March 10, 1925. 1,528,982

E. C. MOLINA

TRUNKING SYSTEM

Filed Feb. 26, 1921 5 Sheets-Sheet 1

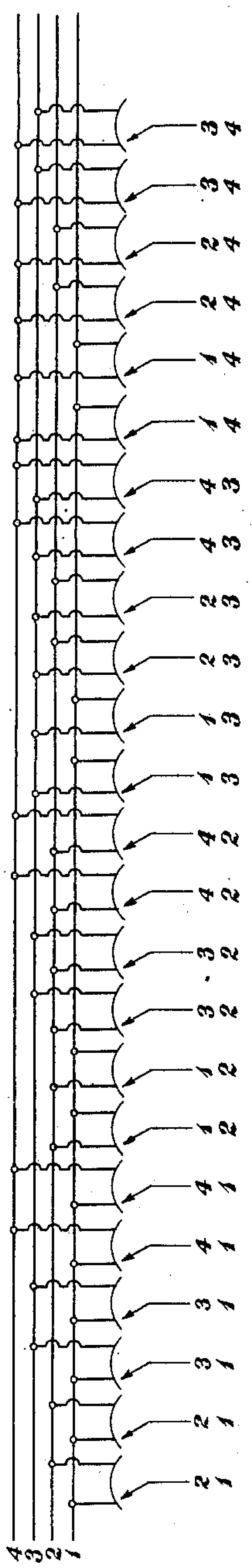

Fig. 3
Permutations of 4 Trunks 2 at a Time

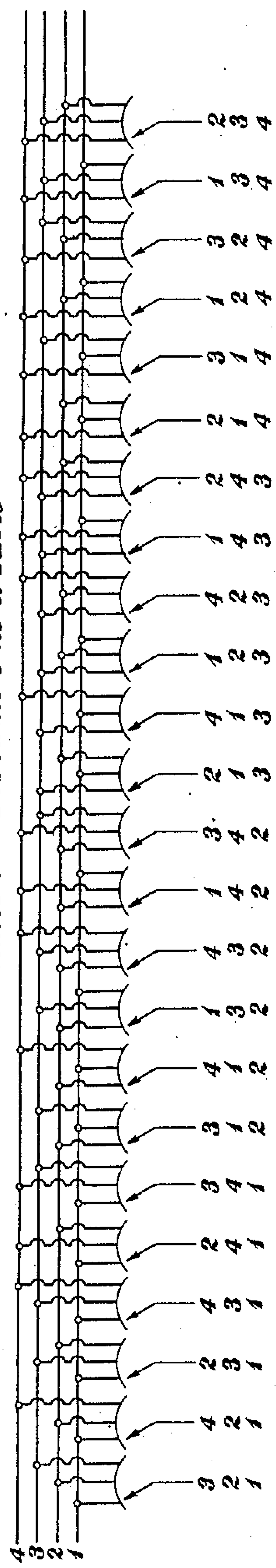

Fig. 2
Permutations of 4 Trunks 3 at a Time

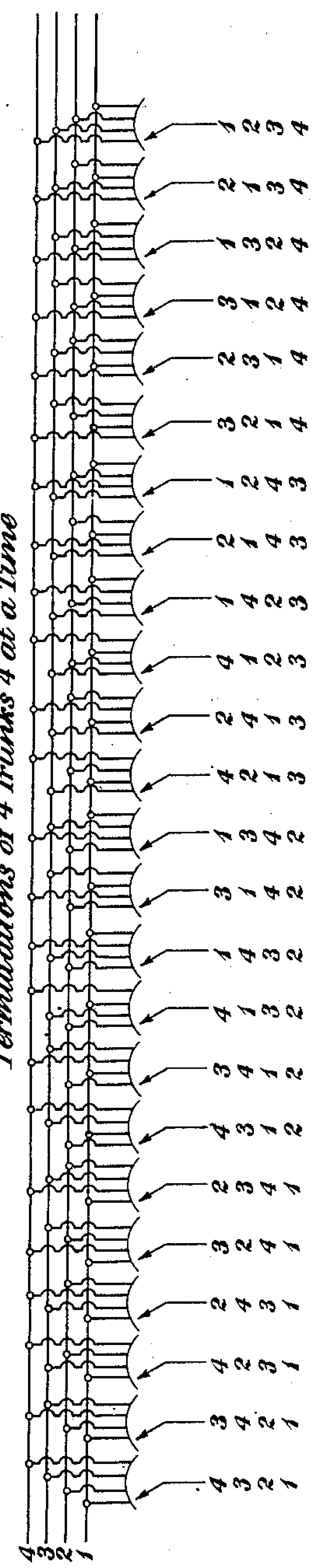

Fig. 1
Permutations of 4 Trunks 4 at a Time

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TRUNKING SYSTEM

