

Jan. 5, 1926.

1,568,143

H. W. ELSASSER

FREQUENCY SELECTIVE CIRCUITS

Filed August 13, 1920

3 Sheets-Sheet 1

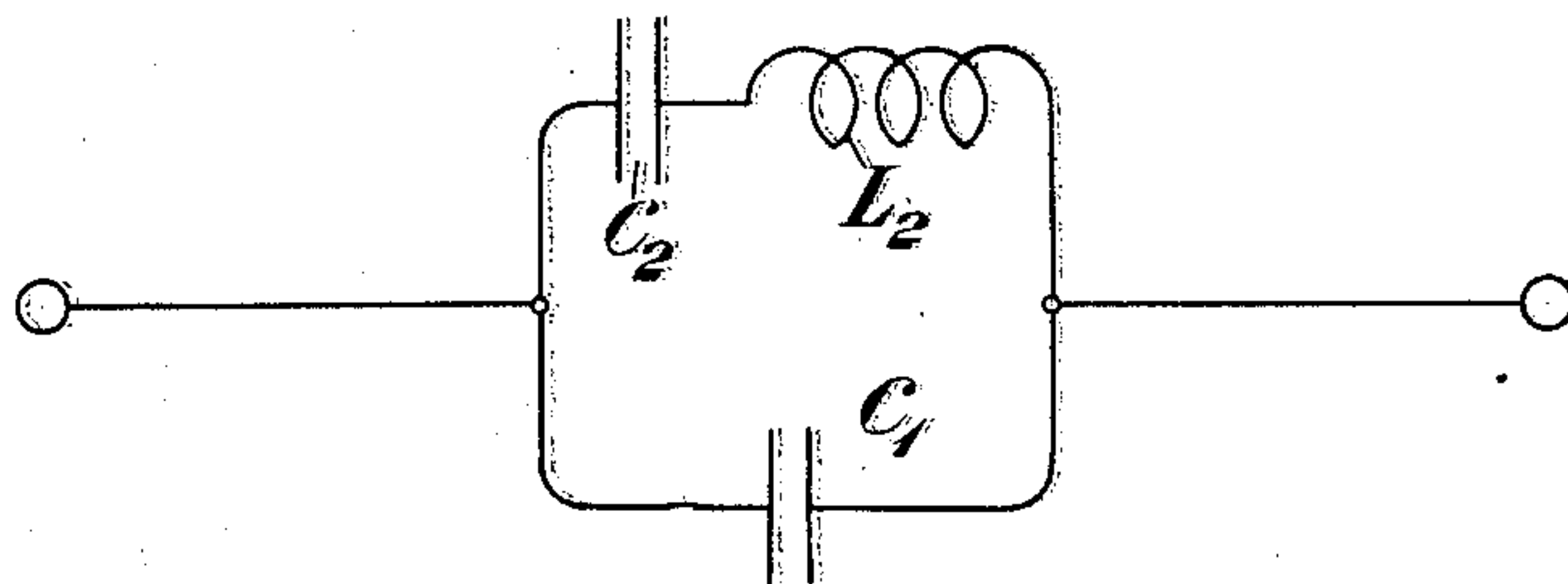


Fig. 1

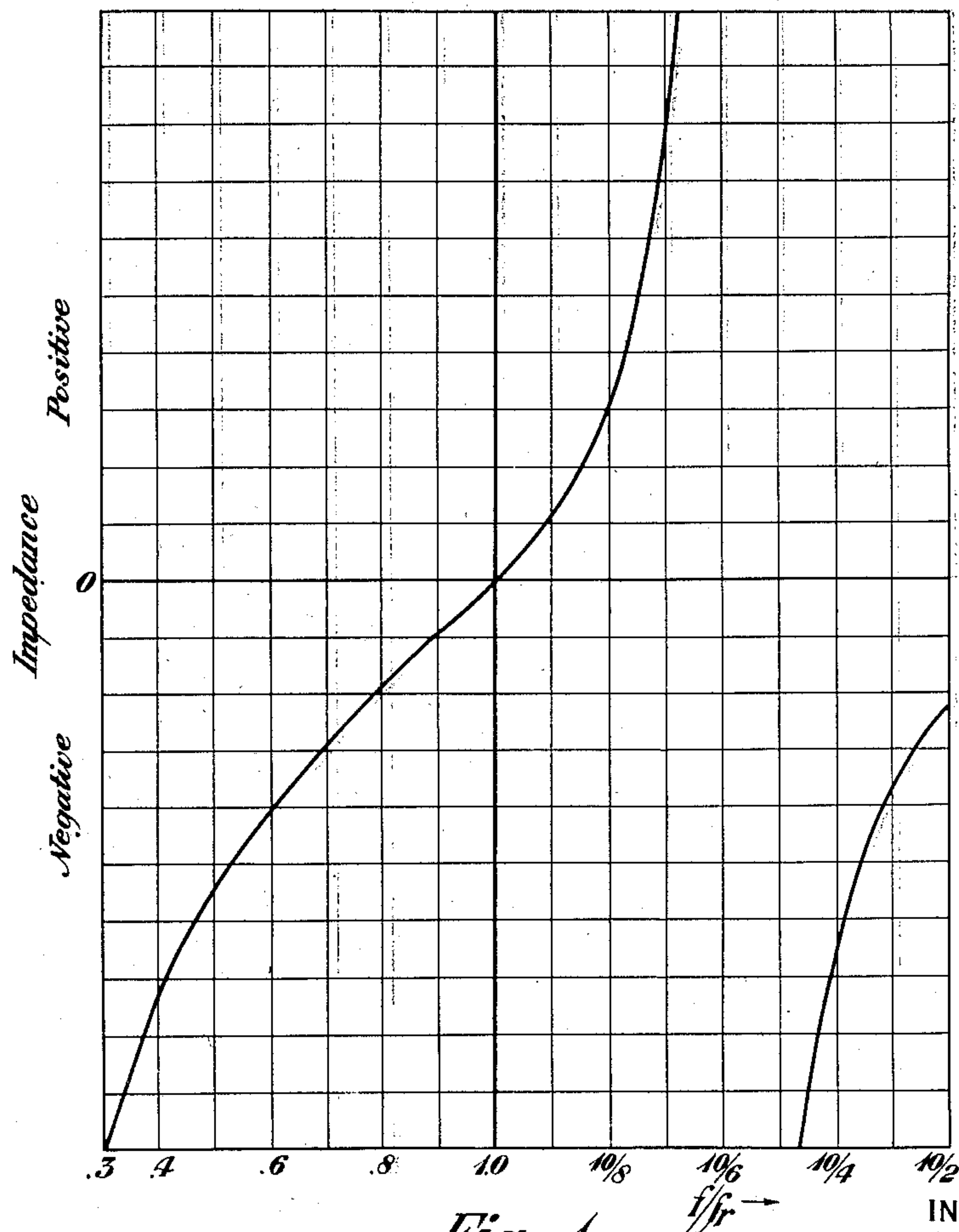


