

March 10, 1925.

1,528,920

R. M. FOSTER

TRUNKING SYSTEM

Filed Feb. 26, 1921

2 Sheets-Sheet 1

Fig. 3

Permutations of 4 Trunks 2 at a Time

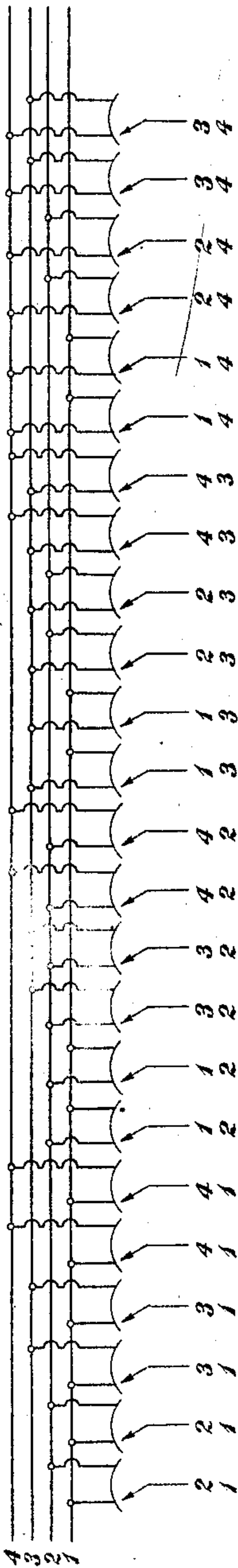


Fig. 2

Permutations of 4 Trunks 3 at a Time

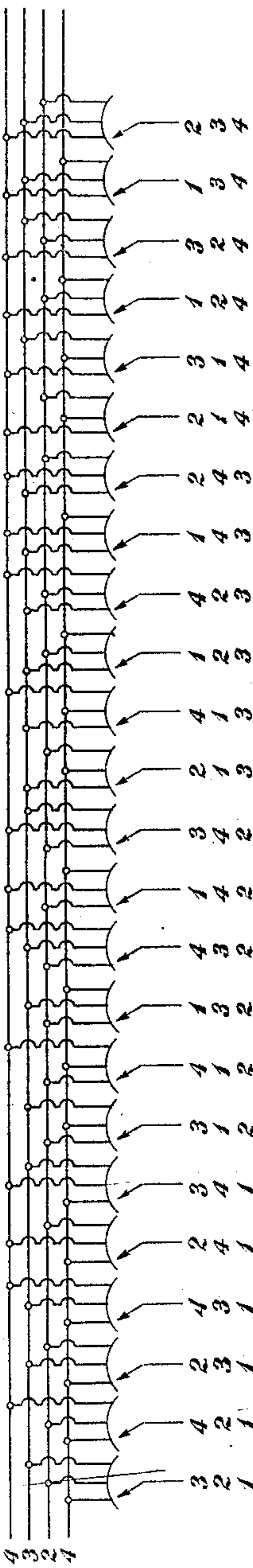
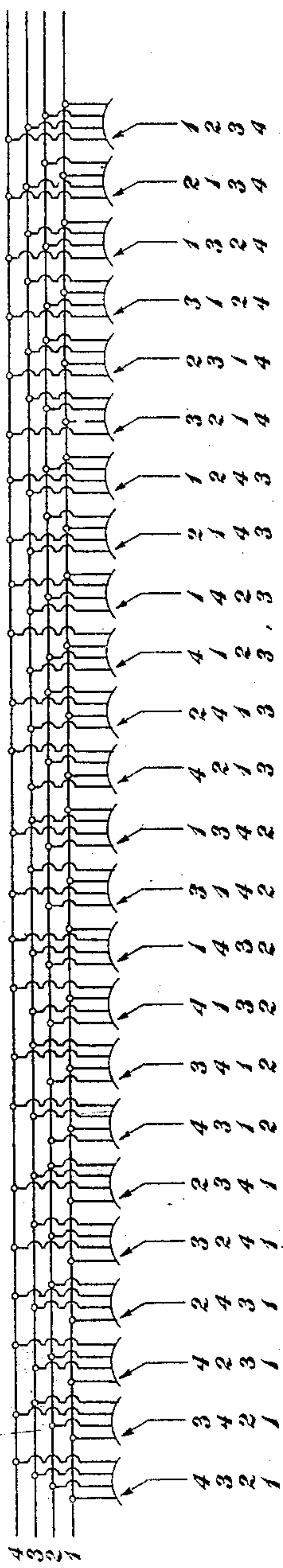


