Jan. 5, 1926. 1,568,142

H. W. ELSASSER

FREQUENCY SELECTIVE CIRCUITS

Filed August 13, 1920 3 Sheets-Sheet 1

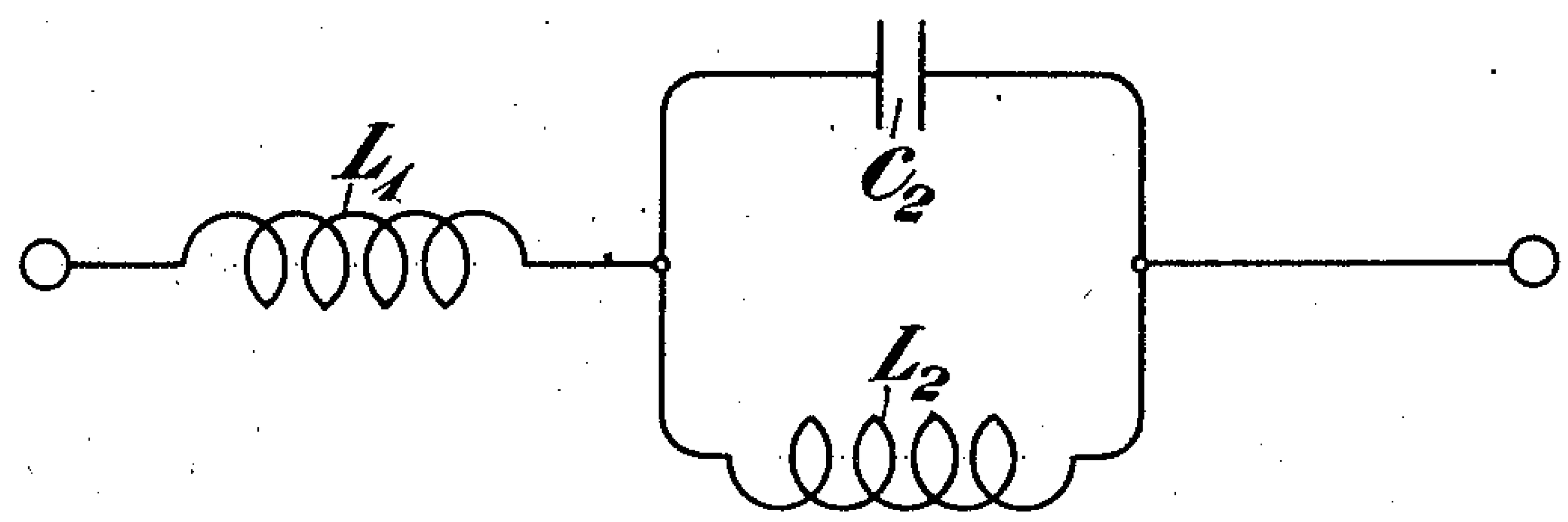

Fig. 1

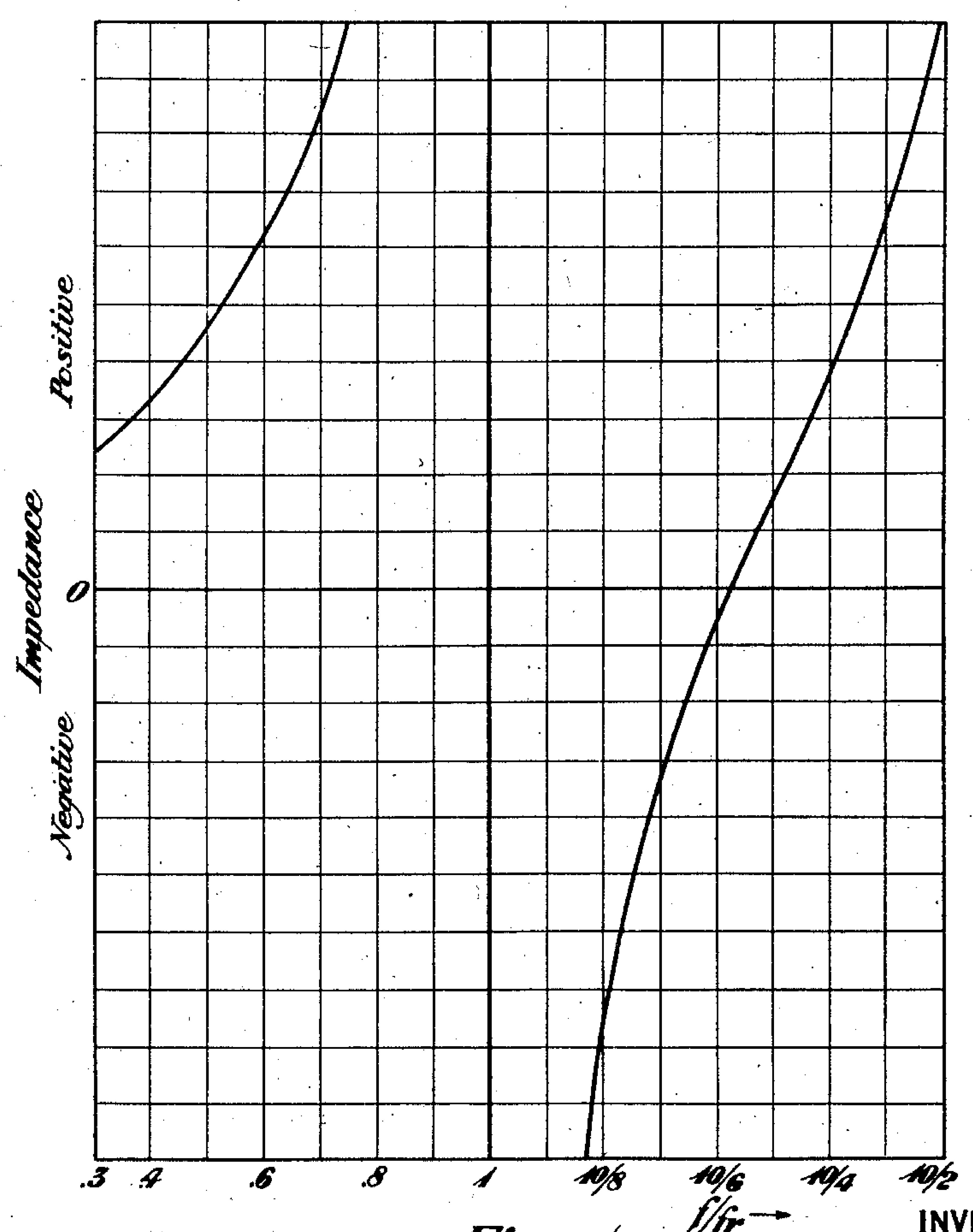

Fig. 1a

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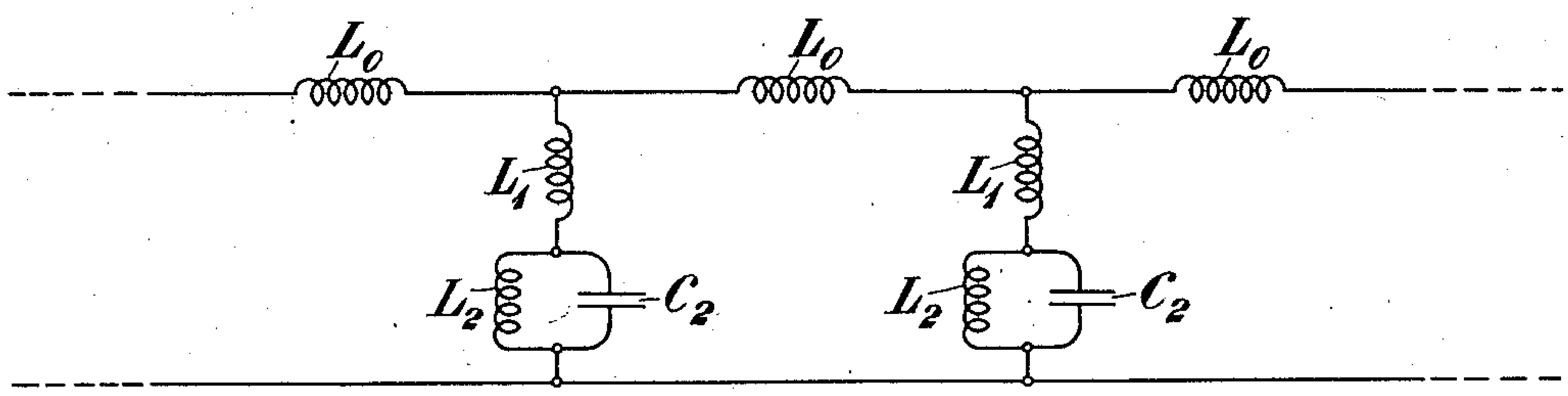

