May 26, 1925. **1,538,964**

O. J. ZOBEL

WAVE FILTER

Filed Jan. 15, 1921 5 Sheets-Sheet 1

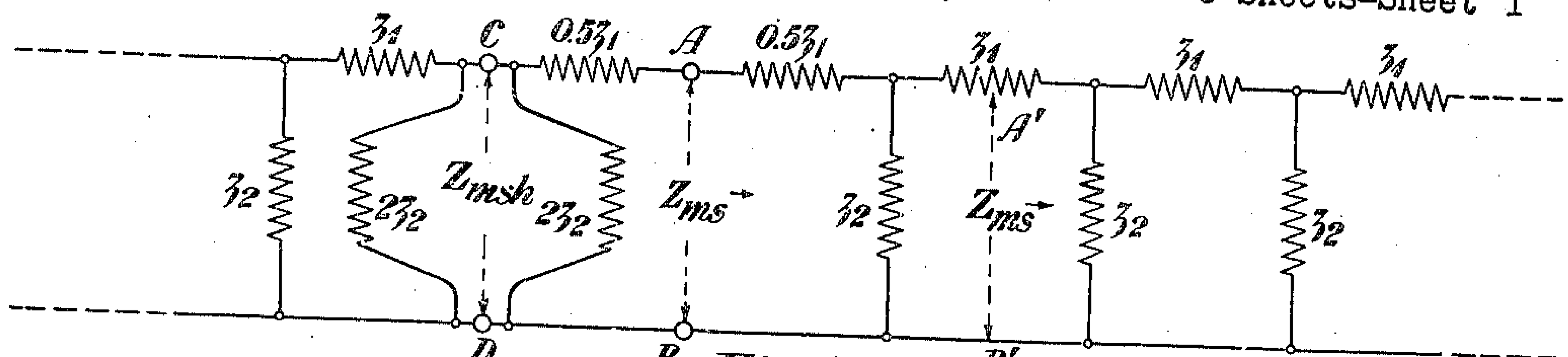

Fig. 1

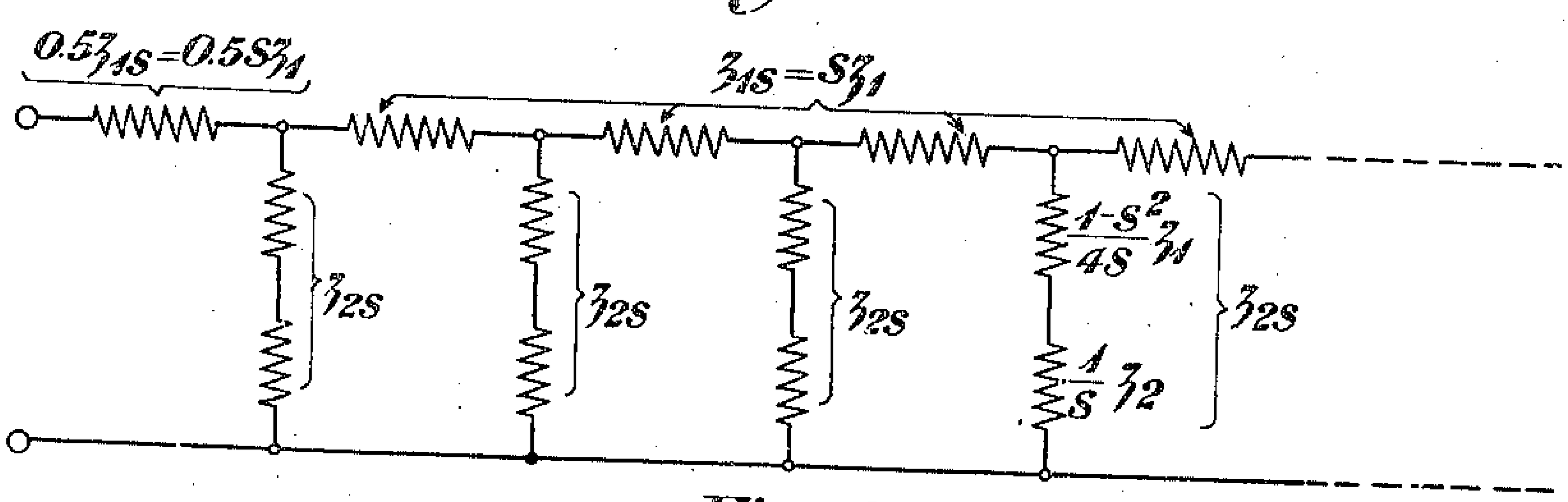

Fig. 2

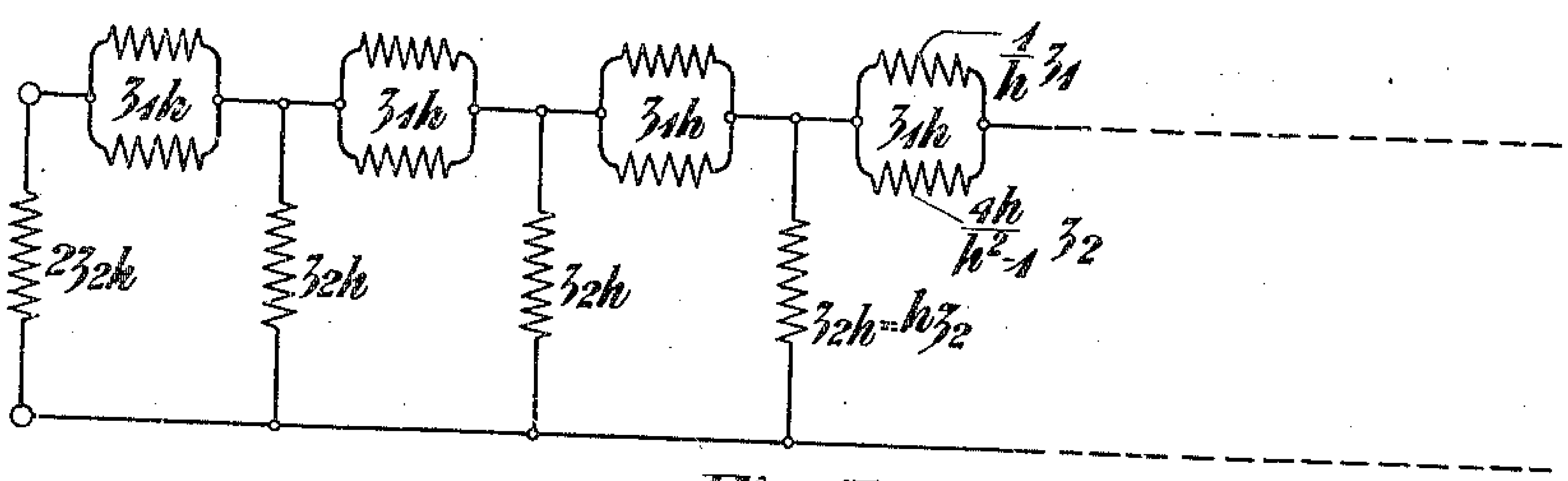

Fig. 3

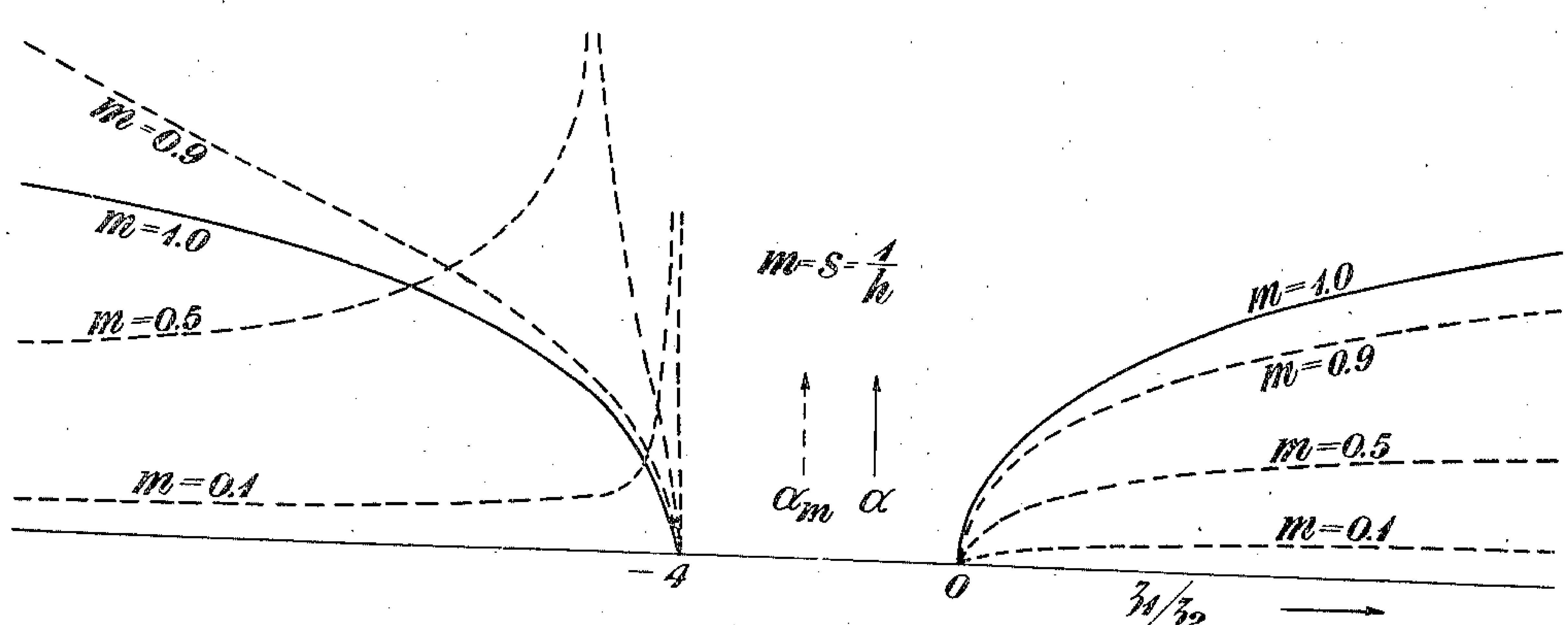