Fig. 1

Permutations of 4 Trunks 4 at a Time



Inventor  
R. M. Foster

By his Attorney

*guy*



**March 10. 1925.**

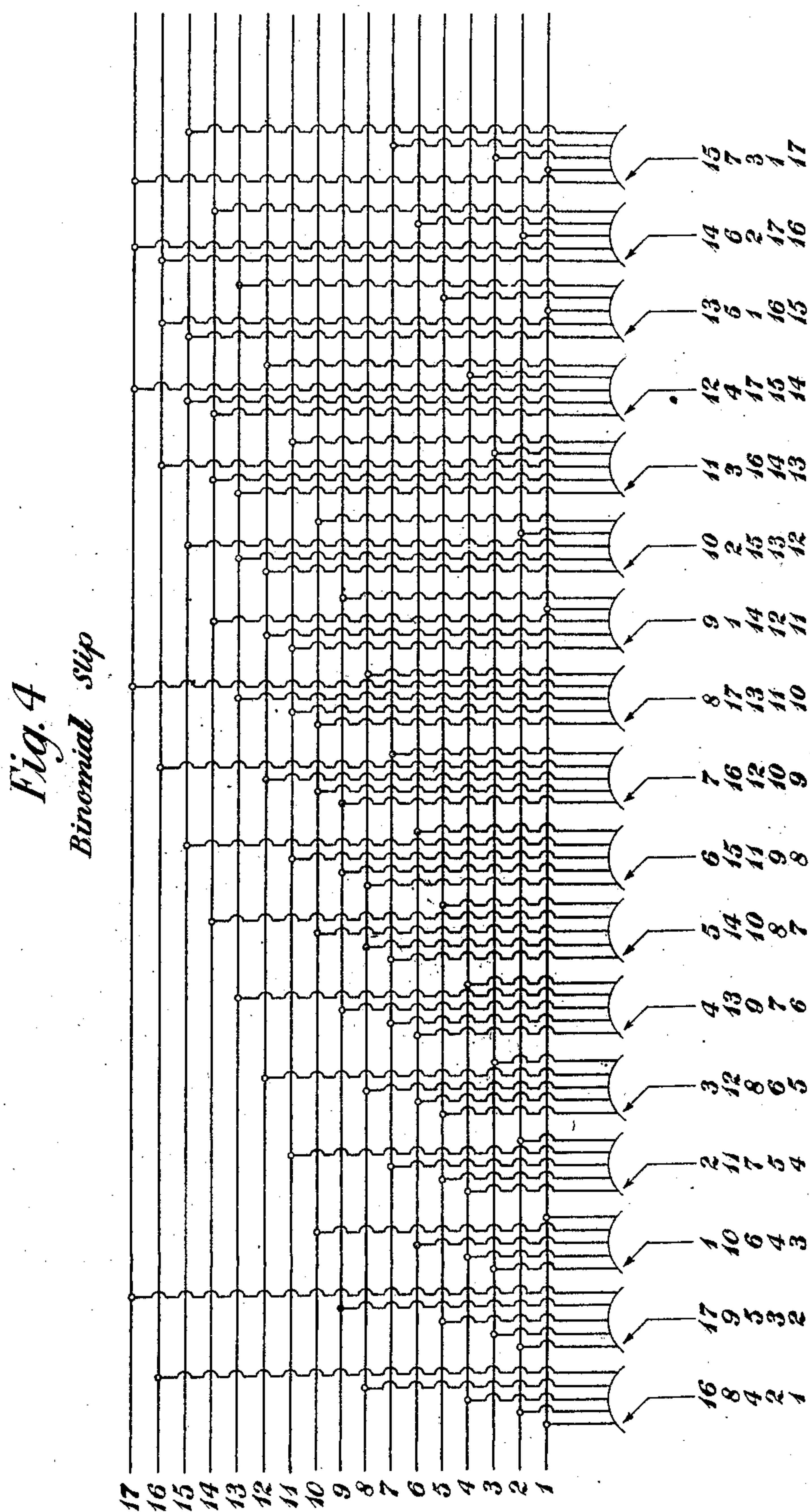
**1,528,920**

**R. M. FOSTER**

## TRUNKING SYSTEM

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2 Sheets-Sheet 2



Inventor

*R. M. Foster*

By his Attorney

94704



## UNITED STATES PATENT OFFICE.

RONALD M. FOSTER, OF BROOKLYN, NEW YORK, ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

## TRUNKING SYSTEM.

Application filed February 26, 1921. Serial No. 448,056.

*To all whom it may concern:*

Be it known that I, RONALD M. FOSTER, residing at Brooklyn, in the county of Kings and State of New York, have invented certain Improvements in Trunking Systems, of which the following is a specification.

This invention relates to signaling circuits and more particularly to methods and means for trunking between different subscribers in a telephone exchange or between subscribers in different exchanges.

Heretofore in providing trunking arrangements for handling the traffic originated by subscribers' lines, particularly where the connections were established by means of switching machinery, two general methods have been proposed. One method involved providing a sufficient number of trunks to handle all of the traffic originated by the subscribers' lines involved and establishing connections between the calling subscribers' lines and the trunks through a switching arrangement equipped with a number of selecting points corresponding to the number of trunks necessary to handle the traffic. The other method consists in dividing the subscribers' lines into groups and providing trunks for each group sufficient in number to handle the traffic originating in the group.

The first of these methods possesses the advantage that the number of trunks necessary to handle the traffic without an unduly large probability of a call being lost due to all of the trunks being busy will be a minimum. The method is subject to the serious disadvantage, however, that each subscriber must have available a switching arrangement having a number of switching terminals sufficient to connect with each and all of the trunks, and where the traffic is so large as to require a great number of trunks the expense involved in providing a switching arrangement having the necessary capacity becomes prohibitive.