Fig. 2

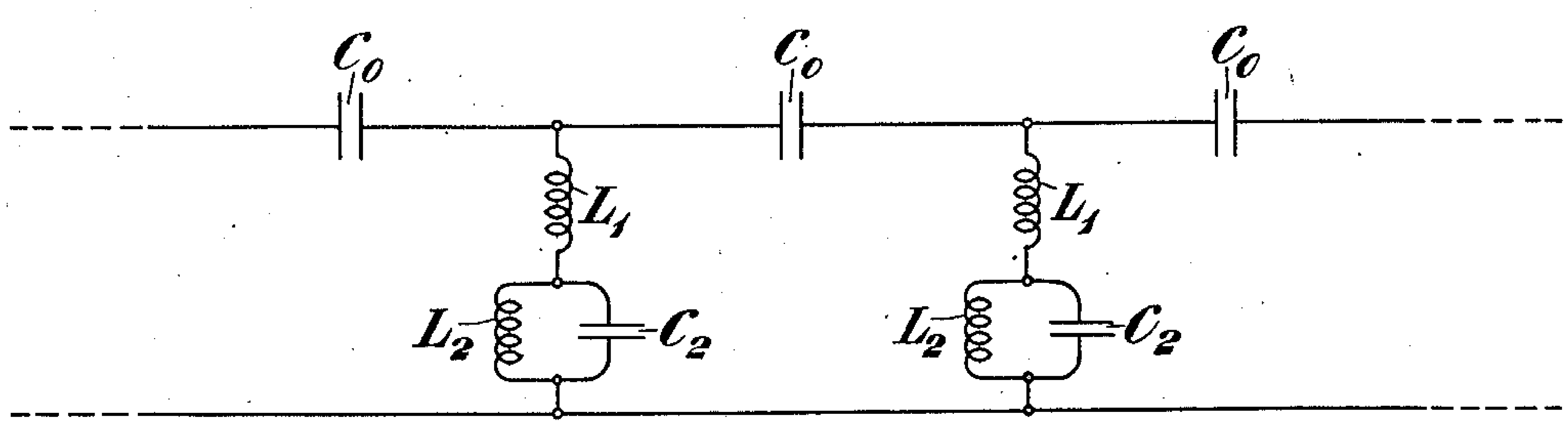

Fig. 3

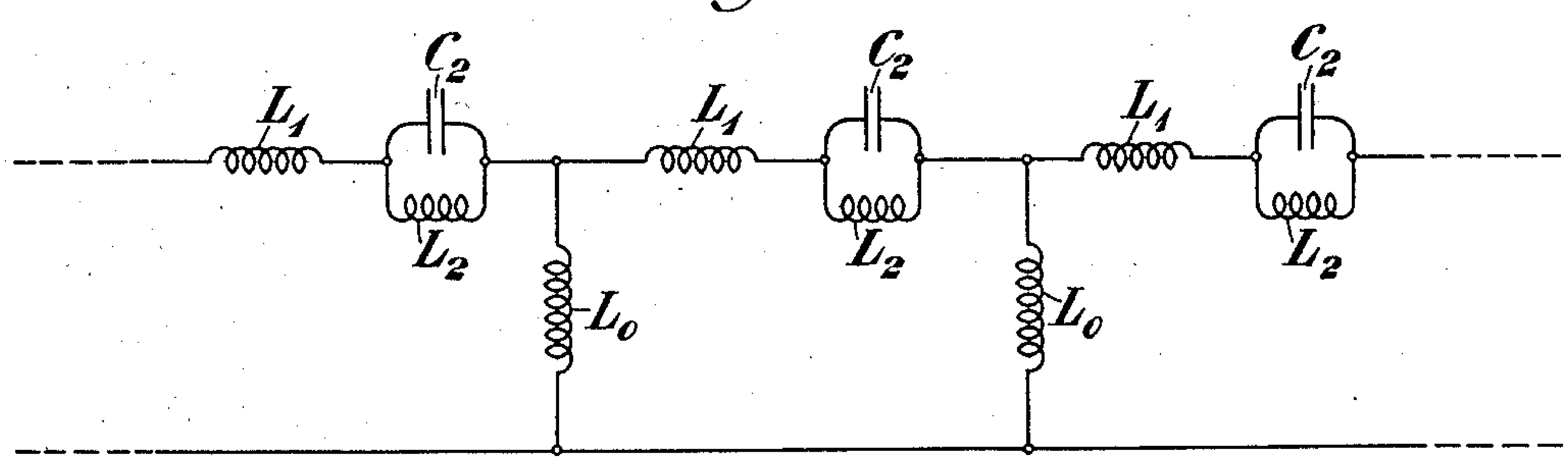

Fig. 4