Fig 4

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O. J. ZOBEL

WAVE FILTER

Filed Jan. 15, 1921 5 Sheets-Sheet 2

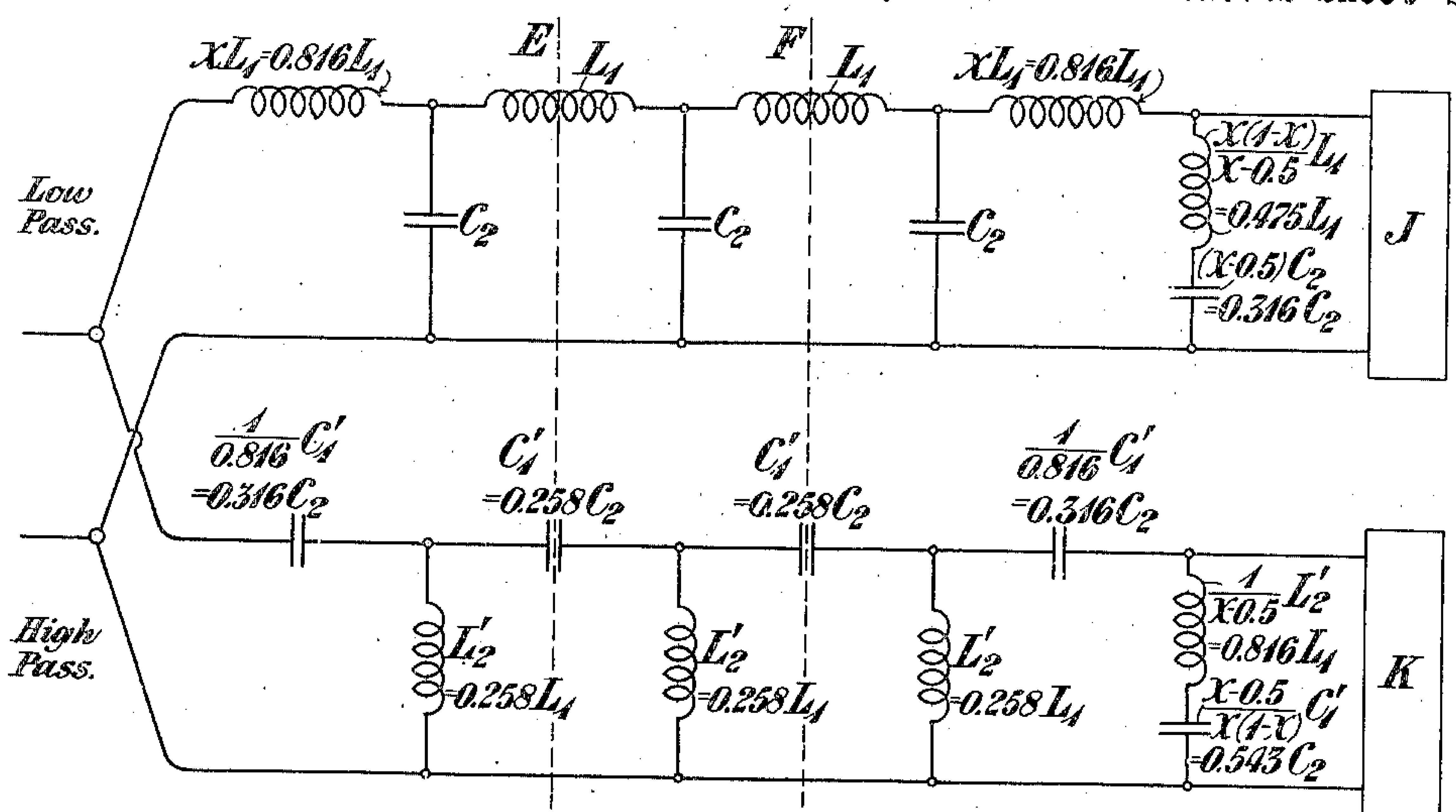


Fig. 5

Fig. 6

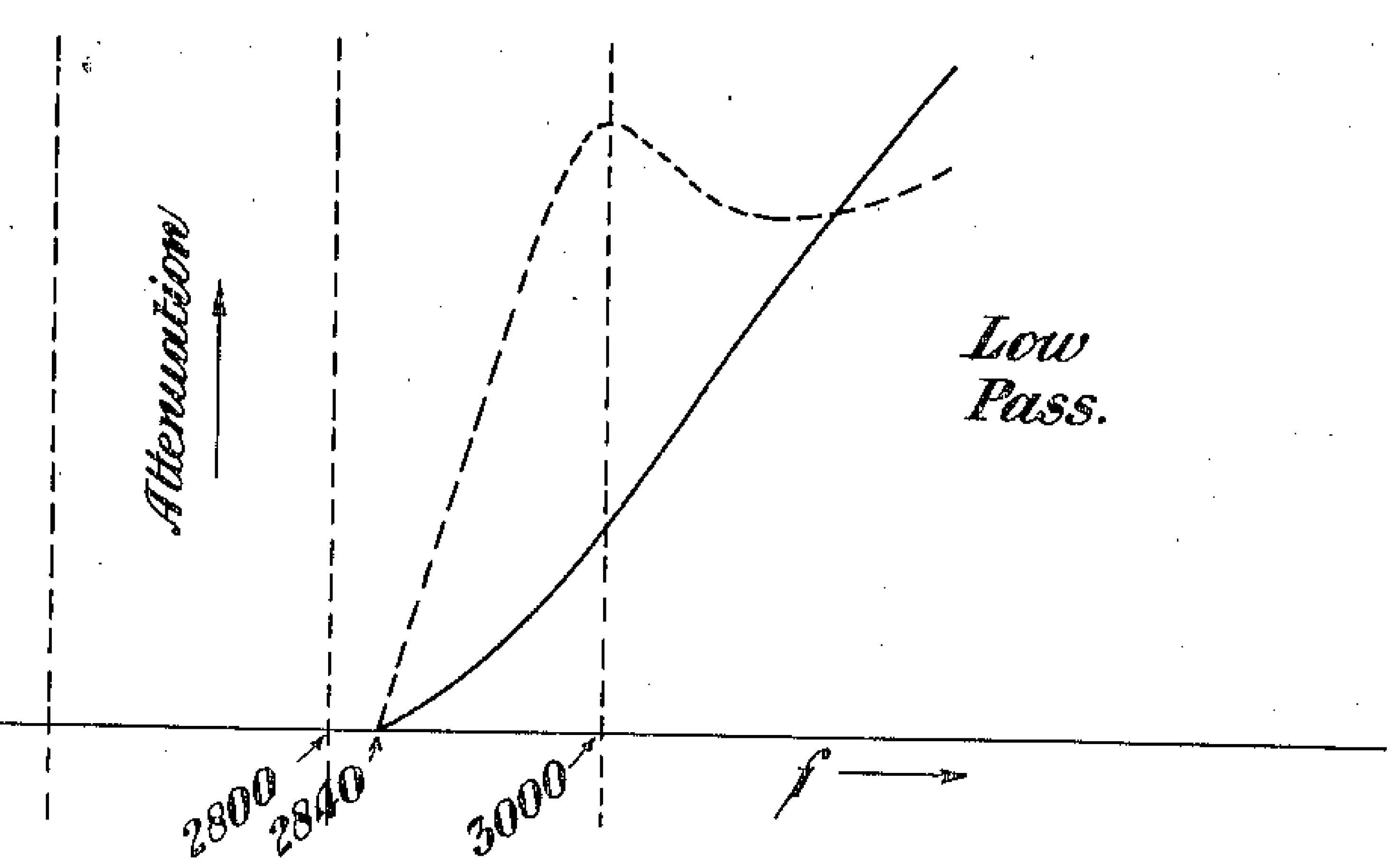
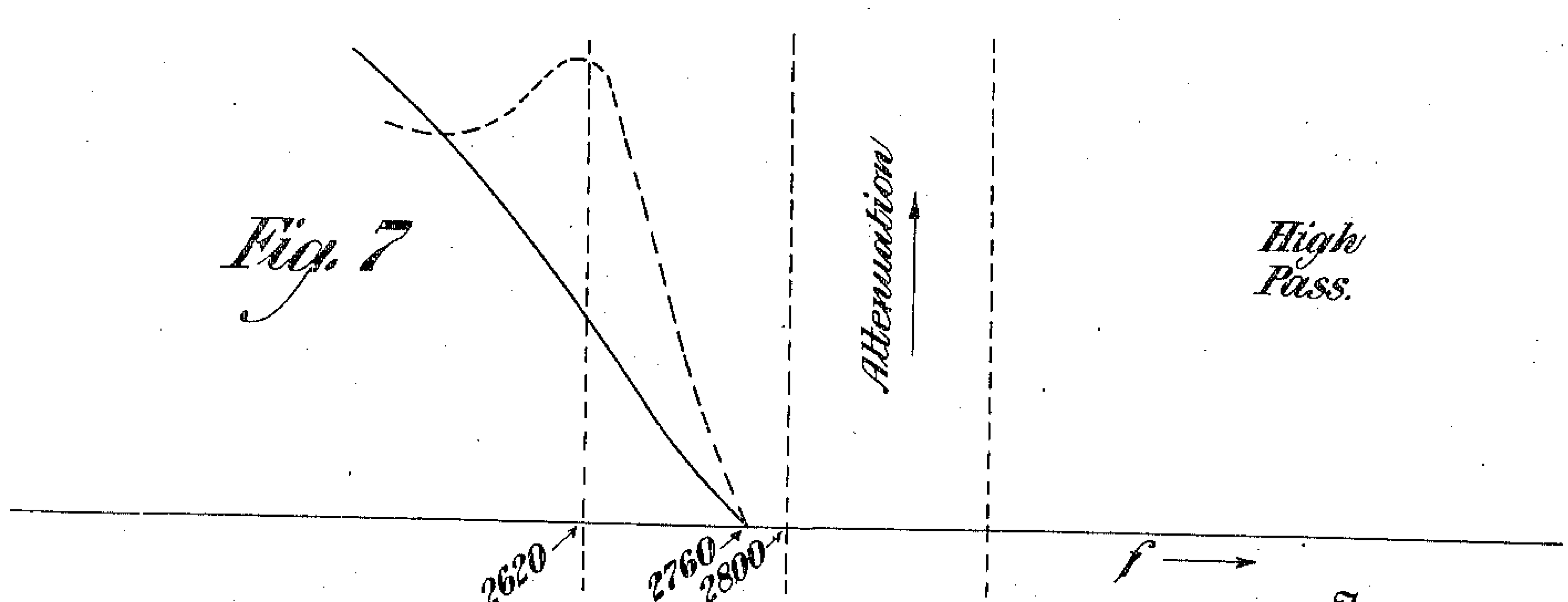


