Jan. 5, 1926. 1,568,141

H. W. ELSASSER

FREQUENCY SELECTIVE CIRCUITS

Filed August 13, 1920 3 Sheets-Sheet 1

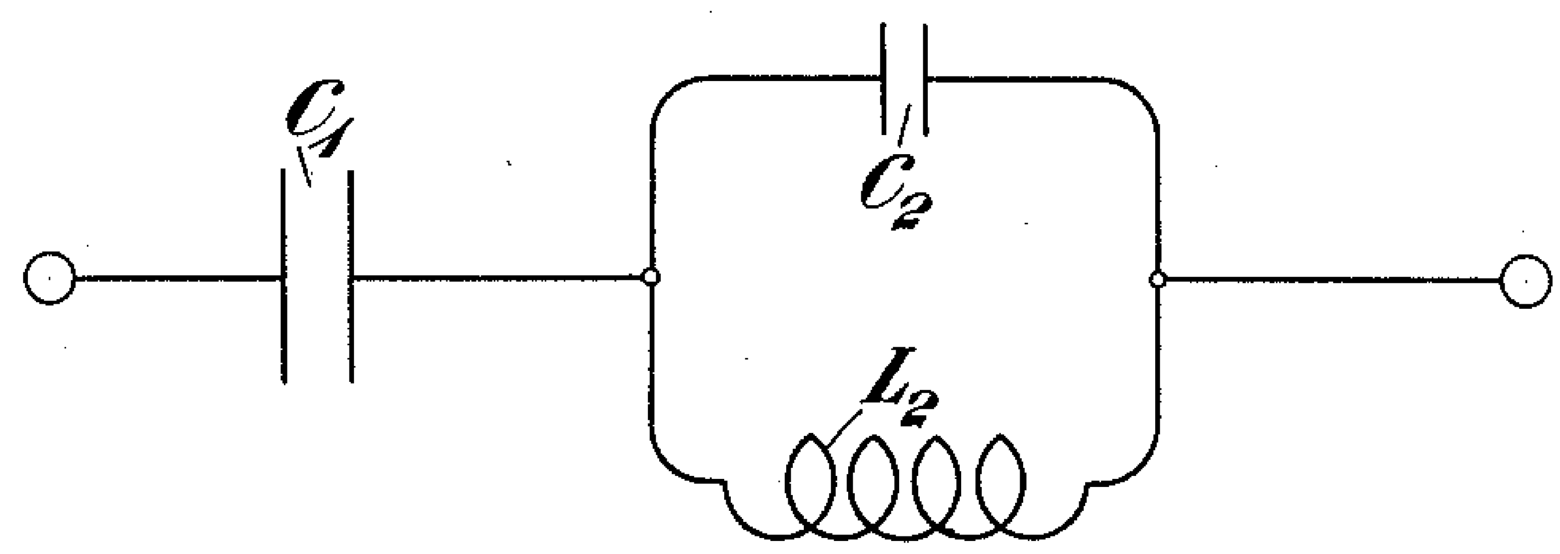

Fig. 1

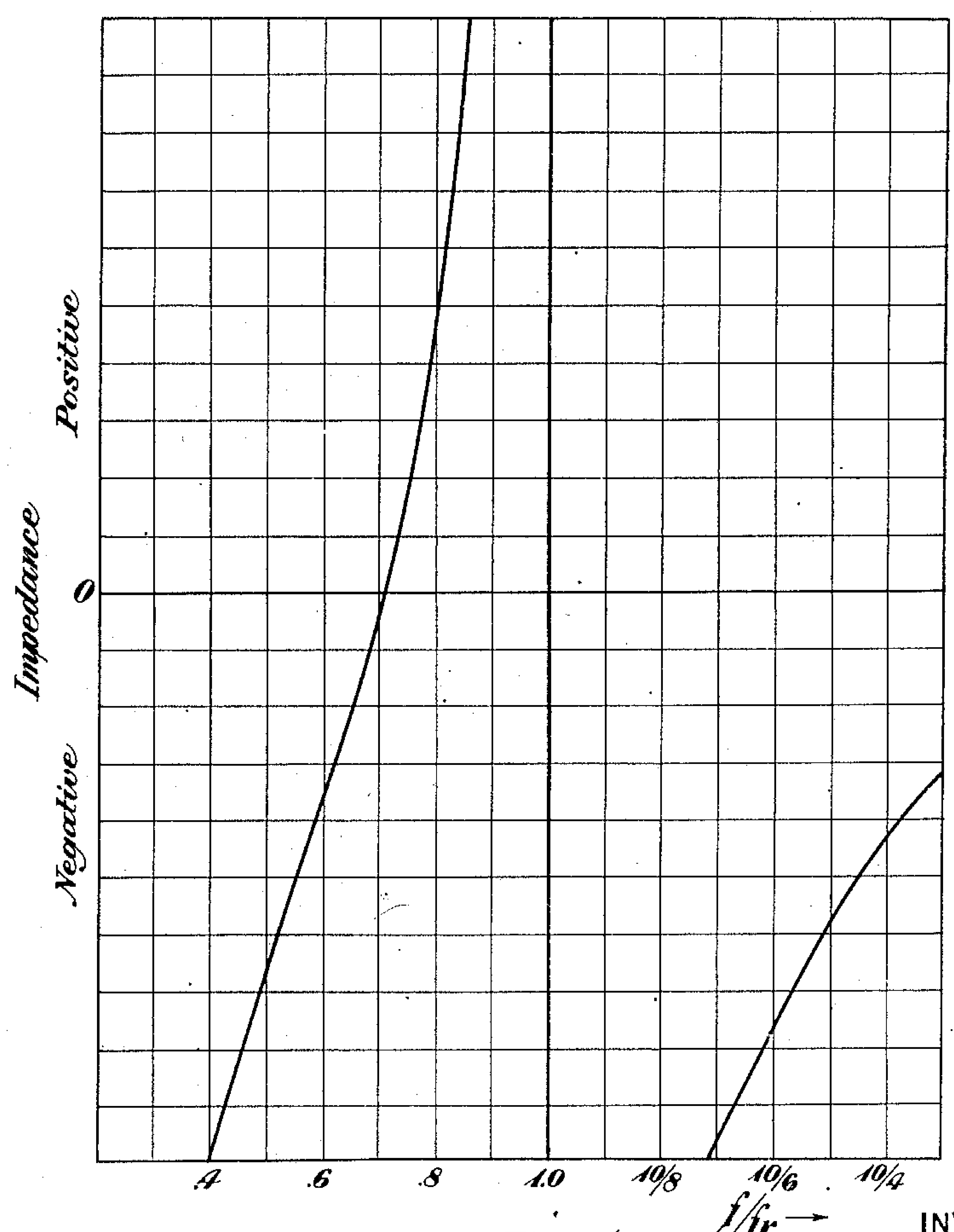

Fig. 1a

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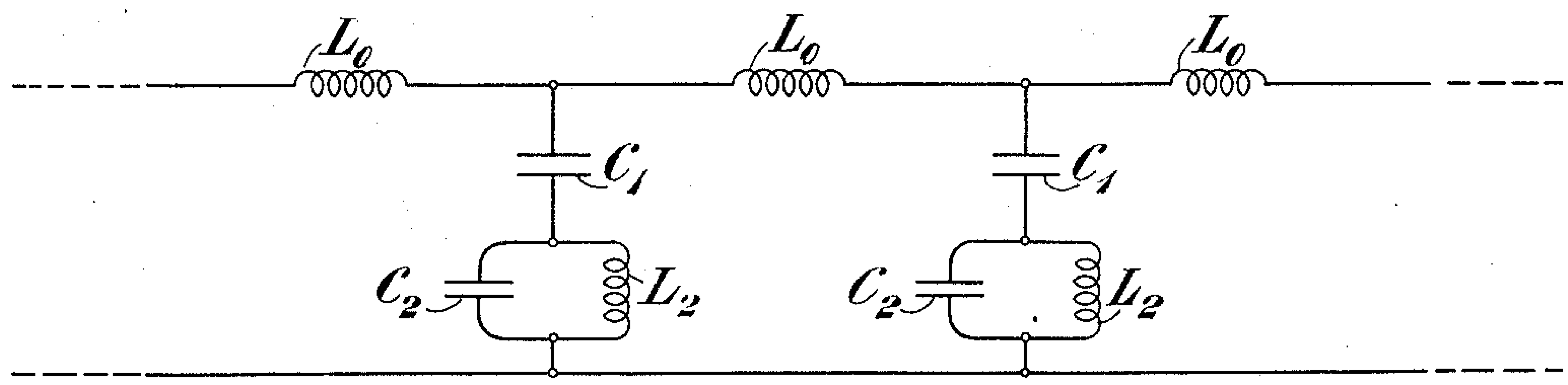