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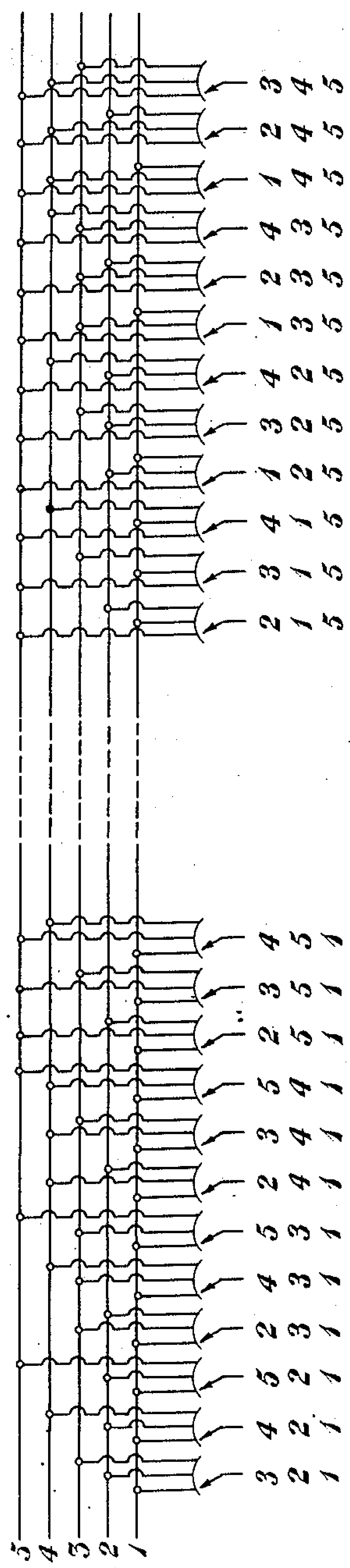

Fig. 5

Permutations of 5 Trunks 3 at a time

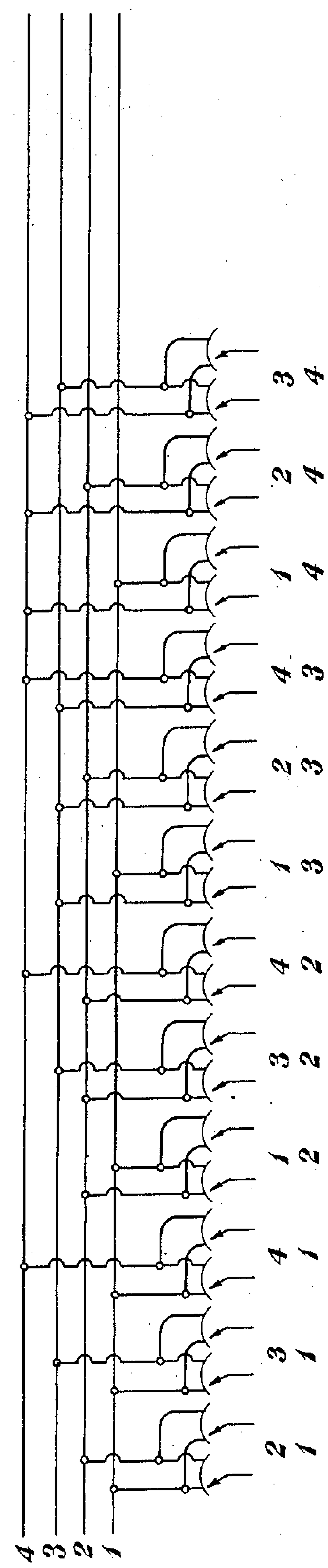

Fig. 4

Permutations of 4 Trunks 2 at a time

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TRUNKING SYSTEM

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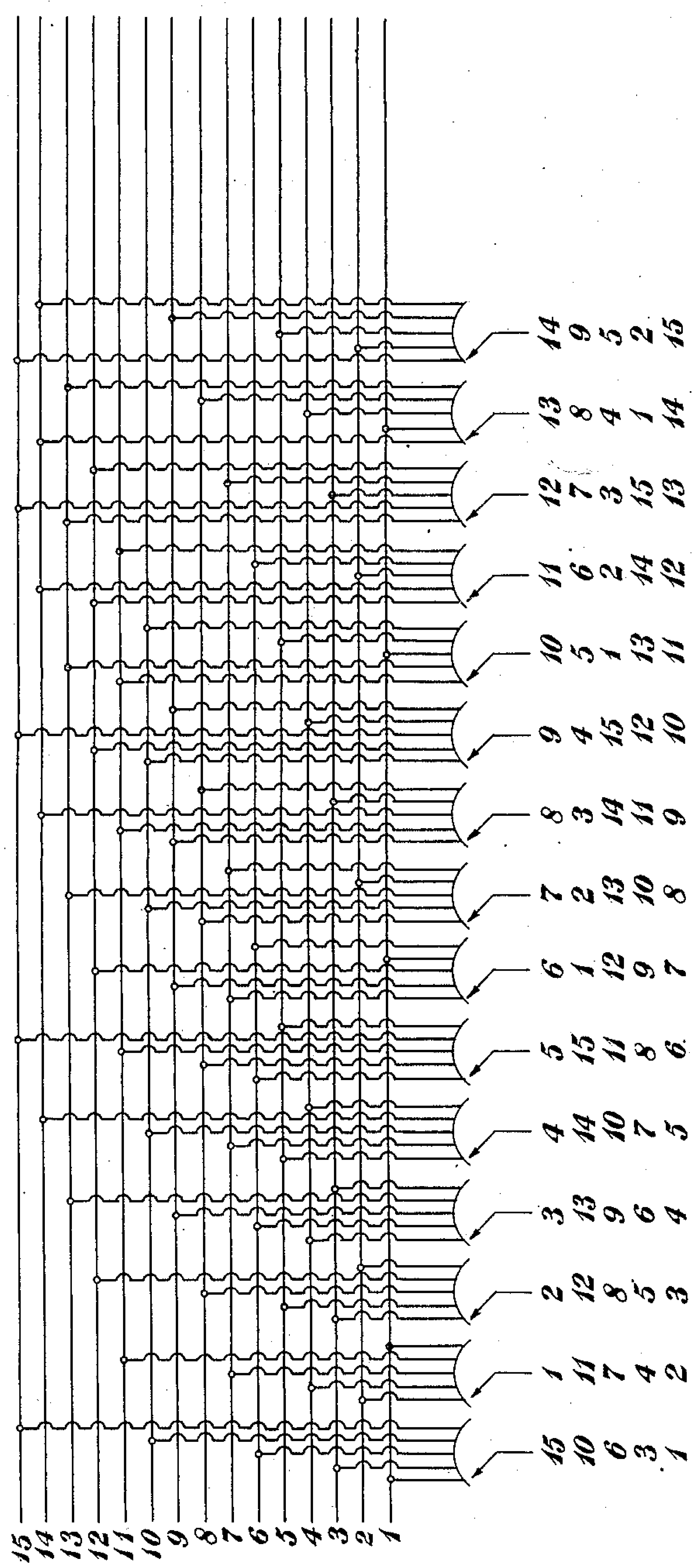
Fig. 6