Fig. 7



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May 26, 1925. 1,538,964

O. J. ZOBEL

WAVE FILTER

Filed Jan. 15, 1921 5 Sheets-Sheet 3

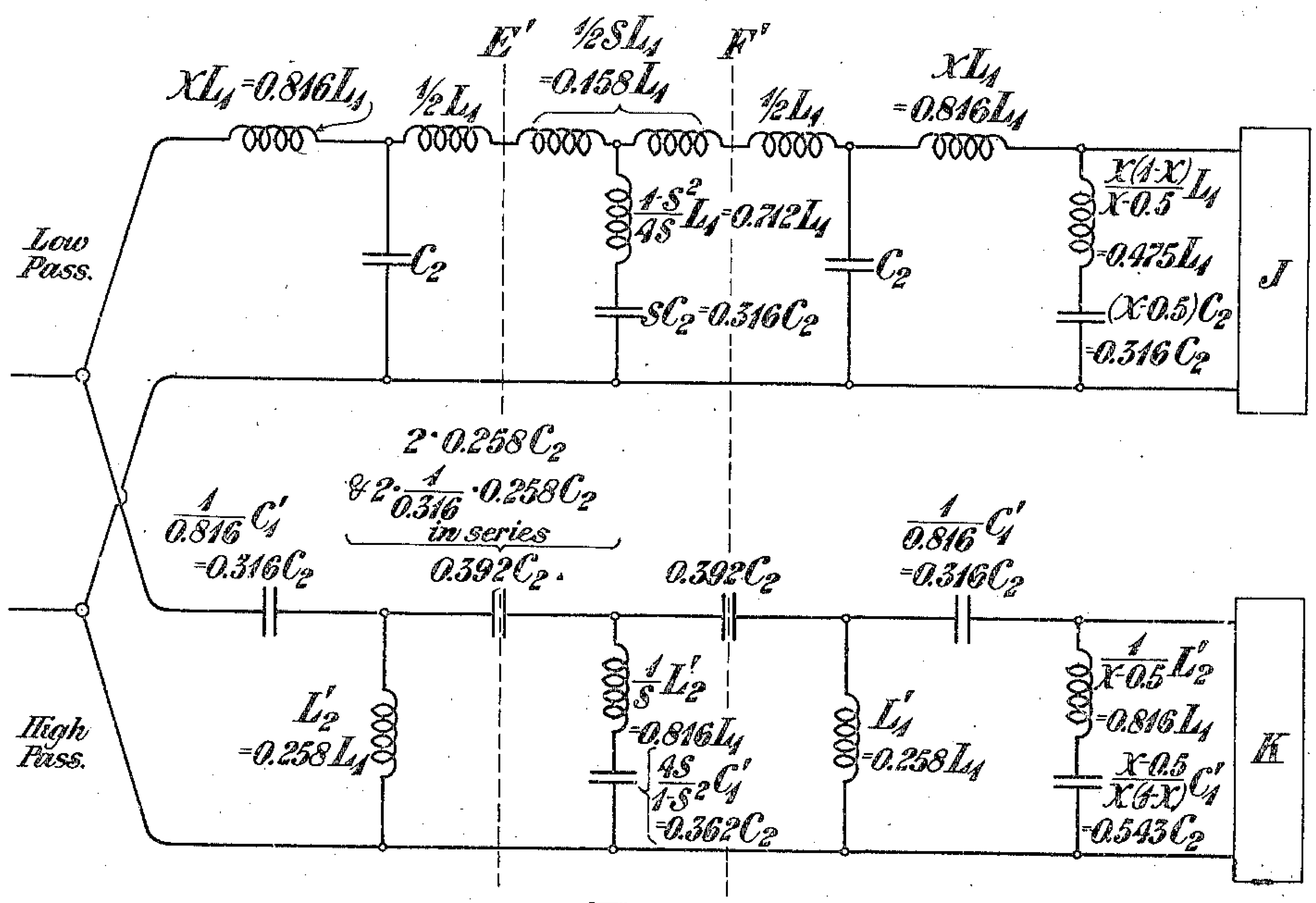

Fig. 8

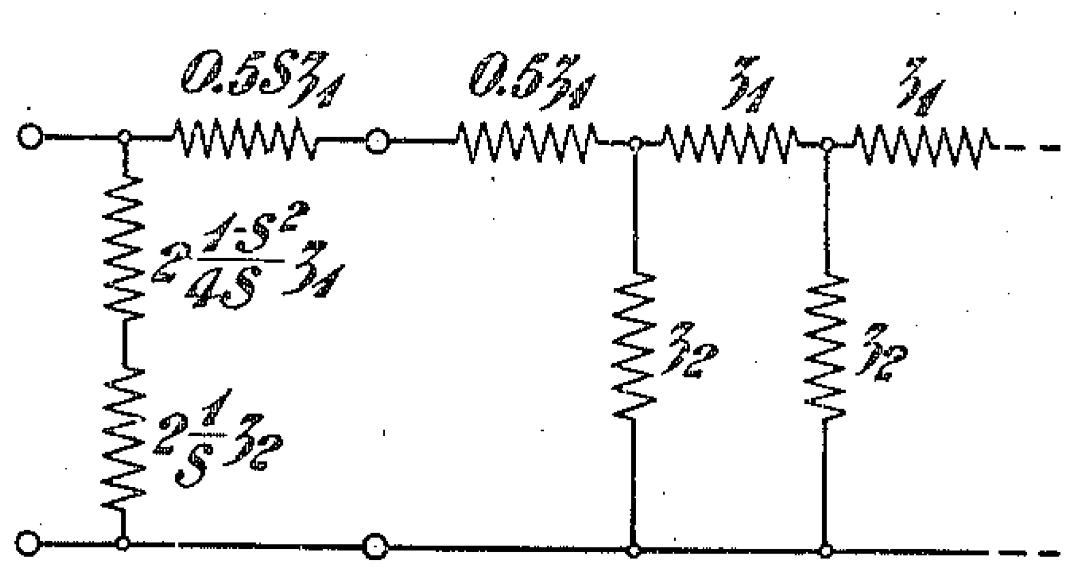

Fig. 9

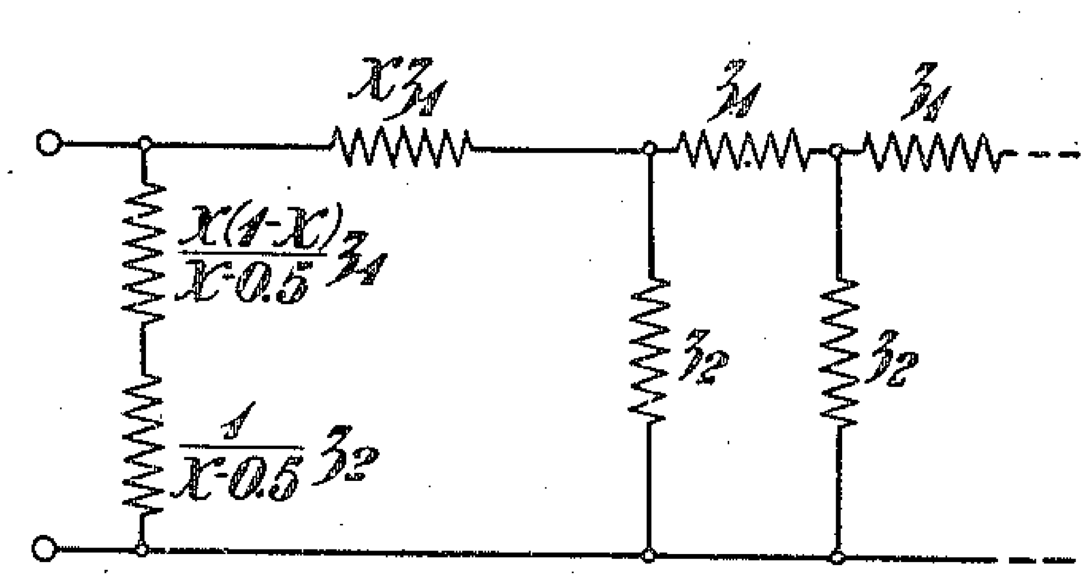

Fig. 10

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May 26, 1925.