Fig. 2

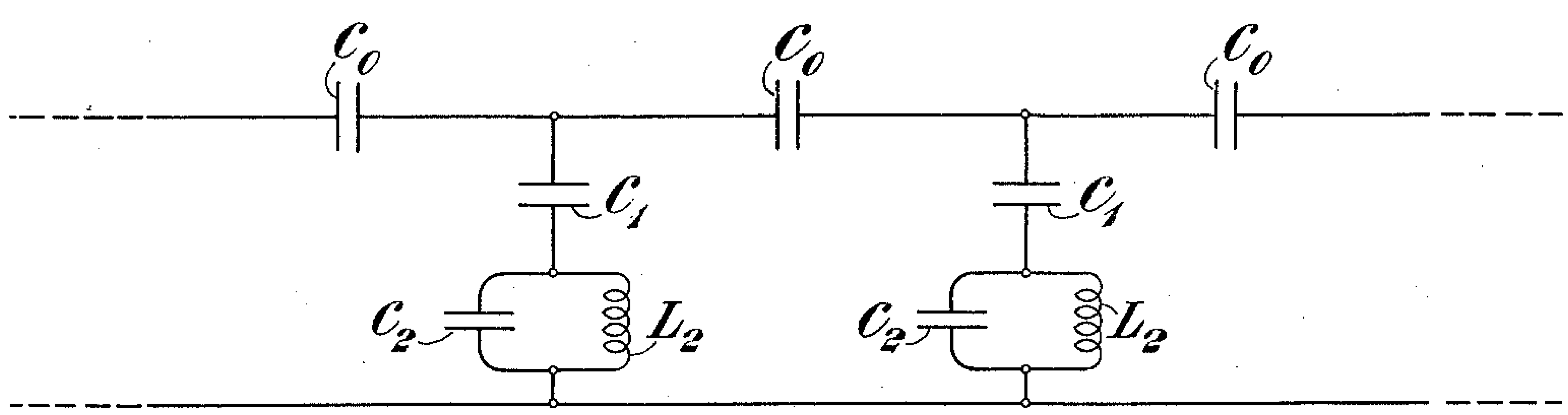

Fig. 3

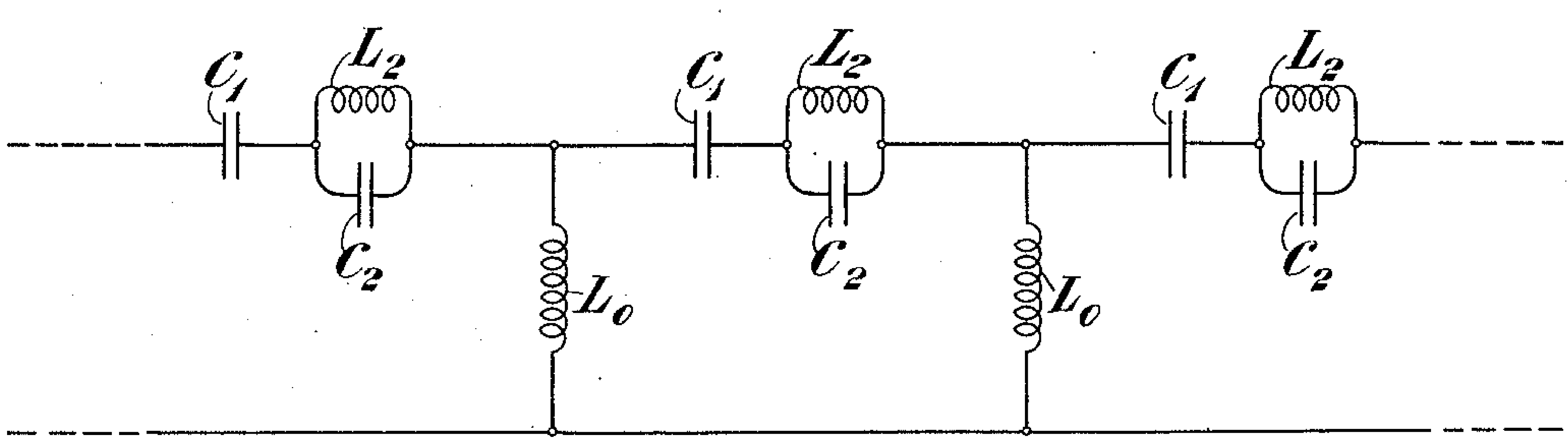

Fig. 4

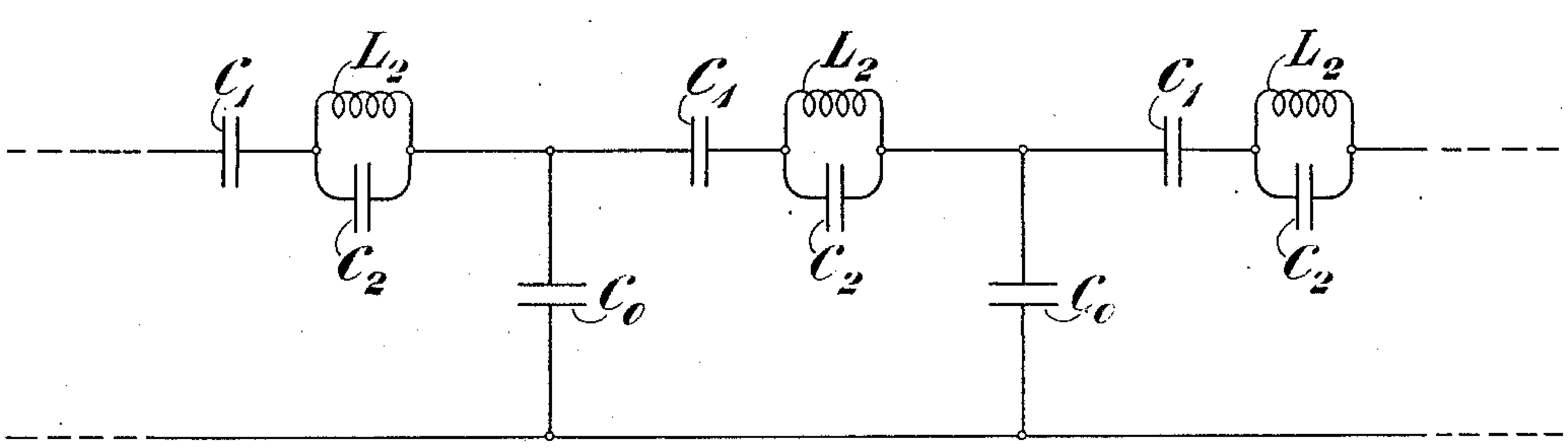

Fig. 5

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FREQUENCY SELECTIVE CIRCUITS