Fig. 1a

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3 Sheets-Sheet 2

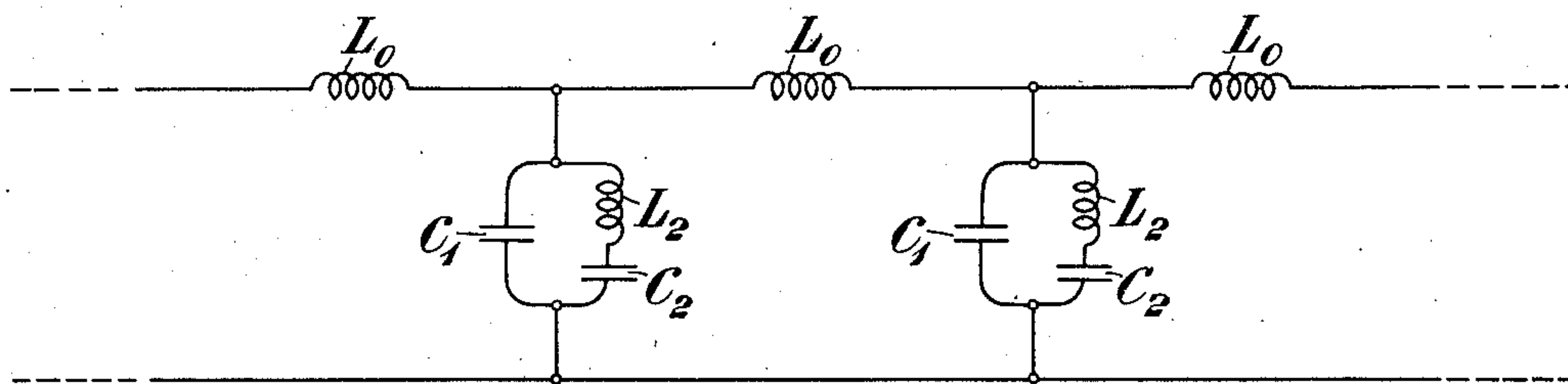


Fig. 2

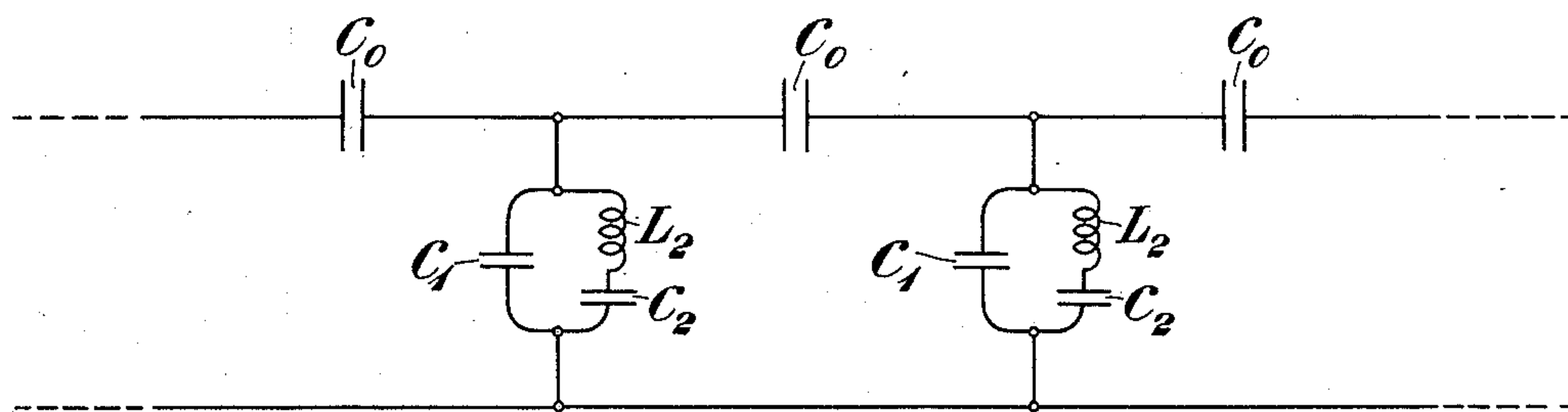


Fig. 3