Triangular Slip



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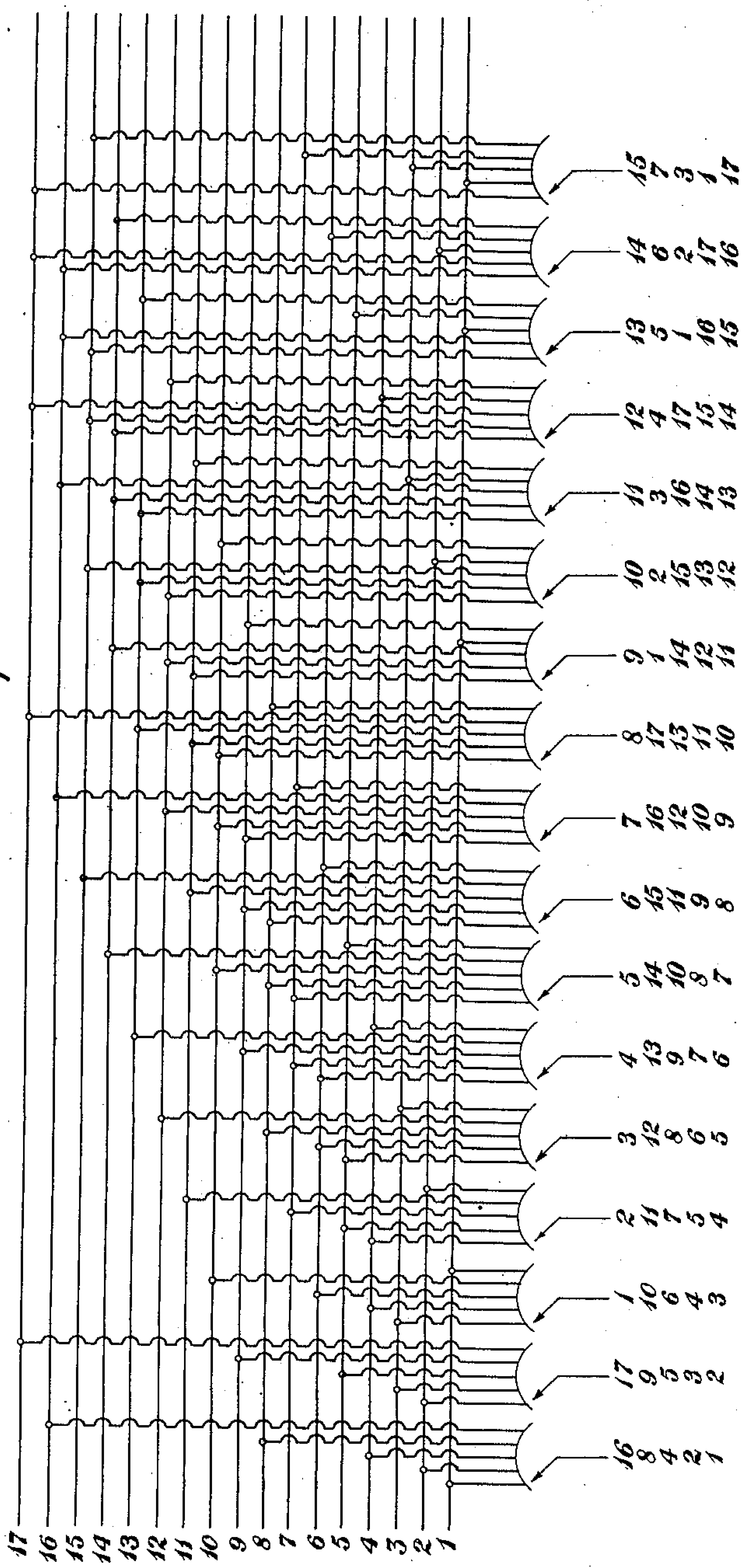