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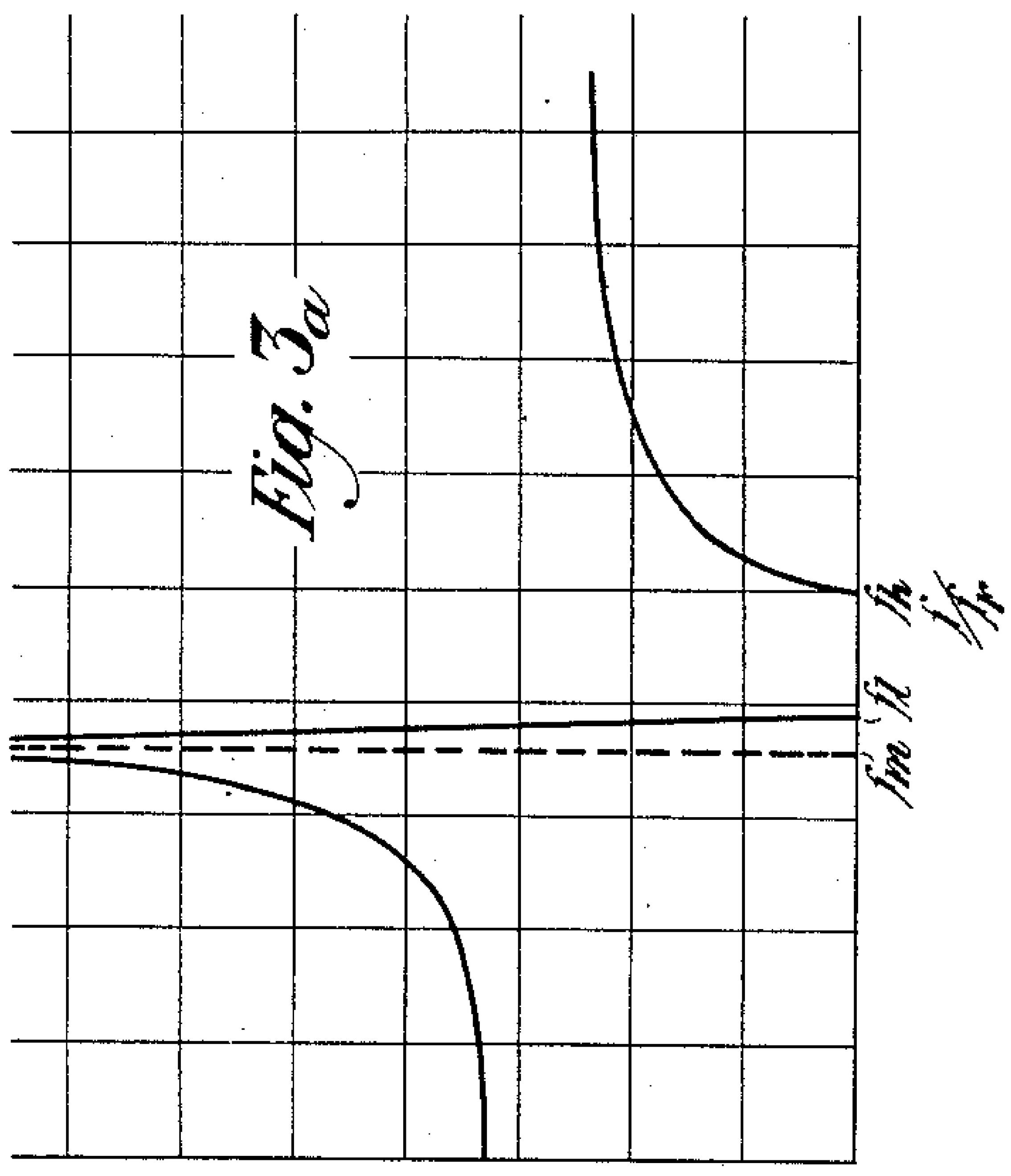