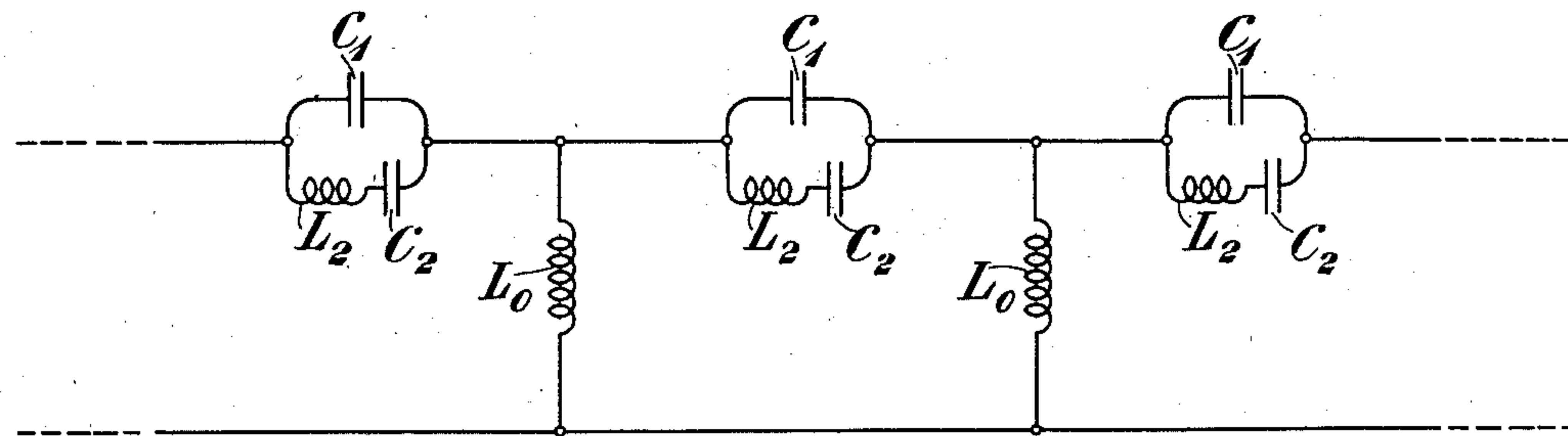


Fig. 4

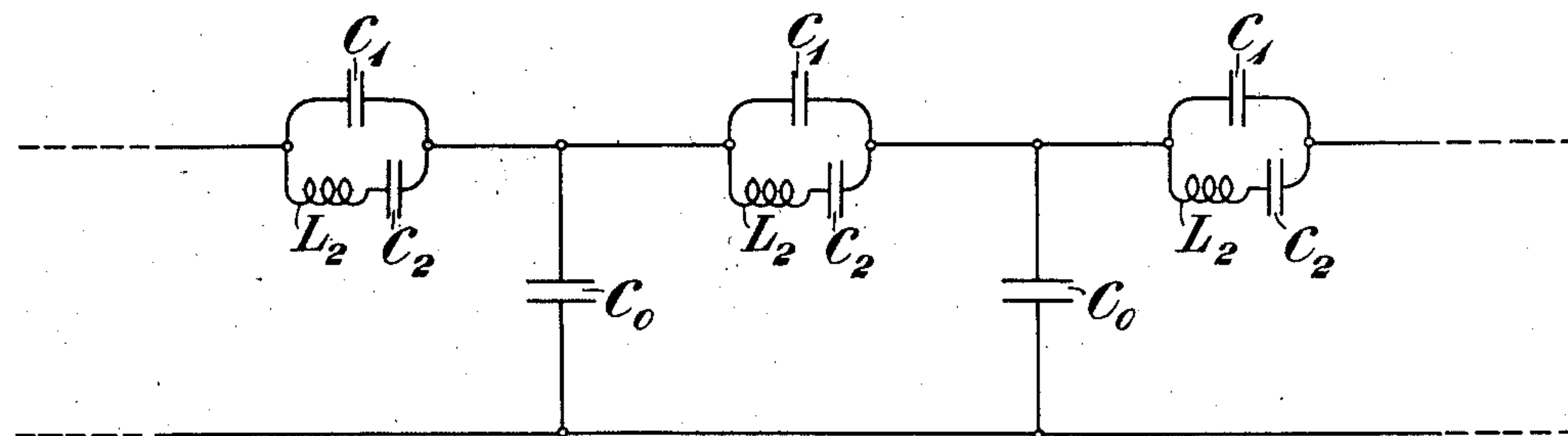


Fig. 5

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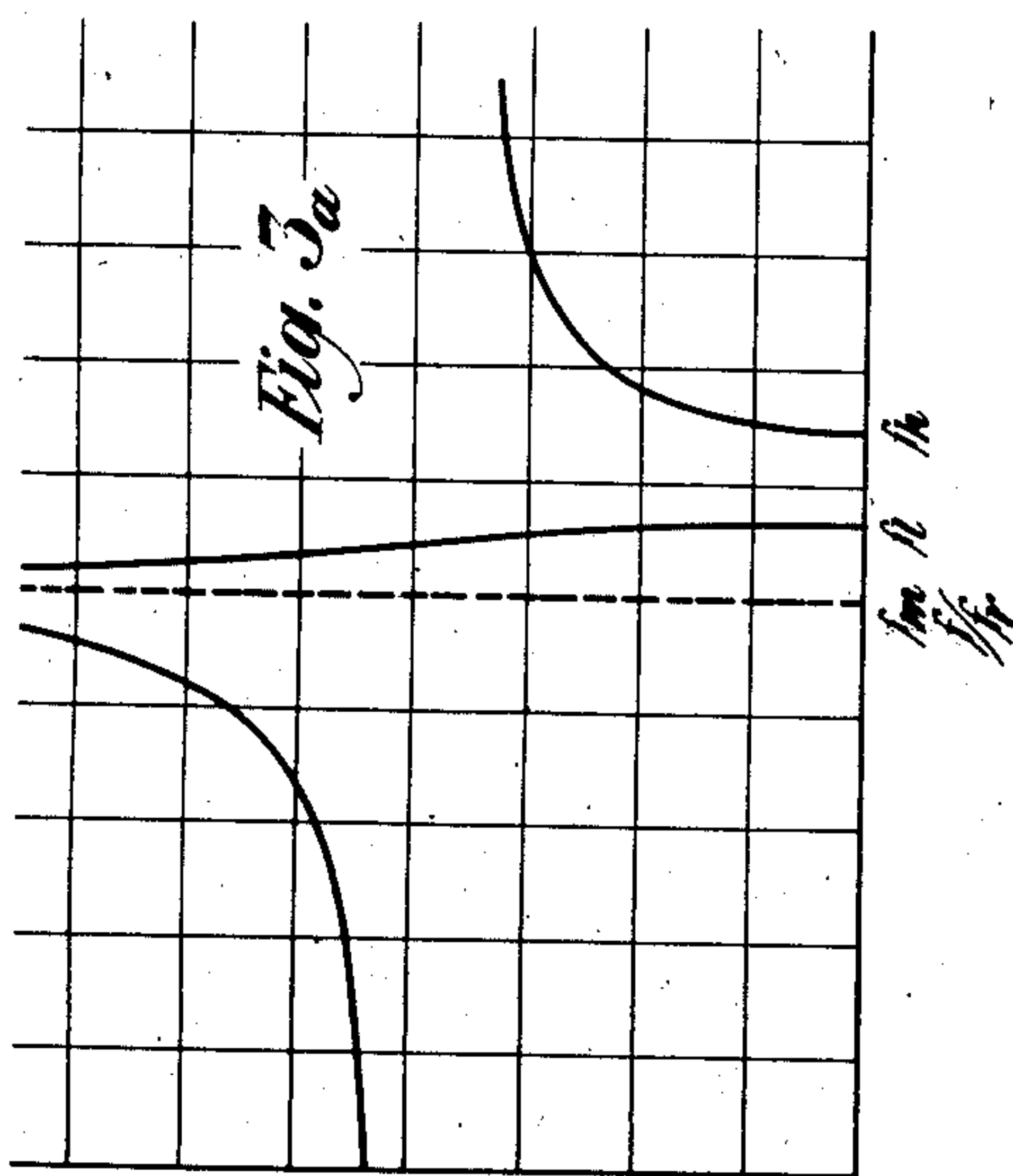