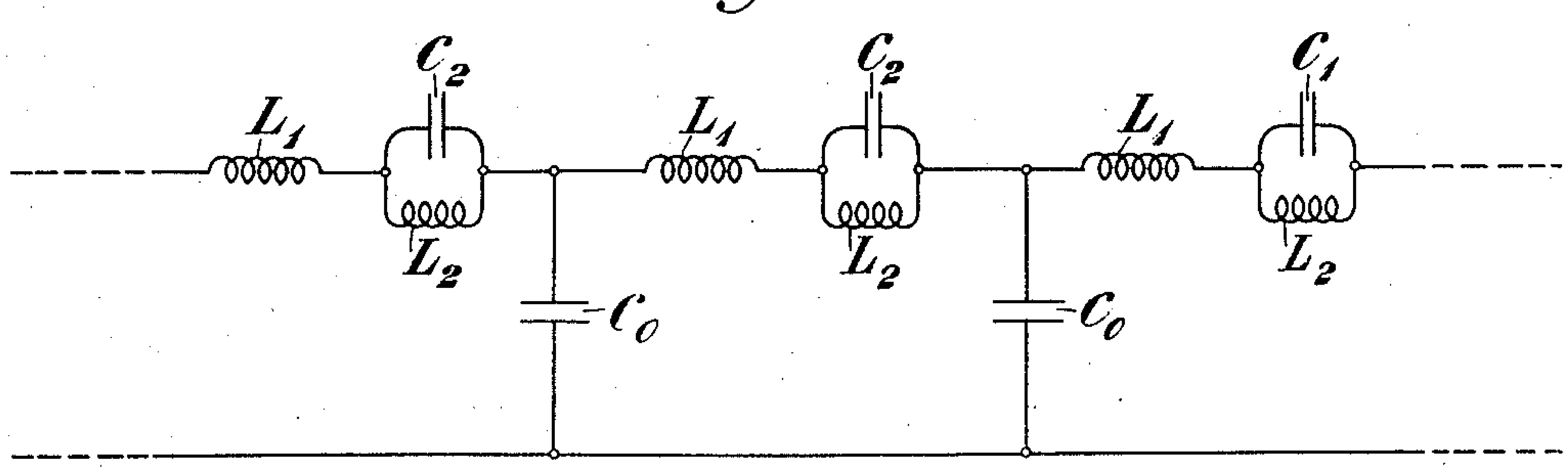

Fig. 5

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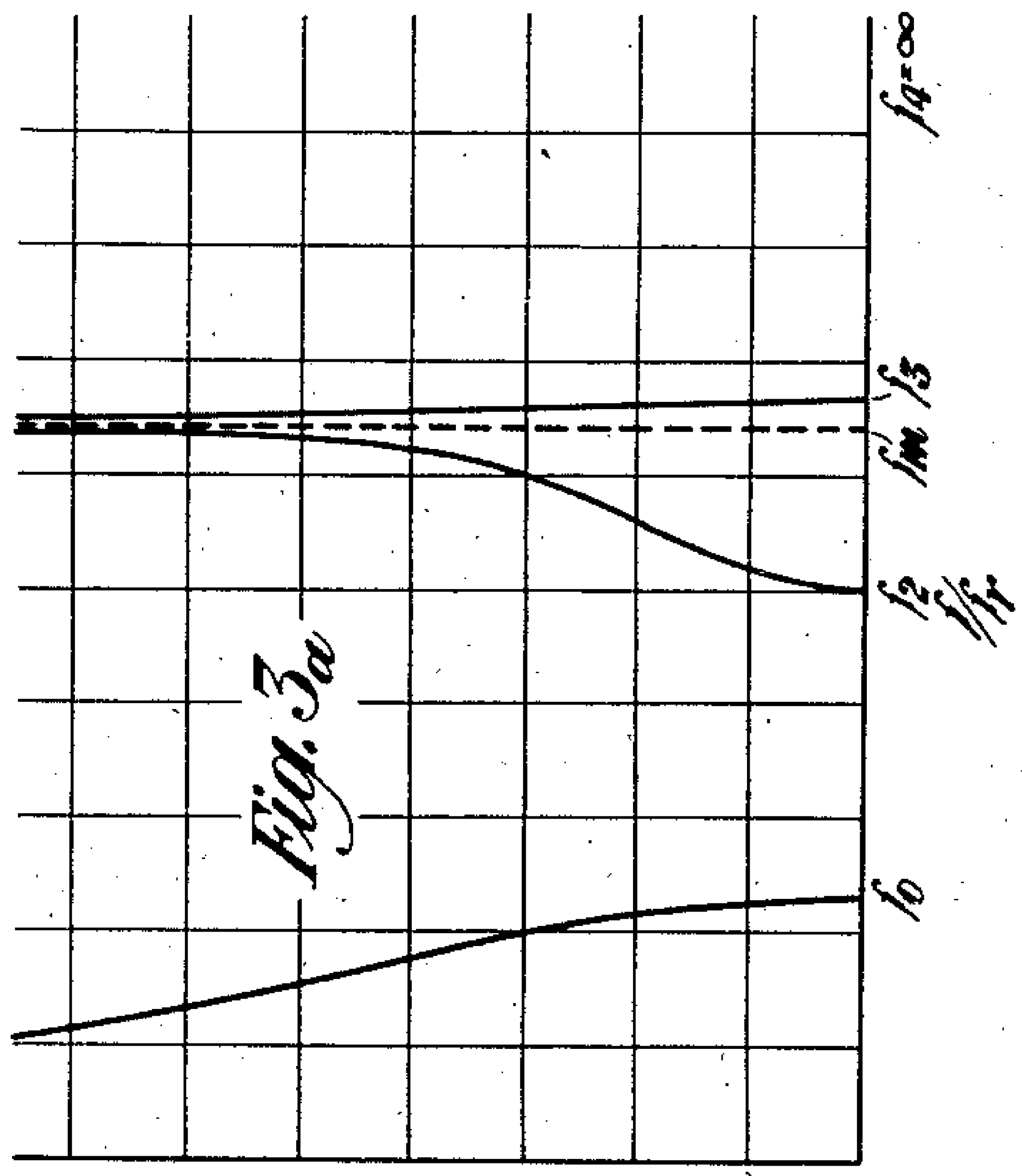