The advantage of the second method is that the subscribers' groups may be made sufficiently small so that the number of trunks necessary to handle the traffic originating in each group will be within the capacity limits of a relatively small and inexpensive switching arrangement; that is, a switching arrangement having a comparatively small number of terminal points. This

method is subject to the disadvantage, however, that while economic in the matter of switching arrangements or machinery it is prodigal in the use of trunks, since the total number of trunks required to handle the traffic from all of the subscribers is greater than would be the case in the first method referred to above, due to the fact that the ratio of trunks to subscribers varies inversely with the number of subscribers having access to the trunks.

It is one of the principal objects of the present invention to provide a scheme for multiplying trunks such that the use of switches having a small number of switching points will be permitted without requiring a large ratio of the total number of trunks to the total number of subscribers.

The above object, as well as other objects more fully appearing hereinafter, are accomplished by the use of what is herein termed a "random slip" multiple. The nature of this multiple will now be clear from the following description of the invention, when read in connection with the accompanying drawing, Figure 1 of which is a schematic diagram of a so-called "straight" multiple, Figs. 2 and 3 inclusive, of which are schematic diagrams of a trunking system in which the multiples are "slipped" upon a purely random basis; while Fig. 4 is a schematic diagram of a modification in which the "slipping" of the multiple is worked out in accordance with a definite law which closely approximates the conditions involved in a perfect "random slip".