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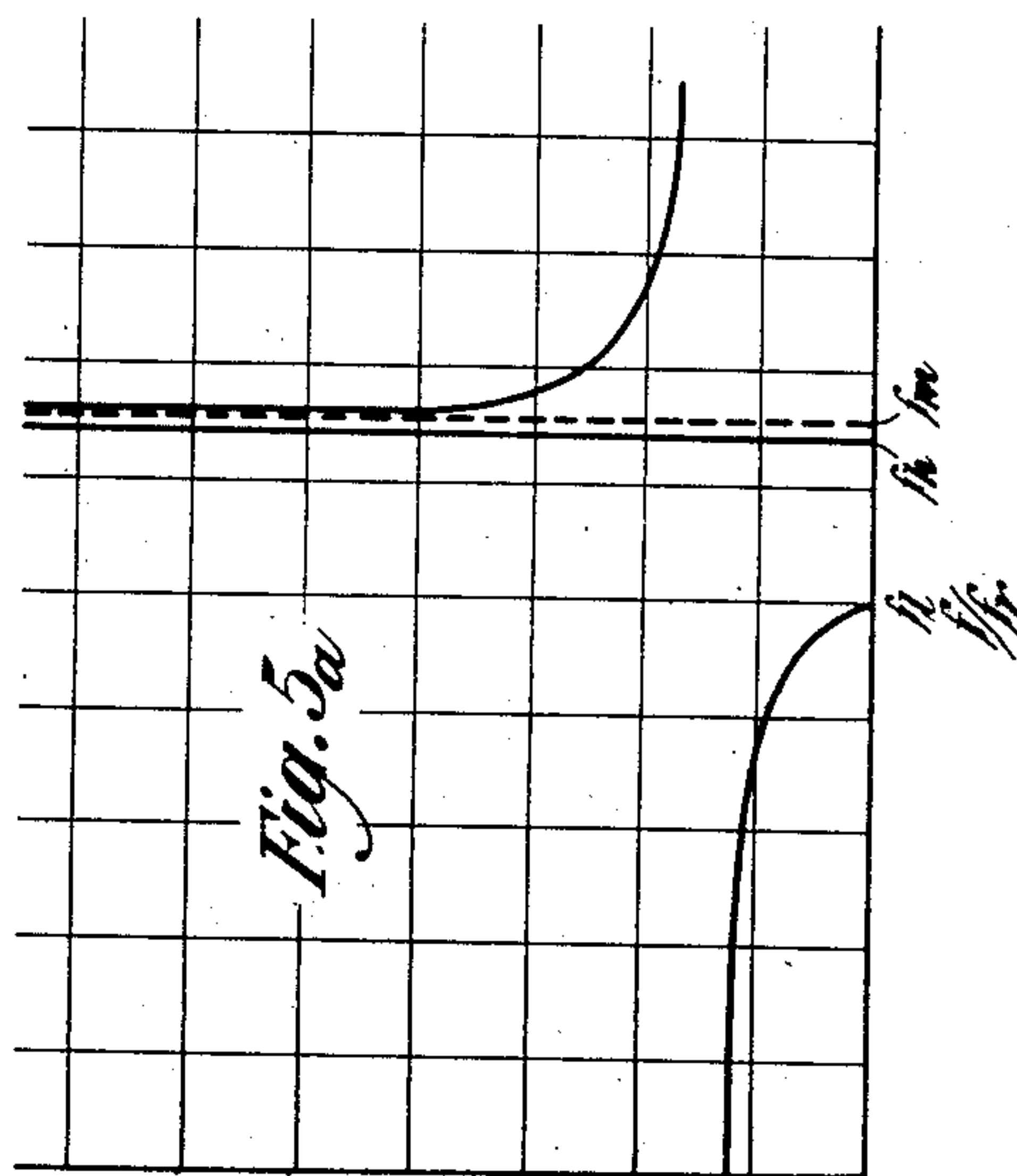
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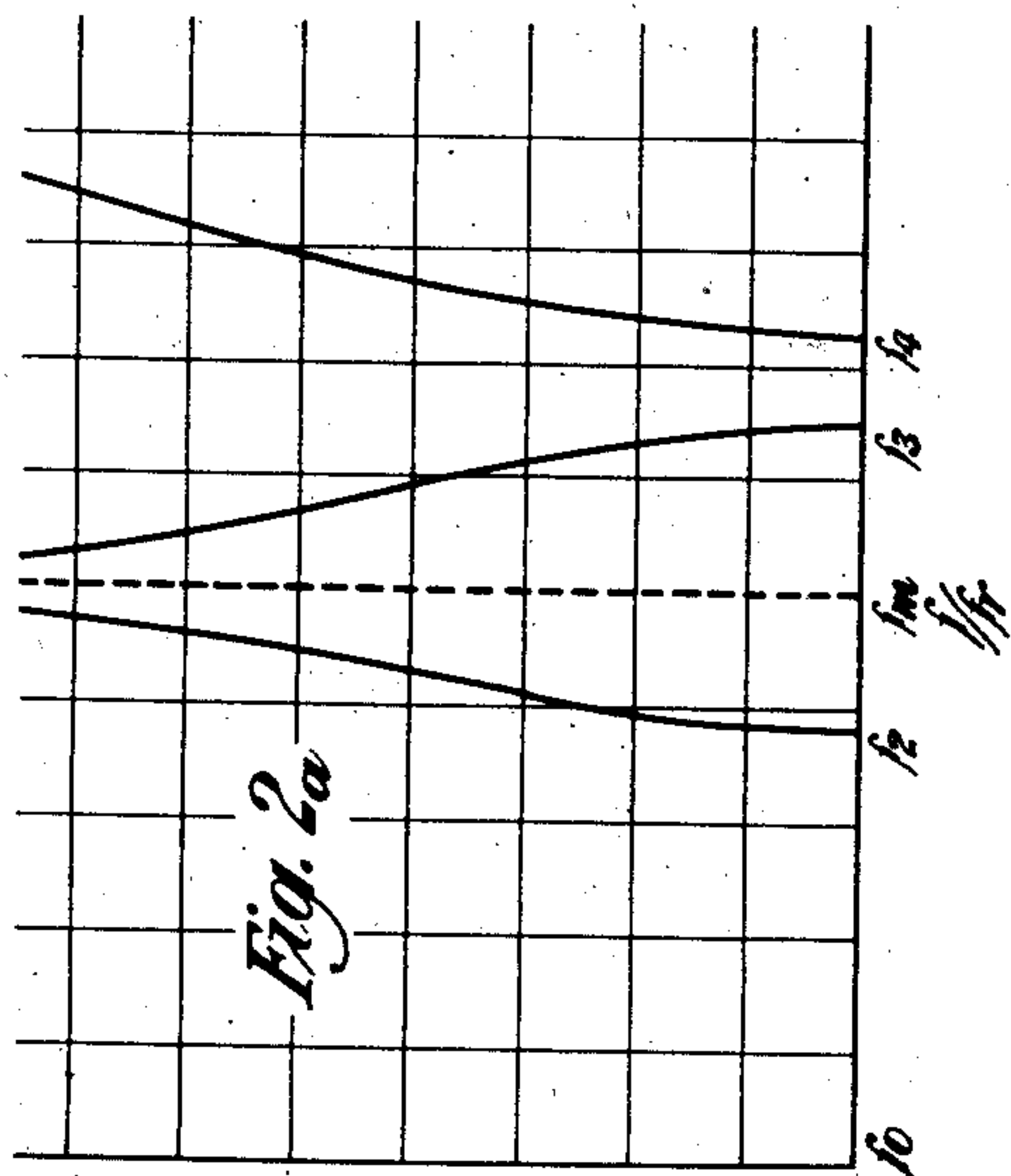
3 Sheets-Sheet 3