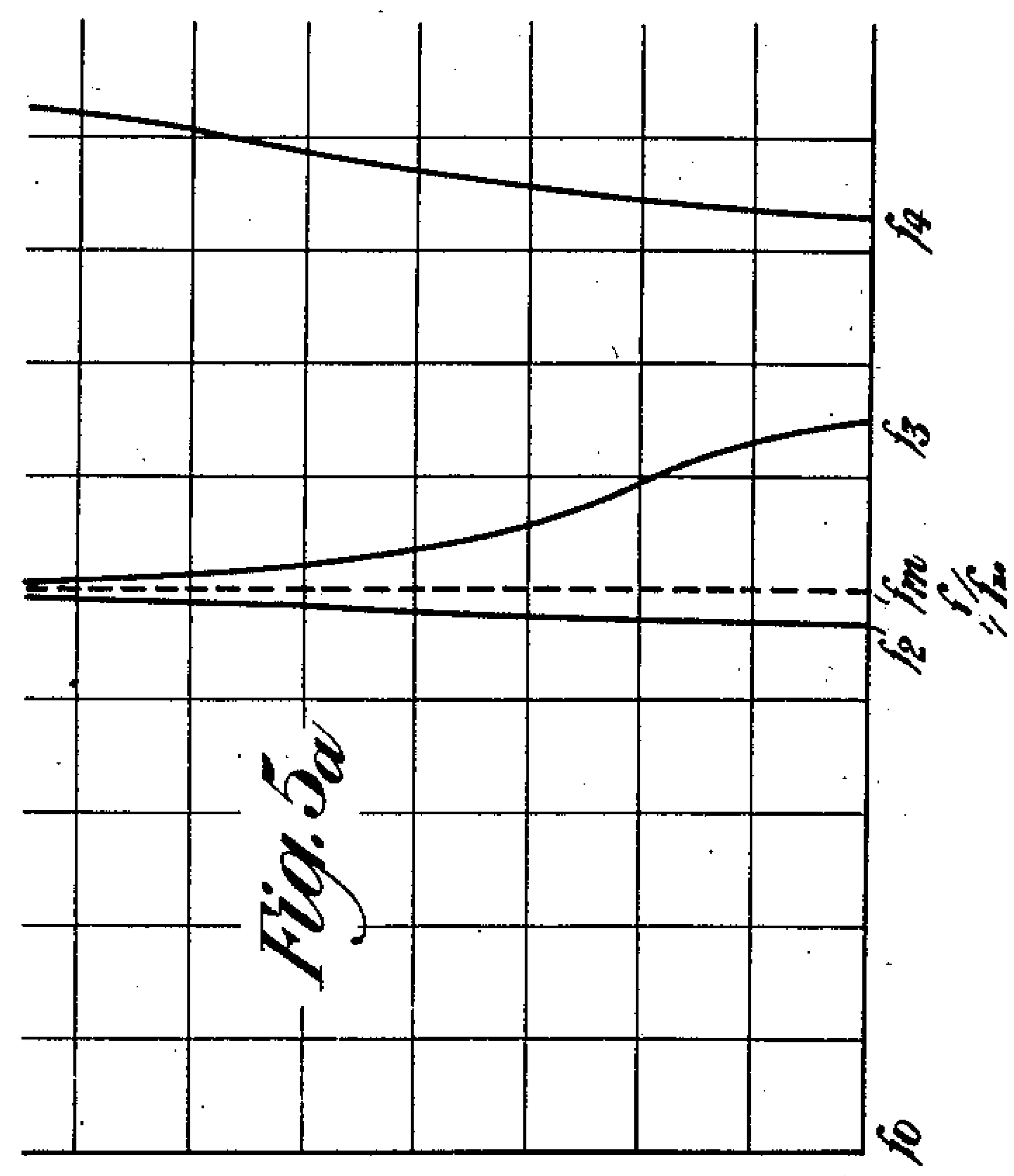

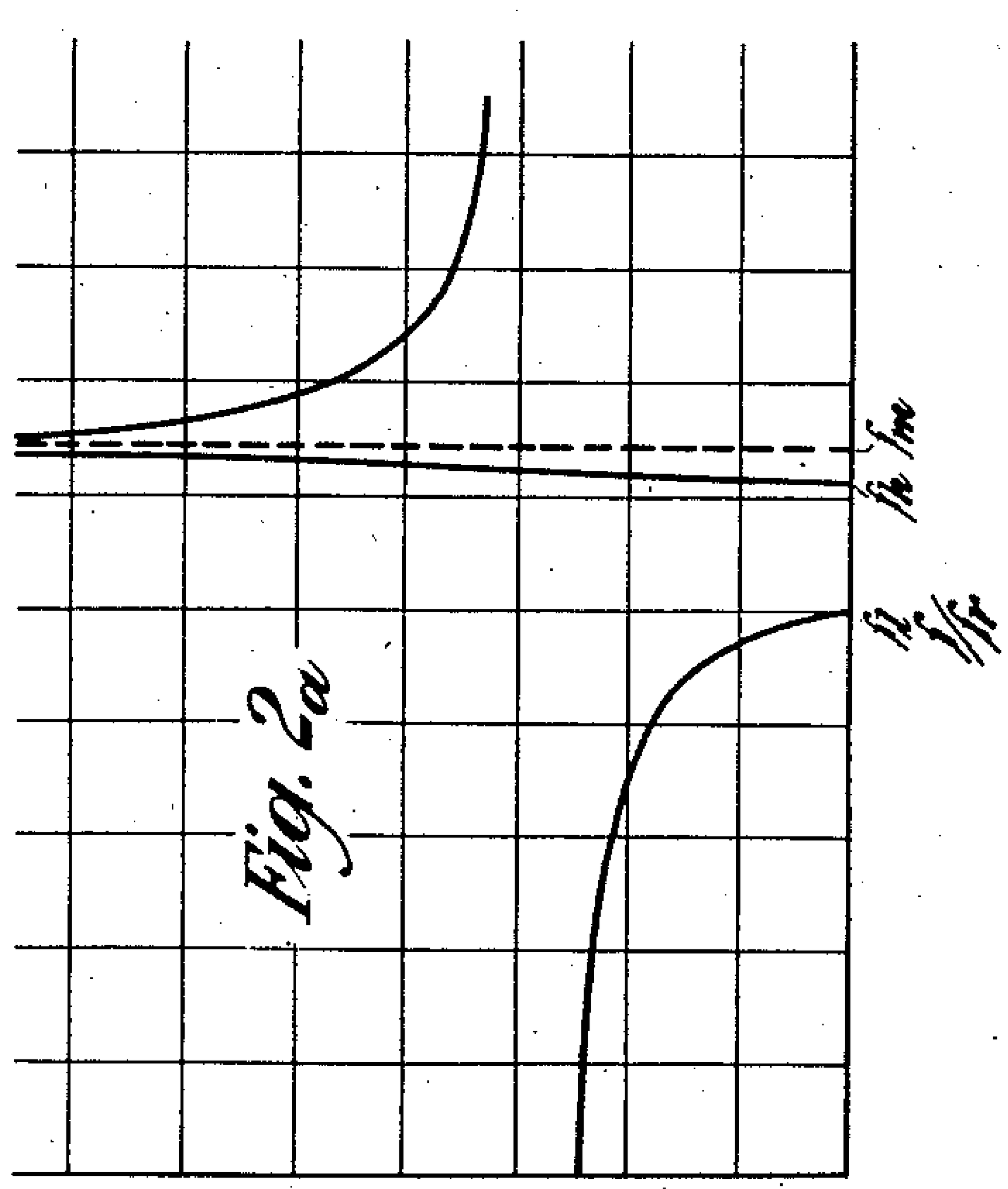

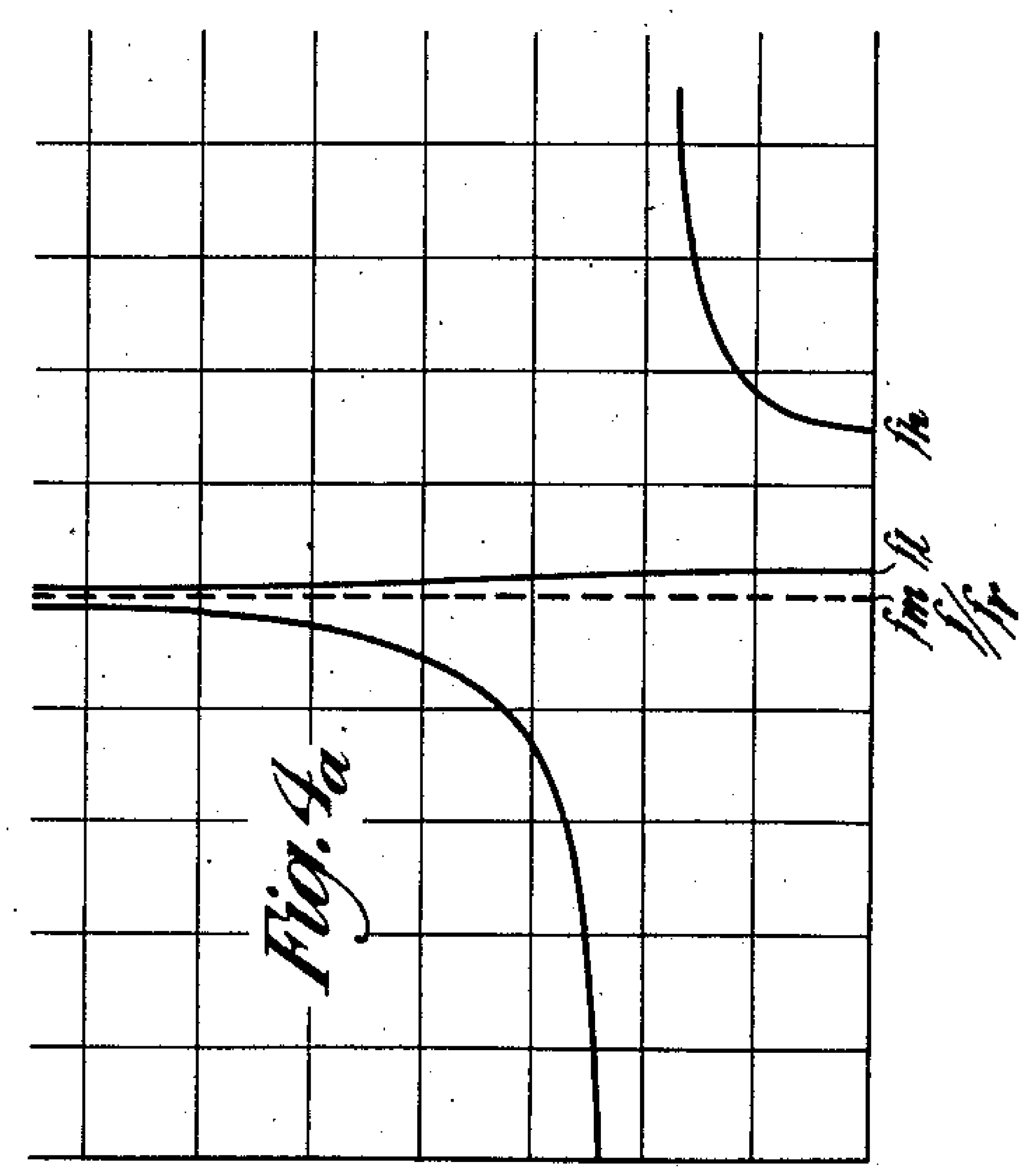

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Patented Jan. 5, 1926. **1,568,142**

UNITED STATES PATENT OFFICE.

HENRY W. ELSASSER, OF NEW YORK, N. Y., ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

FREQUENCY SELECTIVE CIRCUITS.

Application filed August 13, 1920. Serial No. 403,368.

To all whom it may concern:

Be it known that I, HENRY W. ELSASSER, a citizen of the United States, residing at New York, in the county of New York and State of New York, have invented certain Improvements in Frequency Selective Circuits, of which the following is a specification.

This invention relates to frequency-selective circuits.

It contemplates a network of impedances having a period of series resonance and a period of parallel resonance, so that its impedance for a certain frequency of current is very low and for another, very high.

The invention proposes, further, the use of a network of this character in combination with other impedance elements in a periodic structure of the type illustrated and described in the patents to G. A. Campbell, 1,227,113 and 1,227,114 of May 22, 1917. Certain new and useful types of wave filters are thus arrived at, the characteristics of which are explained hereinbelow.

This application is related to certain copending cases, Serial Numbers 403,367, 403,369, 403,370, filed of even date herewith.

A good understanding of the invention may now be had from the following description of certain specific embodiments thereof, having reference to the accompanying drawing, in which,

Fig. 1 is a diagrammatic view showing one form of network embodying the invention;

Figs. 2 to 5 inclusive are diagrammatic views showing various types of filters comprising the network of Fig. 1;