In order to understand the principle underlying a "random slip" multiple let us consider, for example, a group of 100 trunks carrying the traffic originated by subscribers' lines. Assume that each subscriber's line is equipped with a 100 point selector, thereby giving each line access to the entire 100 trunks. Let the order in which the trunks are connected to the different selectors be varied as much as possible. A perfectly heterogeneous arrangement would be one such that when 37 (for example) of the 100 trunks are busy a specified set of 37 trunks are as likely to be the trunks actually busy as any other set of 37 trunks.

The arrangement so far described corresponds to the first method of trunking outlined above and has the advantages and dis-



advantages previously discussed. Let us now calculate for such a promiscuously connected multiple the probability that more than 20 terminals (for example) will have to be hunted over by a selector before an idle trunk is obtained. The load to be carried may be adjusted so that the probability comes out any desired value as, for example .001. It is evident that under these conditions the last 80 terminals of every group may be disconnected from the trunks without appreciably changing the efficiency of the group of 100 trunks. Consequently each partially wired selector may be replaced by a 20 point switch having access to the same trunks in the same order as the first 20 trunks hunted over by the corresponding 100 point selector. Such an arrangement will involve what is herein termed a "random slip" multiple and has the obvious advantage that a 20 point switch may be used having access to 20 trunks previously selected at random from the total number of trunks without increasing the total number of trunks required to handle the traffic from all of the subscribers.

Considering the same arrangement from another view point, assume 5 independent groups of lines equipped with selectors. Let each group of selectors be wired to a group of 20 trunks. The total number of trunks required will therefore be 100. If we bring together these isolated groups of selectors and groups of trunks by a partial interchange of trunks, or in other words, by arranging matters so that each one of the 5 groups of selectors exchanges some of its trunks for some of the trunks assigned to each of the other 4 groups, we will have a promiscuously connected multiple of 100 trunks to some 20 of which each selector will have access. If this interchanging were done thoroughly the load carried by the total number of trunks with a given probability of lost calls would be appreciably greater than the load carried by the same number of trunks when divided into five isolated groups each of which is accessible to a 20 point selector.

In order that the advantages inherent in the "random slip" multiple may be more apparent a mathematical solution of the problem involved will now be given. The problem may be stated as follows:

A group of T trunks handles the traffic originated by n sub-groups of lines or switches.

Sub-group #1 of lines or switches has access to a specified number  $t_1$  of the T trunks.

Sub-group #2 of lines or switches has

access to a specified number  $t_2$  of the T trunks. Some of these  $t_2$  trunks are not included in the  $t_1$  trunks assigned to sub-group #1.

Sub-group #3 of lines or switches has access to a specified number  $t_3$  of the T trunks. Some of these  $t_3$  trunks are not included in the  $t_1$  trunks assigned to sub-group #1 and some are not included in the  $t_2$  trunks assigned to sub-group #2. The trunks  $t_1$  and  $t_2$  together may, however, include all the  $t_3$  trunks.

Sub-group #n of lines or switches has access to a specified number  $t_n$  of the T trunks. These  $t_n$  trunks are not identical with either the  $t_1, t_2, t_3, \dots$  or  $t_n-1$  trunks but they are all included in the totality of  $t_1+t_2+\dots+t_n-1$  trunks.

Thus, no sub-group of lines or switches has the exclusive use of a set of trunks chosen from among the T trunks and on the other hand no two or more sub-groups use identically the same trunks.

It is not assumed above that numerically  $t_1=t_2=t_3=\dots=t_n-1=t_n$  although this will probably be the case in practice.

Such a group of T trunks giving service to several over-lapping or inter-linking sub-groups of lines, each of which sub-groups makes partial use of the T trunks, may be defined as a "random slip" multiple provided the following statement can be made with reference to it . . . when S of the T trunks are busy they are as likely to be one specified set of S trunks as another specified set of S trunks.