1,538,964

O. J. ZOBEL

WAVE FILTER

Filed Jan. 15, 1921

5 Sheets-Sheet 4

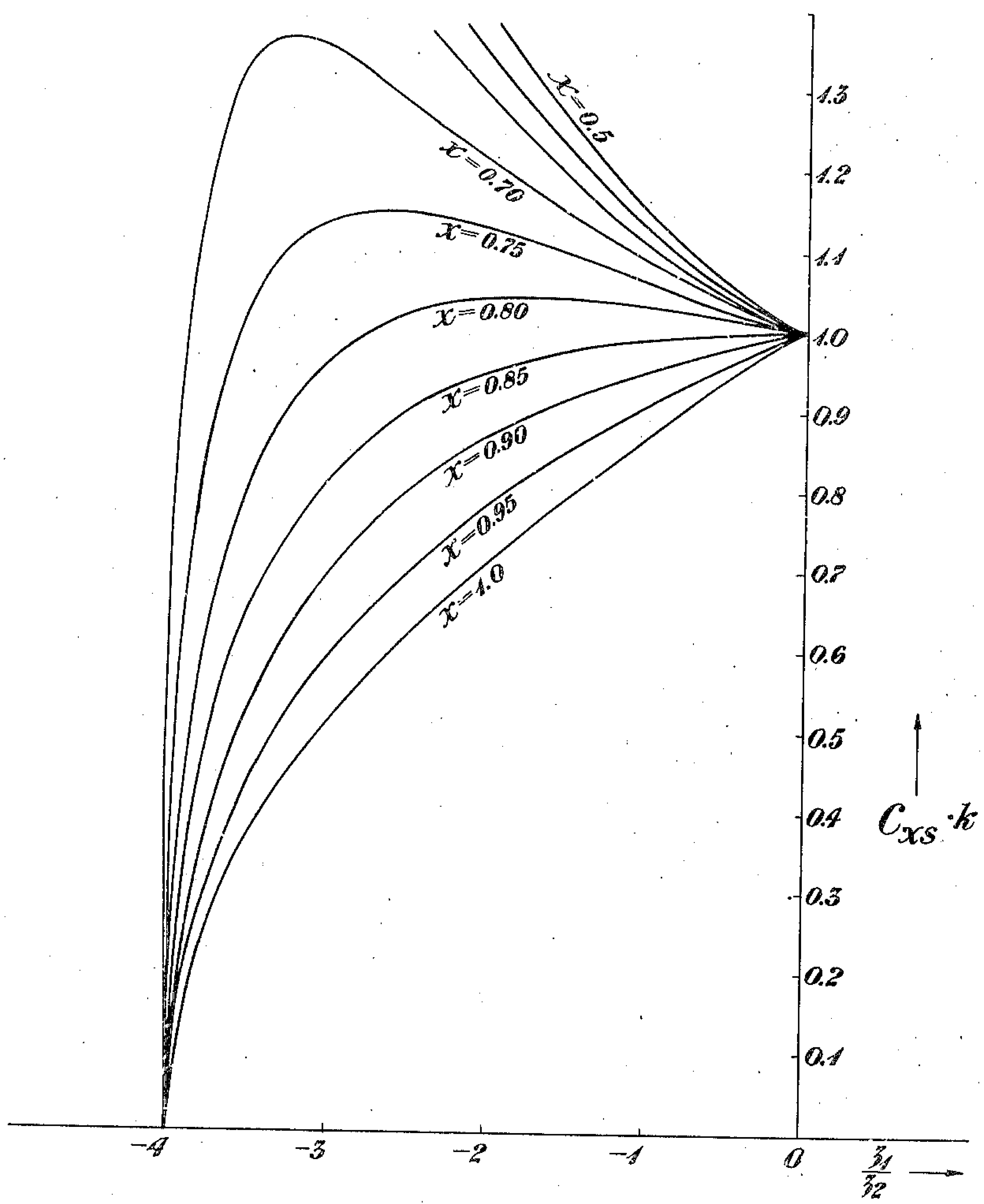

Fig. 11

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May 26, 1925. 1,538,964

O. J. ZOBEL

WAVE FILTER

Filed Jan. 15, 1921 5 Sheets-Sheet 5

Fig. 12 $t>1$

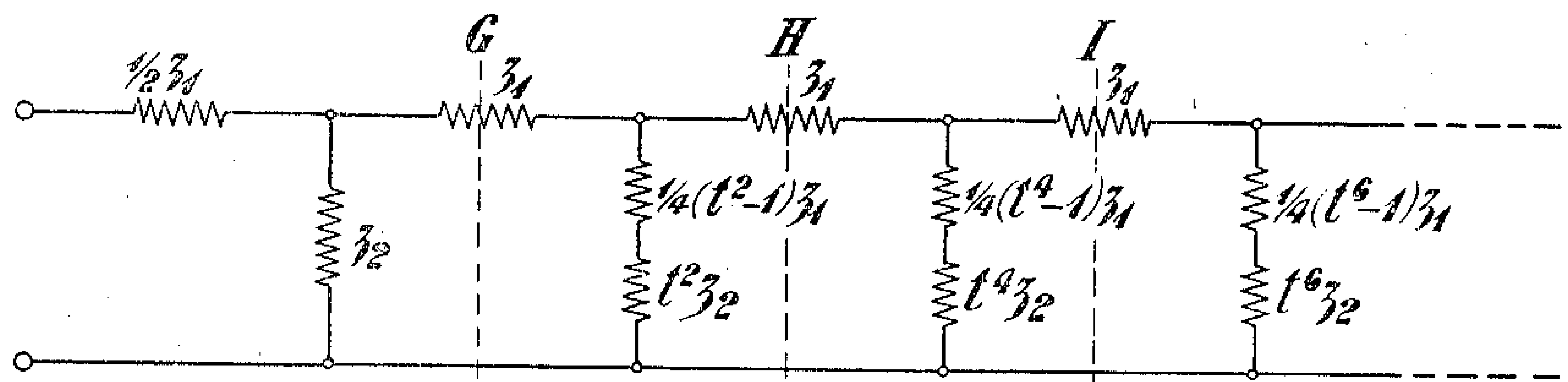


Fig. 13

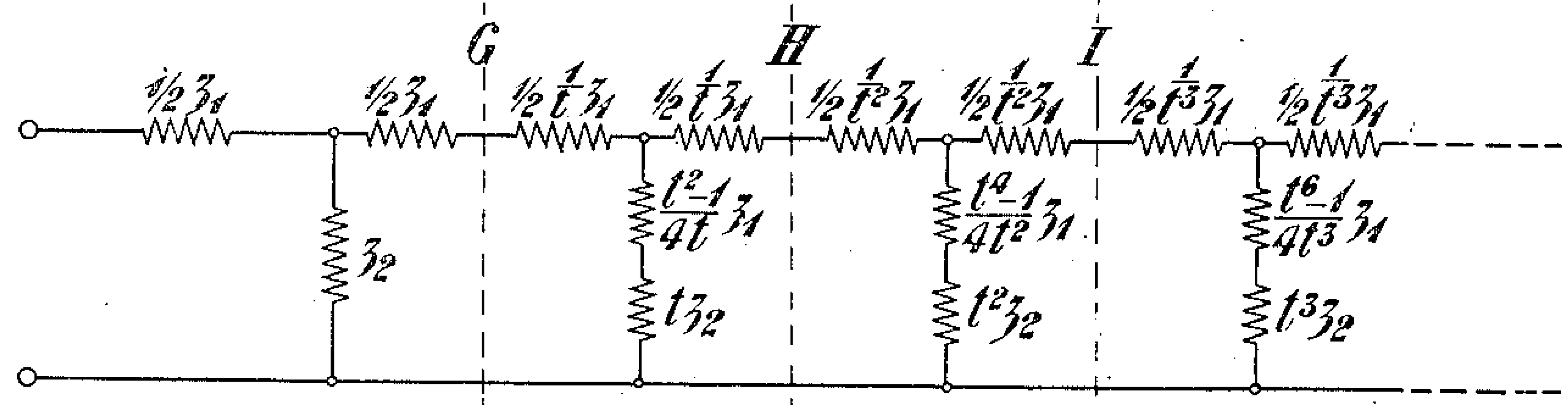