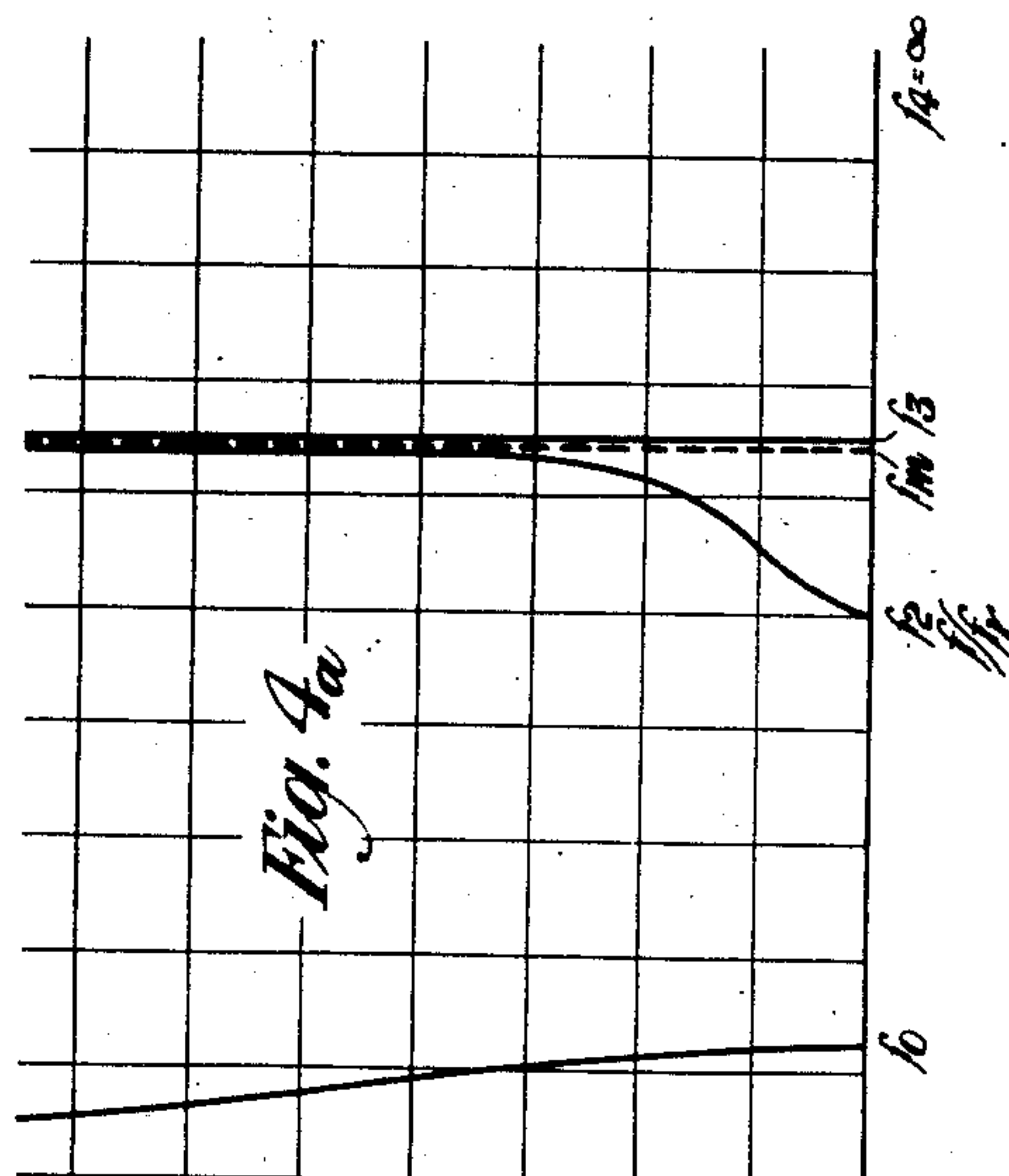
Attenuation per Section



Attenuation per Section



Attenuation per Section



Attenuation per Section

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UNITED STATES PATENT OFFICE.

HENRY W. ELSASSER, OF NEW YORK, N. Y., ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

FREQUENCY SELECTIVE CIRCUITS.

Application filed August 13, 1920. Serial No. 403,369.

To all whom it may concern:

Be it known that I, HENRY W. ELSASSER, a citizen of the United States, residing at New York, in the county of New York and State of New York, have invented certain Improvements in Frequency Selective Circuits, of which the following is a specification.

This invention relates to frequency-selective circuits.