Fig. 7
Binomial Slip

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TRUNKING SYSTEM

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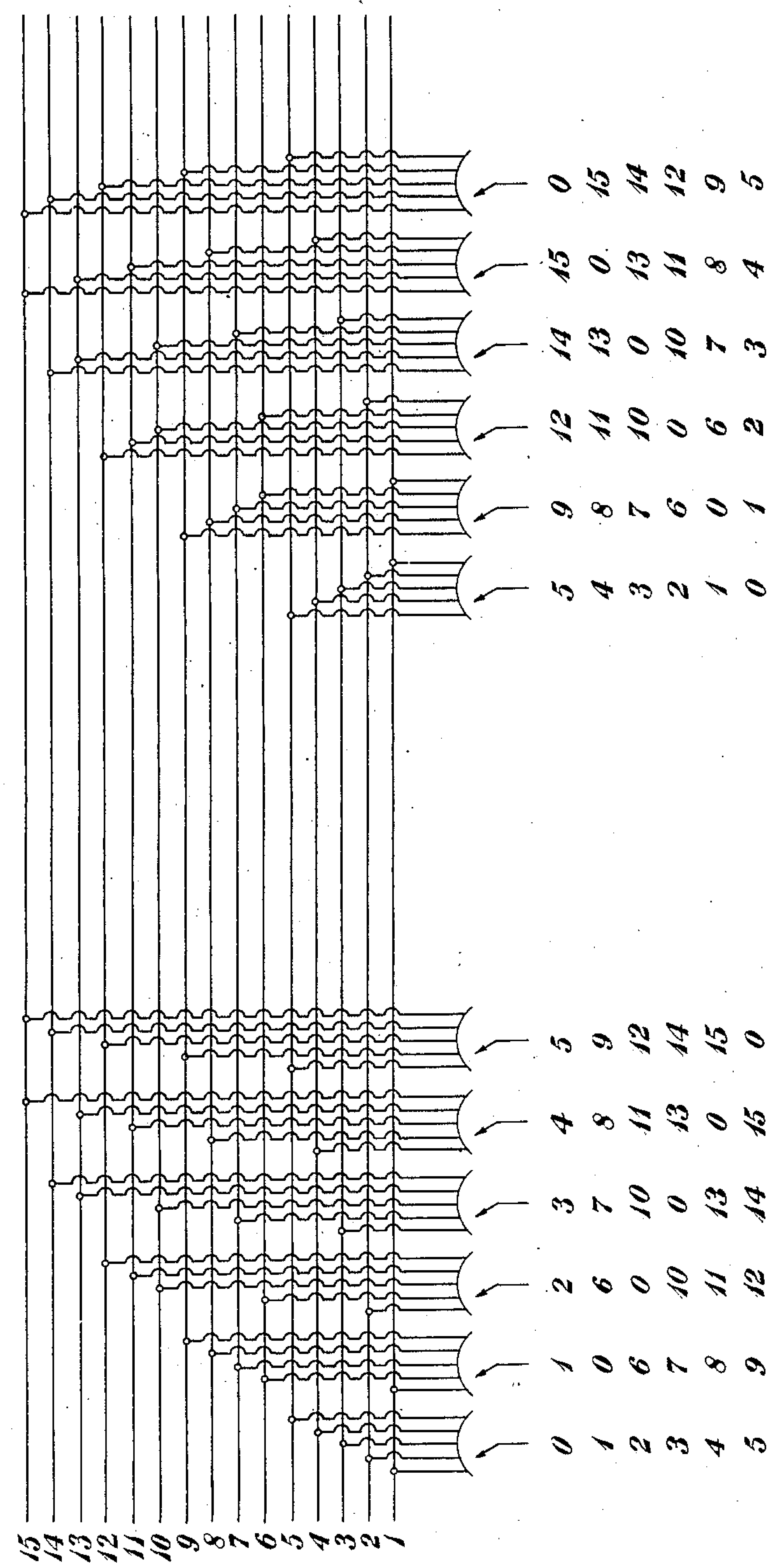
Fig. 8

Axi-Symmetric Slip



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Patented Mar. 10, 1925. **1,528,982**

UNITED STATES PATENT OFFICE.

EDWARD C. MOLINA, OF EAST ORANGE, NEW JERSEY, ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

TRUNKING SYSTEM.

Application filed February 26, 1921. Serial No. 447,966.

To all whom it may concern:

Be it known that I, EDWARD C. MOLINA, residing at East Orange, in the county of Essex and State of New Jersey, have invented certain Improvements in Trunking Systems, of which the following is a specification.

This invention relates to signaling circuits and more particularly to methods and means for trunking between different subscribers in a telephone exchange or between subscribers in different exchanges.