Fig. 14

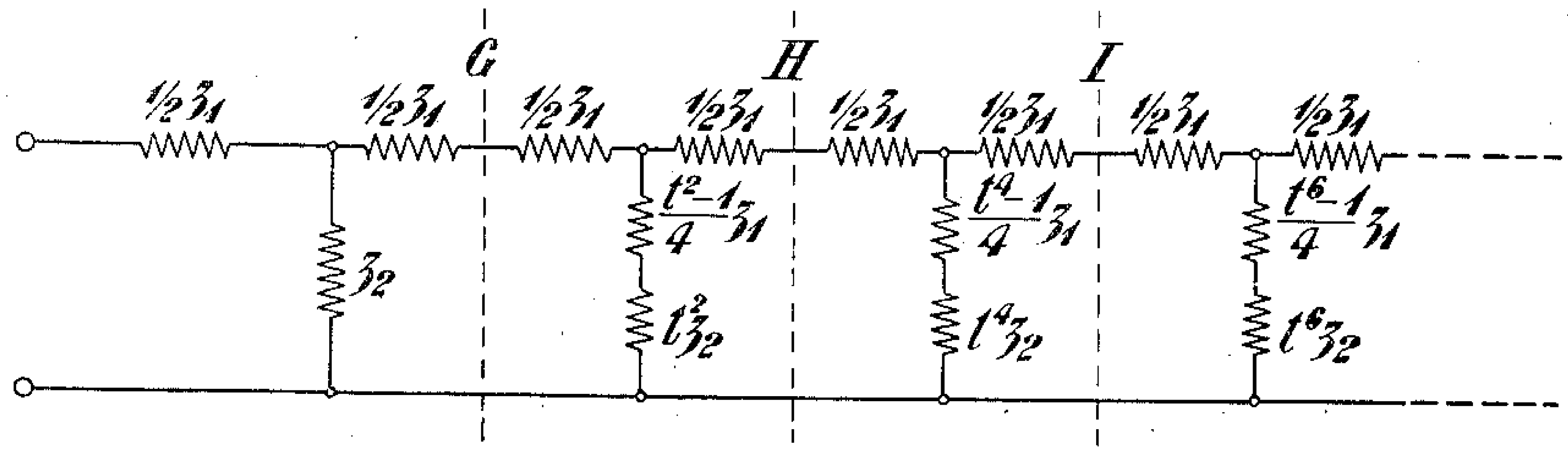


Fig. 15 $t<1$

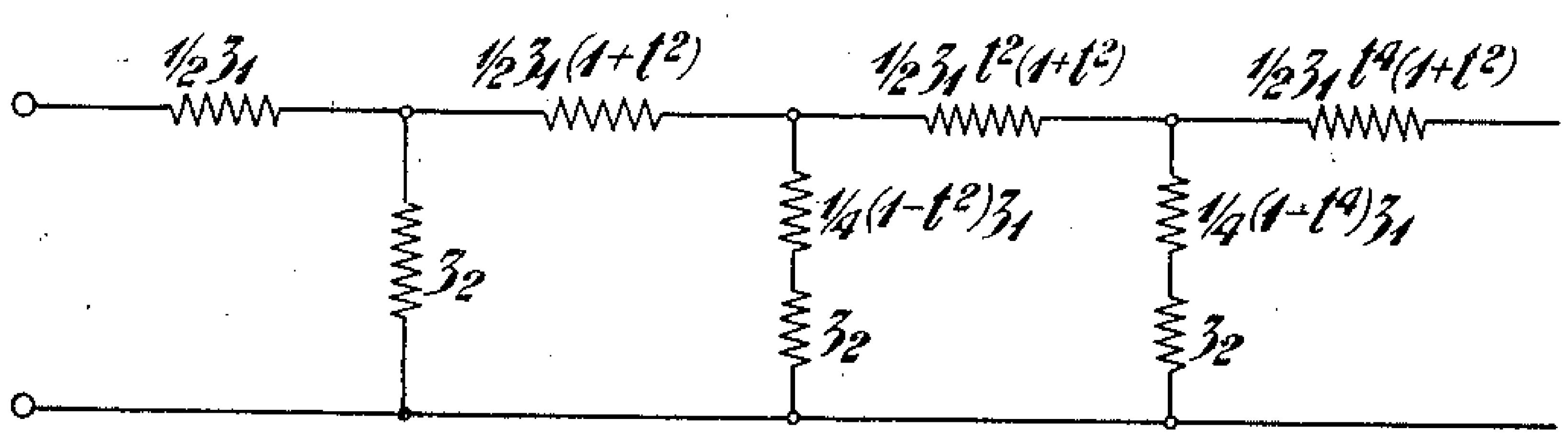


Fig. 16 $t<1$

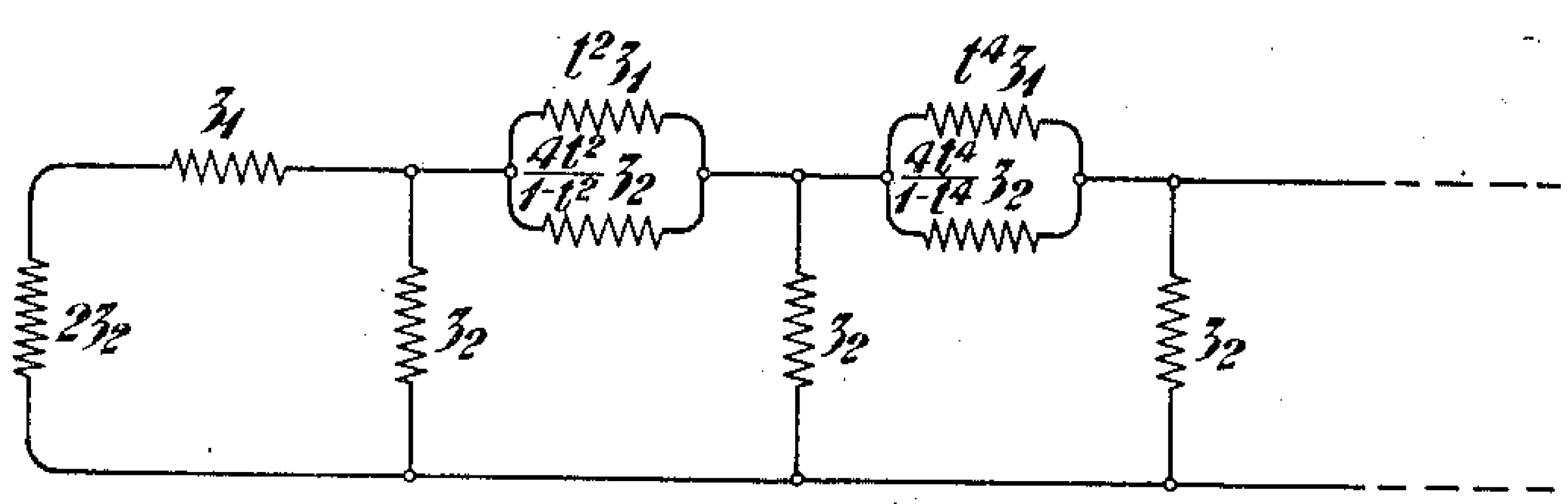
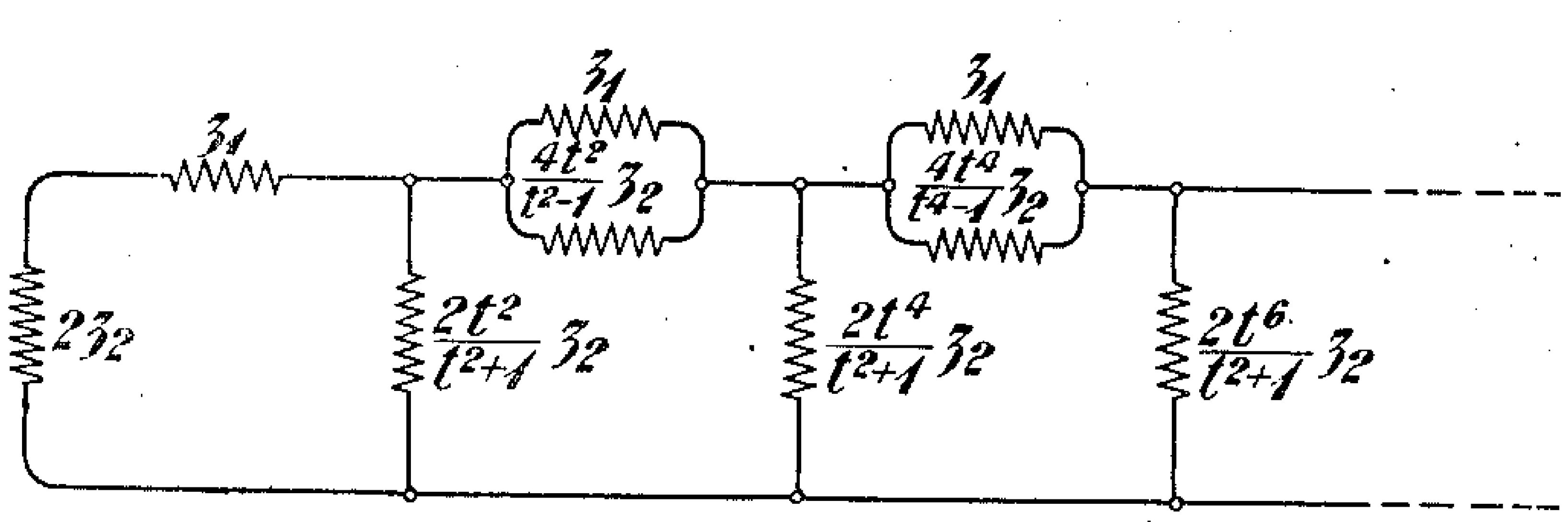


Fig. 17 $t>1$



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Patented May 26, 1925. 1,538,964

UNITED STATES PATENT OFFICE.

