W. H. MARTIN.

WAVE FILTER.

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1,434,555. Patented Nov. 7, 1922.

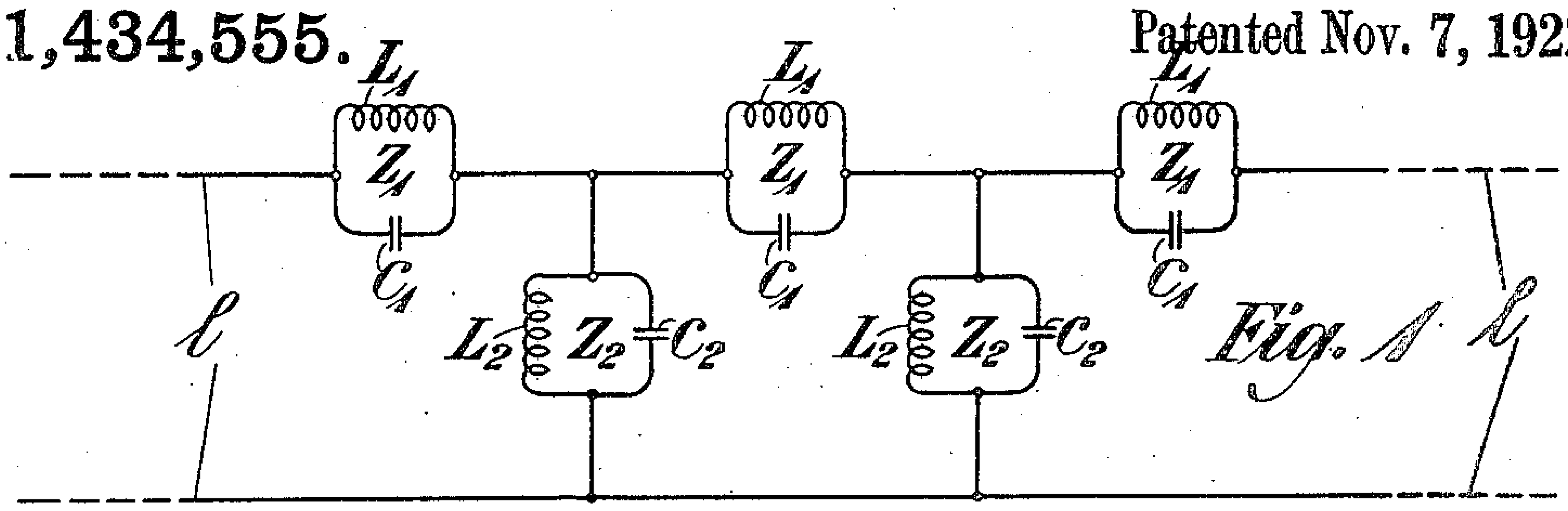

Fig. 1

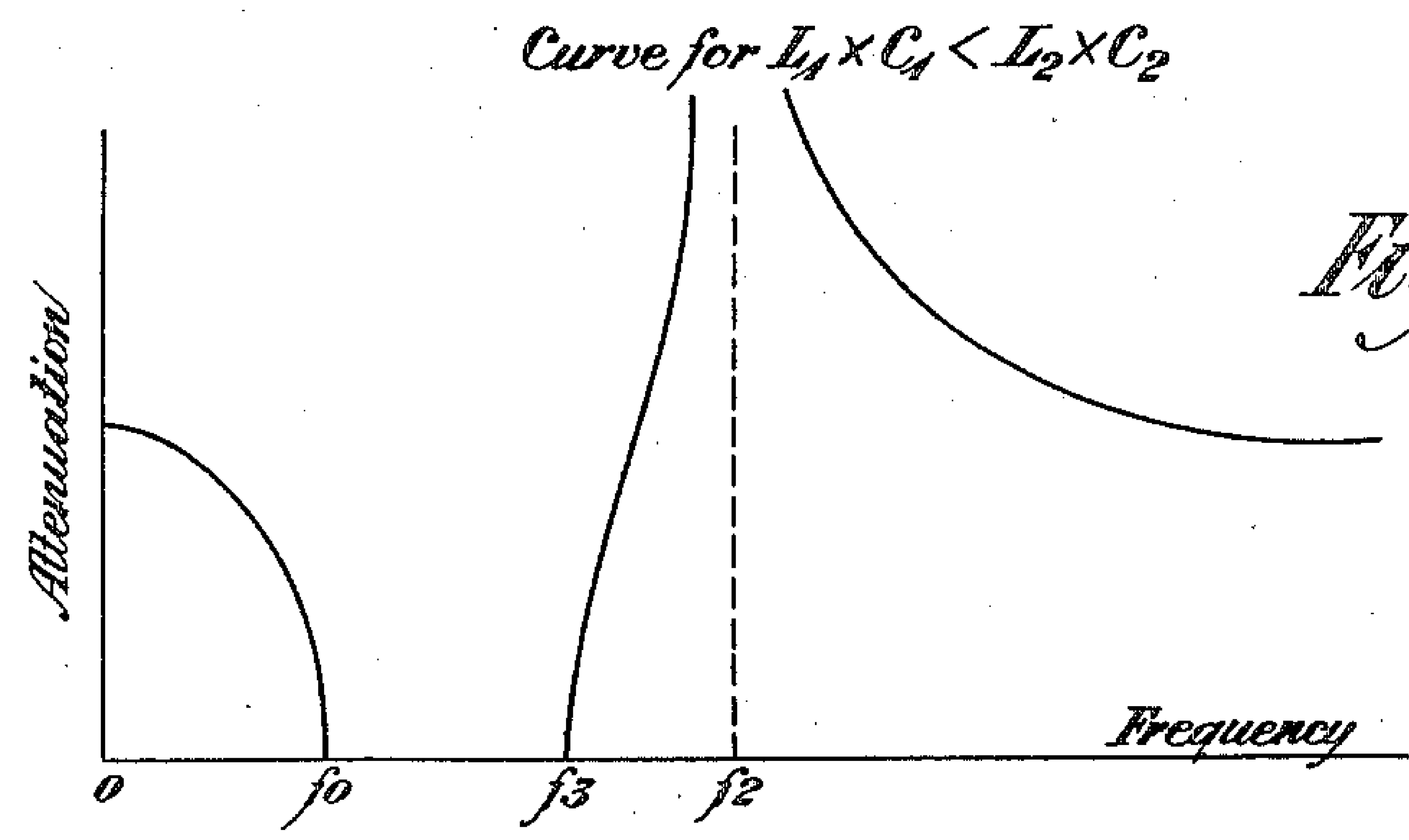

Fig. 2

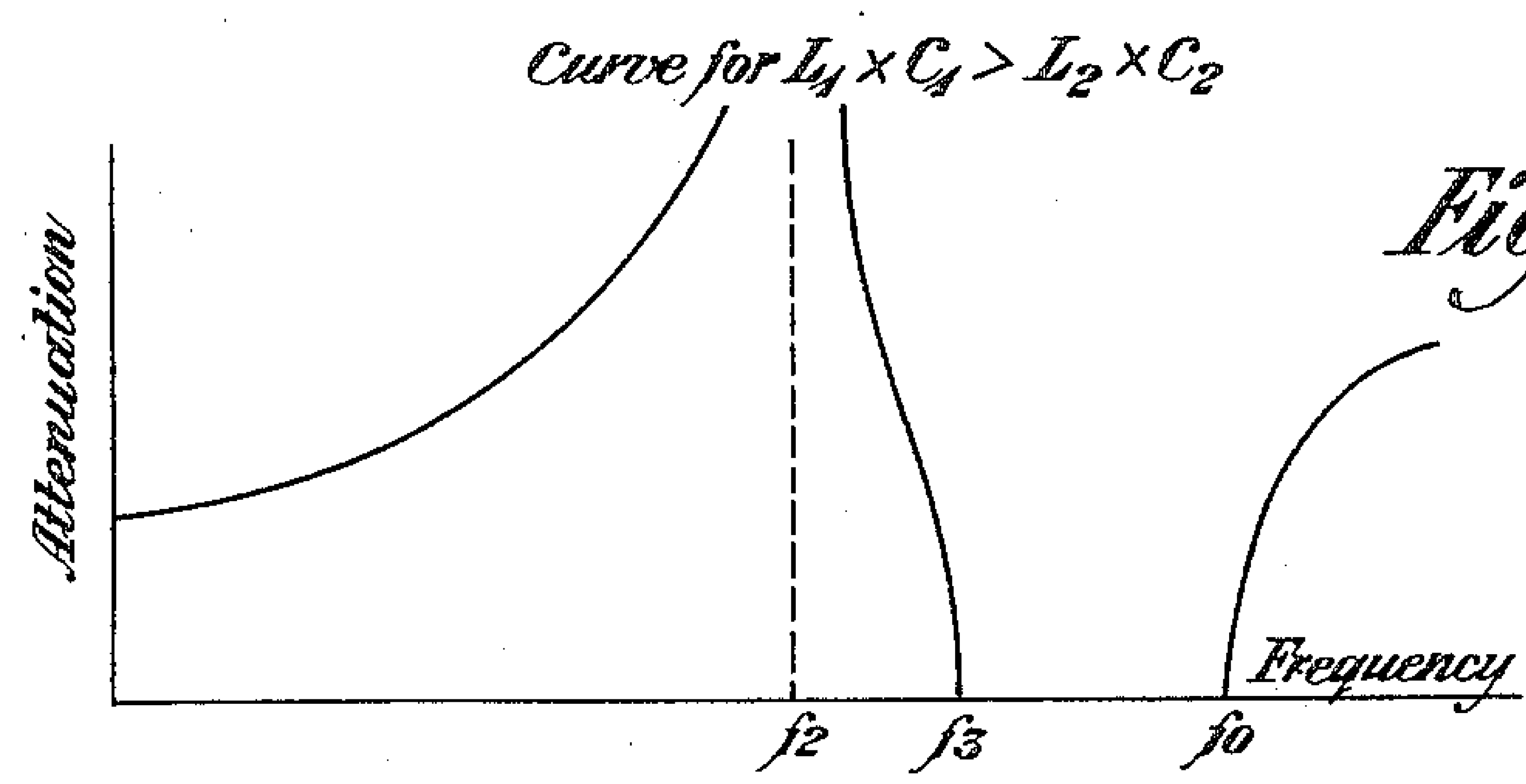

Fig. 3

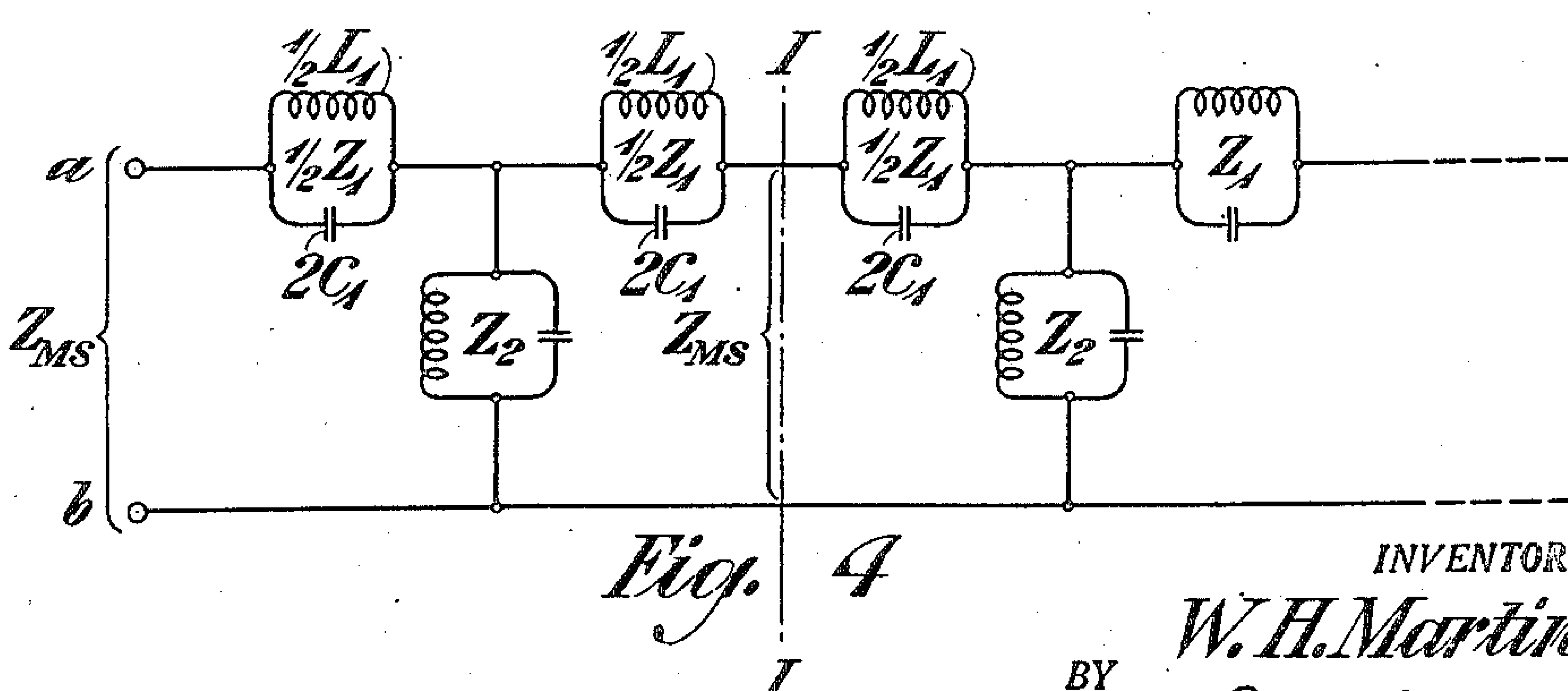

Fig. 4

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UNITED STATES PATENT OFFICE.

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WAVE FILTER.

Application filed April 26, 1920. Serial No. 376,764.

To all whom it may concern:

Be it known that I, WILLIAM H. MARTIN, residing at New York, in the county of Bronx and State of New York, have invented certain Improvements in Wave Filters, of which the following is a specification.