Heretofore in providing trunking arrangements for handling the traffic originated by subscribers' lines, particularly where the connections were established by means of switching machinery, two general methods have been proposed. One method involved providing a sufficient number of trunks to handle all of the traffic originated by the subscribers' lines involved and establishing connections between the calling subscribers' lines and the trunks through a switching arrangement equipped with a number of selecting points corresponding to the number of trunks necessary to handle the traffic. The other method consists in dividing the subscribers' lines into groups and providing trunks for each group sufficient in number to handle the traffic originating in the group.

The first of these methods possesses the advantage that the total number of trunks necessary to handle the traffic without an unduly large probability of a call being lost due to all of the trunks being busy will be a minimum. The method is subject to the serious disadvantage, however, that each subscriber must have available a switching arrangement having a number of switching terminals sufficient to connect with each and all of the trunks, and where the traffic is so large as to require a great number of trunks the expense involved in providing a switching arrangement having the necessary capacity becomes prohibitive.

The advantage of the second method is that the subscribers' groups may be made sufficiently small so that the number of trunks necessary to handle the traffic originating in each group will be within the capacity limits of a relatively small and inexpensive switching arrangement; that is, a switching arrangement having a comparatively small number of terminal points. This method is subject to the disadvantage, however, that while economic in the matter of switching arrangements or machinery it is prodigal in the use of trunks, since the total number of trunks required to handle the traffic from all of the subscribers is greater than would be the case in the first method referred to above, due to the fact that the ratio of trunks to subscribers varies inversely with the number of subscribers having access to the trunks.

It is one of the principal objects of the present invention to provide a scheme for multipling trunks such that the use of switches having a small number of switching points will be permitted without requiring a large ratio of the total number of trunks to the total number of subscribers.

The above object, as well as other objects more fully appearing hereinafter, are accomplished by the use of what is herein termed a "random slip" multiple. The nature of this multiple will now be clear from the following description of the invention when read in connection with the accompanying drawing, Fig. 1 of which is a schematic diagram of a so-called "straight" multiple, Figs. 2 to 5 inclusive of which are schematic diagrams of a trunking system in which the multiples are "slipped" upon a purely random basis, while Figs. 6, 7 and 8 are schematic diagrams of several modifications in which the slipping of the multiples is worked out in accordance with a definite law which closely approximates the conditions involved in a perfect "random slip."

In order to understand the principle underlying a "random slip" multiple let us consider, for example, a group of 100 trunks carrying the traffic originated by subscribers' lines. Assume that each subscriber's line is equipped with a 100 point selector, thereby giving each line access to the entire 100 trunks. Let the order in which the trunks are connected to the different selectors be varied as much as possible. A perfectly heterogeneous arrangement would be one such that when 37 (for example) of the 100 trunks are busy a specified set of 37 trunks are as likely to be the trunks actually busy as any other set of 37 trunks.

The arrangement so far described corresponds to the first method of trunking outlined above and has the advantages and disadvantages previously discussed. Let us now calculate for such a promiscuously connected multiple the probability that more than 20 terminals (for example) will have to be hunted over by a selector before an idle trunk is obtained. The load to be carried may be adjusted so that the probability comes out any desired value as, for example, .001. It is evident that under these conditions the last 80 terminals of every group may be disconnected from the trunks without appreciably changing the efficiency of the group of 100 trunks. Consequently each partially wired selector may be replaced by a 20 point switch having access to the same trunks in the same order as the first 20 trunks hunted over by the corresponding 100 point selector. Such an arrangement will involve what is herein termed a "random slip" multiple and has the obvious advantage that a 20 point switch may be used having access to 20 trunks previously selected at random from the total number of trunks without increasing the total number of trunks required to handle the traffic from all of the subscribers.

Considering the same arrangement from another view point, assume 5 independent groups of lines equipped with selectors. Let each group of selectors be wired to a group of 20 trunks. The total number of trunks required will therefore be 100. If we bring together these isolated groups of selectors and groups of trunks by a partial interchange of trunks, or in other words, by arranging matters so that each one of the 5 groups of selectors exchanges some of its trunks for some of the trunks assigned to each of the other 4 groups, we will have a promiscuously connected multiple of 100 trunks to some 20 of which each selector will have access. If this interchanging were done thoroughly the load carried by the total number of trunks with a given probability of lost calls would be appreciably greater than the load carried by the same number of trunks when divided into five isolated groups each of which is accessible to a 20 point selector.

In order that the advantages inherent in the "random slip" multiple may be more apparent a mathematical solution of the problem involved will now be given. The problem may be stated as follows:

A group of T trunks handles the traffic originated by n sub-groups of lines or switches.

Sub-group #1 of lines or switches has access to a specified number t_1 of the T trunks.

Sub-group #2 of lines or switches has access to a specified number t_2 of the T trunks. Some of these t_2 trunks are not included in the t_1 trunks assigned to sub-group #1.

Sub-group #3 of lines or switches has access to a specified number t_3 of the T trunks. Some of these t_3 trunks are not included in the t_1 trunks assigned to sub-group #1 and some are not included in the t_2 trunks assigned to sub-group #2. The trunks t_1 and t_2 together may, however, include all the t_3 trunks.