OTTO J. ZOBEL, OF NEW YORK, N. Y., ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

WAVE FILTER.

Application filed January 15, 1921. Serial No. 437,527.

To all whom it may concern:

Be it known that I, OTTO J. ZOBEL, residing at New York, in the county of New York and State of New York, have invented certain Improvements in Wave Filters, of which the following is a specification.

The principal object of my invention is to provide a new and improved network for the purpose of transmitting electric currents having their frequency within a certain range and attenuating currents of frequency within a different range. My improvement belongs to the class of devices commonly called wave-filters and I shall refer to them by that name. Another object of my invention is to provide a wave-filter with recurrent sections not all of which are alike, and having certain advantages over a wave-filter with all its sections alike. Another object of my invention is to provide a class of wave-filters which shall be alike as to the transition frequencies between the free transmitting ranges and the attenuating ranges, but shall be unlike in other respects, so that a choice may be made among them as to the one most desirable for the attainment of a given object for which these wave-filters may be employed. Another object of my invention is to provide a tapered wave-filter which may go in between lines or apparatus of different characteristic impedances so as to transmit currents without serious reflection losses. These and other objects of my invention will be made apparent by the following discussion of a few specific embodiments of the inventive idea.

The following specification relates specifically to examples chosen by way of illustration, and the definition of my invention will be given in the appended claims.

Referring to the drawings, Figure 1 is a diagram illustrating a wave-filter of known type with the recurrent sections all having their corresponding component impedance alike. Fig. 2 is a diagram showing a derived S-type wave-filter for which the prototype is given in Fig. 1. (The use of the term "S-type" and similar terms will be explained later.) Fig. 3 is a diagram illustrating an H-type wave filter derived from Fig. 1 as the prototype. Fig. 4 is a curve diagram exhibiting attenuation characteristics for the wave-filters of Figs. 2 and 3, each of which includes Fig. 1 as a limiting case. Fig. 5 is a diagram illustrating two complementary wave-filters on a single input line. Figs. 6 and 7 are diagrams giving attenuation characteristics for the two wave-filters of Fig. 5. Fig. 8 is a diagram corresponding to Fig. 5, but exhibiting the replacement of certain of the wave-filter sections of Fig. 5 by S-type sections, for a purpose that will be explained later. Figs. 9, 10 and 11 are diagrams illustrating the introduction of a half S-type section at the input termination of a wave-filter to serve as a reactance annulling network, and to give a substantially constant resistance over the free transmitting range. Fig. 12 is a tapered wave-filter whose sections are all derived as S-types from a common initial prototype. Figs. 13 and 14 are diagrams illustrating steps in the development of the theory of the tapered wave-filter shown in Fig. 12. Figs. 15, 16 and 17 are diagrams of other forms of tapered wave-filters.

The known general type of wave-filter, of which my improved wave-filter may be regarded as a development, is illustrated in Fig. 1, which shows a periodically recurring network with the series impedances z_1 and the periodically recurring shunt impedances z_2. One series impedance z_1 has been divided into two equal parts with the point A between them to facilitate definition of the "mid-series" termination. If the wave-filter is cut off at A and B, its impedance looking to the right is called the mid-series characteristic impedance, and is represented by Z_{ms}. In this case the wave-filter is said to have mid-series termination.

One shunt impedance z_2 has been substituted by the equivalent combination of two shunt impedances in parallel, each of value $2z_2$, on either side of the points C and D. This affords an opportunity to explain the term "mid-shunt" termination. If the wave-filter is cut off at the points C and D, the part looking to the right is said to have mid-shunt termination, and its impedance Z_{msh} is called the mid-shunt characteristic impedance.

The theory of this wave-filter is usually developed most readily on the assumption that the number of sections to the right of the termination, whether mid-series at AB or mid-shunt at CD, is infinite. A moderate finite number of sections with a suitable termination at the other end is found to give results that approximate closely to the ideal performance of the infinite wave-filter. The impedances z_1 and z_2 can be made to have very small dissipation loss, and hence may be regarded without sensible error as being pure reactances.

Regarding the wave-filter of Fig. 1 as a prototype, the wave-filter of Fig. 2 is of S-type, where $0<s<1$. Representing the series elements of the S-type wave-filter as z_{1s} and the shunt elements as z_{2s}, the S-type wave-filter is completely defined with reference to its prototype by the equations.

$$z_{1s}=sz_1 \quad (1)$$

and

$$z_{2s}=\frac{1-s^2}{4s}z_1+\frac{1}{s}z_2 \quad (2)$$

Before entering upon a discussion of the properties and uses of S-type wave-filters and wave-filter sections, I will define the H-type. With Fig. 1 as the prototype, an H-type wave-filter is shown in Fig. 3, where $1<h<\infty$. With reference to the prototype in Fig. 1, the H-type wave-filter is completely defined by the equations

$$z_{1h}=\frac{1}{\frac{h}{z_1}+\frac{h^2-1}{4hz_2}} \quad (3)$$

$$z_{2h}=hz_2 \quad (4)$$

Before exhibiting the use that may be made of wave-filters and wave-filter sections of S- and H-types, I will direct attention to some of their properties upon which their utility depends.

With mid-series termination, the prototype and all the S-types have the same characteristic impedance over the entire frequency range. To prove this, I will first derive a formula for the mid-series impedance of the prototype and show that the mid-series impedance of the S-type is the same.

Since the wave-filter may be assumed to extend without limit to the right from AB in Fig. 1, the impedance across the mid-series section points A′ and B′ will be the same as across the points A and B. Accordingly we have the equation

$$z_{ms}=\frac{z_1}{2}+\cfrac{1}{\cfrac{1}{z_2}+\cfrac{1}{z_1+\cfrac{1}{\cfrac{z_1}{2}+Z_{ms}}}}$$

The solution of this equation gives

$$z_{ms}=\sqrt{z_1z_2+\frac{1}{4}z_1^2} \quad (5)$$

This is a general formula and accordingly for the S-type wave-filter we have the equation

$$z^1_{ms}=\sqrt{z_{1s}z_{2s}+\frac{1}{4}z^2_{1s}}$$

and substituting from equations (1) and (2), the right-hand member reduces to the same expression as in equation (5). This demonstrates the equality of mid-series characteristic impedance for all S-types and their prototype at all frequencies.

Another important property of S-type wave-filters is that, with their prototype, they all have the same critical frequencies. It is known from the fundamental theory of wave-filters that the critical frequencies are given by solving for f the two equations

$$\frac{z_1}{z_2}=0 \quad (6)$$

and

$$\frac{z_1}{z_2}=-4 \quad (7)$$

It is easily shown that if we let

$$\frac{z_{1s}}{z_{2s}}=0$$

and

$$\frac{z_{1s}}{z_{2s}}=-4$$

and substitute from equations (1) and (2), and simplify, we shall get equations (6) and (7) as the results. Thus we show that the equations that define the critical frequencies are the same for the S-types as for the prototype.

By a parallel course of reasoning which is clearly suggested by the foregoing, it may be shown that all H-type wave-filters and their prototype have the same mid-shunt characteristic impedance over the whole frequency range, and that they all have the same critical frequencies.

Having the same critical frequencies, and assuming as usual that there is no dissipation, the attenuation constant is of course 0 for the same frequency range in all S-type or H-type wave-filters and their prototype; but the attenuation characteristic may be widely different for frequencies outside the free transmitting range. This fact will become apparent upon a consideration of the comparative attenuation characteristics of a set of S-type wave-filters and their prototype. It is a fundamental equation of wave-filter theory that

$$\cosh \Gamma=1+\frac{1}{2}\frac{z_1}{z_2}, \quad (8)$$

where Γ is the "propagation constant", defined as the natural logarithm of the ratio of the current in any series element to the current in the succeeding series element. Let $\Gamma=a+i\beta$, with the conventional notation. It is to be remembered that z_1 and z_2 are pure reactances, hence their quotient is real. Expanding equation (8) gives

$$\cosh\alpha\cos\beta+i\sinh\alpha\sin\beta=1+\frac{1}{2}\frac{z_1}{z_2},$$

whence

$$\cosh\alpha\cos\beta=1+\frac{1}{2}\frac{z_1}{z_2}$$

and $\sinh\alpha\sin\beta=0$. It follows that if $\sinh\alpha=0$, $\alpha=0$ and $\cosh\alpha=1$ and

$$\cos\beta=1+\frac{1}{2}\frac{z_1}{z_2}.$$

This last equation gives the phase relation within the free transmitting range. Since $\cos\beta$ lies between -1 and $+1$ it follows that

$$\frac{z_1}{z_2}$$

lies between -4 and 0 for the free transmitting range. On the other hand, if $\sin\beta=0$, $\beta=0$ or $\pm\pi$, and $\cos\beta=\pm1$ and

$$\cosh\alpha=\pm\left(1+\frac{1}{2}\frac{z_1}{z_2}\right)$$

whence

$$\sinh\alpha=\sqrt{\frac{z_1}{z_2}+\frac{1}{4}\left(\frac{z_1}{z_2}\right)^2}\quad(9)$$

in the attenuation range. The full line in Fig. 4 is a plot of equation 9, showing α as a function of

$$\frac{z_1}{z_2};$$

that is, the full line gives the attenuation characteristic of the prototype wave-filter for values of

$$\frac{z_1}{z_2}$$

corresponding to frequencies outside the free transmitting range.