Fig. 1^A is a graph showing the variation with frequency in the impedance of the network of Fig. 1, and

Figs. 2^A to 5^A inclusive, are graphs showing the variation in attenuation of the filters of Figs. 2 to 5, respectively.

Similar characters of reference designate similar parts in each of the several views.

The network of Fig. 1 consists of an inductance L_1 in series with a pair of parallel paths, one of which contains an inductance L_2 and the other of which comprises a condenser C_2. The impedance of the inductance L_1 is jwL_1, where j is written for the $\sqrt{-1}$ and w equals $2\pi f$, f being the frequency of the current. The impedance of the parallel paths is

$$\frac{jwL_2}{1-w^2L_2C_2}.$$

Hence, the impedance Z of the entire network is

$$Z=j\left\{\frac{wL_2}{1-w^2L_2C_2}+wL_1\right\} \quad (1)$$

Place for convenience

$$f_1=\frac{f}{f_r} \quad (2)$$

where f_r is the frequency at which L_2 and C_2 are in resonance with each other. Substitute equation (2) in equation (1) and simplify. Then

$$Z=jw_rL_2\left\{\frac{f_1}{1-f_1^2}+f_1\frac{L_1}{L_2}\right\} \quad (3)$$

Where

$$w_r=2\pi f_r.$$

The expression within the brackets in the above equation may be placed equal to K. Then

$$Z=jw_rL_2K \quad (4)$$

where

$$K=\frac{f_1}{1-f_1^2}+f_1\frac{L_1}{L_2} \quad (5)$$

The variation in the value of K with frequency is shown by the curves in Fig. 1^A, in which the values of K are ordinates and the ratios of f to f_r are abscissæ. These curves indicate also the manner in which the impedance of the network changes with frequency, as may be seen by an inspection of equation 4. At low frequencies, the impedance of the network is a small, positive value, which increases as the frequency is raised until at the point of resonance of L_2 and C_2, the value thereof is infinite. In