In order to solve this problem let

$P_t$  = probability that a subscriber originating a call from a sub-group having access to a  $t$  of T trunks fails to get an idle trunk.

Assume that at the moment the call is made S of the T trunks are busy. The probability of the existence of this assumed condition is

$$\frac{A^S e^{-A}}{S!}$$

where A is the average load carried by the group T.

The probability that the assumed S busy trunks embrace the particular  $t$  trunks to which the subscriber under consideration has access is

$$\frac{1/T-t}{1/S-t \cdot 1/T-S} \bigg/ \frac{1/T}{1/S \cdot 1/T-S} = \frac{1/S \cdot 1/T-t}{1/S-t \cdot 1/T}$$

Therefore, the compound probability that S of the T trunks are busy and that these S trunks include the  $t$  trunks under consideration is

$$\left( \frac{A^S e^{-A}}{S!} \right) \left( \frac{1/S \cdot 1/T-t}{1/S-t \cdot 1/T} \right) = \left( \frac{A^t \cdot 1/T-t}{1/T} \right) \left( \frac{A^{S-t} e^{-A}}{(S-t)!} \right)$$



Therefore

$$P_t = \left( \frac{A^t / T - t}{/T} \right) \sum_{S=t}^{S=T-1} \left( \frac{A^{S-t} e^{-A}}{/S-t} \right) + P(T,A)$$

Where

$$P(T,A) = e^{-A} \left[ \frac{A^T}{/T} + \frac{A^{T+1}}{/T+1} + \frac{A^{T+2}}{/T+2} \cdot \cdot \cdot \text{ad infinitum} \right]$$

=probability that all T trunks are busy.

Now

$$\sum_t^{T-1} \frac{A^{S-t} e^{-A}}{/S-t} = \sum_0^{T-t-1} \frac{A^X e^{-A}}{/X} = 1 - P(T-t,A)$$

Therefore, finally,

$$P_t = \frac{A^t / T - t}{/T} \left[ 1 - P(T-t,A) \right] + P(T,A)$$

The preceding equation gives the probability of lost calls in terms of the average load carried by the group of trunks, the total number of trunks involved, and the number of trunks assigned to each group. With this equation as a basis Tables I and II have been prepared to show the saving involved in a "random slip" multiple as compared with a "straight" multiple, . . . that is, a multiple in which the trunks are divided into independent groups, each group accessible to a switch having a correspondingly small number of terminal points.

In a system where 20 point sender selectors give access to the senders of an automatic switching exchange the saving in senders when the "random slip" arrangement is used instead of the "straight" multiple will be as indicated in Table I. It can be readily seen from the table that with the "random slip" arrangement the number of senders required to carry a given load is considerably smaller than the number of senders necessary for the same load with the "straight" multiple. For example, it is apparent that if the allowable probability of lost calls is .001 and the trunks are divided into ten groups of 20 each the "random slip" multiple arrangement will require only 137 senders as against 200 senders required by the "straight" multiple arrangement for an average load of 89.6, the expression "average load" being here taken to mean the average number of connections existing at any one time for the entire group of trunks. The saving just referred to amounts to 31.5%. On the other hand, in a case where but one group is involved and the average load is 8.96, 20 senders will be required in both cases and the "random slip" multiple presents no advantages over the "straight" multiple, as in this limiting case the "random slip" multiple becomes a "straight" multiple.

Table II shows the saving in final switches which would result from a "random" arrangement of trunks to finals in a panel system where the incoming multiple is divided into groups of 24 trunks each. Here also a smaller number of switches can carry the same load as a larger number with the "straight" multiple or inversely, the same number of switches in the "random" arrangement can take care of a larger load than in the "straight" multiple arrangement.