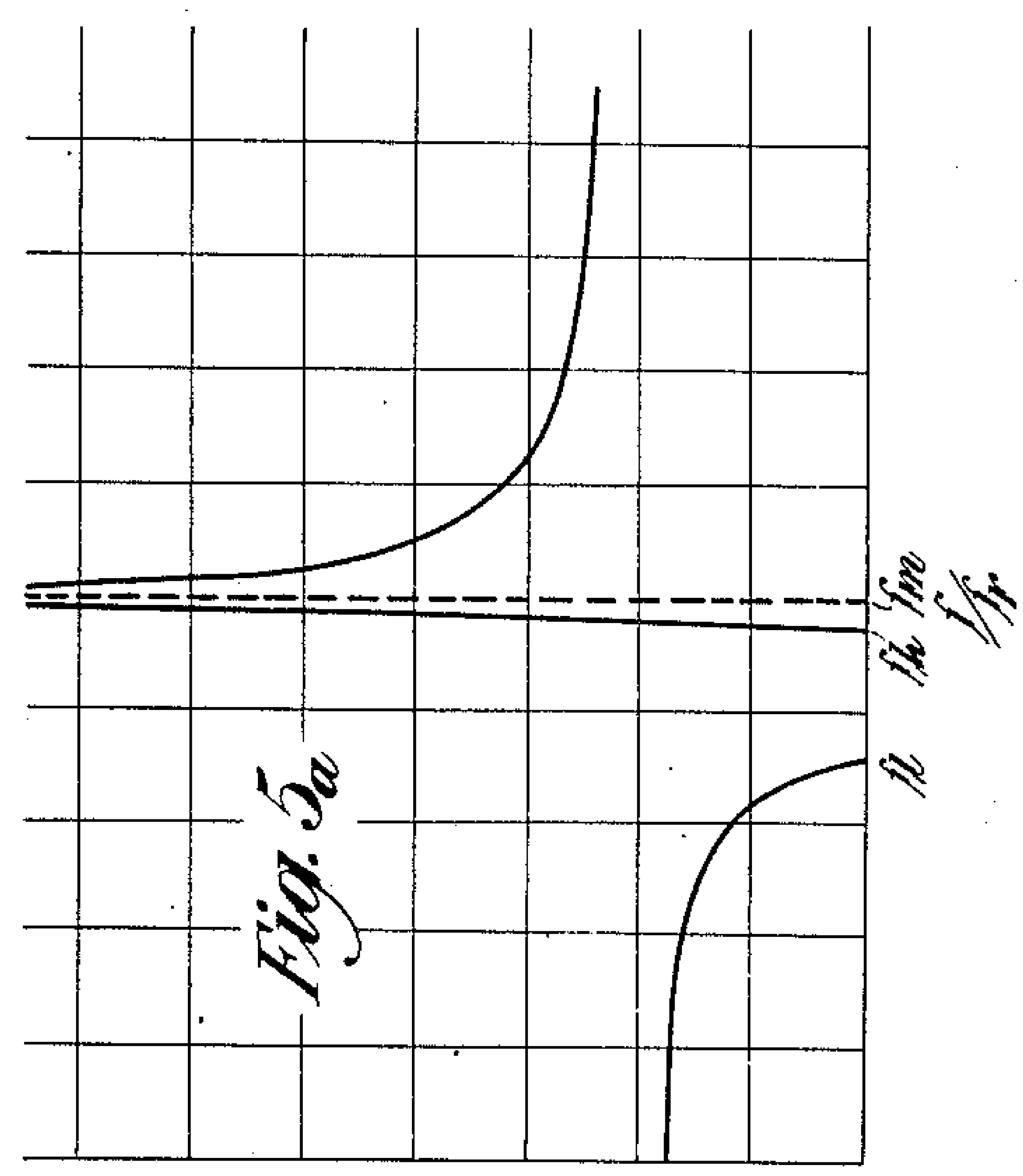
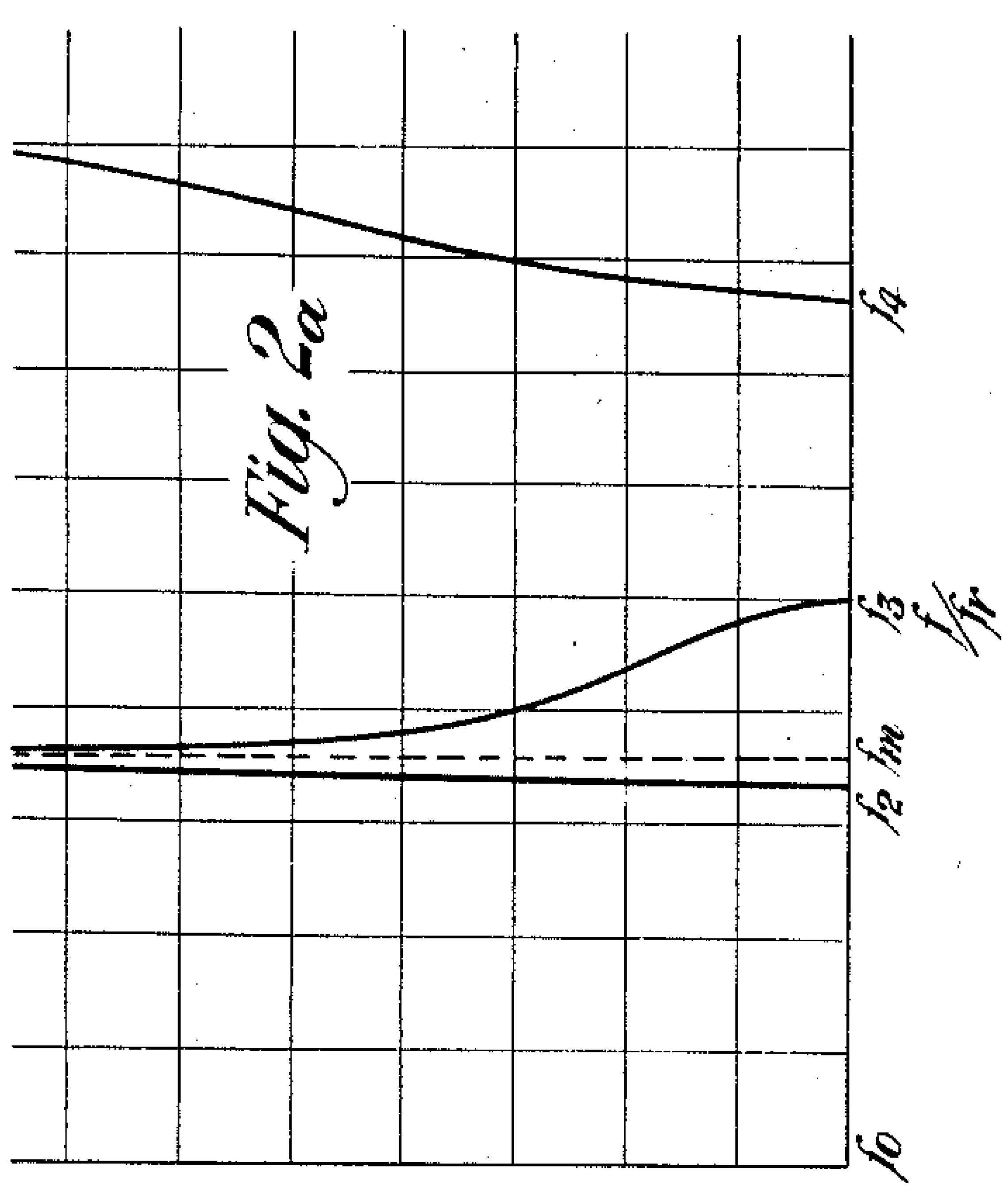