passing through the resonant point the impedance changes sign and thereafter decreases with rise in frequency until at the point of series resonance of the network, it becomes zero. The impedance then increases until its value is again infinite at infinite frequency. It is thus seen that the network of Fig. 1 has two periods of resonance, a period of parallel resonance at one frequency and a period of series resonance at a higher frequency. The point of parallel resonance is dependent upon the relative values of only two of the impedance elements, namely L_2 and C_2, and the point of series resonance is governed by the relative proportions of all three of the impedance elements. The curves of Fig. 1^A are drawn for an ideal network containing no resistance or other dissipative element, but in any actual case, the resistance may be made so small that its effect is practically negligible. It thus appears that the network of Fig. 1 may be used as a selective network—for passing current of series resonant frequency and preventing the passage of current of parallel resonant frequency.

I have found, moreover, that by employing the network as a shunt and series impedance in a periodic structure like that discussed in the Campbell patents hereinbefore mentioned, certain new types of wave filters are arrived at, which filters have certain new and valuable characteristics which I shall now describe.

Figs. 2, 3, 4 and 5 illustrate four types of filters, employing the network of Fig. 1, the first two of these views showing the network as a shunt impedance element and the last two as the series impedance element of the filter section. Figs. 2 and 3 show an inductive and a capacity reactance respectively as the series impedance element, and Figs. 4 and 5 show the same reactances, respectively, as the shunt impedance element.