It contemplates a network of impedances having a period of series resonance and a period of parallel resonance, so that its impedance for a certain frequency of current is very low and for another, very high.

The invention proposes, further the use of a network of this character in combination with other impedance elements in a periodic structure of the type illustrated and described in the patents to G. A. Campbell, 1,227,113 and 1,227,114 of May 22, 1917. Certain new and useful types of wave filters are thus arrived at, the characteristics of which are explained hereinbelow.

This application is related to certain co-pending cases, Serial Numbers 403,367, 403,368, 403,370, filed of even date herewith.

A good understanding of the invention may now be had from the following description of certain specific embodiments thereof, having reference to the accompanying drawing, in which,

Fig. 1 is a diagrammatic view showing one form of network embodying the invention;

Figs. 2 to 5 inclusive are diagrammatic views showing various types of filters comprising the network of Fig. 1;

Fig. 1^A is a graph showing the variation with frequency in the impedance of the network of Fig. 1, and

Figs. 2^A to 5^A inclusive, are graphs showing the variation in attenuation of the filters of Figs. 2 to 5, respectively.

Similar characters of reference designate similar parts in each of the several views.

The network of Fig. 1 consists of a condenser C_1 in parallel with an inductance L_2 and a condenser C_2 which are in series with each other and thus constitute a series res-

onant path. The impedance of the condenser C_1 is

$$\frac{-j}{wC_1}, \quad 55$$

where j is written for $\sqrt{-1}$ and w equals $2\pi f$, f being the frequency of the current. The impedance of the series resonant path is

$$jwL_2 - \frac{j}{wC_2}, \quad 60$$

hence the impedance value of the entire network is

$$Z = \frac{\left(jwL_2 + \frac{1}{jwC_2}\right) \left(\frac{j}{wC_1}\right)}{jwL_2 + \frac{1}{jwC_2} + \frac{1}{jwC_1}} \quad (1) \quad 65$$

Place for convenience 70

$$f_1 = \frac{f}{f_r} \quad (2)$$

where f_r is the frequency at which L_2 and C_2 are resonant. Substitute equation 2 in equation 1 and simplify. Then 75

$$Z = jw_r L_2 \left(\frac{\frac{f_1^2 - 1}{f_1}}{1 - f_1^2 \frac{C_1}{C_2} \left(\frac{f_1^2 - 1}{f_1} \right)} \right) \quad (3) \quad 80$$

where

$$w_r = 2\pi f_r \quad 85$$

The expression in the brackets of the above equation may be placed equal to K . Then

$$Z = jw_r L_2 K \quad (4)$$

where 90

$$K = \frac{\frac{f_1^2 - 1}{f_1}}{1 - f_1^2 \frac{C_1}{C_2} \left(\frac{f_1^2 - 1}{f_1} \right)} \quad (5) \quad 95$$

The variation in the value of K with frequency is shown by the curves of Fig. 1^A, in which the values of K are ordinates and the ratios of f to f_r are abscissæ. These curves indicate also the manner in which the 100

impedance of the network changes with frequency as may be seen by an inspection of equation 4. At low frequencies, the impedance of the network is negative, but as the frequency is raised, the impedance changes to positive, the point of crossing of the axis of abscissæ denoting series resonance of the path containing C_2 and L_2 , or zero impedance of the network. The impedance then increases until, at the frequency at which the elements of the network are in parallel resonance, its value is infinite. The impedance then changes sign and thereafter decreases, approaching zero at infinite frequency. The network therefore, has two periods of resonance, a period of series resonance at one frequency and a period of parallel resonance at a higher frequency. The period of series resonance depends on the relative value of only two impedance elements, namely L_2 and C_2 , and the period of parallel resonance is governed by the values of all three reactances. The curves of Fig. 1^A are drawn for an ideal network containing no resistance or other dissipative elements, but in an actual case, the resistance may be made so small that its effect is practically negligible. It thus appears that the network of Fig. 1 may be used as a selective circuit—for passing current of series resonant frequency and preventing the passage of current of parallel resonant frequency.

I have found, moreover, that by employing the network as a shunt and series impedance in a periodic structure like that disclosed in the Campbell patents hereinbefore mentioned, certain new types of wave filters are arrived at, which filters have certain new and valuable characteristics which I shall now describe.

Figs. 2, 3, 4, and 5 illustrate four types of filters employing the network of Fig. 1, the former two of these views showing the network as a shunt impedance element and the latter two, as a series impedance element of the filter section. Figs. 2 and 3 show an inductive and a capacity reactance respectively as the series impedance element, and Figs. 4 and 5 show the same reactances respectively as the shunt impedance elements.

The properties of the above filters may be determined from certain mathematical expressions which set forth the relations existing between the frequency of current and the impedance elements of the filters. In the Campbell patents hereinbefore mentioned, it was shown (equation 2) that for a periodic structure of the type now under

consideration, in which the series impedance per section is Z_1 and the shunt impedance per section is Z_2 , the attenuation per section of the filter may be derived from the relation

$$\cosh \gamma = \frac{1}{2} \frac{Z_1}{Z_2} + 1 \quad (6)$$

in which γ denotes the propagation constant of the structure. The variation of the attenuation of any filter with frequency of current may, therefore, be deduced from equation 6, when the corresponding values of Z_1 and Z_2 are substituted therein. For the filter shown in Fig. 2, the value of Z_1 is

$$Z_1 = j\omega L_0 \quad (7)$$

and (according to equation 4 above),

$$Z_2 = j\omega_r L_2 K \quad (8)$$

The resultant equation for $\cosh \gamma$ is, therefore,

$$\cosh \gamma = \frac{1}{2} \frac{j\omega L_0}{j\omega_r L_2 K} + 1 \quad (9)$$

Fig. 2^A is a graph showing the variation of the attenuation of the filter of Fig. 2, as computed from equation 9. The axis of the abscissæ is laid off in ratios of f to f_r and the axis of the ordinates in values of the attenuation constant per filter section. An inspection of the curves shows that the attenuation is nil for two ranges of frequencies, f_0 to f_2 and f_3 to f_4 . The filter, in other words, passes without attenuation, the two ranges of frequencies. The filter is, therefore, a combined low-pass and band filter and performs the functions of both. It is characterized, moreover, by having infinite attenuation at a frequency f_m which is close to f_2 , and it discriminates, consequently, with particular sharpness against frequencies just above the upper limit of the low-pass range.