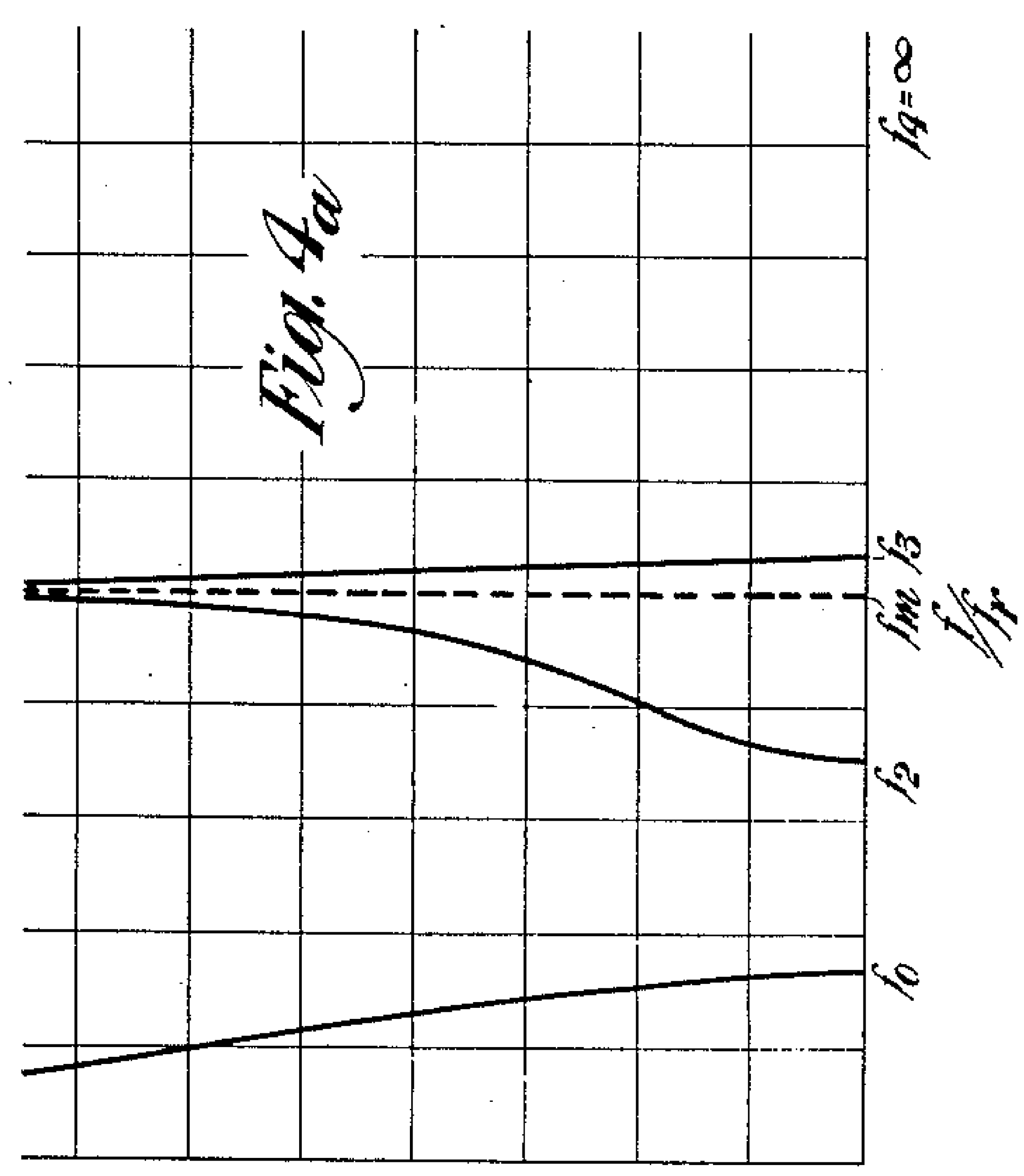
Attenuation per Section



Attenuation per Section



Attenuation per Section



Attenuation per Section

INVENTOR
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BY
[signature]
ATTORNEY

Patented Jan. 5, 1926. **1,568,141**

UNITED STATES PATENT OFFICE.