My invention relates to wave filters for use in electric circuits where it is desired to transmit alternating current of selected frequencies and to suppress, or partly suppress, currents of other frequencies. More particularly it concerns wave filters of the general type illustrated and described in the patents to George A. Campbell, 1,227,113 and 1,227,114 issued May 22, 1917.

The present invention proposes a filter which freely transmits all frequencies in a range between certain pre-assigned limits, neither of which is zero or infinity, and which attenuates frequencies outside this range. The filter of the present invention is characterized by having theoretically infinite attenuation at a finite frequency other than zero and lying in the attenuated range. It may consequently be so designed that it will effect substantially complete suppression of any desired frequency outside the transmitted band, or it may be so proportioned, that the frequency of infinite attenuation is close to either limit of the transmitted band, so that a very sharp cut-off action may be attained at the said limit. As in the Campbell filters, all frequencies within the attenuated range can be suppressed to any desired degree by use of a sufficiently large number of sections. The special feature of this filter, however, lies in the fact that the attenuation per section is substantially infinite for one frequency which can be freely chosen within the specified limits.

A good understanding of the invention may now be had from the following description of one form of embodiment thereof, reference being had to the accompanying drawing in which,

Figure 1 is a diagrammatic view showing one form of embodiment of the invention;

Figs. 2 and 3 are diagrams showing graphically the attenuation characteristics of the filter of Fig. 1; and

Fig. 4 is a diagrammatic view showing one form of termination of the filter.

Similar characters of reference designate similar parts in each of the several views.

In the form shown in Fig. 1 the filter comprises a number of sections or elements each of which consists of a lumped impedance Z_1 in series with the line 1 and a lumped impedance Z_2 in shunt to the line. The impedances Z_1 and Z_2 are shown as comprised of inductances L_1 and L_2 in parallel with capacities C_1 and C_2, respectively. Both series and shunt impedances are thus constituted of parallel resonant circuits, a feature which distinguishes them from the filters disclosed in the Campbell patents hereinbefore referred to, in certain of which filters the series impedances consist of series resonant circuits and the shunt impedances, of parallel or anti-resonant circuits.