The frequencies f_2 , f_3 and f_4 may be evaluated as follows. It was shown in the said Campbell patents, that for unattenuated transmission γ must be a pure imaginary, and that, therefore, the value of $\cosh \gamma$ must lie between ± 1 . The frequencies which limit the ranges of free transmission may consequently be determined by placing equation 9 equal to $+1$ and -1 respectively, and solving for f . When this is done, it will be found that the roots are respectively, $f_0=0$

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{L_0 C_1}{L_2 C_2} + \frac{L_0}{L_2} + 4 - \sqrt{\left(\frac{L_0 C_1}{L_2 C_2} + \frac{L_0}{L_2} + 4\right)^2 - 16 \frac{L_0 C_1}{L_2 C_2}}} \quad (10)$$

$$f_3 = \frac{1}{2\pi} \sqrt{\frac{C_1 + C_2}{C_1 C_2 L_2}} \quad (11)$$

$$f_4 = \frac{1}{2\pi} \sqrt{\frac{\frac{L_0 C_1}{L_2 C_2} + \frac{L_0}{L_2} + 4 + \sqrt{\left(\frac{L_0 C_1}{L_2 C_2} + \frac{L_0}{L_2} + 4\right)^2 - 16 \frac{L_0 C_1}{L_2 C_2}}}{2 L_0 C_1}} \quad (12)$$

The frequency f_m , at which the attenuation is infinite, may be evaluated by placing equation 9 equal to ∞ and solving for f , whence

$$f_m = \frac{1}{2\pi \sqrt{L_2 C_2}} \quad (13)$$

$$f_h = \frac{1}{2\pi} \sqrt{\frac{C_1 + C_2}{C_1 C_2 L_2}} \quad (16) \quad 35$$

When equation 14 is placed equal to ∞ and solved for f , the frequency of attenuation, f_m , is found to be 40

$$f_m = \frac{1}{2\pi \sqrt{L_2 C_2}} \quad (17)$$

The attenuation characteristics of the remaining filters may be arrived at in a similar manner. The curves of Fig. 3^A show that the filter of Fig. 3 passes without substantial attenuation only a single band, the limiting values of which are f_1 and f_h . The filter is further characterized by having infinite attenuation per section at a frequency f_m below f_1 . This frequency may be chosen within the lower attenuated range, but preferably so as to lie close to f_1 , so that the filter has a sharp "cut-off" at the lower limit of the transmitted band.

The curves of Fig. 4^A show that the filter of Fig. 4 is similar to that of Fig. 2 in that it has two ranges of free transmission. It differs therefrom, however, in that it passes all frequencies above a certain value without substantial attenuation, and serves consequently as a combined band and high-pass filter. The frequency f_m , at which the attenuation is infinite, may be so chosen as to lie close to f_3 , thus giving the filter a sharp "cut-off" at the lower limit of the high-pass band. These curves have been derived from the expression, 45

$$\cosh \gamma = \frac{1}{2} \frac{j\omega_r L_2 K}{j\omega L_0} + 1 \quad (18) \quad 60$$

The frequencies f_1 , f_h , and f_m may be evaluated similarly as the limiting frequencies of the filter of Fig. 2. The expression for $\cosh \gamma$ in the present case is, since Z_1 is a capacity reactance, 50

$$\cosh \gamma = -\frac{1 + j\omega C_0}{2j\omega_r L_2 K} + 1 \quad (14) \quad 25$$

Placing equation 14 equal respectively to $+1$ and -1 , and solving for f , the roots will be found to be 55

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{4C_0 + C_1 + C_2}{L_2 C_2 (4C_0 + C_1)}} \quad (15)$$

which is similar to equation 9 except that the values of Z_1 and Z_2 are interchanged, the shunt impedance of Fig. 2 becoming the series impedance of Fig. 4, and vice versa. The limiting frequencies are obtained, as before, by placing $\cosh \gamma$ equal to $+1$ and -1 respectively and solving for f . Hence 65

$$f_0 = \frac{1}{4\pi} \sqrt{\frac{4 \frac{L_0 C_1}{L_2 C_2} + 4 \frac{L_0}{L_2} + 1 - \sqrt{\left(4 \frac{L_0 C_1}{L_2 C_2} + 4 \frac{L_0}{L_2} + 1\right)^2 - 16 \frac{L_0 C_1}{L_2 C_2}}}{2 L_0 C_1}} \quad (19)$$

$$f_2 = \frac{1}{2\pi \sqrt{L_2 C_2}} \quad (20)$$

$$f_3 = \frac{1}{4\pi} \sqrt{\frac{4 \frac{L_0 C_1}{L_2 C_2} + 4 \frac{L_0}{L_2} + 1 + \sqrt{\left(4 \frac{L_0 C_1}{L_2 C_2} + 4 \frac{L_0}{L_2} + 1\right)^2 - 16 \frac{L_0 C_1}{L_2 C_2}}}{2 L_0 C_1}} \quad (21)$$

$$f_4 = \infty$$

The expression for the frequency of maximum attenuation, f_m , is obtained by placing $\cosh \gamma$ equal to ∞ and solving for f . Then 70

$$f_m = \frac{1}{2\pi} \sqrt{\frac{C_1 + C_2}{C_1 C_2 L_2}} \quad (22) \quad 75$$

The curves of Fig. 5^A show that the filter of Fig. 5 is of the single band type and similar to that of Fig. 3, differing therefrom, however, in that the frequency of infinite attenuation lies in the upper attenuated range. This filter, therefore, may be caused to have a sharp "cut-off" at the upper limit 80

of the band. The attenuation curve of Fig. 5^A is derived from the expression,

$$\cosh \gamma = -\frac{1}{2} \frac{j\omega_r L_2 K}{j \frac{1}{\omega C_0}} + 1 \quad (23)$$

and the values of f_1 and f_h and f_m are obtained by placing the above expression for $\cosh \gamma$ equal to +1, -1 and ∞ , respectively.

$$f_1 = \frac{1}{2\pi \sqrt{L_2 C_2}} \quad (24)$$

$$f_h = \frac{1}{2\pi \sqrt{L_2 C_2 (C_0 + 4C_1 + 4C_2)}} \quad (25)$$

$$f_m = \frac{1}{2\pi \sqrt{\frac{C_1 + C_2}{C_1 C_2 L_2}}} \quad (26)$$

It should be noted that the attenuation curves herein illustrated refer to the ideal structure in which the resistance of the impedance units is zero. In a practical filter there is a departure from these curves, owing to energy dissipation. In any case, however, the resistance may be made so small that the departure from the ideal is practically negligible.