HENRY W. ELSASSER, OF NEW YORK, N. Y., ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

FREQUENCY SELECTIVE CIRCUITS.

Application filed August 13, 1920. Serial No. 403,367.

To all whom it may concern:

Be it known that I, HENRY W. ELSASSER, a citizen of the United States, residing at New York, in the county of New York and State of New York, have invented certain Improvements in Frequency Selective Circuits, of which the following is a specification.

This invention relates to frequency-selective circuits.

It contemplates a network of impedances having a period of series resonance and a period of parallel resonance, so that its impedance for a certain frequency of current is very low and for another, very high.

The invention proposes, further, the use of a network of this character in combination with other impedance elements in a periodic structure of the type illustrated and described in the patents to G. A. Campbell, 1,227,113 and 1,227,114 of May 22, 1917. Certain new and useful types of wave filters are thus arrived at, the characteristics of which are explained hereinbelow.

This application is related to certain copending cases, Serial Numbers 403,368, 403,369, 403,370, filed of even date herewith.

A good understanding of the invention may now be had from the following description of certain specific embodiments thereof, having reference to the accompanying drawing, in which,

Figure 1 is a diagrammatic view showing one form of network embodying the invention,

Figs. 2 to 5 inclusive, are diagrammatic views showing various types of filters comprising the network of Fig. 1,

Fig. 1^A is a graph showing the variation with a frequency in the impedance of the network of Fig. 1, and

Figs. 2^A to 5^A, inclusive are graphs showing the variation in attenuation of the filters of Figs. 2 to 5, respectively.

Similar characters of reference designate similar parts of the several views.

The network of Fig. 1 consists of a condenser C_1 in series with a pair of parallel paths, one of which contains an inductance L_2 and the other of which a condenser C_2. The impedance of the condenser C_1 is—

$$-\frac{j}{wC_1},$$

where j is written for $\sqrt{-1}$ and w equals $2\pi f$, f being the frequency of the current. The impedance of the parallel resonant path is

$$\frac{jwL_2}{1-w^2L_2C_2},$$

hence the impedance value of the entire network is

$$Z=j\left[\frac{wL_2}{1-w^2L_2C_2}-\frac{1}{wC_1}\right] \quad (1)$$

Place for convenience

$$f_1=\frac{f}{f_r} \quad (2)$$

where f_r is the frequency at which L_2 and C_2 are resonant. Substitute equation 2 in equation 1 and simplify. Then

$$Z=jw_rL_2\left[\frac{f_1}{(1-f_1^2)}-\frac{1}{f_1\frac{C_1}{C_2}}\right] \quad (3)$$

where

$$w_r=2\pi f_r=\frac{1}{\sqrt{L_2C_2}}$$

The expression in the brackets of the above equation may be placed equal to K. Then

$$Z=jw_rL_2K \quad (4)$$

where

$$K=\frac{f_1}{(1-f_1^2)}-\frac{1}{f_1\frac{C_1}{C_2}} \quad (5)$$

The variation in the value of K with frequency is shown by the curves of Fig. 1^A, in which the values of K are ordinates and the ratios of f to f_r are abscissæ. These curves indicate also the manner in which the impedance of the network changes with frequency, as may be seen by an inspection of equation 4. At low frequencies the impedance of the network is negative, but as the frequency is raised, the impedance changes from negative to positive, the point of crossing of the axis of abscissæ denoting series resonance of C_1 and the combination of C_2 and L_2, or zero impedance of the network. The impedance then increases until, at the frequency at which L_2 and C_2 are in parallel resonance, its value is infinite. The impedance then changes sign and thereafter decreases, approaching zero at infinite frequency. The network, therefore, has two periods of resonance, a period of series resonance at one frequency and a period of parallel resonance at a higher frequency. The period of parallel resonance depends on the relative value of only two impedance elements, namely L_2 and C_2, and the period of series resonance is governed by the values of all three reactances. The curves of Fig 1^A are drawn for an ideal network containing no resistance or other dissipative elements, but in an actual case, the resistance may be made so small that its effect is practically negligible. It thus appears that the network of Fig. 1 may be used as a selective circuit for passing current of series resonant frequency and preventing the passage of current of parallel resonant frequency.

I have found, moreover that by employing the network as a shunt and series impedance in a periodic structure like that discussed in the Campbell patents hereinbefore mentioned, certain new types of wave filters are arrived at, which filters have certain new and valuable characteristics which I shall now describe.

Figs. 2, 3, 4 and 5 illustrate four types of filters employing the network of Fig. 1, the first two of these views showing the network as a shunt impedance element and the last two as a series impedance element of the filter section. Figs. 2 and 3 show an inductive and a capacity reactance, respectively as the series impedance element, and Figs. 4 and 5 show the same reactances, respectively, as the shunt impedance element.