Sub-group #n of lines or switches has access to a specified number t_n of the T trunks. These t_n trunks are not identical with either the t_1, t_2, t_3 . . . or t_{n-1} trunks but they are all included in the totality of $t_1+t_2+\ . \ . \ . \ +t_{n-1}$ trunks.

Thus, no sub-group of lines or switches has the exclusive use of a set of trunks chosen from among the T trunks and on the other hand no two or more sub-groups use identically the same trunks.

It is not assumed above that numerically $t_1=t_2=t_3=\ . \ . \ . \ t_{n-1}=t_n$ although this will probably be the case in practice.

Such a group of T trunks giving service to several over-lapping or inter-linking sub-groups of lines, each of which sub-groups makes partial use of the T trunks, may be defined as a "random slip" multiple provided the following statement can be made with reference to it—when S of the T trunks are busy they are as likely to be one specified set of S trunks as another specified set of S trunks.

In order to solve this problem let $P_t=$ probability that a subscriber originating a call from a sub-group having access to t of T trunks fails to get an idle trunk.

Assume that at the moment the call is made S of the T trunks are busy. The probability of the existence of this assumed condition is

$$\frac{A^{S}e^{-A}}{\underline{|S}}$$

where A is the average load carried by the group T.

The probability that the assumed S busy trunks embrace the particular t trunks to which the subscriber under consideration has access is

$$\frac{\underline{|T-t}}{\underline{|S-t}\;\underline{|T-S}} \Bigg/ \frac{\underline{|T}}{\underline{|S}\;\underline{|T-S}} = \frac{\underline{|S}\;\underline{|T-t}}{\underline{|S-t}\;\underline{|T}}$$

Therefore, the compound probability that S of the T trunks are busy and that these S trunks include the t trunks under consideration is

$$\left(\frac{A^S e^{-A}}{\underline{|S}}\right)\left(\frac{\underline{|S}\;\underline{|T-t}}{\underline{|S-t}\;\underline{|T}}\right)=\left(\frac{A^t\,\underline{|T-t}}{\underline{|T}}\right)\left(\frac{A^{S-t}e^{-A}}{\underline{|S-t}}\right)$$

Therefore

$$T_t=\left(\frac{A^t\,\underline{|T-t}}{\underline{|T}}\right)\sum_{S=t}^{S=T-1}\left(\frac{A^{S-t}e^{-A}}{\underline{|S-t}}\right)+P(T,A)$$

Where

$$P(T,A)=e^{-A}\left[\frac{A^T}{\underline{|T}}+\frac{A^{T+1}}{\underline{|T+1}}+\frac{A^{T+2}}{\underline{|T+2}}\cdots\cdots\text{ ad infinitum}\right]$$

$$=\text{probability that all T trunks are busy.}$$

Now

$$\sum_{t}^{T-1}\frac{A^{S-t}e^{-A}}{\underline{|S-t}}=\sum_{0}^{T-t-1}\frac{A^X e^{-A}}{\underline{|X}}=1-P(T-t,A)$$

Therefore, finally

$$P_t=\frac{A^t\,\underline{|T-t}}{\underline{|T}}\Big[1-P(T-t,A)\Big]+P(T,A)$$

The preceding equation gives the probability of lost calls in terms of the average load carried by the group of trunks, the total number of trunks involved, and the number of trunks assigned to each group. With this equation as a basis Tables I and II have been prepared to show the saving involved in a "random slip" multiple as compared with a "straight" multiple,—that is, a multiple in which the trunks are divided into independent groups, each group accessible to a switch having a correspondingly small number of terminal points.

In a system where 20 point sender selectors give access to the senders of an automatic switching exchange the saving in senders when the "random slip" arrangement is used instead of the "straight" multiple will be as indicated in Table I. It can be readily seen from the table that with the "random slip" arrangement of the number of senders required to carry a given load is considerably smaller than the number of senders necessary for the same load with the "straight" multiple. For example, it is apparent that if the allowable probability of lost calls is .001 and the trunks are divided into ten groups of 20 each the "random slip" multiple arrangement will require only 137 senders as against 200 senders required by the "straight" multiple arrangement for an average load of 89.6, the expression "average load" being here taken to mean the average number of connections existing at any one time for the entire group of trunks. The saving just referred to amounts to 31.5%. On the other hand, in a case where but one group is involved and the average load is 8.96, 20 senders will be required in both cases and the "random slip" multiple represents no advantages over the "straight" multiple, as in this limiting case the "random slip" multiple becomes a "straight" multiple.

Table II shows the saving in final switches which would result from a "random" arrangement of trunks to finals in a panel system where the incoming multiple is divided into groups of 24 trunks each. Here also a smaller number of switches can carry the same load as a larger number with the "straight" multiple or inversely, the same number of switches in the "random" arrangement can take care of a larger load than in the "straight" multiple arrangement.

Table I.