For the S-type wave-filter the equation corresponding to equation (9) is

$$\sinh\alpha_s=\sqrt{\frac{z_{1S}}{z_{2S}}+\frac{1}{4}\left(\frac{z_{1S}}{z_{2S}}\right)^2}\quad(10)$$

Substituting from equations (1), (2), and (9) this reduces to

$$\sinh\alpha_s=\frac{4s\sqrt{z_1z_2+1/4z_1^2}}{(1-s^2)\,z_1+4z_2}=\frac{4sz_2}{(1-s^2)z_1+4z_2}\sinh\alpha\quad(11)$$

Plotting the foregoing expression for α_s as a function of

$$\frac{z_1}{z_2}$$

for various values of s, we get the dotted line curves as shown in Fig. 4.

Thus it appears that S-type wave-filter gives a sharper attenuation transition than the prototype, on the side beyond

$$\frac{z_1}{z_2}=-4,$$

the point of maximum attenuation being brought nearer the critical frequency. The attenuation is less sharp, however, beyond

$$\frac{z_1}{z_2}=0.$$

However, for the simple low-pass wave-filter, the simple high-pass wave-filter, and in general all "constant k" types (for which $z_1z_2=k^2$, and k is a constant, independent of frequency), the attenuation ranges correspond only to

$$\frac{z_1}{z_2}$$

less than -4. Therefore S-types derived from these classes of wave-filters always give this sharper attenuation, which may be more important near the critical frequencies than beyond. Hence as a general proposition if one has a wave-filter giving a certain attenuation characteristic, and if he desires to have a sharper transition of attenuation as z_1/z_2 passes from the value -4 to values less than -4, he can accomplish this to any desired degree by substituting an S-type wave-filter for the prototype.

Moreover, since the characteristic impedance is exactly the same, he can remove a mid-series section or several mid-series sections from the prototype and replace them by S-type sections, thus making the change of attenuation more abrupt to a degree by combining the attenuation effect of the prototype and its S-type.

On the other hand, if we have a given S-type wave-filter, we may desire that its attenuation shall be less sharp, or that at a remote part of the attenuation range, the attenuation shall be increased; to secure these results we can change the value of s in appropriate manner.

I will now disclose a specific example in which the substitution of S-type sections improves the operation of certain wave-filters. It is well known to make a long telephone circuit available not only for the ordinary telephoning frequencies but also for "carrier currents" of higher frequency which may be modulated for additional telegraph or telephone uses. At the receiving station it becomes necessary to separate the frequencies so that those of the ordinary telephone range may go to an ordinary telephone receiving instrument and those of higher frequency may go to proper modulating apparatus. In Fig. 5 the incoming line from the left branches to two wave-filters in parallel which lead respectively to the apparatus J and K, J for ordinary telephone frequencies, K for higher frequencies. Leading to J is a low-pass wave-filter and to K is a high-pass wave-filter. It is desirable that frequencies below 2800 per second go to J, and that higher frequencies go to K. To insure that frequencies in one range shall not go into the other apparatus from that for which they are intended, there must necessarily be an intermediate band of "lost frequencies", which it is desirable to make as narrow as possible. It is practicable with my apparatus to put frequencies up to 2600 into J and above 3000 into K, thus having a lost frequency band of only 400 cycles.

The low-pass wave-filter leading to J has series impedances z_1 represented by the inductances L_1. At the input end this low-pass wave-filter has "x-series" termination, which means that the terminal series impedence is x times that of the sections that follow, x being less than 1. At the drop end is a reactance annulling network, which comprises a half of an S-type section. I shall refer to this kind of reactance annulling network more fully in connection with Figs. 9 and 10.

The high-pass wave-filter leading to apparatus K has the series impedances z'_1 represented by the reactances of condensers C'_1 except that at the input end the value is modified to give the same x-series termination as for the low-pass wave-filter, and at the drop end to give a similar reactance annulling network. In order that each of the two wave-filters shall co-operate for the intended purpose in this case, they are made "complementary", which means that, with the primed letters referring to the high-pass wave-filter.

$$z'_1 = a z_2$$

and

$$z'_2 = \frac{1}{a} z_1 \quad (12)$$

Equations (12) give the condition for complementary constant-k wave-filters in general. In the case of low-pass and high-pass wave-filters, these equations reduce to the following:

$$C'_1 = \frac{1}{a} C_2$$

and

$$L'_2 = \frac{1}{a} L_1 \quad (13)$$

Equations (13) afford the data by which the values of C'_1 and L'_2 are expressed in terms of C_2 and L_1 in the legends of Fig. 5. In this particular case I let $a=3.878$.

Though the example now under consideration relates to a case of a simple low-pass wave-filter and a simple high-pass wave-filter, it is only an example of a general method of treatment which is good for any pair of complementary constant-k filters. It will be seen that the values for z'_1 and z'_2 in equations (12) satisfy the definition of constant K, namely $K^2 = z_1 z_2$.

The attenuation characteristics for the low-pass wave-filter and high-pass wave-filter of Fig. 5 are shown respectively in Figs. 6 and 7 by means of full lines. It is at once apparent that it will be advantageous if these characteristics can be made steeper for each wave-filter. This is accomplished by substituting S-type sections between the dotted lines E and F in Fig. 5. The resultant pair of wave-filters is illustrated in Fig. 8, where the substituted S-type sections stand between the lines E′ and F′. It will not be necessary to explain all the details of my design. I have chosen certain parameters so as to make the number of different sizes of inductance coils and condensers as low as practicable, and I have been influenced by other considerations. Fig. 8 shows the structural modification made in Fig. 5 to get a sharper cut-off for each wave-filter, and the dotted curves in Figs. 6 and 7 show the improvement that has been gained in this way. The values of the inductances and capacities that go to make up the reactances of Fig. 8 are labelled thereon, and they may readily be verified by means of the equations that have been given earlier in this specification. For both wave-filters I have let $s=0.316$. Accordingly equations 1 and 2 give $z_{1s}=0.316\ z_1$ and $z_{2s}=0.712\ z_1+3.16\ z_2$ for the low pass wave-filter. For the high pass wave-filter $z'_{1s}=0.316\ z'_1$ and $z'_{2s}=0.712\ z'_1+3.16\ z'_2$, whence the values appearing as legends on Fig. 8 may be derived.

The foregoing example illustrates how upon inspection of the attenuation characteristic of a wave-filter, I can remove a section from it and substitute an S-type section to sharpen the cut-off, if so desired. The effect may be enhanced by substituting a plurality of S-type sections. The value of s can be determined with reference to the desired degree of steepness of attenuation characteristic, or with reference to convenience of structural design, or with reference to other conditions, or two or more of these conditions.

I will now turn to another example of the application of the theory of S-types for improving the operative characteristic of a wave-filter. From equation 5 it is apparent that in the free transmitting range, the mid-series characteristic impedance of a wave-filter is a pure resistance, remembering that the impedances z_1 and z_2 may be regarded practically as pure reactances. Inasmuch as the characteristic impedance of most lines is practically indistinguishable from a pure resistance, the property just mentioned would indicate that a mid-series termination is a proper one for a wave-filter so that it may fit a line without reflection losses. However, it is likely to be the case that the mid-series characteristic impedance, even though a pure resistance over the free transmitting range of frequency, is widely variable over that range. Obviously it would be desirable to have it nearly constant over at least the major part of the range. If I should substitute a full S-type section for any section of the wave-filter, it would not help matters, because as I have already explained, the salient property of S-type sections is that they have the same characteristic impedance as the prototype for all frequencies. Accordingly, I try the effect of adding at the end of the wave-filter a half S-type section as illustrated at the left in Fig. 9, thus terminating the wave-filter at mid-shunt S-type. Now a mid-shunt termination gives a pure resistance for the characteristic impedance of the wave-filter in the free transmitting range. This may readily be proved according to a procedure that is suggested by the preceding derivation of equation 5. Hence the wave-filter of Fig. 9 has its impedance reduced to a pure resistance, and it remains to investigate the variability of that resistance over the free transmitting range. To do this, the parameter s must be given some particular value between 0 and 1. A somewhat different way of looking at the diagram of Fig. 9 will be convenient. The two series impedances $0.5\ sz_1$ and $0.5\ z_1$ at the left may be looked upon as the x-series impedance of an x-series termination with the first shunt member as a susceptance annulling network. This leaves in the free transmitting range a resistance $1/C_{xs}$, where C_{xs} is the characteristic conductance of the x-series termination. Accordingly we have the relation

$$xz_1=\frac{1}{2}sz_1+\frac{1}{2}z_1$$

whence

$$s=2x-1. \quad (14)$$

Substituting for s in terms of x from equation (14) we get the legends of Fig. 10, for which the undetermined parameter is now x instead of s.

The characteristic conductance C_{xs}, which is also the reciprocal of the mid-shunt characteristic impedance of the S-type, can readily be computed from elementary network theory for any particular value of x, which may vary from $\frac{1}{2}$ to 1 consistently with the limits for s. Plotting the conductance coefficient $C_{xs}.k$, where

$$k=\sqrt{z_1z_2},$$

against

$$\frac{z_1}{z_2}$$

over the free transmitting range, which we know is terminated at 0 and −4, we get a variety of curves in Fig. 11 corresponding to different values of x, and we can choose that one which makes C_{xs} most nearly constant. I am sketching the procedure only in outline. I mention it here merely as one of several examples of applications of my improved S-type method. It will be sufficient for the present purpose, that for "constant k" types the most available value of x is approximately 0.809. It may be added that this end half section, besides giving impedance correction in the free transmitting range, has the steeper attenuation characteristics of an S-type in the attenuation range.