I have found that the type of filter of this invention has certain advantageous characteristics which I shall presently explain with reference to Figs. 2 and 3, which show graphically the attenuation of filters similar to that of Fig. 1, the abscissae representing frequencies and the ordinates, values of attenuation. The curve of Fig. 2 shows the attenuation for the general case in which the numerical product of L_1 times C_1 is smaller than that of L_2 times C_2 and that of Fig. 3, the attenuation when the reverse is true. It will be observed that the attenuation is nil only for the band of frequencies lying between f_0 and f_3, this being the transmitted band. The attenuation is a maximum at the frequency f_2, being theroretically equal to infinity. When f_2 is close to f_3 the slope of the attenuation curve through f_3 is very steep showing that the filter discriminates sharply against frequencies just outside the transmitted band. By so designing the filter that f_2 is above f_3, (Fig. 2) the sharp-cut-off may be attained for the upper limit of the transmitted band, and by so proportioning the impedances that f_2 is below f_3 (Fig. 3) the same property may be secured for the lower limit. Moreover, where it is desired to discriminate particularly against some pre-determined frequency outside the transmitted band, the filter may be so designed that it has infinite attenuation for this frequency, provided there is no energy dissipation in the filter.

I shall now give certain convenient mathematical formulæ by means of which any one skilled in the art may design a filter of the type disclosed herein, to meet the requirements in any particular practical case.

The design of the filter involves the derivation of the values of the four constants L_1, L_2, C_1 and C_2. These values may of course, be chosen at will, and the properties of the filter computed therefrom, but in the general case the reverse procedure will be more convenient, i. e., to choose the properties of the filter (according to the conditions under which it is to be employed) and to calculate the values which the constants of the filter must have to give it these properties. Since there are four independent constants to be determined, it follows that any four properties of the filter, dependent upon the constants but independent of each other, may be chosen at will. Two of these may be taken as the frequencies f_0 and f_3, thus defining the range of free transmission, the third as f_2, the frequency of infinite attenuation, and the fourth, as the impedance of the filter, this factor being of importance because it is desirable to make the same substantially equal to that of the circuit into which the filter is placed, in order that reflection losses may be avoided. The relations which exist between the above-mentioned properties of the filter and the quantities L_1, L_2, C_1 and C_2, will now be set out in mathematical form.

In the Campbell patents hereinbefore mentioned, it was shown (equations 3 and 4) that for a periodic structure of the type now under consideration, in which the series impedance per section is Z_1 and the shunt impedance per section is Z_2, there is unattenuated transmission for all frequencies of current for which the value of

$$1/2\left(\frac{Z_1}{Z_2}\right)+1 \text{ lies between } \pm 1, \quad (1)$$

so that the limiting values (f_3 and f_0) of the frequencies for free transmission may be determined from the following equations:

$$\frac{Z_1}{Z_2}=O \quad (2)$$

$$Z_1=-4Z_2$$

In the present case the values of Z_1 and Z_2 in terms of frequency are

$$Z_1=i\frac{pL_1}{1-p^2L_1C_1} \quad (3)$$

$$Z_2=i\frac{pL_2}{1-p^2L_2C_2} \quad (4)$$

where i is the imaginary operator $\sqrt{-1}$ and p, the angular velocity, is written for $2\pi f$, for sake of convenience.