The properties of the above filters may be determined from certain mathematical expressions which set forth the relations existing between the frequency of current and the impedance elements of the filters. In the Campbell patents hereinbefore mentioned, it was shown (equation 2) that for a periodic structure of the type now under consideration, in which the series impedance per section is Z_1 and the shunt impedance per section is Z_2, the attenuation per section of the filter may be derived from the relation

$$\cosh \Gamma = \frac{1}{2}\frac{Z_1}{Z_2} + 1 \quad (6)$$

in which Γ denotes a propagation constant of the structure. The variation of the attenuation of any filter with frequency of current may, therefore, be deduced from equation 6, when the corresponding values of Z_1 and Z_2 are substituted therein. For the filter shown in Fig. 2, the value of Z_1 is

$$Z_1 = jwL_0 \quad (7)$$

and (according to equation 4 above),

$$Z_2 = jw_rL_2K \quad (8)$$

The resultant equation for $\cosh \Gamma$ is, therefore

$$\cosh \Gamma = \frac{1}{2}\frac{jwL_0}{jw_rL_2K} + 1 \quad (9)$$

Fig. 2^A is a graph showing the variation of the attenuation of the filter of Fig. 2, as computed from equation 9. The axis of the abscissæ is laid off in ratios of f to f_r and the axis of the ordinates in values of the attenuation constant per filter section. An inspection of the curves shows that the attenuation is nil for a range of frequencies extending between f_1 and f_h. At a frequency f_m in the upper attenuated range, the attenuation is infinite and when this frequency is chosen close to f_h, as shown in Fig. 2^A, the filter has a sharp "cut-off" for frequencies lying just above f_h.

The frequencies f_1, f_h and f_m may be evaluated as follows: It was shown in the said Campbell patents that for unattenuated transmission Γ must be a pure imaginary, and that, therefore, the value of $\cosh \Gamma$ must lie between ± 1. The frequencies which limit the ranges of free transmission may consequently be determined by placing equation 9 equal to $+1$ and -1 respectively and solving for f. When this is done, it will be found that the roots are respectively,

$$f_1 = \frac{1}{2\pi\sqrt{L_2C_2}} \quad (10)$$

$$f_h = \frac{1}{2\pi}\sqrt{\frac{L_0 + 4L_1 + 4L_2}{L_2C_2(L_0 + 4L_1)}} \quad (11)$$

The frequency f_m, at which the attenuation is infinite, may be evaluated by placing equation 9 equal to ∞ and solving for f, whence

$$f_m = \frac{1}{2\pi}\sqrt{\frac{L_1 + L_2}{L_1L_2C_2}} \quad (12)$$

The attenuation characteristics of the remaining filters may be arrived at in a similar manner. The curves of Fig. 3^A show that the filter of Fig. 3 passes without substantial attenuation two ranges of frequencies, one of which extends between f_0 and f_2, and the other from f_3 to ∞. The filter has, therefore, the combined characteristics of a high-pass and a band filter, and serves the purpose of both. It has, moreover, a frequency f_m at which the attenuation is infinite. This frequency may be chosen close to f_3 thus giving the filter a sharp "cut-off" for frequencies just below the high-pass range.

The frequencies f_0, f_2, f_3, and f_m may be evaluated similarly as the limiting frequencies of the filter of Fig. 2. The expression for $\cosh \diagup\overline{\quad}$ in the present case is, since C_1 is a capacity reactance,

$$\cosh \diagup\overline{\quad} = -\frac{1}{2}\frac{j\dfrac{1}{wC_0}}{jw_rL_2K}+1 \quad (13)$$

Placing equation 13 equal respectively to +1 and −1, and solving for f, the roots will be found to be

$$f_0=\frac{1}{4\pi}\sqrt{\frac{\dfrac{C_2}{C_0}+4\dfrac{L_1}{L_2}+4-\sqrt{\left(\dfrac{C_2}{C_0}+4\dfrac{L_1}{L_2}+4\right)^2-16\dfrac{C_2}{C_0}\dfrac{L_1}{L_2}}}{2L_1C_2}} \quad (14)$$

$$f_2=\frac{1}{2\pi\sqrt{L_2C_2}} \quad (15)$$

$$f_3=\frac{1}{4\pi}\sqrt{\frac{\dfrac{C_2}{C_0}+4\dfrac{L_1}{L_2}+4+\sqrt{\left(\dfrac{C_2}{C_0}+4\dfrac{L_1}{L_2}+4\right)^2-16\dfrac{C_2}{C_0}\dfrac{L_1}{L_2}}}{2L_1C_2}} \quad (16)$$

$$f_4=\infty$$

When equation 13 is placed equal to ∞ and solved for f, the frequency of maximum attenuation is found to be

$$f_m=\frac{1}{2\pi}\sqrt{\frac{L_1+L_2}{L_1L_2C_2}} \quad (17)$$

The curves of Fig. 4^A show that the filter of Fig. 4 is similar to that of Fig. 2 in that it has a single band of free transmission. It differs therefrom, however, in that the frequency f_m, at which the attenuation is infinite, lies in the lower attentuated range, thus giving this filter a sharp "cut-off" for frequencies, lying just below f_1. These curves have been derived from the expression

$$\cosh \diagup\overline{\quad} = \frac{1}{2}\frac{jw_rL_2K}{jwL_0}+1 \quad (18)$$

which is similar to equation 9 above, except that the values of Z_1 and Z_2 are interchanged, the shunt impedance of Fig. 2 being the series impedance of Fig. 4, and vice versa. The limiting frequencies are obtained, as before, by placing $\cosh \diagup\overline{\quad}$ equal to +1 and −1 respectively, and solving for f. Then

$$f_1=\frac{1}{2\pi}\sqrt{\frac{4L_0+L_1+L_2}{L_2C_2(4L_0+L_1)}} \quad (19)$$

and

$$f_h=\frac{1}{2\pi}\sqrt{\frac{L_1+L_2}{L_1L_2C_2}} \quad (20)$$

The expression for the frequency of maximum attenuation, f_m, is obtained by placing $\cosh \diagup\overline{\quad}$ equal to ∞ and solving for f. Then

$$f_m=\frac{1}{2\pi\sqrt{L_2C_2}} \quad (21)$$

The curves of Fig. 5^A show that the filter of Fig. 5 is of the double band type and similar to that of Fig. 3, differing therefrom, however, in that one of the ranges of free transmission extends from 0 to f_2, thus making the filter a combined low-pass and band filter. The frequency f_m of infinite attenuation lies close to the frequency f_2, thus giving this filter a sharp "cut-off" for the low pass range.

