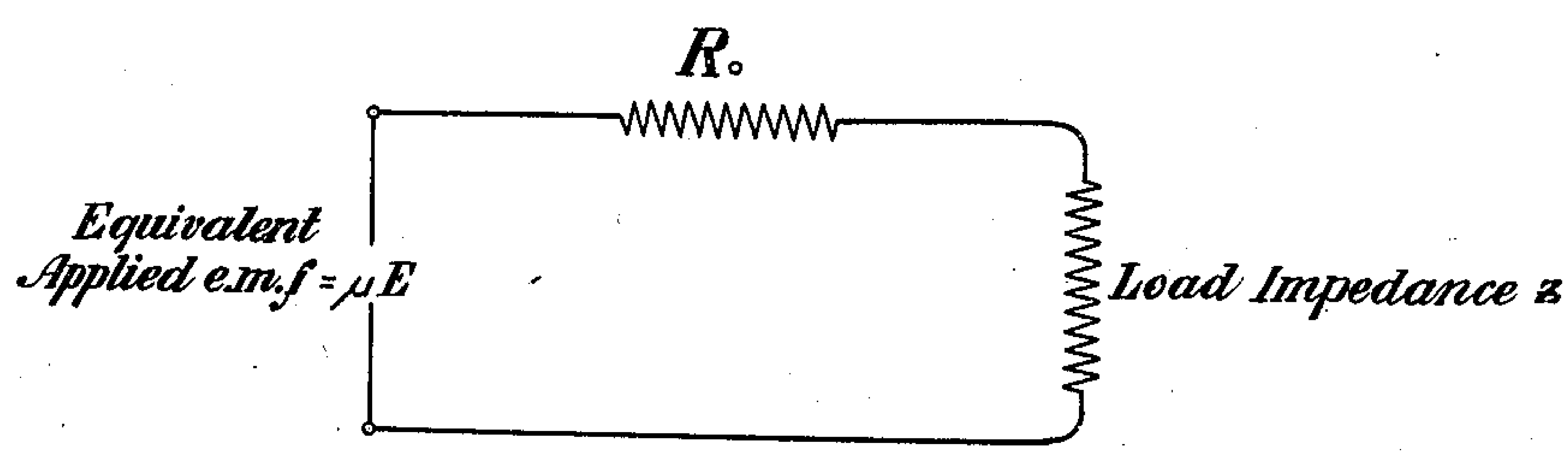
Mar. 13, 1923.