When, therefore, the values of Z_1 and Z_2, as given by (3) and (4), are substituted in equation (2) the resultant expression may be solved for p, one of the roots, p_3, determining one limiting frequency, f_3, and the other, p_0, the other limiting frequency, f_0:

$$f_0=\frac{p_0}{2\pi}=\frac{1}{2\pi\sqrt{L_2C_2}} \quad (5)$$

and

$$f_3=\frac{p_3}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{L_1+4L_2}{L_1L_2(C_2+4C_1)}} \quad (6)$$

The relation which exists between the values of the constants of the filter and the frequency, f_2, at which the attenuation is infinite, may be determined by placing the expression (equation 2 of the hereinbefore-mentioned Campbell patent) for cosh Γ, where Γ is the propagation constant, equal to infinity:

$$\operatorname{Cosh}\Gamma=1/2\frac{Z_1}{Z_2}+1=\infty$$

or

$$1/2\frac{L_1}{L_2}\,\frac{1-p^2L_2C_2}{1-p^2L_1C_1}+1=\infty$$

When solved for f_2, this gives

$$f_2=\frac{p_2}{2\pi}=\frac{1}{2\pi\sqrt{L_1C_1}} \quad (7)$$

The frequencies f_0, f_3 and f_2 are thus expressed in terms of the constants of the filter. It remains to evolve an expression for the impedance of the filter.

This impedance varies with frequency, and it has been found convenient in the art to specify in design the impedance at the frequency, termed mid-frequency, which is the geometric mean of the limiting frequencies of the transmitted range, i. e., $\sqrt{f_0f_3}$. The impedance of a filter or any similar periodic structure varies also according to the termination of the structure and it therefore is important to specify the termination which is to be used. I shall for sake of illustration employ the mid-series termination for the present design formulæ. It is to be expressly understood, however, that the invention is not limited to any particular type of termination.

A filter is defined to have mid-series termination when it is terminated by a series impedance of half that of the normal series section. This is illustrated in Fig. 4 in which the normal series impedance is Z_1 and the left terminal impedance is $\frac{1}{2}Z_1$, i. e., composed of $\frac{1}{2}L_1$ and $2C_1$. The filter may be assumed to have an infinite number of sections toward the right so that the impedance at any other mid-series section, such as that which extends to the right from line I—I is the same as the impedance measured at the terminals a—b. Consequently, if Z_{ms} designates the mid-series section impedance

$$Z_{ms}=1/2\,Z_1+\cfrac{1}{\cfrac{1}{Z_{ms}+1/2\,Z_1}+\cfrac{1}{Z_2}} \quad (8)$$

or

$$Z^2_{ms}=Z_1Z_2+\frac{Z_1^2}{4} \quad (9)$$

substituting for Z_1 and Z_2, their values as given by equations (3) and (4), and simplifying, it will be found that the impedance at the frequency $\sqrt{f_o f_3}$ is given by the expression

$$Z_{ms}=\frac{\sqrt{1+\frac{4C_1}{C_2}}}{2\left(\frac{\sqrt{L_2}}{L_1}\sqrt{\frac{C_2+4C_1}{L_1+4L_2}}-\frac{C_1\sqrt{C_2}}{C_2\ \ L_2}\right)} \quad (10)$$

Although the above expression is derived for a filter having an infinite number of sections, it is substantially correct for one having only a few sections provided the same is shown so as to be approximately the same as that of the line to which the filter is connected. For mid-series termination the two terminal sections of the filter should, however, be alike i. e. in the present instance both terminations should be mid-series.

Equations (5), (6), (7) and (10) are four simultaneous equations involving the factors p_2, p_o, p_3 and Z_{ms}, the values of which may be assumed, and the factors L_1, L_2, C_1 and C_2, the values of which are to be determined. These equations may therefore be solved for the values of the last mentioned quantities, and when this is done it will be found that

$$L_1=\frac{2Z_{ms}(p_2^2-p_op_3)}{p_2^2p_3}\sqrt{\frac{p_2^2-p_3^2}{p_2^2-p_o^2}} \quad (11)$$

$$L_2=\frac{Z_{ms}(p_2^2-p_op_3)\,(p_3^2-p_o^2)}{2p_3p_o^2\sqrt{(p_2^2-p_o^2)(p_2^2-p_3^2)}} \quad (12)$$

$$C_1=\frac{p_3}{2Z_{ms}(p_2^2-p_op_3)}\sqrt{\frac{p_2^2-p_o^2}{p_2^2-p_3^2}} \quad (13)$$

$$C_2=\frac{2p_3\sqrt{(p_2^2-p_o^2)\,(p_2^2-p_3^2)}}{Z_{ms}(p_2^2-p_op_3)\,(p_3^2-p_o^2)} \quad (14)$$

The constants of the filter may consequently be determined by merely substituting the assumed values of p_o, p_2, p_3 and Z_{ms} in the above equations.