Table I.

| Average load. | Straight multiple. |                         | Random multiple.        | Per cent saving in senders. | Probability. |
|---------------|--------------------|-------------------------|-------------------------|-----------------------------|--------------|
|               | Number of groups.  | Total senders required. | Total senders required. |                             |              |
| 8.96.....     | 1                  | 20                      | 20                      | 0                           | 0.001        |
| 17.92.....    | 2                  | 40                      | 34                      | 15.0                        |              |
| 26.88.....    | 3                  | 60                      | 47                      | 21.7                        |              |
| 35.84.....    | 4                  | 80                      | 60                      | 25.0                        |              |
| 44.80.....    | 5                  | 100                     | 73                      | 27.0                        |              |
| 89.60.....    | 10                 | 200                     | 137                     | 31.5                        |              |
| 134.40.....   | 15                 | 300                     | 199                     | 33.7                        |              |
| 179.20.....   | 20                 | 400                     | 262                     | 34.5                        |              |
| 224.....      | 25                 | 500                     | 326                     | 34.8                        |              |

Table II.

| Average load. | Straight multiple. |                        | Random multiple.       | Per cent saving in finals. | Probability. |
|---------------|--------------------|------------------------|------------------------|----------------------------|--------------|
|               | Number of groups.  | Total finals required. | Total finals required. |                            |              |
| 14.1.....     | 1                  | 24                     | 24                     | 0                          | 0.01         |
| 28.2.....     | 2                  | 48                     | 43                     | 10.4                       |              |
| 42.3.....     | 3                  | 72                     | 60                     | 16.7                       |              |
| 56.4.....     | 4                  | 96                     | 78                     | 18.8                       |              |
| 70.5.....     | 5                  | 120                    | 96                     | 20.0                       |              |
| 11.65.....    | 1                  | 24                     | 24                     | 0                          | .001         |
| 23.30.....    | 2                  | 48                     | 41                     | 14.6                       |              |
| 34.95.....    | 3                  | 72                     | 57                     | 20.8                       |              |
| 46.60.....    | 4                  | 96                     | 73                     | 22.9                       |              |
| 58.25.....    | 5                  | 120                    | 90                     | 25.0                       |              |

It is obvious, of course, that multiple arrangements involving the "random slip"



principle may be embodied in various forms. In Figs. 2 and 3 are shown perfect "random slips" based on all of the 24 possible permutations of the numbers 1, 2, 3 and 4, the permutations being shown in Fig. 1, which illustrates in schematic form the permutations of four trunks taken four at a time in accordance with the first method outlined at the beginning of this specification. In all three figures a group of  $T=4$  trunks is shown, together with  $n=24$  sub-groups of subscribers' lines. Each sub-group is represented by one line equipped with a selector giving the line access to four trunks in the case of Fig. 1, three trunks in the case of Fig. 2 and two trunks in the case of Fig. 3. In Fig. 1 and the succeeding figures the arrows may therefore be taken to represent the wipers of the switches, each of which is available to one or more subscribers and each switch having terminal points equal in number to the vertical lines connecting the arc of a circle representing the contact points of the switch with the horizontal lines representing the trunks. The numbers below the arrows representing the wipers of the switch indicate the order in which the switches obtain access to the trunks.

If, in the case of the arrangement of Fig. 1 a determination is made of the probability that more than 3 terminals will have to be hunted over by the selector before an idle trunk is obtained and it be found that this probability is within the allowable probability of lost calls, the connection leading to the last trunk by each switch may be omitted from the switches in Fig. 1, in which case we get the perfect "random slip" arrangement of Fig. 2, which is based upon permutations of 4 trunks taken 3 at a time.

Similarly, if it be found that the allowable probability will permit of dispensing with connections to the last two trunks selected by each switch in Fig. 1, we may obtain the perfect "random slip" multiple illustrated in Fig. 3 which involves permutations of 4 trunks taken two at a time.