1,448,702

J. R. CARSON

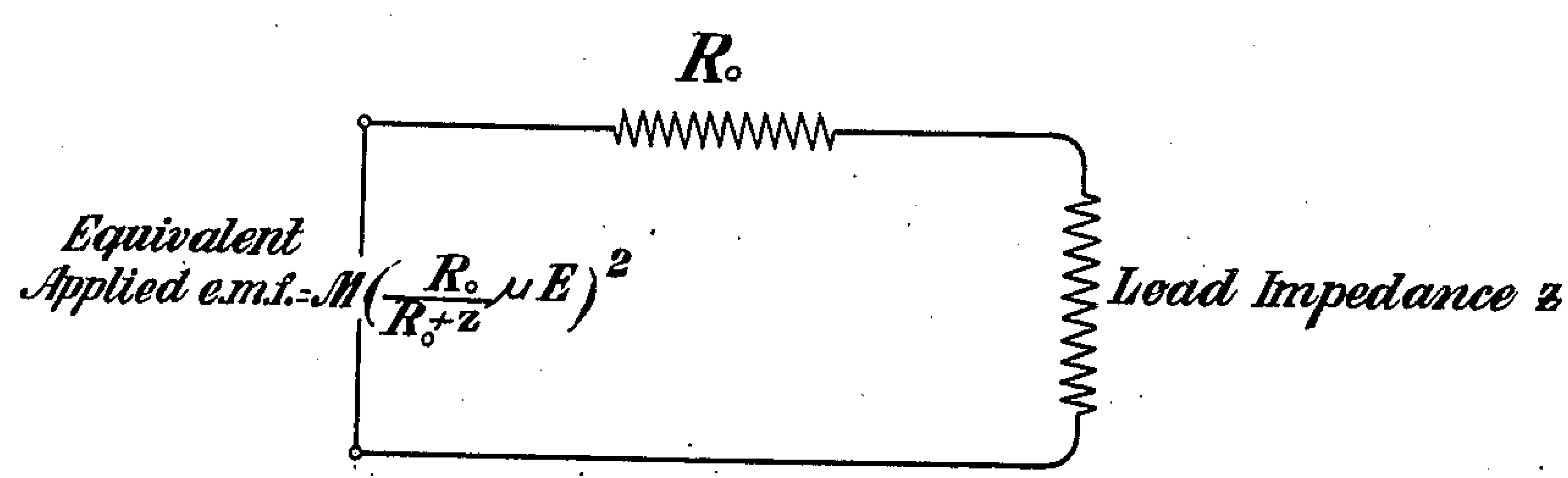
TRANSLATING CIRCUITS

Filed July 10, 1920



Equivalent Amplifier Circuit

Fig. 1



Equivalent Modulator Circuit

Fig. 2

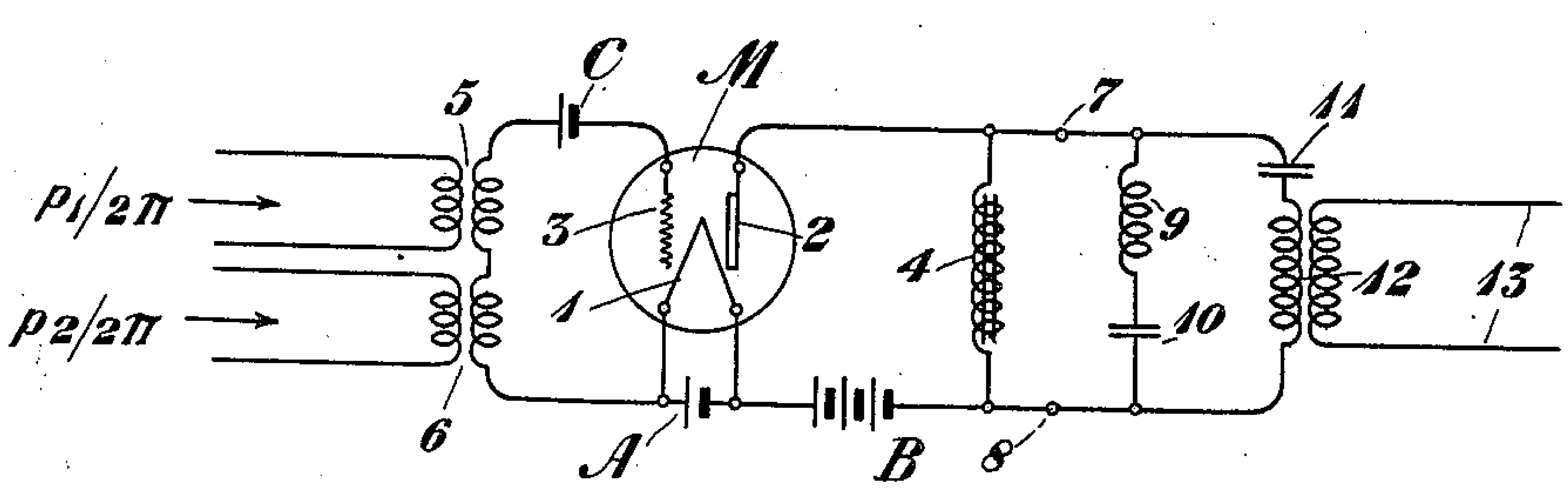


Fig. 3

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UNITED STATES PATENT OFFICE.

JOHN R. CARSON, OF MONTCLAIR, NEW JERSEY, ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH CO., A CORPORATION OF NEW YORK.

TRANSLATING CIRCUITS.

Application filed July 10, 1920. Serial No. 395,308.

To all whom it may concern:

Be it known that I, JOHN R. CARSON, residing at Montclair, in the county of Essex and State of New Jersey, have invented certain Improvements in Translating Circuits, of which the following is a specification.

This invention relates to translating devices, and more particularly to translating devices in the nature of modulators or detectors.