The formulæ 10-13, 15-17, 19-22, and 24-26, given above, may be used in designing filters to meet any specified sets of conditions. Since there are four independent impedance elements in each filter section, any four properties of the filter dependent upon the values of the impedance elements but independent of each other may be chosen at will. For example, in the design of a filter of the type illustrated in Fig. 2, two of the design conditions may be taken as the frequencies f_3 , f_4 , and the third as f_2 , thus de-

fining the ranges of free transmission. This leaves one condition open to choice, and this may be taken as the impedance of the filter at any desired frequency, or as the value of any one of the elements of the filter section. In the design of a filter of the type of Fig. 3, two of the design conditions may be chosen as the frequencies f_1 , and f_h , and the third as f_m , thus leaving the fourth to be chosen in accordance with any other condition that may prevail. Similar considerations apply to the remaining types of filters.

As an example of the application of the formulæ, let it be required to design a filter of the type illustrated in Fig. 5, which shall transmit frequencies between 400 and 2500 cycles, and which shall have maximum, ideally infinite, attenuation at 2750 cycles, so that it has a particularly sharp cut-off at the upper limit of the freely transmitted range. Frequencies f_1 , f_h and f_m are thus specified as 400, 2500 and 2750 cycles respectively. As a fourth design factor, let it be assumed that certain considerations dictate that the value of C^2 shall be 3.00 microfarads. Applying formula (24), we find that

$$400 = \frac{1}{2\pi \sqrt{3 \times 10^{-6} L_2}} \quad (70)$$

hence

$$L_2 = .0528 \text{ henries}$$

Substituting in (26), we have

$$2750 = \frac{1}{2\pi \sqrt{\frac{C_1 + 3 \times 10^{-6}}{C_1 \times 3 \times 10^{-6} \times .0528}}} \quad (75)$$

Hence

$$C_1 = .0649 \text{ microfarads}$$

Finally according to (25)

$$2500 = \frac{1}{2\pi \sqrt{\frac{C_0 + 4 \times .0649 \times 10^{-6} + 4 \times 3 \times 10^{-6}}{.0528 \times 3 \times 10^{-6} (C_0 + 4 \times .0649 \times 10^{-6})}}}$$

Therefore

$$C_0 = .0559 \text{ microfarads}$$

All the constants of the filter are thus determined. It will readily be seen that, instead of the above-mentioned set of conditions, any others involving the filter impedances may be imposed, it being understood that the above example is merely a simple illustration, and in no way limits the invention.

Although only certain forms of filters embodying the invention are shown and described herein, it is easily understood that various changes and modifications may be made therein within the scope of the following claims, without departing from the spirit and scope of the invention.

While it is assumed, for the purpose of

computation, that the impedance elements of a filter are pure reactances, it is, of course, understood that these reactances, in the forms of coils or condensers, must inevitably have a little resistance. In this respect the procedure is no different from that in other applications of mathematics to engineering. In a problem in dynamics, friction may be neglected. In problems in statics, strains are often neglected. Similarly, in this case, for the purposes of computation, the resistance of the coils and condensers is neglected.

The unavoidable resistance is greater in a coil than in a condenser. Hence, for some purposes, a filter which has its reactance elements principally in the form of condensers instead of coils, will involve less departure from theory.

Referring to the drawings of the present

case, it will be seen that in Figs. 3 and 5 each filter section is made up of three condensers and only one coil. These filters will have the advantage just pointed out.

5 What is claimed is:

1. A wave filter of the type having similar recurrent sections, each section having a series element and a shunt element, one of those elements in each section consisting of
10 a single lumped reactance and the other element in each section being a network comprising a capacity reactance in parallel with a path comprising an inductive reactance and a capacity reactance in series with each
15 other.

2. A filter for an electric circuit, consisting of a plurality of recurrent sections, each such section comprising an impedance in series with the circuit and an impedance in
20 shunt thereto, one of said impedances consisting of a single reactance element and the other, of a network comprising a capacity reactance in parallel with a path comprising a capacity reactance in series with an inductive reactance.

3. A wave filter of periodic structure comprising sections each having four and only four reactances of which three are embodied in condensers and one in a coil, said three
30 condensers constituting three different capacities adapted to function in different phases.

4. A single band filter with recurrent sections having a frequency of maximum attenuation close to one side of the band
35 whereby the filter will have a sharp cut-off at that side of the band and having only one coil per section, the remaining impedance elements of the sections being condensers, the said coil and one of the said
40 condensers in combination being resonant at the said frequency of maximum attenuation.

5. A wave filter having recurrent sections with four reactance elements in each section
45 and having a single transmission band, said four reactances being determined as functions of (1) one limiting frequency, (2) the other limiting frequency, (3) the frequency for maximum attenuation close to one of said limiting frequencies outside the transmission range and (4) the "mid-frequency"

characteristic impedance of the filter within the transmission range.

6. A wave filter of the type having recurrent sections, each section consisting of a
55 series impedance and a shunt impedance, one of these impedances being made up of a network consisting of three reactance elements, and the other such impedance consisting of only a single reactance element, said
60 filter having a single continuous free transmitting range between finite limiting frequencies and infinite attenuation at a frequency outside that range.

7. A wave filter of the type having recurrent sections, each section consisting of a series impedance and a shunt impedance, one of these impedances being a simple capacity reactance, the other such impedance being embodied in a three-element network,
65 the filter having a single free transmitting range between two finite frequencies and having infinite attenuation at a frequency outside that range.

8. The method of discriminating among
75 various alternating current frequency components from zero frequency up to a high frequency, which consists in attenuating the components from zero up to a certain finite limiting frequency with a maximum of attenuation at a frequency only a little less than this finite limit, then passing currents from this finite limit up to a higher limiting frequency and then attenuating all currents higher than this last mentioned limiting frequency.

9. The method of discriminating among a plurality of alternating current components of various frequencies, which consists in attenuating all components of frequency from zero to a certain finite limiting frequency, then passing all components from this frequency to a certain higher finite limiting frequency and then attenuating all components above said last mentioned frequency with a maximum of attenuation close to one of said two limiting frequencies.

In testimony whereof, I have signed my name to this specification this 10th day of August 1920.

HENRY W. ELSASSER.