The properties of the above filters may be determined from certain mathematical expressions which set forth the relations existing between the frequency of current and the impedance elements of the filters. In the Campbell patents hereinbefore mentioned, it was shown (equation 2) that for a periodic structure of the type now under consideration, in which the series impedance per section is Z_1 and the shunt impedance per section is Z_2, the attenuation per section of the filter may be derived from the relation

$$\cosh \Gamma = 1/2 \left(\frac{Z_1}{Z_2}\right) + 1 \quad (6)$$

in which Γ denotes the propagation constant of the structure. The variation of the attenuation of any filter with frequency of current may, therefore, be deduced from equation 6, when the corresponding values of Z_1 and Z_2 are substituted therein. For the filter shown in Fig. 2, the value of Z_1 is

$$Z_1 = jwL_0 \quad (7)$$

and (according to equation 4 above),

$$Z_2 = jw_rL_2K \quad (8)$$

The resultant equation for cosh Γ is, therefore,

$$\cosh \Gamma = \frac{1}{2}\ \frac{jwL_0}{jw_rL_2K} + 1 \quad (9)$$

Fig. 2^A is a graph showing the variation of the attenuation of the filter of Fig. 2, as computed from equation 9. The axis of the abscissæ is laid off in ratios of f to f_r and the axis of the ordinates in values of the attenuation constant per filter section. An inspection of the curves shows that the attenuation is nil for two ranges of frequencies, f_0 to f_2 and f_3 to f_4. The filter, in other words, passses without attenuation, only such frequencies as lie below f_2 or between f_3 and f_4. It is, therefore, a combined low-pass and band filter and performs the functions of both. It is characterized, moreover, by having infinite attenuation at a frequency f_m which is close to f_2, and it discriminates, consequently, with particular sharpness against frequencies just above the upper limit of the low-pass range.

The frequencies f_2, f_3 and f_4 may be evaluated as follows: it was shown in the said Campbell patents, that for unattenuated transmission, Γ must be a pure imaginary, and that, therefore, the value of cosh Γ must lie between ± 1. The frequencies which limit the ranges of free transmission may consequently be determined by placing equation 9 equal to $+1$ and -1 respectively, and solving for f. When this is done, it will be found that the roots are respectively, $f_0 = 0$.

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{\frac{L_0C_1}{L_2C_2} + 4\frac{C_1}{C_2} + 4 - \sqrt{\left(\frac{L_0C_1}{L_2C_2} + 4\frac{C_1}{C_2} + 4\right)^2 - 16\frac{L_0C_1}{L_2C_2}}}{2L_0C_1}} \quad (10)$$

$$f_3 = \frac{1}{2\pi\sqrt{L_2C_2}} \quad (11)$$

$$f_4 = \frac{1}{2\pi}\sqrt{\frac{\frac{L_0C_1}{L_2C_2} + 4\frac{C_1}{C_2} + 4 + \sqrt{\left(\frac{L_0C_1}{L_2C_2} + 4\frac{C_1}{C_2} +,4\right)^2 - 16\frac{L_0C_1}{L_2C_2}}}{2L_0C_1}} \quad (12)$$

The frequency f_m, at which the attenuation is infinite, may be evaluated by placing equation 9 equal to ∞ and solving for f, whence

$$f_m=\frac{1}{2\pi}\sqrt{\frac{1}{L_2(C_1+C_2)}}\quad(13)$$

The attenuation characteristics of the remaining filters may be arrived at in a similar manner. The curves of Fig. 3^A show that the filter of Fig. 3 passes without substantial attenuation only a single band, the limiting values of which are f_1 and f_h. The filter is further characterized by having infinite attenuation per section at a frequency f_m below f_1. This frequency may be chosen within the lower attenuated range, but preferably so as to lie close to f_1, so that the filter has a sharp "cut-off" at the lower limit of the transmitted band.

The frequencies f_1, f_h, and f_m may be evaluated similarly as the limiting frequencies of the filter of Fig. 2. The expression for cosh Γ in the present case is, since Z_1 is a capacity reactance,

$$\cosh\Gamma=-1/2\frac{+j\frac{1}{wC_0}}{jw_rL_2K}+1\quad(14)$$

By placing equation 14 equal, respectively, to $+1$ and -1, simplifying, and solving for f, the roots will be found to be

$$f_1=\frac{1}{2\pi}\sqrt{\frac{(4C_0+C_1)}{L_2(C_1C_2+4C_0C_1+4C_0C_2)}}\quad(15)$$

$$f_h=\frac{1}{2\pi\sqrt{L_2C_2}}\quad(16)$$

When equation 14 is placed equal to ∞ and solved for f, the frequency of maximum attenuation, f_m, is found to be

$$f_m=\frac{1}{2\pi}\sqrt{\frac{1}{L_2(C_1+C_2)}}\quad(17)$$

The curves of Fig. 4^A show that the filter for Fig. 4 is similar to that of Fig. 2 in that is has two ranges of free transmission. It differs therefrom, however, in that it passes all frequencies above a certain value, f_3, without substantial attenuation, and serves consequently as a combined band and high-pass filter. The frequency f_m, at which the attenuation is infinite, may be so chosen as to lie close to f_3, thus giving the filter a sharp discrimination at the lower limit of the high-pass band. These curves have been derived from the expression.

$$\cosh\Gamma=1/2\frac{jw_rL_2K}{jwL_0}+1\quad(18)$$

which is similar to equation 9 except that the value of Z_1 and Z_2 are interchanged, the shunt impedance of Fig. 2 being the series impedance of Fig. 4, and vice versa. The limiting frequencies are obtained, as before, by placing cosh Γ equal to $+1$ and -1 respectively and solving for f. Hence

$$f_0=\frac{1}{4\pi}\sqrt{\frac{4\frac{L_0C_1}{L_2C_2}+\frac{C_1}{C_2}+1-\sqrt{\left(4\frac{L_0C_1}{L_2C_2}+\frac{C_1}{C_2}+1\right)^2-16\frac{L_0C_1}{L_2C_2}}}{2L_0C_1}}\quad(19)$$

$$f_2=\frac{1}{2\pi}\sqrt{\frac{1}{L_2(C_1+C_2)}}\quad(20)$$

$$f_3=\frac{1}{4\pi}\sqrt{\frac{4\frac{L_0C_1}{L_2C_2}+\frac{C_1}{C_2}+1+\sqrt{\left(4\frac{L_0C_1}{L_2C_2}+\frac{C_1}{C_2}+1\right)^2-16\frac{L_0C_1}{L_2C_2}}}{2L_0C_1}}\quad(21)$$

$$f_4=\infty$$

The expression for the frequency of maximum attenuation, f_m, is obtained by placing cosh Γ equal to ∞ and solving for f. Then