One of the objects of this invention is to increase the output energy of a modulator, preferably of the vacuum tube type, although the invention is equally applicable to a vacuum tube detector.

Other and further objects of the invention will be clear from the following description when read in connection with the accompanying drawing, Figures 1 and 2 of which are schematic diagrams indicating the electrical equivalents of certain types of vacuum tube circuits, while Fig. 3 is a diagram of a circuit arrangement of a vacuum tube modulator embodying the principles of this invention.

It is a well-known fact that if an electromotive force E is impressed on the input circuit of a three-element vacuum tube amplifier of amplification constant μ and an external or load impedance z, its behavior as an amplifier may be correctly described by postulating an electromotive force μE impressed on the load impedance z through a resistance R_o, where R_o is the "internal resistance" of the tube, and is given by the formula

$$\frac{1}{R_o}=\frac{\theta 1}{\theta E_b} \quad (1)$$

I being the plate current and E_b being the steady plate-filament potential difference. (See articles by H. J. van der Bijl in the Physical Review of September, 1918, and in The Proceedings of the Institute of Radio Engineers of April, 1919, as well as an article by John R. Carson in The Proceedings of the Institute of Radio Engineers of April, 1919).

This law has led to the concept of the equivalent circuit of the vacuum tube amplifier, indicated schematically in Fig. 1, and has been a very valuable aid in interpreting the behavior of amplifiers of this character and in designing the associated circuits. It may also be shown that a vacuum tube, when functioning as a modulator, may be represented by an "equivalent circuit" of almost equal simplicity. The equivalent modulator circuit may be inferred at once from the following theorem:

If a voltage E is impressed on the input terminals of a three-element vacuum tube feeding into a load impedance z, the operation of the device as a modulator may be correctly described by postulating an E. M. F.

$$M\left(\frac{\mu R_o}{R_o+z}E\right)^2 \quad (2)$$

impressed on the load impedance z through a resistance R_o. In equation (2) M is the modulation factor, a physical parameter of the tube, and is given by the equation

$$M=\left(\frac{1}{2R_o}\right)\cdot\left(\frac{\theta R_o}{\theta E_b}\right) \quad (3)$$

The equivalent circuit is shown schematically in Fig. 2.

The law formulated by this theorem and the character of the approximations involved are easily deduced from the treatment of the problem developed in the above mentioned article by John R. Carson in The Proceedings of the Institute of Radio Engineers, and for full details of the solution reference may be had to said article. For the purposes of the present application it will be sufficient to merely outline the method of solution.

Let J represent the variable output current of a three-element vacuum tube, e the variable electromotive force impressed on the input terminals, and v the variable potential difference across the load impedance z. It follows at once that J is a function of the expression $\mu e+v$ and this function may be expressed in a power series expansion as follows:

$$J=P_1(\mu e+v)+P_2(\mu e+v)^2+\text{--} \quad (4)$$

in which P_1, P_2; P_3 and P_4 are different parameters of the tube.

Let us now set

$$\begin{aligned}J&=J_1+J_2+J_3+\text{--}\\ v&=v_1+v_2+v_3+\text{--}\end{aligned} \quad (5)$$

The significance of the series formulation of

J and v in the above equation will become apparent later. At present it is sufficient to observe that the component terms of the series (5) are unrestricted except by the necessity of satisfying equation (4) and the relations obtaining between J and v,—namely, that v is the potential drop with sign reversed of the current J through the load impedance z.

Now let the components of the J and v series of equation (5) be determined to satisfy an assumed system of equations as follows:

$$\begin{aligned} J_1 &= P_1(\mu e+v_1) \\ J_2 &= P_1 v_2 + P_2(\mu e+v_1)^2 \\ J_3 &= P_1 v_3 + 2P_2(\mu e+v_1)v_2 + P_3(\mu e+v_1)^3 \quad (6) \\ J_4 &= P_1 v_4 + 2P_2\left[(\mu e+v_1)v_3+\frac{1}{2}v_2^2\right] \\ &\quad + 3P_3(\mu e+v_1)^2 v_2 \\ &\quad + P_4(\mu e+v_1)^4 \end{aligned}$$

Direct addition by columns of equation (6), regard being had to the identities of equation (5), shows that equation (6) satisfies the requirements of equation (4), and is, therefore, a formal solution.