Briefly stated, the following aspect of the theory of Figs. 9 and 10 may be helpful. By introducing a half S-type section at the end I lose nothing as compared with ordinary mid-series termination of the wave-filter, and I introduce a parameter s, substituted by x, with freedom to choose for x the value which gives the most nearly constant impedance over the whole free transmitting range.

Still another example of my improved method of modifying the operative characteristics of a wave-filter is involved in the design of a tapered wave-filter, which I will now discuss. To prevent reflection losses it is always desirable that when a wave-filter is interposed between two lines or pieces of apparatus, the input characteristic impedance, the characteristic impedance of the wave-filter, and the output characteristic impedance shall be equal or nearly equal. But it does not always happen that the two lines or pieces of apparatus to be united by the wave-filter have their characteristic impedances equal. For example, it may be desired to receive current from a circuit in a cable, through a wave-filter, and transmit it thence to an open wire line. In such a case the input and output impedances will be unequal. One way of meeting this difficulty is to employ a tapered wave-filter, one in which the characteristic impedance of the wave-filter changes progressively from section to section. It might seem upon first consideration that there would be no advantage in substituting a series of small reflections for a single large reflection; but, when it is remembered that, in general, the currents in the successive sections of the wave-filter differ in phase, it will at once be appreciated that the various little reflections differing in phase will to a large extent neutralize each other, just as the vector sum of a plurality of non-parallel vectors will be much less than if they have the same direction. The improvement is a matter of degree and is small in certain special cases, namely at frequencies for which the phase constant is zero or nearly zero for successive sections of the wave-filter. But in general, the advantage of a tapered wave-filter will be realized. However, the change in characteristic impedance from section to section must be accomplished in some such way as to avoid changing the critical frequencies or otherwise spoiling the wave-filter action.

Let the input characteristic impedance be Z_a and the output characteristic impedance be Z_b, and assume that there will be n mid-series sections of tapered wave-filter interposed between the terminals. Accordingly there will be $n+1$ points of discontinuity; and the continued product of the impedance ratios across these $n+1$ points should be Z_b/Z_a, and these ratios should be as nearly equal as practicable. A tapered wave-filter that answers to the requirements of the situation here sketched is shown in Fig. 12, where the series impedances are equal and have the value z_1, except for the half value at the mid-series termination; and the shunt impedances have the values appearing as legends in the diagram, z_2 being the initial shunt impedance. It should be understood that z_1 and z_2 have the proper values so that in a wave-filter of uniform sections they would give it a suitable characteristic impedance.

To present the theory of the tapered wave-filter of Fig. 12 I will first direct attention to a hypothetical design shown in Fig. 13 that stands in an intermediate relation between Figs. 1 and 12. The first mid-series section of Fig. 13 is the same as of Fig. 1, beginning at AB in Fig. 1. With this as a prototype, the mid-series section in Fig. 13 between G and H is of S-type, where

$$s=\frac{1}{t},$$

and t has a suitable value based on the ratio Z_b/Z_a, so that $t^{n+1}=Z_b/Z_a$, and the total ultimate impedance gradation corresponds to said ratio. The next section from H to I is of S-type, where

$$s=\frac{1}{t^2}.$$

For the next section

$$s=\frac{1}{t^3},$$

and so on. According to the principles that have been enunciated, the mid-series characteristic impedance of the wave-filter of Fig. 13 is throughout the same as of the prototype Fig. 1.

It is well understood that if all the impedances of a net connecting two points are multiplied by the same real factor, the resultant impedance of the net beween those two points will be multiplied by that same factor. Accordingly the mid-series characteristic impedance of Fig. 13 will be stepped up by the factor t at the mid-series point G if all the impedance elements to the right of G, both series and shunt, are multiplied by t. Again, the mid-series characteristic impedance will be stepped up at H by multiplying all the impedances to the right of H by t, superposing this operation on the result of the operation of the preceding sentence. Let this procedure be followed clear through to the end of the wave-filter of Fig. 13, and it will be transformed into the wave-filter of Fig. 14, which is at once seen to be the same as Fig. 12. This derivation of the tapered wave-filter of Fig. 12 through Figs. 13 and 14 from Fig. 1 proves that they have the same points of critical frequency and that the resultant wave-filter answers to the tapering requirement of the situation; by a proper choice of t, its resultant impedance ratio will correspond to the given ratio Z_b/Z_a. The resultant wave-filter has the advantage that it keeps the number of different sizes of coils and condensers as low as practicable. It will be noticed that the series impedances are all equal. It also gives a sharper cut-off for at least one end of the free transmitting range than the prototype on which it is based. This will be especially advantageous in case of simple low-pass or high-pass filters. It will be noticed that the geometrical relation obtained by keeping the same ratio t throughout is not necessary. The ratio might vary from section to section, but a constant ratio is convenient and appropriate.

Other tapered wave-filters are shown in Figs. 15, 16 and 17. In Fig. 15, $t<1$, and the wave-filter is derived from its prototype through the S-type construction, taking $s=t$ initially. In Fig. 16 $t<1$, but the theory of H-type is employed in this case. The prototype wave-filter with the appropriate mid-shunt termination is shown in Fig. 1, extending to the right from the points C D. The result of passing to H-type sections after the full mid-shunt section, taking

$$h=\frac{1}{t},$$

gives the series admittances in order,

$$\frac{1}{z_1},$$

$$\frac{1}{tz_1}+\frac{1-t^2}{4tz_2},$$

$$\frac{1}{t^2z_1}+\frac{1-t^4}{4t^3z_2},$$

etc.; and the shunt admittances in order,

$$\frac{1}{2z_2},$$

$$\frac{1}{2z_2}+\frac{t}{2z_2},$$

$$\frac{t}{2z_2}+\frac{t^2}{2z_2},$$

etc. Now successively multiplying all the impedances to the right of each mid-shunt division point by t, we get the resultant wave-filter of Fig. 16. Its properties and advantages will be apparent upon comparison and contrast with the wave-filter of Fig. 12.

In view of the explanation that has been given for other examples, it is thought that the derivation of the wave-filter of Fig. 16 will be apparent; it is based on H-types, with $h=1/t$ initially.

Wave-filters or wave filter sections of S-type are based on mid-series points as the division points between sections; and H-type wave-filters and wave-filter sections are based on mid-shunt points as the division points between sections. These are the only mid-points at which a wave-filter can be divided into sections. These are the only points that divide the wave-filter symmetrically. They are the only points that will divide it so that in the simple wave-filter of Fig. 1 the impedance is the same looking either way from the division point. Both the S-type and the H-type may be looked upon as species under a more generic concept mid-type, which I call the M-type.

I define the class M-type to comprehend the classes S-type and H-type, and no others; having precisely defined the S-type and the H-type in connection with the foregoing equations 1, 2, 3 and 4, this statement gives an exact definition of the M-type.

It may be noted that the prototype itself is a member of the whole of a class of M-types. For example, the prototype is an S-type for which $s=1$ and the prototype is an H-type for which $h=1$. Thus when I refer to a set of wave-filters or wave-filter sections which are all S-types or H-types or M-types of a common prototype, it will be seen that the prototype itself may be one of the set. It can readily be shown that with a given prototype, the S-type and H-type wave-filters have identical propagation constants provided $sh=1$.

I claim:

1. A wave-filter having one or more half-sections of a certain kind and one or more other half-sections that are M-types thereof, M being different from unity.

2. A wave-filter having its sections and half-sections so related that they comprise different M-types of a common prototype, M having several values for respectively different sections and half-sections.

3. A wave-filter having one or more half-sections of a certain kind and one or more half-sections introduced from a different wave-filter having the same characteristic impedance and the same critical frequencies and a different attenuation characteristic outside the free transmitting range.

4. A wave filter having recurrent sections and comprising means to give it a steep attenuation characteristic, said means comprising at least one section with component impedances different from the others and with the same critical frequencies.

5. A wave filter having recurrent sections and having means to make it cut off the transmission abruptly at an end of a transmitting range consisting of at least one-half section of M-type, M being different from unity.

6. A wave filter of the kind having recurrent sections and comprising means to give it a steep attenuation characteristic consisting of a section of M-type, M being different from unity.

7. A tapered wave-filter having its sections all M-types of a common prototype, modified by successively increasing the component impedance elements in the same ratio beyond successive points of junction of the wave-filter sections.

8. A wave-filter having its characteristic impedance at successive mid-section points graded between different terminal values, and having certain impedance elements of constant value throughout the sections.

9. A wave-filter of tapering impedance having its sections all M-types of a common prototype, except for the variation of impedance from one to another.

10. Two complementary wave-filters connected to the same line, and each comprising a respective M-type section, M being different from unity.

11. In combination, a transmission line,

a high pass wave-filter connected thereto, a low pass wave-filter connected thereto, said two wave-filters having nearly the same critical frequency with a narrow lost frequency range between them, and each said wave-filter comprising an M-type section to steepen its attenuation characteristic adjacent to its critical frequency, M being different from unity.

In testimony whereof, I have signed my name to this specification this 13th day of January, 1921.

OTTO J. ZOBEL.