The attenuation curve of Fig. 5^A is derived from the expression

$$\cosh \diagup\overline{\quad} = -\frac{1}{2}\frac{jw_rL_2K}{j\dfrac{1}{wC_0}}+1 \quad (22)$$

and the values of f_0, f_2, f_3, f_4 and f_m are obtained by placing the above expression for $\cosh \diagup\overline{\quad}$ equal to $+1$, -1 and ∞ respectively. Thus

$$f_0=0$$

$$f_2=\frac{1}{2\pi}\sqrt{\frac{4\dfrac{C_2}{C_0}+\dfrac{L_1}{L_2}+1\sqrt{\left(4\dfrac{C_2}{C_0}+\dfrac{L_1}{L_2}+1\right)^2-16\dfrac{C_2L_1}{C_0L_2}}}{2L_1C_2}} \quad (23)$$

$$f_3=\frac{1}{2\pi}\sqrt{\frac{L_1+L_2}{L_1L_2C_2}} \quad (24)$$

$$f_4=\frac{1}{2\pi}\sqrt{\frac{4\dfrac{C_2}{C_0}+\dfrac{L_1}{L_2}+1+\sqrt{\left(4\dfrac{C_2}{C_0}+\dfrac{L_1}{L_2}+1\right)^2-16\dfrac{C_2L_1}{C_0L_2}}}{2L_1C_2}} \quad (25)$$

$$f_m=\frac{1}{2\pi\sqrt{L_2C_2}} \quad (26)$$

It should be noted that the attenuation curves illustrated herein refer to the ideal structure in which the resistance of the impedance units is zero. In a practical filter there is a departure from these curves, owing to energy dissipation. In any case, however, the resistance may be made so small that the departure from the ideal is practically negligible.

The formulæ 10—12, 14—17, 19—21, and 23—26, given above, may be used in designing filters to meet any specified sets of conditions. Since there are four independent impedance elements in each filter section, any four properties of the filter dependent upon the values of the impedance elements but independent of each other may be chosen at will. For example, in the design of a filter of the type illustrated in Fig. 3, two of the design conditions may be taken as the frequencies f_0 and f_2, and a third as f_3, thus defining the ranges of free transmission. This leaves one condition open to choice, and this may be taken as the impedance of the filter at any desired frequency, or as the value of any one of the elements of the filter section. In the design of a filter of the type of Fig. 2, two of the design conditions may be chosen as the frequencies f_1 and f_h, and the third as f_m, thus leaving the fourth to be chosen in accordance with any other condition that may prevail. Similar considerations apply to the remaining types of filters.

As an example of the application of the formulæ, let it be required to design a filter of the type illustrated in Fig. 5, which shall transmit all frequencies between 5,000 and 20,000 cycles, and which shall pass also all frequencies below 3,000 cycles. Frequencies f_2, f_3 and f_4 are thus specified as 3,000, 5,000 and 20,000 cycles respectively. As a fourth design factor, let it be assumed that certain considerations dictate that the value of C_0 shall be .025 microfarads. This leaves three constants of the filter section, namely L_1, L_2 and C_2 to be determined. It will be noted that formulæ 23, 24, and 25 are three simultaneous equations involving the three unknown constants. When, therefore, the values of f_2, f_3, f_4 and C_0, assumed above, are substituted in these equations, they may solved for the three unknowns. When this is done it will be found that

$$L_1 = .0106 \text{ henries.}$$
$$L_2 = .0176 \text{ henries.}$$
$$C_2 = .154 \text{ microfarads.}$$

All the constants of the filter are thus determined. It will readily be seen that, instead of the above-mentioned set of conditions, any others involving the filter impedances may be imposed, it being understood that the above example is merely a simple illustration, and in no way limits the invention.

Although only certain forms of filters embodying the invention are shown and described herein, it is readily understood that various changes and modifications may be made therein within the scope of the following claims, without departing from the spirit and scope of the invention.

What is claimed is:

1. In a wave filter of the type having like recurrent sections, each section consisting of a series element and a shunt element, a complete element consisting of a network of impedances so arranged and so proportioned that at low frequencies the reactance thereof has a small positive value which increases as the frequency is raised to a large positive value, then undergoes a sudden change from a large positive to a large negative value, and thereafter decreases in negative value, finally becoming increasingly positive.

2. In a wave filter of the type having like recurrent sections, each section consisting of a series element and a shunt element, a complete element consisting of a network comprising an inductive reactance in series with a pair of parallel paths, one of which comprises a capacity reactance and the other of which comprises an inductive reactance.

3. A filter for an electric circuit consisting of a plurality of sections, each section consisting of an impedance in series with the circuit and an impedance in shunt thereto, said impedances being determined as functions of certain definite parameters so that the filter transmits without substantial attenuation all frequencies lying above a limiting frequency and such frequencies as lie in a band below said limiting frequency, the limits of said band being neither zero nor said limiting frequency, while attenuating and approximately suppressing currents of neighboring frequencies lying outside the above-defined limits, the attenuation being approximately a maximum at a frequency lying between said limiting frequency and the upper limit of the said band, said parameters being said limiting frequency and the upper and lower limits of said band and the mid-section characteristic impedance of the filter at an extreme frequency.

4. A filter for an electric circuit, consisting of a plurality of sections, each section comprising an impedance in series with the circuit and an impedance in shunt thereto, one of said impedances consisting of a single reactance element and the other of a network comprising an inductive reactance in series with a pair of parallel paths, one of which comprises a capacity reactance and the other an inductive reactance.

5. The method of discriminating among alternating current components according to their frequency which consists in attenuating currents from zero frequency to a certain finite frequency, then passing currents from that frequency to another higher finite frequency, then attenuating currents from the last mentioned frequency up to a third finite frequency still higher, and then passing all currents of higher frequency than said third finite frequency and developing a maximum of attenuation at a frequency a little less than said third finite frequency.

In testimony whereof, I have signed my name to this specification this 10th day of August 1920.

HENRY W. ELSASSER.