The coefficients in equation (4) will have a clear physical significance if we express P_1 in terms of the first differential; thus

$$P_1=\frac{1}{1'}\left(\frac{\theta 1}{\theta E_b}\right) \quad (7)$$

Comparing this expression with the first term of equation (4), it will be seen that by definition it is equal to $\frac{1}{R_o}$, so that we may write:

$$P_1=\left(\frac{\theta 1}{\theta E_b}\right)_o=\frac{1}{R_o} \quad (8)$$

which is equation (1). Writing in a similar manner expression P_2 in terms of the second differential, we have

$$P_2=\frac{1}{2'}\left(\frac{\theta^2 1}{\theta E_b^2}\right)=-\frac{1}{2}\frac{1}{R_o^2}\frac{\theta R_o}{\theta E_b}=\frac{M}{R_o} \quad (9)$$

In equations (7), (8) and (9) E_b is the voltage of the battery in the plate circuit of the tube, and M has already been defined as the modulation factor of the tube. It will be seen that, disregarding the sign which in this instance has no significance, the value of M as expressed in equation (3) follows at once from equation (9).

Combining equations (5), (6), (8) and (9), we have:

$$\begin{aligned} R_oJ_1-v_1 &= \mu e \\ R_oJ_2-v_2 &= R_oP_2(\mu e+v_1)^2 \quad (10) \\ R_oJ_3-v_3 &= 2R_oP_2(\mu e+v_1)v_2+R_oP_3(\mu e+v_1)^3 \end{aligned}$$

Bearing in mind the assumed significance of the v series of equation (5), it follows at once that any equation, as, for example, the n^{th} equation of group (10), is simply that of the current J_n in a circuit of impedance R_o+z in response to the voltage given by the right hand side of the equation. Now writing $v_1=-zJ_1$ and $v_2=-zJ_2$ (relations which follow at once from the above physical considerations), the first equation of group (10) may be written:

$$(R_o+z)J_1=\mu e \quad (11)$$

and the second equation of the group (10) may be written:

$$(R_o+z)J_2=M\left(\frac{R_o}{R_o+z}\mu e\right)^2 \quad (12)$$

Equation 11 is simply the equation for the current J_1 flowing in the circuit of impedance R_o+z in response to the impressed electromotive force μe, and therefore, identifying J_1 with the amplification current, leads at once to the law of the equivalent amplifier circuit as expressed in the early part of this specification, and as illustrated in Fig. 1. Similarly, equation (6) is simply the equation for the current J_2 flowing in a circuit of impedance R_o+z in response to an impressed E. M. F.

$$M\left(\frac{R_o}{R_o+z}\cdot\mu e\right)^2$$

As shown in the paper by John R. Carson already referred to, the component current J_2 is that which represents the modulated or detected output current; and consequently equation (12) is simply a statement of the law of the equivalent modulator circuit set forth in the early part of this specification, and illustrated schematically in Fig. 2.

The above outline of the method of reasoning presented by John R. Carson in the Radio Institute article already referred to will be sufficient to indicate to one skilled in the art how the law of the modulator formulated by the theorem set forth in the early part of this specification may be obtained. A simple problem will now be dealt with to illustrate the application of the concept of the equivalent modulator circuit in deducing certain principles which have a direct and important bearing on the design of the circuits of the vacuum tube modulator of this invention.

Let the impressed electromotive force E be the sum of two sinusoidal functions; thus,

$$E=E_1\sin p_1t+E_2\sin p_2t \quad (13)$$

In this equation the first component of E may be regarded as the carrier wave and the second component as the signal wave to be impressed upon a modulator. In substituting this expression for E in equation (2) to evaluate the fictitious electromotive force supposed to be acting in the equivalent modulator circuit of Fig. 2, it is essential that E be written down as a complete real time function, since the operation of squaring is in effect a frequency transformation. By virtue of equation (13) we have:

$$\frac{\mu R_o}{R_o+z}E=\mu R_o\left(\frac{E_1}{Z(p_1)}\sin(p_1t-\theta_1)+\frac{E_2}{Z(p_2)}\sin(p_2t-\theta_2)\right)\quad(14)$$