Average load.	Straight multiple.		Random multiple.	Per cent saving in senders.	Probability.
	Number of groups.	Total senders required.	Total senders required.		
8.96	1	20	20	0	.001
17.92	2	40	34	15.0	----------
26.88	3	60	47	21.7	----------
35.84	4	80	60	25.0	----------
44.80	5	100	73	27.0	----------
89.60	10	200	137	31.5	----------
134.40	15	300	199	33.7	----------
179.20	20	400	262	34.5	----------
224	25	500	326	34.8	----------

Table II.

Average load.	Straight multiple.		Random multiple.	Per cent saving in finals.	Probability.
	Number of groups.	Total finals required.	Total finals required.		
14.1	1	24	24	0	.01
28.2	2	48	43	10.4	----------
42.3	3	72	60	16.7	----------
56.4	4	96	78	18.8	----------
70.5	5	120	96	20.0	----------
11.65	1	24	24	0	.001
23.30	2	48	41	14.6	----------
34.95	3	72	57	20.8	----------
46.60	4	96	73	22.9	----------
58.25	5	120	90	25.0	----------

It is obvious, of course, that multiple arrangements involving the "random slip" principle may be embodied in various forms. In Figs. 2 and 3 are shown perfect "random slips" based on all of the 24 possible permutations of the numbers 1, 2, 3 and 4, the permutations being shown in Fig. 1, which illustrates in schematic form the permutations of four trunks taken four at a time in accordance with the first method outlined at the beginning of this specification. In all three figures a group of T=4 trunks is shown, together with n=24 sub-groups of subscribers' lines. Each sub-group is represented by one line equipped with a selector giving the line access to four trunks in the case of Fig. 1, three trunks in the case of Fig. 2 and two trunks in the case of Fig. 3. In Fig. 1 and the succeeding figures the arrows may therefore be taken to represent the wipers of the switches, each of which is available to one or more subscribers and each switch having terminal points equal in number to the vertical lines connecting the arc of a circle representing the contact points of the switch with the horizontal lines representing the trunks. The numbers below the arrows representing the wipers of the switch indicate the order in which the switches obtain access to the trunks.

If, in the case of the arrangement of Fig. 1 a determination is made of the probability that more than 3 terminals will have to be hunted over by a selector before an idle trunk is obtained, and it be found that this probability is within the allowable probability of lost calls, the connection leading to the last trunk selected by each switch may be omitted from the switches in Fig. 1, in which case we get the perfect "random slip" arrangement of Fig. 2, which is based upon permutations of 4 trunks taken 3 at a time.

Similarly, if it be found that the allowable probability will permit of dispensing with connections to the last two trunks selected by each switch in Fig. 1, we may obtain the perfect "random slip" multiple illustrated in Fig. 3 which involves permutations of 4 trunks taken two at a time. It will be observed that in this instance each pair of adjacent sub-groups has access to the same two trunks and in the same order. This permits of modifying the cabling between the switches and the trunks as illustrated in Fig. 4.

Fig. 5 shows a random arrangement in which each sub-group of lines has access to t=3, out of a total of T=5 trunks. This arrangement is obtained by taking every permutation of five things, three at a time. The break between the two halves of the figure indicates that a part of the permutations or possible connections is not illustrated, owing to lack of space. Assuming that the permutations are worked out in logical order, the connections shown at the left represent a small group of permutations beginning with the first, and the connections at the right indicate a similar group ending with the last permutation of the series.

The slip arrangements illustrated in Figs. 2, 3, 4 and 5 are based upon all possible permutations of the numbers involved. Consequently, each of these slips is a perfect random slip, since obviously no particular set of trunks is more likely to be busy than any other set involving an equal number of trunks. In practice, however, it would not be feasible to use slip arrangements based on all possible permutations, where the total number of trunks T is large. For example, a slip multiple of T=25 trunks, in which each line is equipped with a t=5 points selector would require the lines to be divided into 25 x 24 x 23 x 22 x 21=6,375,600 sub-groups if every possible permutation of 25 things taken 5 at a time is to appear. It therefore becomes desirable to devise a system in which only a part of all of the possible permutations will be used and in which the selected permutations will be so chosen that no particular set of trunks is more likely to be busy than any other set involving the same number of trunks. In selecting a limited number of the total possible permutations, it is preferable to select a minimum amount of overlapping i. e. permutations such that any trunks having a given number will appear in as few chosen permutations as possible.

Figs. 6, 7 and 8 illustrate practical forms of slip arrangements obtained by following the principles just discussed and in which a very thorough intermixture of trunks is obtained without dividing the lines into an abnormal number of sub-groups. Fig. 6, for example, shows what is herein termed a "triangular" slip arrangement in which T, the total number of trunks, is 15, and each selector is provided with five points. The first sub-group of lines to the left has assigned to it trunks numbered 1, 3, 6, 10 and 15, and the selector points corresponding to these points will be selected in the order stated. These particular numbers are technically known as "triangular" numbers because they belong to the series of numbers which indicate the number of balls in successive layers (counting from the top) in a triangular pile of cannon balls. The numbers of the trunks to be assigned to the other groups are obtained by writing the numbers from 2 to 15 as the numbers of the first trunks to be selected by the groups to the right of that having assigned to it the numbers 1, 3, 6, 10 and 15. Similarly, the numbers of the second set of trunks to be selected is obtained by writing successive numbers beginning with 4, while the third set is obtained by writing successive numbers beginning with 7, etc. In each instance, when the number 15 is reached, the succeeding number will be 1. An analysis of this trunk arrangement shows that while it is not a perfect random slip arrangement, it conforms very closely to the requirements of the random slip multiple.