For example, where, it is desired to construct a filter whose band of free transmission shall extend from 400 to 2000 cycles, whose maximum (theoretically infinite) attenuation shall be at 2300 cycles and whose impedance at mid-frequency shall be 800 ohms (when terminated by a mid-series section) so that the filter may be inserted in a line of #22 gage cable (the characteristic impedance of which is roughly, 800 ohms) we have, according to the notation used herein:

$$p_o=2\pi\ (400)\ =2513$$
$$p_3=2\pi\ (2000)=12570$$
$$p_2=2\pi\ (2300)=14451$$
$$Z_{ms}=800$$

Substituting these values in equations (11) to (14), and solving gives:

L_1= .0542 henries
L_2=1. 33 henries
C_1= .0884 microfarads
C_2= .119 microfarads

as the constants of the normal filter section.

With the mid-series termination assumed herein the values of the first and last series inductance and capacity are, respectively, .0271 henries and .1768 microfarads.

Although I have herein shown and described only one form and arrangement of apparatus embodying my invention, it is readily understood that various changes and modifications may be made therein within the scope of the following claims without departing from the spirit and scope of the invention.

What I claim is:

1. A wave filter for an electric circuit, said filter having a plurality of like recurrent sections, each section comprising lumped impedence consisting of a parallel resonant circuit in series with the said electric circuit and lumped impedance, consisting of a parallel resonant circuit, adjacently in shunt thereto, said impedances being so proportioned that the filter approximately suppresses all frequencies lying outside a preassigned band extending between two frequencies other than zero or infinity and substantially completely suppresses a certain preassigned finite frequency other than zero.

2. A wave filter for an electric circuit, said filter having a plurality of like recurrent sections, each section comprising a parallel resonant circuit in series with the said circuit and a parallel resonant circuit adjacently in shunt thereto, the impedances of said circuits being so proportioned that the filter transmits freely all frequencies lying within two limiting frequencies other than zero or infinity but approximately suppresses the frequencies outside of the said limiting frequencies and discriminates sharply against frequencies close to one of said limits.

3. A wave filter for an electric circuit, said filter having a plurality of like recurrent sections, each section comprising a lumped inductance and capacity in parallel with each other inserted in series with the circuit and a lumped inductance and capacity in parallel with each other connected in

shunt to the circuit, adjacently to said first-mentioned inductance and capacity.

4. The combination with a line, of a filter composed of a plurality of periodically recurring sections, each of which consists of a parallel resonant circuit connected in series with the line and a parallel resonant circuit connected in shunt thereto.

5. A wave filter for an electric circuit, said filter having a plurality of like recurrent sections, each section comprising a resonant circuit in series with the circuit and a like type of resonant circuit in shunt to the circuit, each of which resonant circuits comprises lumped inductance and capacity connected in parallel with each other, the values of which are determined by a finite frequency other than zero at which the attenuation of the filter is theoretically infinite, the impedance of the filter and the limiting frequencies of a preassigned band of free transmission, which limiting frequencies are other than zero or infinity.

6. A wave filter for an electric circuit, said filter having a plurality of like recurrent sections, each section comprising lumped impedance in series with the circuit and lumped impedance in shunt thereto, the said impedances consisting of parallel resonant circuits so proportioned that the attenuation of the filter has a finite value at a chosen finite frequency and decreases, as the frequency is changed in one direction, until it is substantially equal to zero at a certain other finite frequency, remains at this value until still another finite frequency is reached, then increases until it becomes theoretically infinite at still another finite frequency and then decreases with further change of frequency in the same direction.

7. A wave filter for an electric circuit, said filter having a plurality of like recurrent sections, each section consisting of a parallel combination connected in series with the line and resonant at a certain frequency, and another parallel combination connected in shunt to the line and resonant at a certain other frequency.

In testimony whereof, I have signed my name to this specification this 23rd day of April 1920.

WILLIAM H. MARTIN.