where Z (p_1) and Z (p_2) are the absolute values of the impedance R_o+z to currents of frequencies $\frac{p_1}{2\pi}$ and $\frac{p_2}{2\pi}$ respectively and θ_1 and θ_2 are the corresponding phase angles. By aid of equation (14) formula (2) becomes, after simplification and a rearrangement

$$\frac{M}{2}\Big\{V_1^2[1-\cos(2p_1t-2\theta_1)]+V_2^2[1-\cos(2p_2t-2\theta_2)]+2V_1V_2[\cos(p_1-p_2)t-(\theta_1-\theta_2)+\cos(p_1+p_2)-(\theta_1-\theta_2)]\Big\}\quad(15)$$

In the above expression

$$V_1=\frac{\mu R_oE_1}{Z(p_1)}\text{ and }V_2=\frac{\mu R_oE_2}{Z(p_2)}$$

Equation (15) represents the effective electromotive force acting in the equivalent modulator circuit and the current corresponding is calculated by the usual methods. The modulated output current is evidently that corresponding to the term of equation (15) which includes the factor V_1V_2; it consists of the following two components:

$$\left(M(\mu R_o)^2\frac{E_1E_2}{Z(p_1)\cdot Z(p_2)}\right)\cdot\left(\frac{1}{Z(p_1-p_2)}\right)\text{of frequency}\frac{(p_1-p_2)}{2\pi}\quad(16)$$

$$\left(M(\mu R_o)^2\frac{E_1E_2}{Z(p_1)\cdot Z(p_2)}\right)\cdot\left(\frac{1}{Z(p_1+p_2)}\right)\text{of frequency}\frac{(p_1+p_2)}{2\pi}\quad(17)$$

In equations (16) and (17) the second factor of each expression represents the absolute values of impedance R_o+z at frequencies

$$\frac{p_1-p_2}{2\pi}\text{ and }\frac{p_1+p_2}{2\pi}$$

respectively. The first factor represents the electromotive force impressed upon said impedance as expressed by the term

$$\frac{MV_1V_2}{2}$$

in equation (15).

Suppose now that by proper design of an ordinary modulating arrangement, the load impedance acts like a pure resistance R at the frequencies corresponding to p_1, p_2, (p_1-p_2), and (p_1+p_2). It is at once apparent, by substituting R_o+R for z in equations (16) and (17), that the modulated currents expressed by each of said equations will be proportional to

$$\frac{R_o^2}{(R_o+R)^3}\quad(18)$$

and the modulated energy W_1 delivered to the load impedance will consequently be proportional to

$$\frac{RR_o^4}{(R_o+R)^6}\quad(19)$$

This expression is a maximum when

$$R=\frac{R_o}{5}$$

and when this consideration is satisfied, becomes

$$\frac{\left(\frac{5}{6}\right)^6}{5R_o}\quad(20)$$

Let us now assume that, in accordance with the present invention, the modulating arrangement be so designed that the load impedance acts like a pure resistance R at the frequencies corresponding to p_1, (p_1-p_2), and (p_1+p_2), but acts like a short circuit to currents of the signal frequency $\frac{p_2}{2\pi}$. This condition may be practically realized when the carrier frequency is sufficiently large as compared with the signal frequency. In this case the term $Z(p_2)$ becomes R_o instead of R_o+R, as in the preceding case, and consequently the modulated currents are proportional to

$$\frac{R_o}{(R_o+R)^2} \quad (21)$$

and the modulated energy W_2 is proportional to

$$\frac{RR_o^2}{(R_o+R)^4} \quad (22)$$

This expression is a maximum when

$$R=\frac{R_o}{3}$$

and when this condition is satisfied, becomes

$$\frac{\left(\frac{3}{4}\right)^4}{3R_o} \quad (23)$$

A comparison of equation (20) and (23) shows that for the same applied electromotive forces the modulated energy outputs W_1 and W_2 are related by

$$\frac{W_1}{W_2}=\left(\frac{3}{5}\right)\left(\frac{4}{3}\right)^4\left(\frac{5}{6}\right)^6 \quad (24)$$
$$=0.635 \text{ or } W_2=1.575W_1.$$

It follows at once that a proper design of the associated circuits to satisfy the condition laid down in the second case results in a gain of 57% in the available modulated energy, as compared with the first arrangement. Another immediate deduction from formulas (16) and (17) is that, although we are concerned only in transmitting radio or high frequency modulated currents, it is necessary to provide a low impedance path in the output circuit for the low frequency signaling currents $\frac{(p_2)}{2\pi}$; otherwise a serious loss in the modulated energy results.