Fig. 7 shows what is herein called a "binomial" slip arrangement, as applied to a system involving 17 trunks and five-point selectors. In accordance with this arrangement, the trunks assigned to the first sub-group of lines to the left are those having the binomial numbers 1, 2, $4=2^2$, $8=2^3$ and $16=2^4$. As in the previous case, the trunks to be assigned to the other sub-groups are determined by writing numbers consecutively from left to right. This arrangement also substantially satisfies the requirements of the random slip multiple, although it is not a perfect random type.

Fig. 8 shows a slip arrangement which, for want of a better term, will be referred to as the "axi-symmetric" slip. At the left of the diagram 6 sub-groups of lines are indicated, each having access to 5 out of a total of 15 trunks. To bring out more clearly which set of 5 trunks is to be assigned to each sub-group of lines, a diagonal set of 6 zeros has been added to the 6 sets of 5 numbers. As will be readily seen, the arrangement of the numbers is such that the numbers upon either side of an axis drawn through the series of zeros will be symmetrical with respect to each other, hence the term "axi-symmetric." An analysis of the arrangement shown at the left of the diagram indicates that this portion of the system, standing by itself, will not conform to the requirements of a random slip, as certain trunks would obviously be used oftener than others. For example, trunk No. 1 is only accessible by lines in the first 2 groups, but is first choice for those particular groups. Upon the other hand, trunk No. 15 is accessible only to lines of the last 2 of the 6 groups, but is the last choice of those groups. Trunk No. 15, would, therefore, not be likely to be as busy as trunk 1. This difficulty is overcome, however, by providing additional groups as shown at the right-hand side of the diagram, these groups employing the same trunk numbers, but in inverse order. Therefore, trunk No. 15 is first choice in 2 groups and last choice in 2 other groups, while the same thing holds true of trunk No. 1. This arrangement as illustrated provides for 12 groups. Additional groups may be arranged for by providing additional sets of 6 groups in which a still different order of selection will be used. For example, the first sub-group of another set of 6 might employ the trunk numbers 5, 3, 1, 4 and 2 in the order given. The numbers of the last choice of trunks of the other 5 sub-groups would then be 2, 4, 1, 3 and 5. The remaining trunk numbers might be arranged just as shown at the left of Fig. 8, or they might be varied in their order as desired, so long as they are symmetrically arranged about the zero axis. This arrangement also possesses substantially all the advantages of the perfect random slip multiple and provides a simple scheme for choosing the permutations to be used.

It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated without departing from the spirit of the invention as defined in the following claims.

What is claimed is:

1. A trunking system in which a plurality of trunks are provided for handling the traffic originating from all of the subscribers' lines involved, the subscribers' lines being divided into subgroups each having access to a number of trunks less than the total number provided, the trunks assigned to the several sub-groups being interchanged so that the sub-groups overlap each other and the order of the selection of the trunks in the various sub-groups and the overlapping of the groups being such that when a given number of the total number of trunks involved are busy, the busy trunks are at least approximately as likely to be one set of trunks of the given number as any other set of trunks of equal number.

2. A trunking system in which a plurality of trunks give service to several overlapping and interlinked sub-groups of lines, each sub-group of lines having access to a part only of the total number of trunks and the numbers of the trunks assigned to the sub-groups and the order in which the numbers are selected being such that when a given number of the total number of trunks are busy, the probability of one set of trunks of the given number being busy will be at least approximately the same as the probability that any other set of equal number will be busy.

3. A trunking system in which a plurality of trunks give service to several overlapping and interlinked sub-groups of lines, each sub-group of lines having access to a part only of the total number of trunks and the numbers of the trunks assigned to each sub-group of a set of sub-groups being so chosen that they will fall into an axi-symmetric arrangement.

4. A trunking system in which a plurality of trunks give service to several overlapping and interlinked sub-groups of lines, each sub-group of lines having access to a part only of the total number of trunks and the

numbers of the trunks assigned to each sub-group of a set of sub-groups being so chosen and the order of selection being such that when the numbers are arranged in columns in accordance with their order of selection, the numbers will be symmetrical upon either side of an axis drawn diagonally across the rectangular space occupied by the numbers.

5. A trunking system in which a plurality of trunks give service to several overlapping and interlinked sub-groups of lines, each sub-group of lines having access to a part only of the total number of trunks, the sub-groups being arranged in sets, the numbers of the trunks assigned to each sub-group of one set and their order of selection being such that the numbers will fall into an axi-symmetric arrangement, and the numbers assigned to the sub-groups of other sets being similarly arranged but their order of selection being changed so that when a given number of the total number of trunks is busy, the probability of one set of trunks of the given number being busy will be at least approximately the same as the probability that any other set of equal number will be busy.

In testimony whereof, I have signed my name to this specification this 23rd day of February, 1921.

EDWARD C. MOLINA.