrier frequencies will be reduced in magnitude.

14. In a multiplex system having a plurality of transmission channels, means for supplying carrier frequencies to each channel, said carrier frequencies bearing the relation of harmonics of a common fundamental, and means for impressing the various carrier frequencies from the several channels upon a common transmission circuit, said last mentioned means including devices for adjusting the relative phases of the individual carrier frequencies, so that the individual components making up the interfering electromotive forces arising from the square term due to intermodulation of the individual carrier frequencies with each other will at least partially oppose one another.

15. In a multiplex signaling system employing different carrier frequencies for a plurality of channels, the method of reducing interference between channels, which consists in generating carrier frequencies for the various channels so related to each other that the second harmonics of the carrier frequencies and sum and difference frequencies resulting from intermodulation of the carrier frequencies will fall substantially midway between adjacent carrier frequencies, and in impressing the carrier frequencies so related upon a transmission circuit in such phase relation with each other that a coincidence of phase condition will occur in less than all of the channels.

16. In a multiplex signaling system employing different carrier frequencies for a plurality of channels, the method of reducing interference between channels, which consists in generating carrier frequencies for the various channels so related to each other that the second harmonics of the carrier frequencies and sum and difference frequencies resulting from intermodulation of the carrier frequencies will fall substantially midway between adjacent carrier frequencies, and in impressing certain of the carrier frequencies thus related upon a common transmission circuit 180 degrees out of phase with the phase relation resulting in phase coincidence of all of the carrier frequencies.

17. In a multiplex signaling system employing different carrier frequencies for a plurality of channels, the method of reducing interference between channels, which consists in generating carrier frequencies for the various channels so related to each other that the second harmonics of the carrier frequencies and sum and difference frequencies resulting from intermodulation of the carrier frequencies will fall substantially midway between adjacent carrier frequencies, and in individually adjusting the phase angles of the carrier frequencies relative to each other so that the component electromotive forces making up the different interfering frequencies will at least partially oppose each other.

18. In a multiplex signaling system employing different carrier frequencies for a plurality of channels, the method of reducing interference between channels, which consists in generating carrier frequencies for the different channels so related to each other that they will be odd harmonics of some fundamental frequency, and in impressing the carrier frequencies of the different channels upon a common transmission circuit in such phase relation with respect to each other that interfering components due to the square term resulting from intermodulation of the individual carriers will be reduced in magnitude.

19. In a multiplex signaling system, a plurality of transmission channels, means for supplying different carrier frequencies to each channel, the frequencies of the channels being so related that the second harmonics of the carriers and sum and difference frequencies resulting from intermodulation of the carrier frequencies will fall substantially midway between adjacent carrier frequencies, and means for impressing the carrier frequencies of the different channels upon a common transmission circuit in such phase relation with each other that a coincidence of phase conditions will occur in less than all of the channels, whereby interfering components due to the square term resulting from intermodulation of the individual carriers will be reduced in magnitude.

20. In a multiplex signaling system, a plurality of transmission channels, means for supplying different carrier frequencies to each channel, the frequencies of the channels being so related that the second harmon-

ics of the carriers and sum and difference frequencies resulting from intermodulation of the carrier frequencies will fall substantially midway between adjacent carrier frequencies, and means for impressing the carrier frequencies from the several channels upon a common transmission circuit in such phase relation that certain of the carrier frequencies will be 180 degrees out of phase with the phase relation producing phase coincidence of all of the carrier frequencies, whereby the interfering frequencies resulting from the square term component due to intermodulation of the individual carrier frequencies will be reduced in magnitude.

21. In a multiplex signaling system, a plurality of transmission channels, means for supplying different carrier frequencies to each channel, the frequencies of the channels being so related that the second harmonics of the carriers and sum and difference frequencies resulting from intermodulation of the carrier frequencies will fall substantially midway between adjacent carrier frequencies, and means for impressing the various carrier frequencies from the several channels upon a common transmission circuit, said last mentioned means including devices for adjusting the relative phases of the individual carrier frequencies, so that the individual components making up the interfering electromotive forces resulting from the square term due to intermodulation of the individual carrier frequencies with each other will at least partially oppose one another.

22. In a multiplex signaling system, a plurality of transmission channels, means for supplying different carrier frequencies to each channel, the frequencies of the channels being so related that they will bear the relation to each other of odd harmonics of a common fundamental frequency, and means for impressing the carrier frequencies of the different channels upon a common transmission circuit, in such phase relation to each other that interfering components due to the square term resulting from intermodulation of the individual carriers will be reduced in magnitude.

In testimony whereof, I have signed my name to this specification this 18th day of March 1922.

HARRY NYQUIST.