As a consequence of these deductions, the modulating circuit may be arranged in accordance with the present invention, as shown in Fig. 3, in which M designates a modulating vacuum tube having a heated filament 1, a plate 2, and a grid 3. The filament is heated by a direct current source A, and a source of plate current B is connected between the filament 1 and the plate 2 through a choke coil 4, which may be designed to have practically an infinite impedance for alternating currents of the order of frequencies involved in modulation. A source of polarizing current C may be provided in the input circuit to render the grid 3 negative with respect to the filament. The wave of carrier frequency $\frac{p_1}{2\pi}$ may be impressed upon the grid 3, through a transformer 5, and the signal frequency $\frac{p_2}{2\pi}$ may be impressed upon the grid circuit through a transformer 6. The load circuit corresponding to z in the preceding theoretical considerations may be connected across the terminals 7 and 8 of the output circuit of the modulating tube M. This load circuit is provided with a by-pass including an inductance 9 and capacity 10 to tune the by-pass to currents of the signal frequency $\frac{p_2}{2\pi}$, thereby making the impedance of the load circuit substantially zero for currents of signal frequency. A second path, including a condenser 11 and a transformer 12, is provided for transmitting currents corresponding to the frequencies p_1, (p_1+p_2), and (p_1-p_2) to the out-going circuit 13.

It will be seen that from the theoretical considerations already given, an increase of approximately 57% in the output energy may be obtained by providing the circuit 9—10 forming a by-pass of very low or substantially zero impedance for currents of signal frequency, thereby obtaining a result of considerable importance in the art of modulation. It will be understood, of course, that where the signal wave is of variable frequency, the by-pass circuit should be tuned to the average signal frequency.

While the preceding discussion presupposes the use of a vacuum tube as a modulator, the invention is equally applicable to the use of the tube as a detector, although in the latter case a slight functional modification of the arrangement is involved. Ordinarily, when the device is operating as a demodulator or detector, a low impedance path is provided for the signal frequency to be received. In accordance with this invention a by-pass would also be provided tuned to the carrier frequency so that the load circuit of the detector will have substantially zero impedance at the carrier frequency. In short, regardless of whether the device is to be used as a modulator or detector, not only should a low impedance path be provided for the frequencies which are to be used, as is now the practice, but a low impedance path should also be provided for the frequencies not to be used.

It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated, without departing from the spirit of the invention as defined in the following claims.

What is claimed is:

1. A modulating arrangement comprising an input and an output circuit, means to impress a carrier frequency and a signal frequency upon the input circuit, and the output circuit being so arranged that its impedance will be substantially zero for currents of the signal frequency, but will act substantially as a pure resistance for frequencies in the neighborhood of the carrier frequency.

2. A modulating arrangement comprising a modulating device, means to impress a carrier frequency and a modulating signal frequency upon said device, a load circuit associated with said device, said load circuit being so designed that its impedance will be substantially zero at the modulating signal frequency.

3. A modulating arrangement comprising a modulating device, means to impress a carrier frequency and a modulating signal frequency upon said device, a load circuit associated with said device, said load circuit being so designed that its impedance will be substantially zero at the modulating signal frequency, and will act substantially like a pure resistance for frequencies in the neighborhood of the carrier frequency.

4. A modulating arrangement comprising a vacuum tube modulator, means to impress a carrier frequency and a modulating signal frequency upon said modulator, and a load circuit for said modulator, said load circuit being so designed as to have substantially zero impedance for currents of signal frequency.

5. A modulating arrangement comprising a vacuum tube modulator, means to impress a carrier frequency and a modulating signal frequency upon said modulator, and a load circuit for said modulator, said load circuit being so designed as to have substantially zero impedance for currents of signal frequency, and to have an impedance which acts substantially like a pure resistance for frequencies in the neighborhood of the carrier frequency.