The slip arrangements illustrated in Figs. 2 and 3 are based upon all possible permutations of the numbers involved. Consequently, each of these slips is a perfect random slip, since obviously no particular set of trunks is more likely to be busy than any other set involving an equal number of trunks. In practice, however, it would not be feasible to use slip arrangements based on all possible permutations, where the total number of trunks  $T$  is large. For example, a slip multiple of  $T=25$  trunks, in which each line is equipped with a  $t=5$  point selector would require the lines to be divided into  $25 \times 24 \times 23 \times 22 \times 21 = 6,375,600$  sub-groups if every possible permutation of 25 things taken 5 at a time is to appear. It

therefore becomes desirable to devise a system in which only a part of all of the possible permutations will be used and in which the selected permutations will be so chosen that no particular set of trunks is more likely to be busy than any other set involving the same number of trunks. In selecting a limited number of the total possible permutations, it is preferable to select a minimum amount of overlapping i. e. permutations such that any trunk having a given number will appear in as few chosen permutations as possible.

Fig. 4 illustrates a practical form of slip arrangement obtained by following the principles above discussed and in which a very thorough intermixture of the trunks is obtained without dividing the lines into an abnormal number of sub-groups. This figure illustrates what is herein called a "binomial" slip arrangement as applied to a system, involving 17 trunks and 5 point selectors. In accordance with this arrangement, the trunks assigned to the first sub-group of lines to the left, are those having the binomial numbers 1, 2,  $4=2^2$ ,  $8=2^3$ , and  $16=2^4$ . The numbers of the trunks to be assigned to the other groups are obtained by writing the numbers from 2 to 17 as the numbers of the first trunks to be selected by the groups to the right of that having assigned to it the numbers 1, 2, 4 etc. Similarly the numbers of the second set of trunks to be selected is obtained by writing successive numbers beginning with 3, while the numbers of the third set are obtained by writing successive numbers beginning with 5, etc. In each instance, when the number 17 is reached the succeeding number will be 1. An analysis of this trunking arrangement shows that while it is not a perfect random slip arrangement, it conforms very closely to the requirements of the random slip multiple.

It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated, without departing from the spirit of the invention as defined in the following claims.

What is claimed is:

1. A trunking system in which a plurality of trunks are provided for handling the calls originating from all of the subscribers' lines involved, the subscribers' lines being divided into sub-groups each having access to a number of trunks less than the total number provided, the trunks assigned to the several sub-groups being interchanged so that the sub-groups overlap each other, the numbers of the trunks assigned to the first sub-group and the order of selection thereof being obtained by assigning to said sub-group trunks having the binomial numbers 1, 2,  $4=2^2$  etc., and the numbers of



the trunks assigned to the other sub-groups and the order of selection thereof being obtained by adding successive numbers to the numbers assigned to the first sub-group.

5 2. A trunking system in which a plurality of trunks gives service to several overlapping and inter-linked sub-groups of lines, each sub-group of lines having access to a part only of the total number of trunks, the  
10 numbers of the trunks assigned to the first sub-group being obtained by assigning to said sub-group the binomial numbers 1, 2, 4= $2^2$  etc., and the numbers of the trunks assigned to the other sub-groups being ob-  
15 tained by adding successive numbers to the numbers assigned to the first sub-group.

3. A trunking system in which a plurality of trunks gives service to several overlapping and inter-linked sub-groups of lines,  
20 each sub-group of lines having access to a part only of the total number of trunks, the

numbers of the trunks assigned to the first sub-group and their order of selection being obtained by assigning to said sub-group the trunks having the binomial numbers 1, 2, 25 4= $2^2$  etc., and the numbers of the trunks assigned to the other sub-groups and their order of selection being determined by assigning to the other sub-groups trunks whose numbers are obtained by adding successive  
30 numbers to the numbers assigned to the first sub-group, whereby when a given number of the total number of trunks are busy, the probability of one set of trunks of the given number being busy will be substantially the  
35 same as the probability that any other set of equal number will be busy.

In testimony whereof, I have signed my name to this specification this 23rd day of February, 1921.

RONALD M. FOSTER.