$$f_m=\frac{1}{2\pi\sqrt{L_2C_2}}\quad(22)$$

The curves of Fig. 5^A show that the filter of Fig. 5 is of the single band type and similar to that of Fig. 3, differing therefrom, however, in that the frequency of infinite attenuation lies in the upper attenuated range. This filter, may be caused to have a sharp "cut-off" at the upper limit of the band. The attenuation curve of Fig. 5^A is derived from the expression,

$$\cosh\Gamma=-1/2\frac{jw_rL_2K}{j\frac{1}{wC_0}}+1\quad(23)$$

and the values of f_1 and f_h and f_m are obtained by placing the above expression for cosh Γ equal to $+1$ and -1 and ∞, respectively:

$$f_1=\frac{1}{2\pi}\sqrt{\frac{1}{L_2(C_1+C_2)}}\quad(24)$$

$$f_h=\frac{1}{2\pi}\sqrt{\frac{C_0+4C_1}{L_2(4C_1C_2+C_0C_1+C_0C_2)}}\quad(25)$$

$$f_m=\frac{1}{2\pi\sqrt{L_2C_3}} \quad (26)$$

It should be noted that the attenuation curves herein illustrated refer to the ideal structure in which the resistance of the impedance units is zero. In a practical filter there is a departure from these curves, owing to energy dissipation. In any case, however, the resistance may be made so small that the departure from the ideal is practically negligible.

The formulæ 10–13, 15–17, 19–22, and 24–26, given above, may be used in designing filters to meet any specified sets of conditions. Since there are four independent impedance elements in each filter section, any four properties of the filter dependent upon the values of the impedance elements but independent of each other may be chosen at will. For example, in the design of a filter of the type illustrated in Fig. 2, two of the design conditions may be taken as the frequencies f_3 and f_4, and the third as f_2, thus defining the ranges of free transmission. This leaves one condition open to choice, and this may be taken as the impedance of the filter at any desired frequency, or as the value of any one of the elements of the filter section. In the design of a filter of the type of Fig. 3, two of the design conditions may be chosen as the frequencies f_1 and f_h, and the third as f_m, thus leaving the fourth to be chosen in accordance with any other condition that may prevail. Similar considerations apply to the remaining types of filters.

As an example of the application of the formulæ, let it be required to design a filter of the type illustrated in Fig. 3, which shall transmit frequencies between 400 and 2500 cycles, and which shall have maximum, ideally infinite, attenuation at 360 cycles, so that it has a particularly sharp cut-off at the lower limit of the freely transmitted range. Frequencies f_1, f_h and f_m are thus specified as 400, 2500 and 360 cycles respectively. As a fourth design factor let it be assumed that certain considerations dictate that the value of L_2 shall be .5 henry. Applying formula (16), we find that

$$2500=\frac{1}{2\pi\sqrt{.5C_2}}$$

hence

$$C_2=.00810 \text{ microfarads.}$$

Substituting in (17), we have

$$360=\frac{1}{2\pi}\sqrt{\frac{1}{.5(C_1+.00810\times10^{-6})}}$$

hence

$$C_1=.383 \text{ microfarads.}$$

Finally, according to formula (15)

$$400=\frac{1}{2\pi}\sqrt{\frac{4C_0+.383\times10^{-6}}{.5(.383\times10^{-6}\times.00810\times10^{-6}+4C_0\times.383\times10^{-6}+4C_0\times.00810\times10^{-6})}}$$

Therefore $C_0=.398$ microfarads.

All the constants of the filter are thus determined. It will readily be seen that, instead of the above-mentioned set of conditions, any others involving the filter impedances may be imposed, it being understood that the above example is merely a simple illustration, and in no way limits the invention.

Although only certain forms of filters embodying the invention are shown and described herein, it is readily understood that various changes and modifications may be made therein within the scope of the following claims, without departing from the spirit and scope of the invention.

What is claimed is:

1. A wave filter of the type having like recurrent sections and in which each section consists of a series element and a shunt element, one of these elements consisting of a capacity reactance in series with a plurality of paths in parallel with each other, one of said paths comprising inductive reactance and the other of said paths comprising capacity reactance.

2. A wave filter of the type having like recurrent sections and in which each section consists of a series element and a shunt element, one of these elements consisting of a reactance in series with a plurality of paths in parallel with each other, one of said paths comprising inductive reactance and the other of said paths comprising capacity reactance.

3. A filter for an electric circuit, consisting of an impedance in series with the circuit and an impedance in shunt thereto, one of said impedances consisting of a single reactance element and the other of a network comprising a capacity reactance in series with a pair of parallel paths, one of which consists of a capacity reactance and the other of an inductive reactance.

4. A filter for an electric circuit consisting of an impedance in series with the circuit and an impedance in shunt thereto, one of said impedances consisting of a single reactance element and the other of a network comprising a reactance in series with a pair of parallel paths, one of which consists of a capacity reactance and the other of an inductive reactance.

5. The method of discriminating among alternating current components according to

their frequency which consists in passing currents of frequency from zero to a certain finite frequency, then attenuating currents of higher frequency up to another certain finite frequency with a maximum of attenuation close to the lower of these two frequencies, and then passing currents whose frequency ranges from the upper limiting frequency previously mentioned to a third frequency still higher and attenuating all currents of frequency higher than the last mentioned frequency.

6. A wave filter of the type having like recurrent sections and having a series element and a shunt element in each section, said filter embodying means in said elements to provide two separate ranges of frequencies for free transmission, one lying between finite bounding frequencies and the other bounded by a finite frequency and extending thence over the whole remaining frequency range on that side.

7. A wave filter of the type having like recurrent sections and having a series element and a shunt element in each section, said filter embodying means in said elements to provide two separate ranges of frequencies for free transmission, one lying between finite bounding frequencies and the other range lying below the first mentioned range and extending between a finite frequency and zero.

8. A filter of recurrent sections having four reactances in each section and giving two free transmission bands, one band having bounds at finite frequencies, the other extending from another finite frequency bound over the whole frequency range on that side, the values of the reactances being determined as functions of the said three finite bounding frequencies and of the characteristic impedance of the filter at the extreme of the free transmission range for which there is only a single finite frequency bound.

In testimony whereof, I have signed my name to this specification this 10th day of August, 1920.

HENRY W. ELSASSER.