6. A modulating arrangement comprising a vacuum tube modulator having an input and an output circuit, means to impress a carrier frequency and a modulating signal frequency upon the input circuit, and a load circuit associated with the output circuit, said load circuit being so designed that its impedance will be substantially zero for currents of signal frequency.

7. A modulating arrangement comprising a vacuum tube modulator having an input and an output circuit, means to impress a carrier frequency and a modulating signal frequency upon the input circuit, and a load circuit associated with the output circuit, said load circuit being so designed that its impedance will be substantially zero for currents of signal frequency, and will act substantially like a pure resistance for frequencies in the neighborhood of the carrier frequency.

8. A modulating arrangement comprising a vacuum tube modulator having an input and an output circuit, means to impress a carrier frequency and a modulating signal frequency upon the input circuit, an outgoing circuit associated with the input circuit, and a by-pass bridged across the output circuit, the impedance of said by-pass being substantially zero for currents of signal frequency.

9. A translating arrangement comprising a device for translating a plurality of frequencies impressed thereon into combinations of said frequencies, means to impress a plurality of frequencies on said device for purposes of translation, and a load circuit associated with said translating device, said load circuit being so designed that its impedance will be substantially zero at one of said frequencies, but will act substantially as a pure resistance for frequencies in the neighborhood of other frequencies impressed upon said translating device.

10. A translating arrangement comprising a translating device, means to impress a plurality of frequencies including a current of low signaling frequency upon said device for purposes of translation, and a load circuit associated with said translating device, said load circuit being so designed that its impedance will be substantially zero at said signaling frequency.

11. A translating arrangement comprising a translating device, means to impress a plurality of frequencies including a current of low signaling frequency upon said device for purposes of translation, and a load circuit associated with said translating device, said load circuit being so designed that its impedance will be substantially zero at said signaling frequency but will act substantially as a pure resistance for frequencies in the neighborhood of other frequencies impressed upon said translating device.

In testimony whereof, I have signed my name to this specification this 8th day of July 1920.

JOHN R. CARSON.

Certificate of Correction.

It is hereby certified that in Letters Patent No. 1,448,702, granted March 13, 1923, upon the application of John R. Carson, of Montclair, New Jersey, for an improvement in "Translating Circuits," errors appear in the printed specification requiring correction as follows: Page 1, line 37, for formula

$$\frac{1}{R_o}=\frac{\theta 1}{\theta E_b} \text{ read } \frac{1}{R_o}=\frac{\delta 1}{\delta E_b};$$

same page, line 75, for equation

$$M=\left(\frac{1}{2R_o}\right) \quad \left(\frac{\theta R_o}{\theta E_b}\right) \text{ read } M=\left(\frac{1}{2R_o}\right) \cdot \left(\frac{\delta R_o}{\delta E_b}\right);$$

page 2, line 31, for equation

$$P_1=\frac{1}{1'}\left(\frac{\theta 1}{\theta E_b}\right) \text{ read } P_1=\frac{1}{1'}\left(\frac{\delta 1}{\delta E_b}\right);$$

same page, line 40, for equation

$$P_1=\left(\frac{\theta 1}{\theta E_b}\right)_o=\frac{1}{R_o} \text{ read } P_1=\left(\frac{\delta 1}{\delta E_b}\right)_o=\frac{1}{R_o};$$

same page, line 46, for equation

$$P_2=\frac{1}{2'}\left(\frac{\theta^2{}_1}{\theta E_b^2}\right)=-\frac{1}{2}\frac{1}{R_o^2}\frac{\theta R_o}{\theta E_b}=\frac{M}{R_o} \text{ read } P_2=\frac{1}{2'}\left(\frac{\delta^2{}_1}{\delta E_b^2}\right)=-\frac{1}{2}\frac{1}{R_o^2}\frac{\delta R_o}{\delta E_b}=\frac{M}{R_o};$$

and that the said Letters Patent should be read with these corrections therein that the same may conform to the record of the case in the Patent Office.

Signed and sealed this 14th day of August, A. D., 1923.

[SEAL.]

KARL FENNING,
Acting Commissioner of Patents.